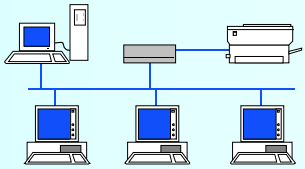


Distributed Systems
600.437
Communication and Knowledge

Department of Computer Science
The Johns Hopkins University

Communication and Knowledge

Lecture 11

Cheating husbands and other stories:
A case study of knowledge, action, and communication.
Y. Moses, D. Dolev and J. Y. Halpern
Distributed Computing (1986) 1:167-176 Springer Verlag.

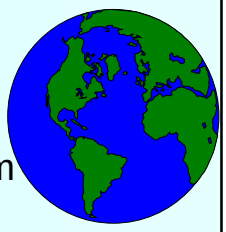
Common Knowledge

A group has common knowledge of a fact *p* if they all know *p*, they all know that they all know *p*, they all know that they .... and so on, infinite number of times.

Communication Knowledge and Action

- In many practical systems, common knowledge is not attainable.
- Sometimes, (remember the generals), a lack of common knowledge means inability to act.
- Sometimes, it does not.

The Cheating Husbands Puzzle



- Once upon a time, there was a matriarchal city.
- The women had to pass a logic exam before being allowed to marry.
- The Queen was not required to take that exam, but it was common knowledge that she was truthful.
- The women were obedient to the queen.
- The city was small, so all women heard every shot fired in the city.

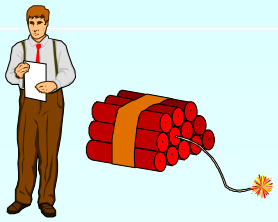
Basic Puzzle

Henrietta I:

- All the women gathered at the city square.
- The queen read the following:
  - **There are (one or more) unfaithful husbands in the city.**

Basic Puzzle (cont.)

- **Although none of you knew before this gathering whether your own husband was faithful, each of you knows which of the other husbands are unfaithful.**
- **I forbid you to discuss it.**
- **Should you discover that your husband is unfaithful, you must shoot him on the midnight of the day you find out about it.**

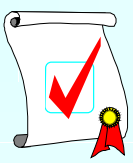


Basic Puzzle (cont..)

Theorem 1.

If there had been n unfaithful husbands, they would all have been shot on the midnight of the n'th day.

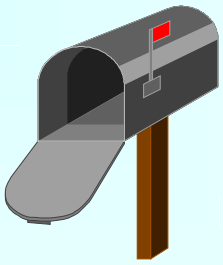
- Henrietta I was highly regarded for her wisdom. She ordered her successors to continue the moral fight against male infidelity.



Asynchronous Communication

Henrietta II:

- To avoid the need for gathering, a mail system was installed.
- The first letter informed all the women that every letter is guaranteed to eventually arrive.
- The second letter was an exact copy of Henrietta I 's original message.

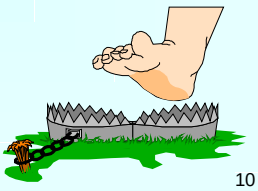


Asynchronous Communication (cont.)

Theorem 2:

If there is more than one unfaithful husband, and an asynchronous channel is used, then no unfaithful husbands are shot.

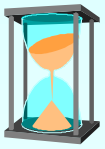
- Henrietta II suffered great disgrace and died in despair. She ordered her successors not to repeat her mistake.

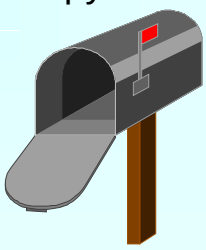


Weakly Synchronous Communication

Henrietta III:

- The mail system was upgraded.
- The first letter informed all the women that every letter is guaranteed to arrive no later than b days after it is sent.
- The second letter was an exact copy of Henrietta I 's original message.





Weakly Synchronous Communication (cont.)

Theorem 3:

A wife that knows of exactly k unfaithful husbands will know that her own husband is unfaithful once kb silent nights pass after the day she receives the letter.

Weakly Synchronous Communication (cont.)

Theorem 3:

A wife that knows of exactly k unfaithful husbands will know that her own husband is unfaithful once kb silent nights pass after the day she receives the letter.

Theorem 4:

The first cheated wives to receive the letter shoot their husbands. All other cheated wives remain forever in doubt.

The Problem

- A letter arrives in no more than 2 days (b=2).
- Susan receives the letter at Monday
- Susan knows that Mary's husband is unfaithful and hears a shot at Tuesday night.
- Susan cannot tell whether:
  - **Mary received the letter at Sunday.**
  - **Mary received the letter at Tuesday.**

If only she told
them to wait for a while...

Bribing the Postman

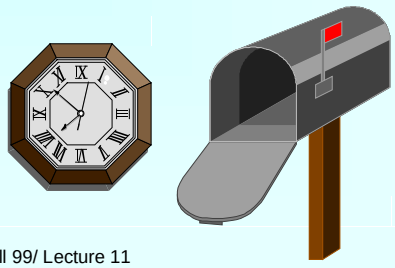
Theorem 7:

A wife that bribes the postman into telling her when the queen had sent the letter, does eventually know whether her own husband is faithful.

Synchronous Communication

Henrietta IV:

- The Queen gathered everyone and announced that, from this day on, every letter will be stamped with the mailing date and will arrive in less than b days.



Synchronous Communication (cont.)

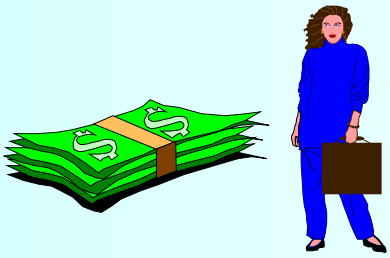
Theorem 9:

If it is common knowledge that there is at least one obedient cheated wife, then all the obedient cheated wives will shoot their husbands.

Comparison

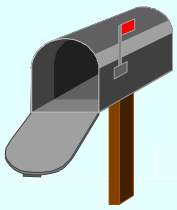
Henrietta III:

It takes (n-1)b+1 to eliminate n>1 cheating husbands



Henrietta IV:

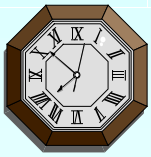
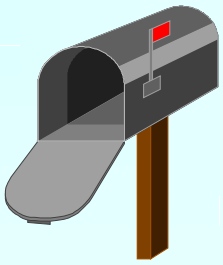
It takes (n-1)+b to eliminate n>1 cheating husbands



Faulty Behavior

Henrietta IV:

- However, due to Henrietta III's injustice, it was no longer clear that the women will obey the Queen.

Faulty Behavior (cont.)

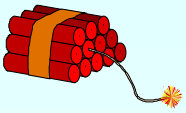
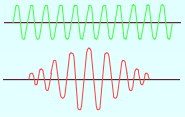
Theorem 8:

If all the cheated wives are disobedient, then all the other wives are in danger of shooting their husbands.

Building on Common Knowledge

Margaret:

- An express mail system was installed. This system guaranteed that each letter will arrive at the same day it is sent.
- Clearly, sending Henrietta I 's original message solves the problem, however, there were too many unfaithful husbands and Queen Margaret did not want to wait...

Building on Common Knowledge (cont.)

Theorem 11:

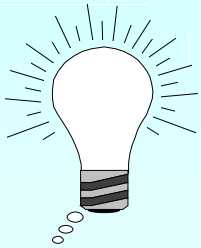
There is a protocol that allows shooting in the air, in which the cheating husbands are all shot by the third night.

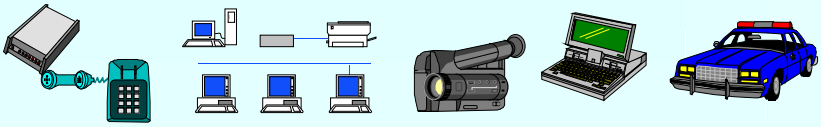
Can you think of it?

A Four Nights Protocol

- If you know of k = 0 (mod 3) cheating husbands, fire on the first night
- If you know of k = 1 (mod 3) cheating husbands, fire on the second night
- If you know of k = 2 (mod 3) cheating husbands, fire on the third night
- If your husband cheats, shoot him on the forth night. (There is at least one cheating husband. So, if shooting occur only on one of the first 3 nights, all of the husbands cheat .)

Based on their existing communication system, could any of the previous queens implement such a protocol ?



Topics

- Network protocols.
- Internet / Web technology.
- Quality of Service.
- Multimedia on the net.
- Portable and mobile operation.
- Security.
- Electronic commerce.
- Metacomputing.
- Any ideas?