Handout 3: Homework 2

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This assignment is due by the start of lecture on October 7.

1. (20 points) Give a complete, formal description of a (basic one-head, one-tape) Turing machine that *decides* the language $L = \{0^i 1^j : 0 \le i < j\}$.

2. (20 points) Show that the collection of decidable languages is closed under the operation of:

   (a) complementation

   (b) concatenation

   (Think about union[1], intersection and star on your own.)

3. (20 points) Show that the collection of Turing-recognizable languages is closed under the operation of:

   (a) star

   (b) intersection

   (Think about union[2] and concatenation on your own.)

4. (20 points) (Sipser 3.18) Show that a language is decidable if and only if some enumerator enumerates the language in lexicographic order. Be sure to prove both directions.

5. (20 points) (Sipser 4.19) Let $S = \{\langle M \rangle | M$ is a DFA that accepts w^R whenever it accepts $w\}$. Show that S is decidable.

6. Bonus: Consider a *flag*-TM, which cannot write on any of its tapes, but can place and move three flags on its work tape. Show that *flag*-TMs can recognize any Turing-recognizable language. Hint: simulate a regular TM using a *flag*-TM.

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[1] Sipser provides a sample solution on page 163.
[2] In Sipser's solution on page 163, the last paragraph should begin "If either M_1 **or** M_2 accepts w, ...".