600.471 Theory of Computation November 11, 2007

Handout 10: Homework 5

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This assignment is due by the start of lecture on Thursday, November 20.

1. **(15 points)** (Sipser 8.8) [$EQ_{REX} \in$ PSPACE]
2. **(15 points)** (Sipser 8.20) [$MULT \in$ L]
3. **(20 points)** (Sipser 8.30) [E_{DFA} is NL-complete]
4. **(20 points)** (Sipser 9.22) [tale of two oracles]
5. **(30 points)** Below we summarize the requirements on the behavior of machines for languages in several probabilistic classes. A language L is in a specified class if and only if there exists a probabilistic polynomial time Turing machine that accepts words in L with the given probability, and rejects words not in L with the given probability.

Class	Probability of accepting $w \in L$	Probability of rejecting $w \notin L$
P	1	1
NP	> 0	1
coNP	1	> 0
BPP	$> \frac{2}{3}$	$> \frac{2}{3}$
RP	$> \frac{2}{3}$	1
coRP	1	$> \frac{2}{3}$
PP	$\geq \frac{1}{2}$	$> \frac{1}{2}$

Recall the definition of PP: A language $L \in$ PP if and only if there exists a probabilistic polynomial time Turing machine such that:

- If $w \in L$, then $\Pr[M \text{ accepts } w] \geq \frac{1}{2}$.
- If $w \notin L$, then $\Pr[M \text{ accepts } w] < \frac{1}{2}$.

Show that:

(a) BPP $\subseteq$ PP.
(b) NP $\subseteq$ PP.
(c) PP $\subseteq$ PSPACE.

Hint for (2): Consider a nondeterministic TM for L, and replace rejections with probabilistic decisions.