

## Handout 8: Homework 4

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This assignment is due by the start of lecture on November 11.

1. **(10 points)** Answer each of the following with TRUE or FALSE. You do not need to justify your answers. (Note: when dealing with sets like  $O(f(n))$ ,  $\Omega(f(n))$ , etc., we use the symbols  $=$  and  $\in$  interchangeably.)

1.  $200 = O(n)$

6.  $\frac{1}{6} = o(1)$

2.  $n^{8/3} = O(n^2 \log^2(n))$

7.  $n - \log_2(n) = \Theta(n)$

3.  $3^n = 2^{O(n)}$

8.  $\log(n) = \Theta(\log(\log(n)))$

4.  $n^n = O(n!)$

9.  $n^5 = \Omega(32^{\log_2(n)})$

5.  $2n = o(n^2)$

10.  $2^{5^n} = \Omega(5^{2^n})$

2. **(15 points)** Prove that P is closed under any two of the following operations (clearly state which two you are proving!):

- (a) union,
- (b) intersection,
- (c) complement,
- (d) concatenation.

P is also closed under the star operation.

3. **(15 points)** Prove that NP is closed under any two of the following operations (clearly state which two you are proving!):

- (a) union,
- (b) intersection,
- (c) concatenation.

NP is also closed under star, but it is not known if NP is closed under complement.

4. **(20 points)** (Sipser 7.27) [3COLOR is NP-complete]
5. **(20 points)** (Sipser 7.29) [A scheduling problem]
6. **(20 points)** (Sipser 7.36) [finding a satisfying assignment if  $P = NP$ ]