600.471 Theory of Computation September 25, 2008

Handout 3: Homework 2

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This assignment is due by the start of lecture on Tuesday, October 7.

1. (20 points) Give a complete, formal description of a (basic one-head, one-tape) Turing machine that *decides* the language

$$L = \{a^i b^i c^i \mid i \geq 0\}.$$

   This should consist of a list of the tuple components of the Turing machine, with the transition function represented by a state transition diagram. (Yes, we know that it's tedious to write Turing machine descriptions, but everyone should do it once.)

2. (20 points) Show that the collection of decidable languages is closed under the operation of:

   (a) complementation

   (b) star

   (Think about union[1], concatenation and intersection on your own.)

3. (20 points) Show that the collection of Turing-recognizable languages is closed under the operation of:

   (a) intersection

   (b) concatenation

   (Think about union[2] and star on your own.)

4. (20 points) (Sipser 3.18) Show that a language is decidable if and only if some enumerator enumerates the language in lexicographic order. Be sure to prove both directions.

5. (20 points) (Sipser 4.28) Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1 \rangle, \langle M_2 \rangle, \ldots\}$, where every M_i is a decider. Prove that some decidable language D is not decided by any decider M_i whose description appears in A. (Hint: You may find it helpful to consider an enumerator for A.)

[1] Sipser provides a sample solution on page 163.

[2] In Sipser's solution on page 163, the last paragraph should begin "If either M_1 **or** M_2 accepts w, ...".