600.471 Theory of Computation November 5, 2007

Handout 11: Homework 5

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This assignment is due by the start of lecture on Monday, November 19.

1. **(2 points, with 1 point bonus if *all* answers are correct.)** Answer each of the following with TRUE or FALSE. You do not need to justify your answers. (Note: when dealing with sets like $O(f(n))$, $\Omega(f(n))$, etc., we use the symbols $=$ and $\in$ interchangeably.)

   1. $2^{10} = O(n)$
   2. $16n = O(n)$
   3. $n^4 = O(n^2 \log^2(n))$
   4. $n \log(n) + 10n = O(n^2)$
   5. $3^n = O(2^n)$
   6. $3^n = 2^{O(n)}$
   7. $2^{2^n} = O(2^{2^n})$
   8. $n^n = O(n!)$
   9. $n = o(n)$
   10. $2n = o(n^2)$
   11. $2^n = o(3^n)$
   12. $1 = o(n)$
   13. $2 \log(n) = o(\log(n))$
   14. $\frac{1}{3} = o(1)$
   15. $log_2(n) = \Theta(log_3(n))$
   16. $2^n = \Theta(4^n)$
   17. $n^5 = \Theta(32^{log_2(n)})$
   18. $n^3 = \Omega(n^3)$
   19. $log(n) = \Omega(log(log(n)))$
   20. $2^{5^n} = \Omega(5^{2^n})$

2. **(1 point)** Prove that the following language is in NP. You may use either the guess-and-check method, or else describe a suitable nondeterministic Turing machine.

   k-PCP (k-Post Correspondence Problem), for a fixed alphabet Σ with $|\Sigma| \geq 2$, is:

   $\{\langle S, k\rangle \mid S$ is a finite set of dominoes over Σ, k is an integer written in unary, and there is a sequence of at most k dominoes (allowing repeats) for which the top and bottom sequences are equal$\}$.

   If k was not written in unary, would your solution to the above still work? Why or why not?

3. **(3 points)** (Sipser 7.20) [undirected paths]

4. **(3 points)** (Sipser 7.26) [puzzle]

5. **(3 points)** (Sipser 7.30) [Minesweeper]

6. **(3 points)** (Sipser 7.37) [factoring if P = NP]