600.471 Theory of Computation October 8, 2007

Handout 7: Homework 3

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This assignment is due by the start of lecture on Wednesday, October 17.

1. Give a complete, formal description of a (basic one-head, one-tape) Turing machine that *decides* the language $L = \{0^i 1^j : 0 \leq i < j\}$. This should consist of a list of the tuple components of the Turing machine, with the transition function represented by a state transition diagram. (Yes, we know that it's tedious to write Turing machine descriptions, but everyone should do it once.)

2. (from Sipser 3.15) Show that the collection of decidable languages is closed under the operation of:

   (a) complementation

   (b) concatenation

   (Think about union[1], star and intersection on your own.)

3. (from Sipser 3.16) Show that the collection of Turing-recognizable languages is closed under the operation of:

   (a) intersection

   (b) star

   (Think about union[2] and concatenation on your own.)

4. (Sipser 3.18) Show that a language is decidable if and only if some enumerator enumerates the language in lexicographic order. Be sure to prove both directions.

5. (Sipser 4.16) Prove that EQ_{DFA} is decidable by testing the two DFAs on all strings up to a certain size. Calculate a size that works.

[1] Sipser provides a sample solution on page 163.

[2] In Sipser's solution on page 163, the last paragraph should begin "If either M_1 **or** M_2 accepts w, ...".