600.363/463 Introduction to Algorithms/Algorithms I November 9, 2009

Handout 11: Homework 6

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This assignment is due by the start of lecture on November 16, 2009. Please clearly indicate your collaborators.

1. (45 points) *Alternative MST algorithms.* (CLRS 23-4) For each of the three algorithms, either prove that T is a minimum spanning tree or prove that T is not a minimum spanning tree.

   Note: you do *not* need to turn in an analysis of the running times of these algorithms, but it is good practice to think about how efficiently you could implement each step.

2. (30 points) *Bellman-Ford.* (extension of CLRS 24.1-4) Modify the Bellman-Ford algorithm so that it sets $v.d$ to $-\infty$ for all vertices v for which there is a negative-weight cycle on some path from the source to v. Informally argue that your algorithm works.

   Next, describe, an original application of Bellman-Ford or this modified version of it (one paragraph is sufficient.) Where would this be useful? You will be judged on your creativity, so come up with the application *on your own.*

3. (25 points) *McDonalds.* You've been appointed as Earth's first Ambassador to Planet Syron. When you first arrive in the capital city of Ofssmour, you are a little concerned about the local cuisine. Fortunately, McDonalds already has several restaurants established throughout the city. Let $G = (V, M, E, L[\cdot])$ be an undirected graph where each edge $e \in E$ has a length $L[e]$. Assume that nodes represent different locations in the city and there is a special set of vertices: McDonalds restaurants $M \subset V$. You would like to know the distance to the closest McDonalds no matter where in the city you are. Design an efficient algorithm that solves this problem by running Dijsktra's shortest path algorithm *only* once. (We mean once in total; not once each time you want to find a McDonalds.) Argue that your algorithm works.