600.363/463 Introduction to Algorithms/Algorithms I	September 16, 2009

Handout 3: Homework 2

Instructor: Susan Hohenberger *TA: Ming Chuang*

This assignment is due by the start of lecture on September 23, 2009. Please clearly indicate your collaborators.

1. (30 points) Let's compare functionality versus running time. Consider the following recursive procedure.

   `Soup`(n):
   If $n = 1$ **Then Return** 1
   Else Return `Soup`($n-1$) + `Soup`($n-1$) + `Soup`($n-1$)

   (a) What function of n does `Soup` compute?
   (b) What is the running time $T(n)$ of `Soup`?
   (c) How do the answers to (a) and (b) change if we replace the last line by "**Else Return** 3·`Soup`($n-1$)"?

2. (40 points) Solve the following recurrences using any method you like. If you use the"master method", use the version from CLRS (which we also discussed in class) and justify why it applies. Assume $T(1) = 2$ and be sure you explain every important step.

   (a) $T(n) = T(9n/10) + n$
   (b) $T(n) = 8T(n/2) + n \lg n$
   (c) $T(n) = 8T(n/2) + 2n^3$
   (d) $T(n) = T(\sqrt{n}) + 1$

3. (30 points) (CLRS 4-5) Professor Smith has n supposedly identical integrated-circuit chips that in principle are capable of testing each other. The professor's test jig accommodates two chips at a time. When the jig is loaded, each chip tests the other and reports whether it is good or bad. A good chip always reports accurately whether the other chip is good or bad, but the professor cannot trust the answer of a bad chip. Thus, the four possible outcomes of a test are as follows:

Chip A says	Chip B says	Conclusion
B is good	A is good	both are good, or both are bad
B is good	A is bad	at least one is bad
B is bad	A is good	at least one is bad
B is bad	A is bad	at least one is bad

   (a) Show that if more than $n/2$ chips are bad, the professor cannot necessarily determine which chips are good using any strategy based on this kind of pairwise test. Assume that the bad chips can conspire to fool the professor.

(b) Consider the problem of finding a single good chip from among n chips, assuming that more than $n/2$ of the chips are good. Show that $\lfloor n/2 \rfloor$ pairwise tests are sufficient to reduce the problem to one of nearly half the size.

(c) Show that the good chips can be identified with $\Theta(n)$ pairwise tests, assuming that more than $n/2$ of the chips are good. Give and solve the recurrence that describes the number of tests.