600.363/463 Introduction to Algorithms/Algorithms I September 9, 2009

Handout 1: Homework 1

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This assignment is due by the start of lecture on September 16, 2009. CLRS refers to the *third* edition of the textbook.

1. **Collaborators? (5 points)** Clearly indicate your collaborators. If none, state that.

2. **Perspective (15 points)** Suppose that one operation takes one microsecond ($= 10^{-6}$ seconds). With one processor, what is the largest value of n such that:

   (a) $\lg n$ operations finish in 1 hour
   (b) n^3 operations finish in 1 day
   (c) 2^n operations finish in 1 week.

3. **Adding binary integers (20 points)** CLRS 2.1-4. Consider the problem of adding two n-bit binary integers, stored in two n-element arrays A and B. The sum of the two integers should be stored in binary form in an $(n+1)$-element array C.

   (a) State the problem formally.
   (b) Write pseudocode for adding the two integers.

4. **Faster insertion sort? (20 points)** CLRS 2.3-6. Observe that the **while** loop of lines 5-7 of the INSERTION-SORT procedure in Section 2.1 uses a linear search to scan (backward) through the sorted subarray $A[1 \ldots j-1]$. Can we use a binary search (see Exercise 2.3-5) instead to improve the overall worst-case running time of insertion sort to $\Theta(n \lg n)$? Explain why or why not.

5. **Sum to x? (20 points)** CLRS 2.3-7.* Describe a $\Theta(n \lg n)$-time algorithm that, given a set S of n integers and another integer x, determines whether or not there exist two elements in S whose sum is exactly x. Briefly justify the running time of your algorithm. If you get stuck, give the best algorithm that you can and analyze its running time.

6. **Order of Growth (20 points)** Answer each of the following with TRUE or FALSE. You do not need to justify your answers. (Note: when dealing with sets like $O(f(n))$, $\Omega(f(n))$, etc., we use the symbols $=$ and $\in$ interchangeably.)

   1. $200 = O(n)$
   2. $3^n = 2^{O(n)}$
   3. $n^n = O(n!)$
   4. $\frac{1}{6} = o(1)$
   5. $2n = o(n^2)$
   6. $\lg(n^2) = \Theta(\lg^2(n))$
   7. $n - \lg(n) = \Theta(n)$
   8. $2n^5 = \omega(n^5)$
   9. $\frac{n^2-100}{4} = \Omega(n^2)$
   10. $2^n \cdot 2^{2n} = \Omega(2^{3n})$