Reinforcement Learning

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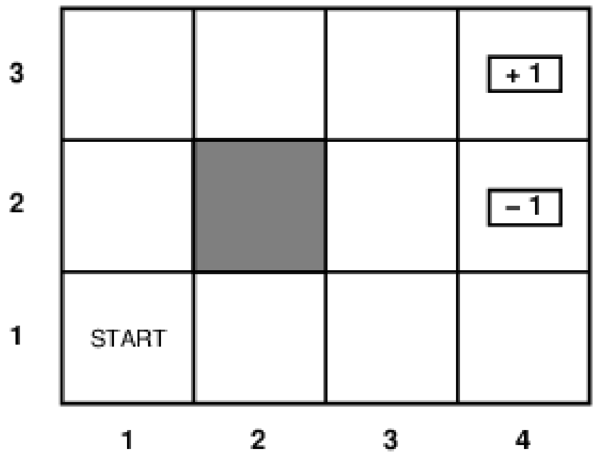
Rewards

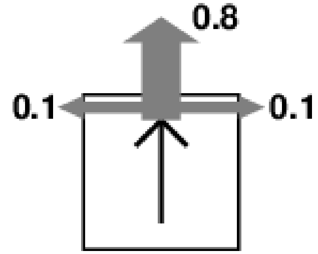
- Agent takes actions
- Agent occasionally receives reward
- Maybe just at the end of the process, e.g., Chess:
  - agent has to decide on individual moves
  - reward only at end: win/lose
- Maybe more frequently
  - Scrabble: points for each word played
  - ping pong: any point scored
  - baby learning to crawl: any forward movement

Markov Decision Process

State Map



Stochastic Movement



- States $s \in S$, actions $a \in A$

- <u>Model</u> $T(s, a, s') \equiv P(s'|s, a)$ = probability that a in s leads to s'

- <u>Reward function</u> $R(s)$ (or $R(s, a)$, $R(s, a, s')$)

$$= \begin{cases} -0.04 & \text{(small penalty) for nonterminal states} \\ \pm 1 & \text{for terminal states} \end{cases}$$

Agent Designs

- Utility based agent
  - needs model of environment
  - learns utility function on states
  - selects action that maximize expected outcome utility

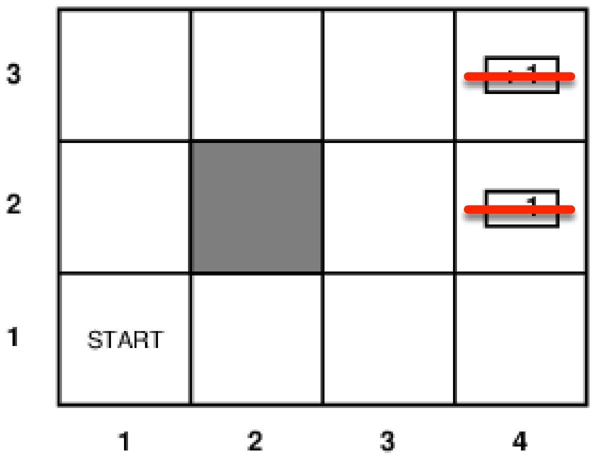
- Q-learning
  - learns action-utility function ($Q(s,a)$ function)
  - does not need to model outcomes of actions
  - function provides expected utility of taken a given action at a given step

- Reflex agent
  - learns policy that maps states to actions

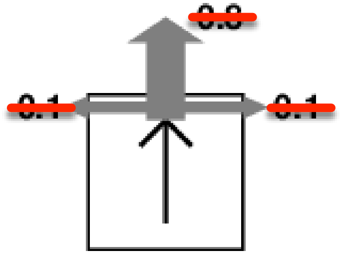
passive reinforcement learning

Setup

State Map



Stochastic Movement



Reward Function

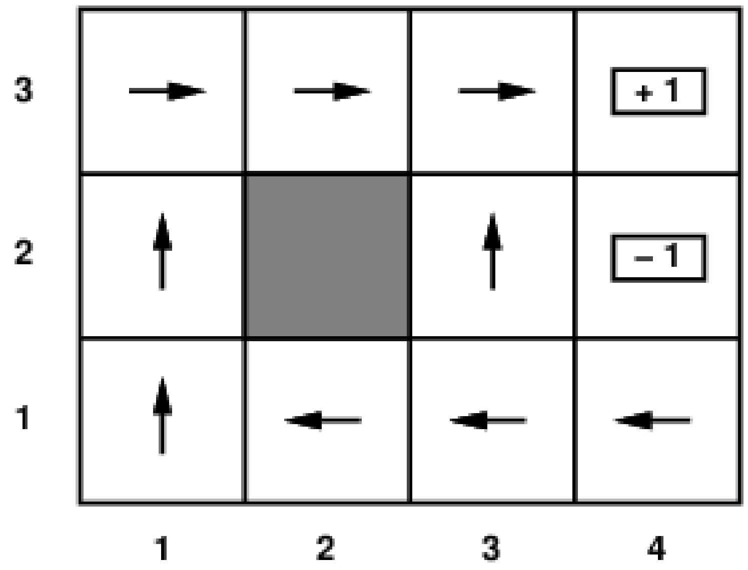
$$R(s) = \begin{cases} +1 & \text{for goal} \\ -1 & \text{for pit} \\ -0.04 & \text{for other} \end{cases}$$

Unknown information

- We know which state we are in (= partially observable environment)
- We know which actions we can take
- But only after taking an action
  → new state becomes known
  → reward becomes known

Passive Reinforcement Learning

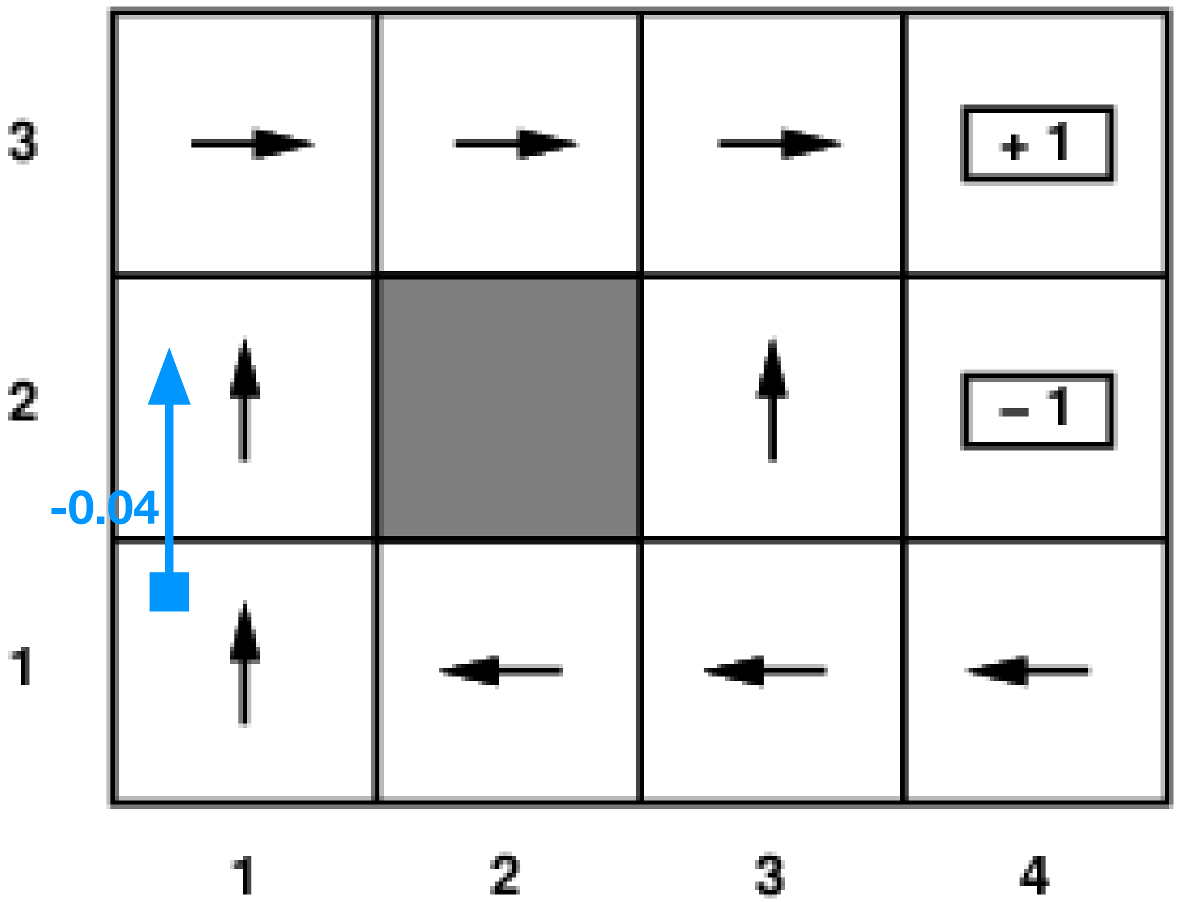
- Given a policy



- Task: compute utility of policy

- We will extend this later to **active** reinforcement learning
  (⇒ policy needs to be learned)

Sampling



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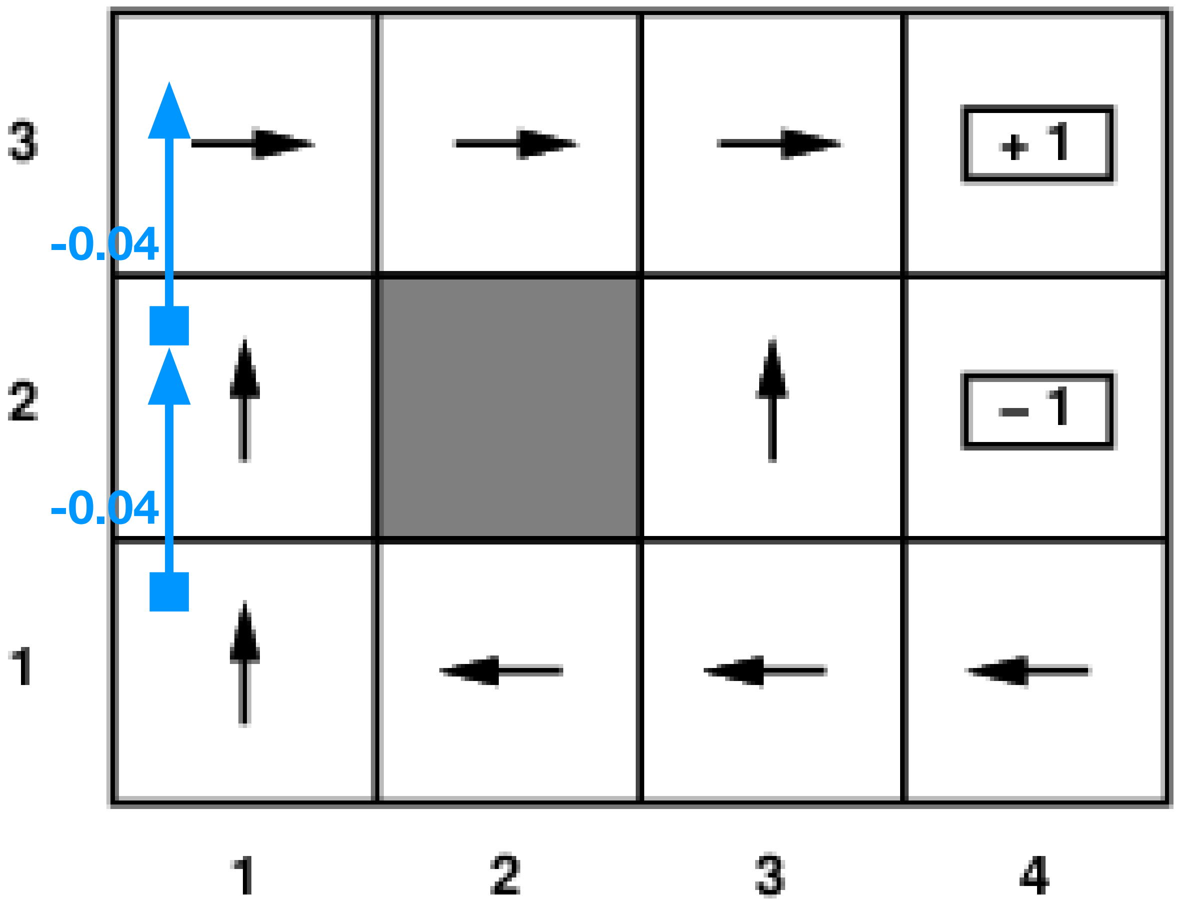

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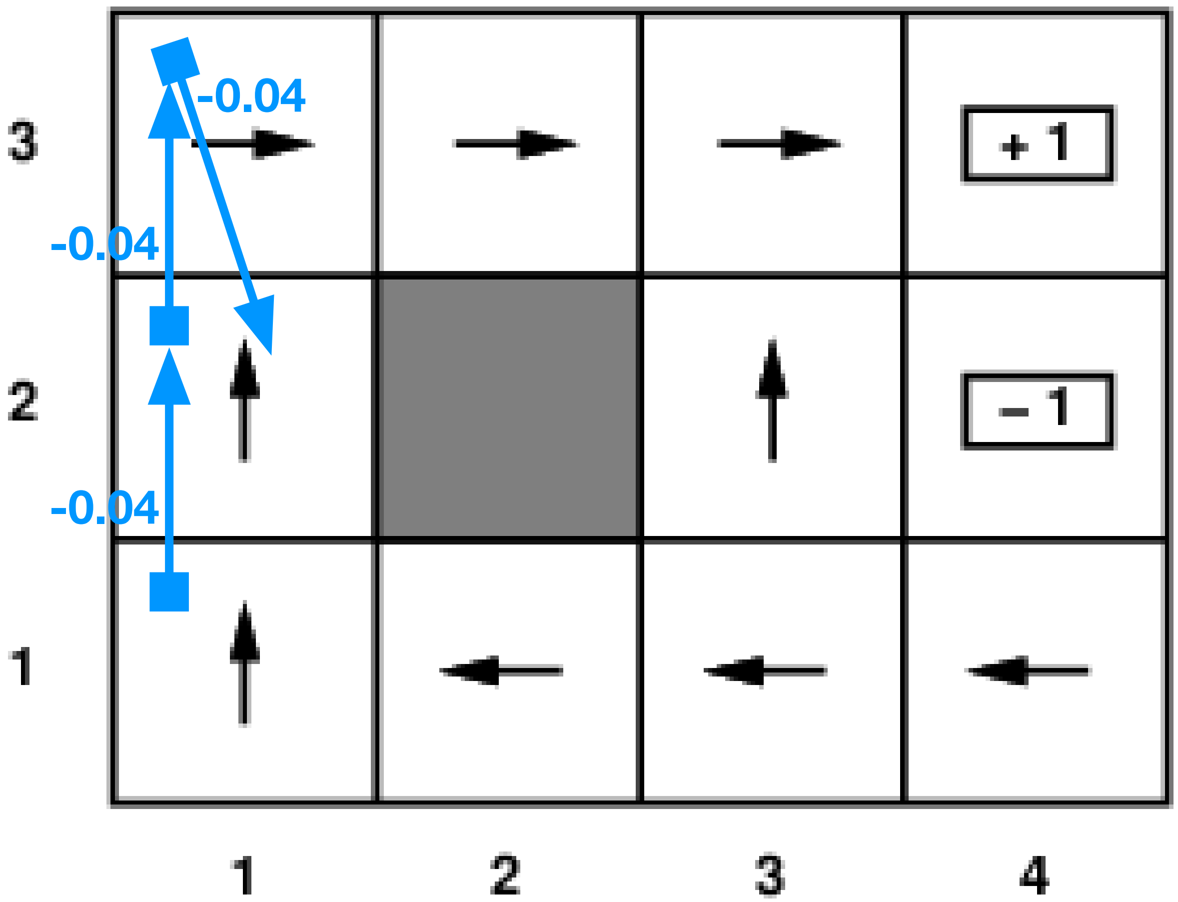

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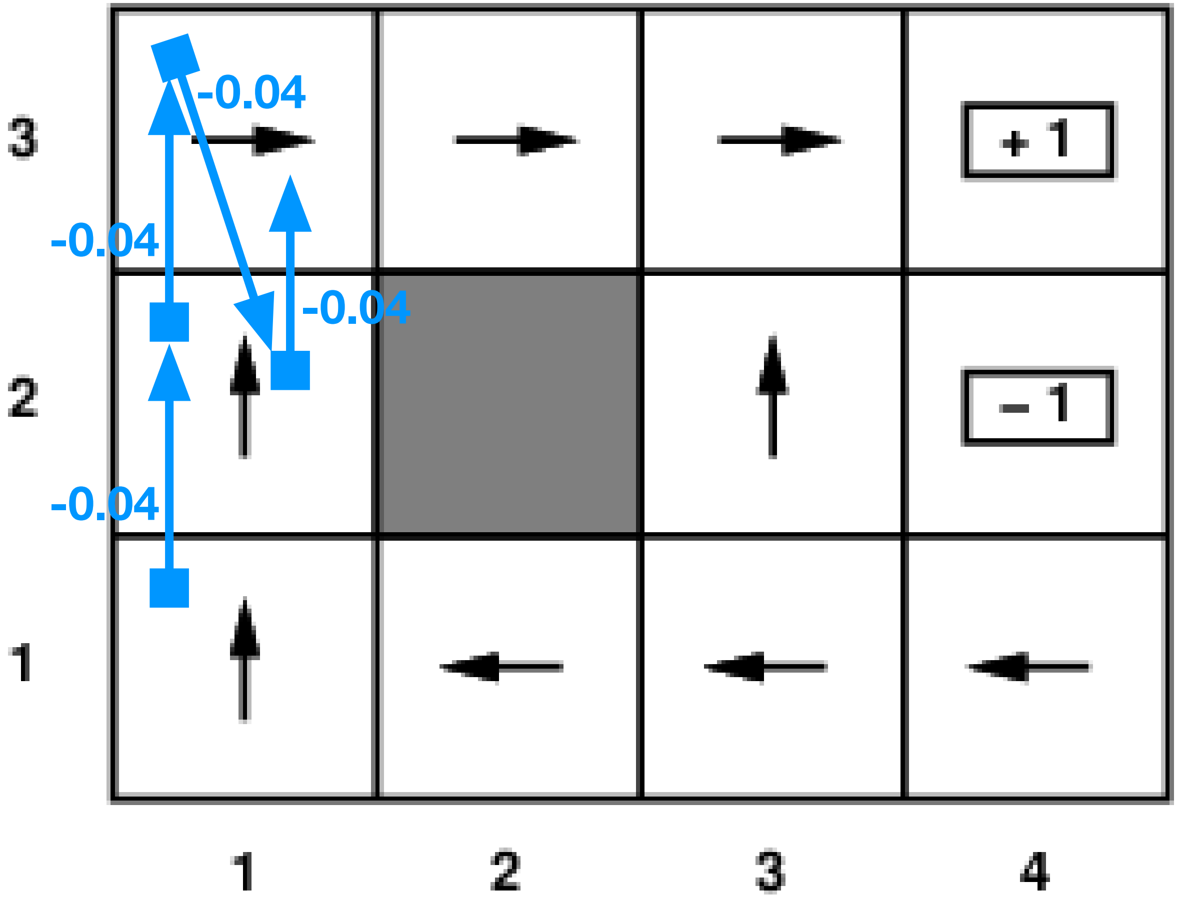

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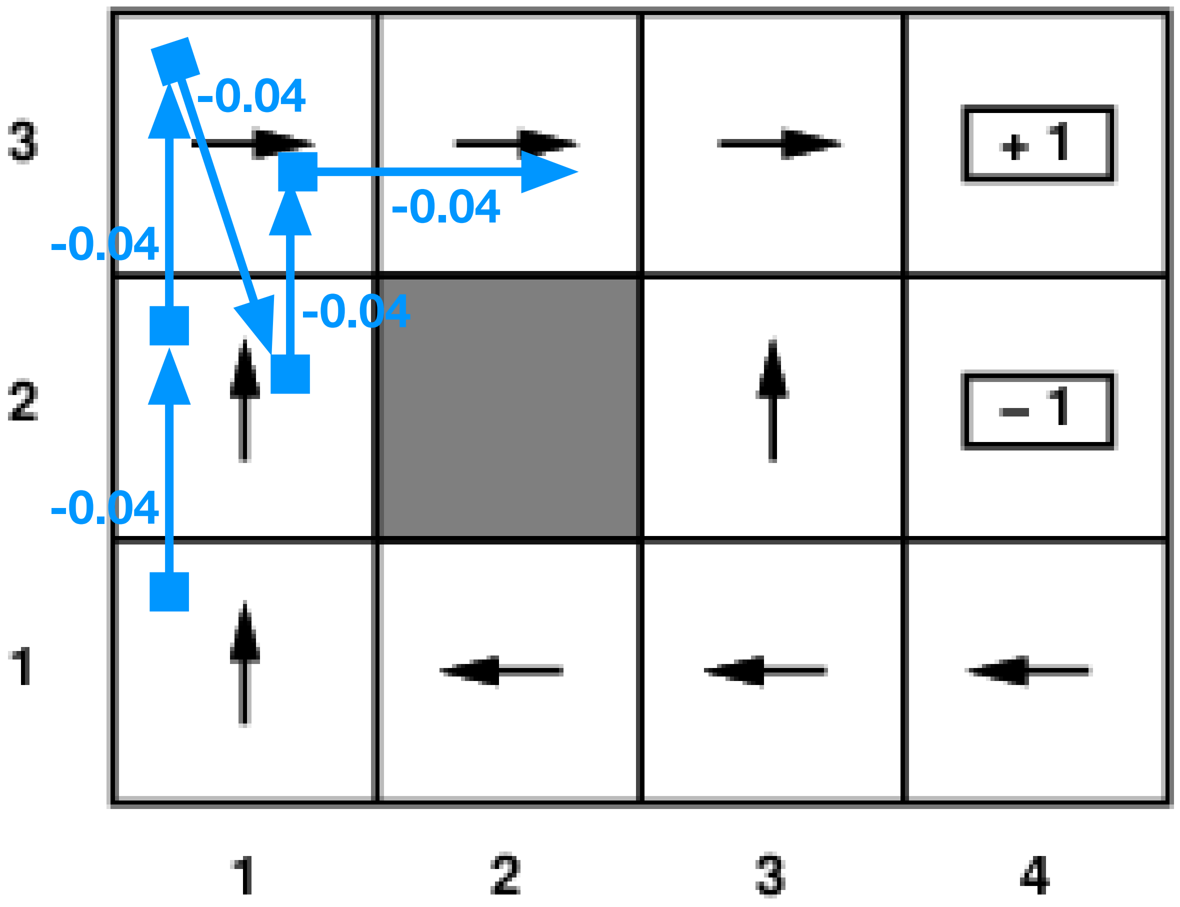

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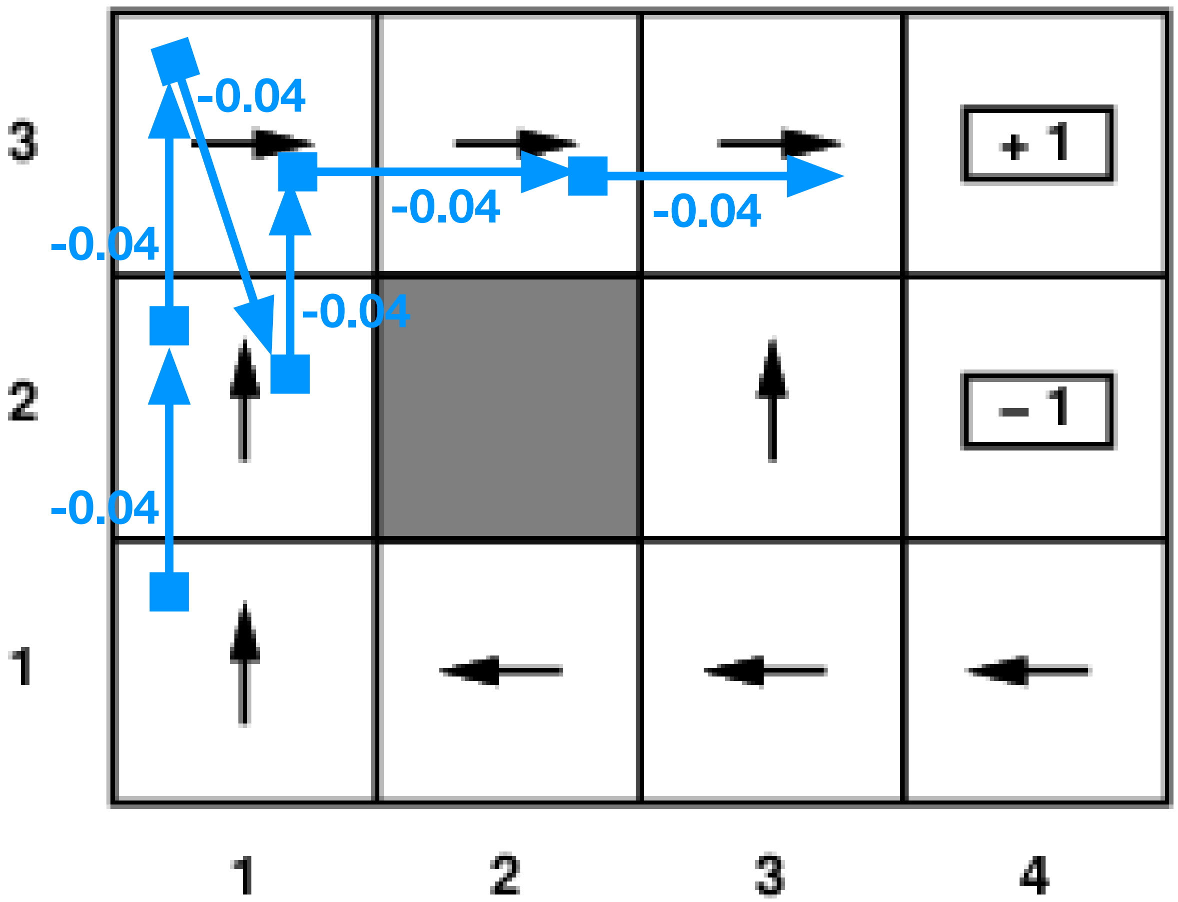

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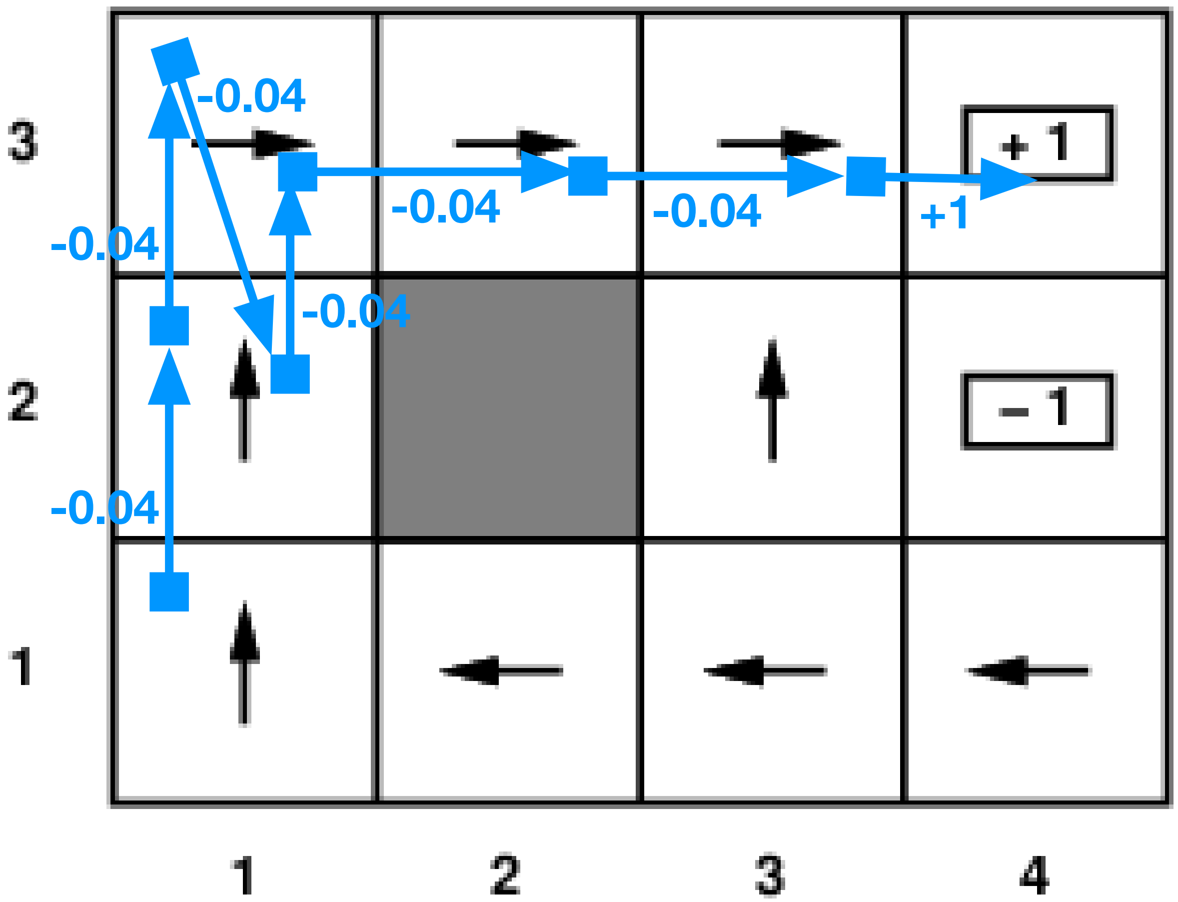

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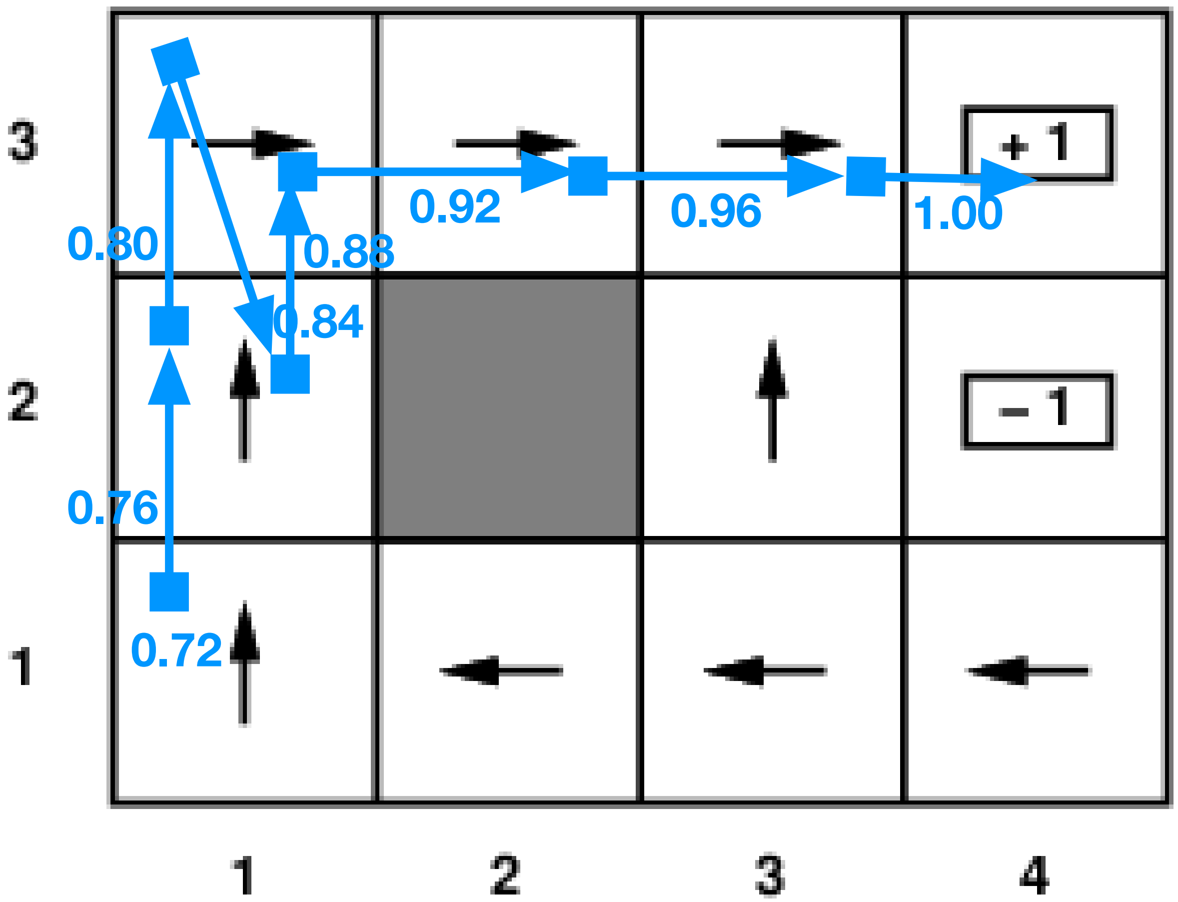
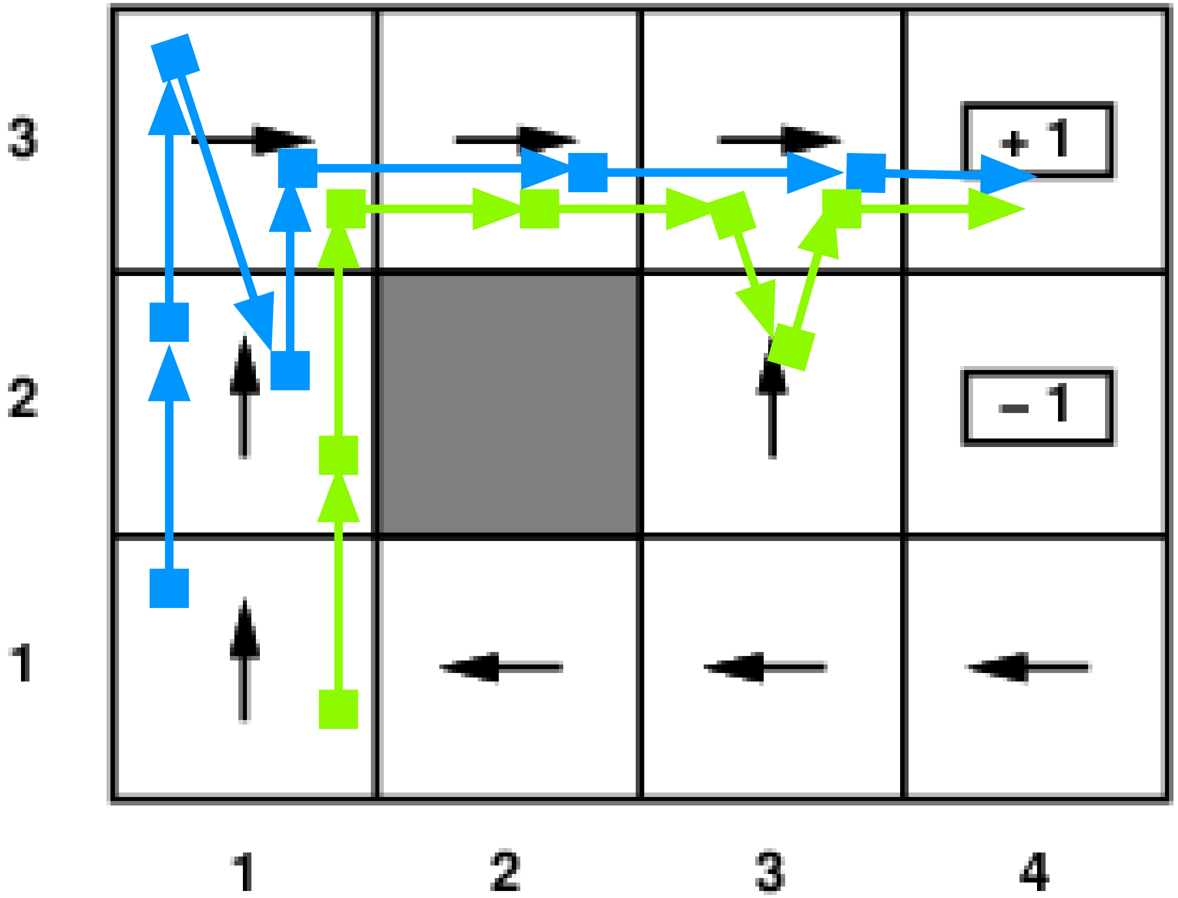


- Sample of reward to go

Sampling



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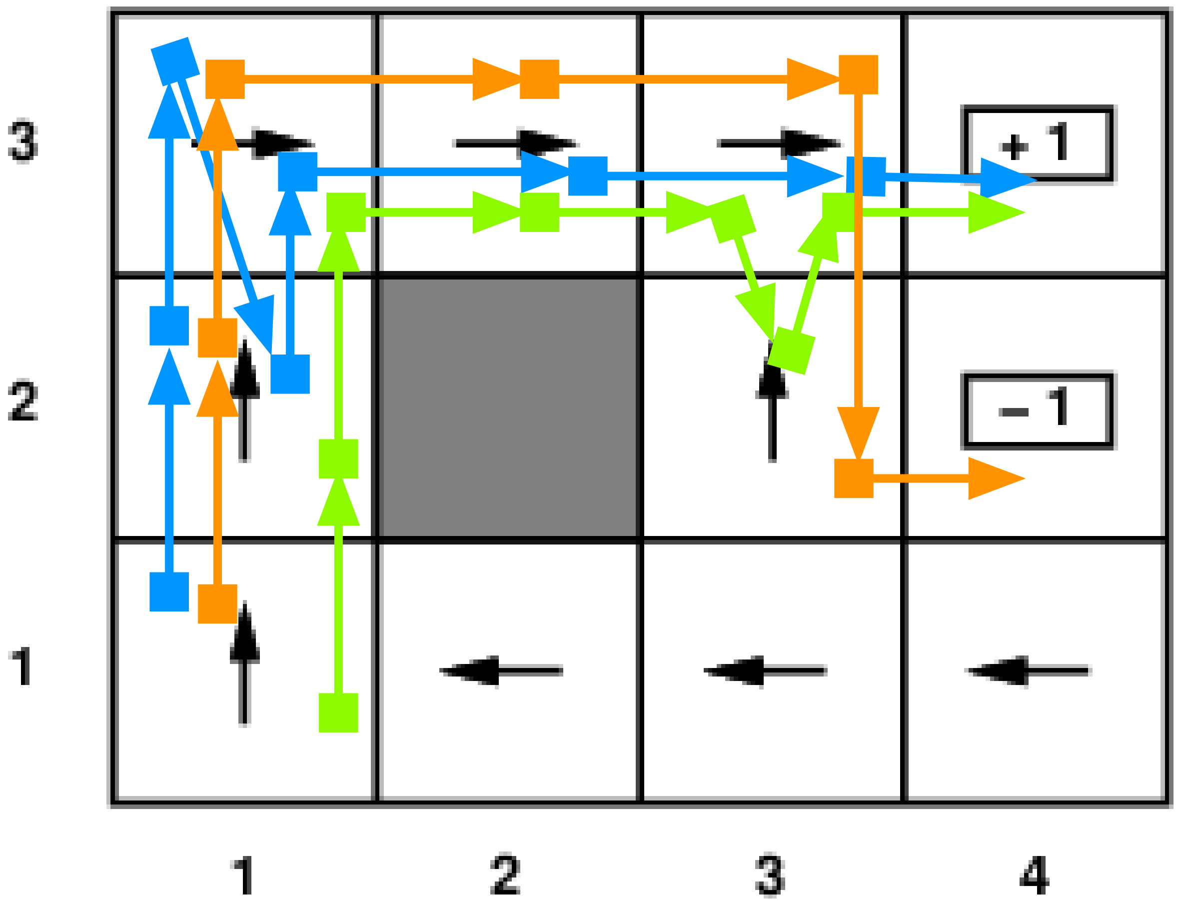


Utility of Policy

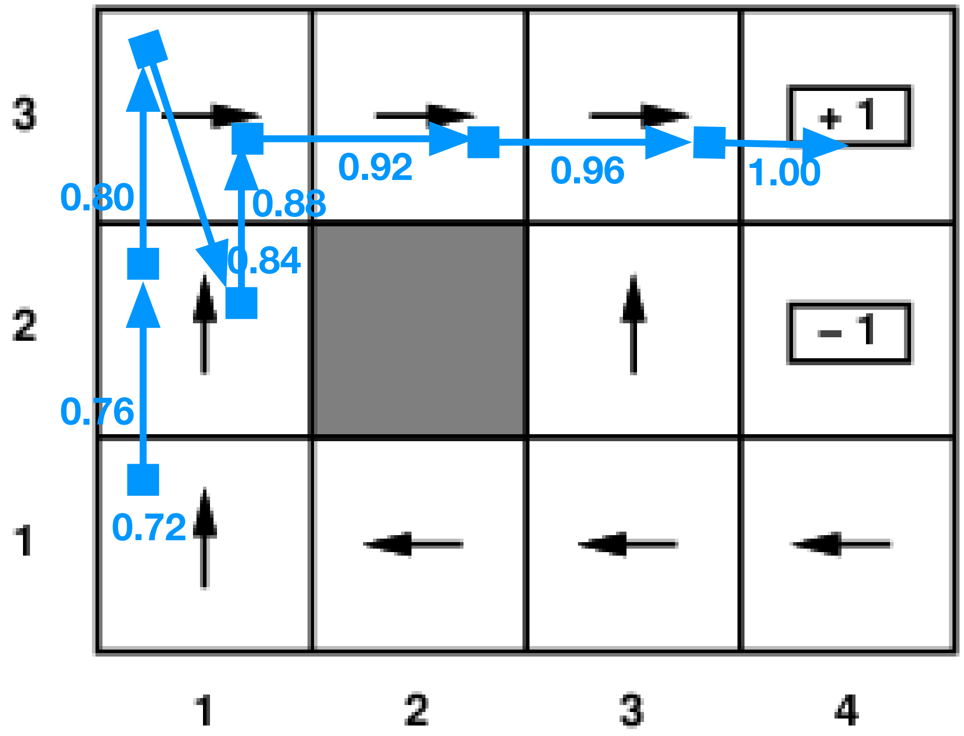
- Definition of utility U of the policy π for state s

$$U^{\pi}(s) = E\left[\sum_{t=0}^{\infty} \gamma^t R(S_t)\right]$$

- Start at state $S_0 = s$
- Reward for each state S_t is $R(S_t)$
- Discount factor γ (we use $\gamma = 1$ in our examples)
- Expected value $E[]$ = average value when sampled infinite times

Direct Utility Estimation

- Learning from the samples
- Reward to go:
  - (1,1) one sample: 0.72
  - (1,2) two samples: 0.76, 0.84
  - (1,3) two samples: 0.80, 0.88
- Reward to go
  will converge to utility of state
- But very slowly — can we do better?



Bellman Equation

- Direct utility estimation ignores dependency between states
- Given by Bellman equation

$$U^\pi(s) = \underbrace{R(s)}_{\text{reward for state}} + \gamma \underbrace{\sum_{s'}}_{\text{all possible ways to get there}} \underbrace{P(s'|s,\pi(s))}_{\text{transition probability}} U^\pi(s')$$

(γ = reward decay)

- Use of this known dependence can speed up learning
- Requires learning of transition probabilities $P(s'|s,\pi(s))$

Adaptive Dynamic Programming

Need to learn:

- State rewards $R(s)$
  - whenever a state is visited, record award (deterministic)
- Transition probabilities for taking action $\pi(s)$ at state s according to policy π
  - collect statistic $\text{count}(s, s')$ that s' is reached from s
  - estimate probability distribution

$$P(s'|s, \pi(s)) = \frac{\text{count}(s, s')}{\sum_{s''} \text{count}(s, s'')}$$

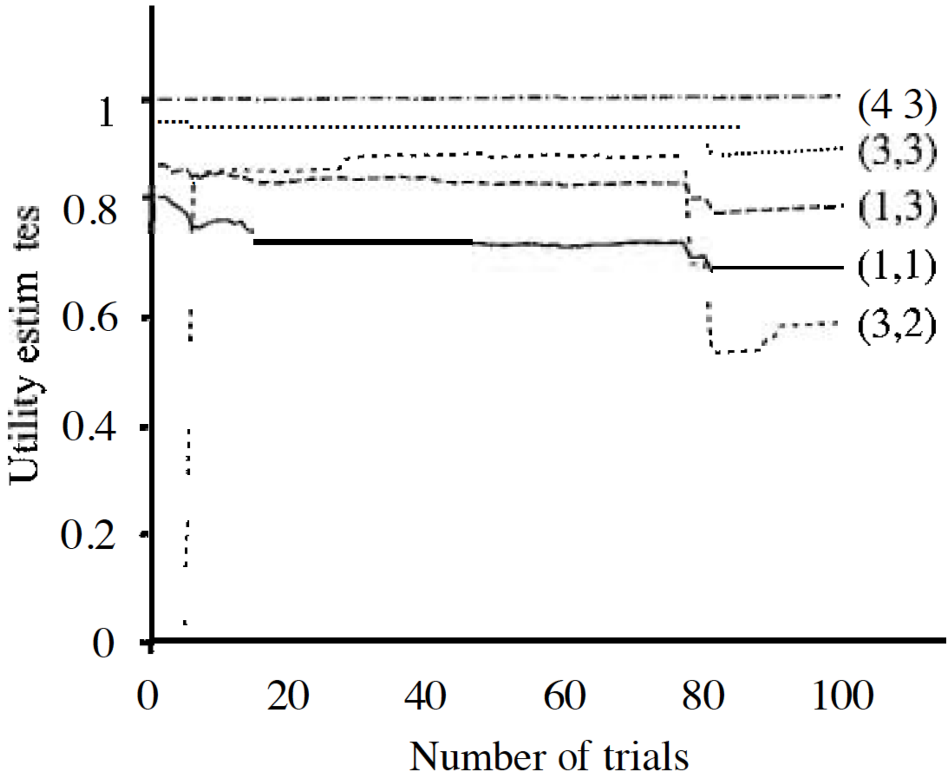
⇒ Ingredients for policy evaluation algorithm

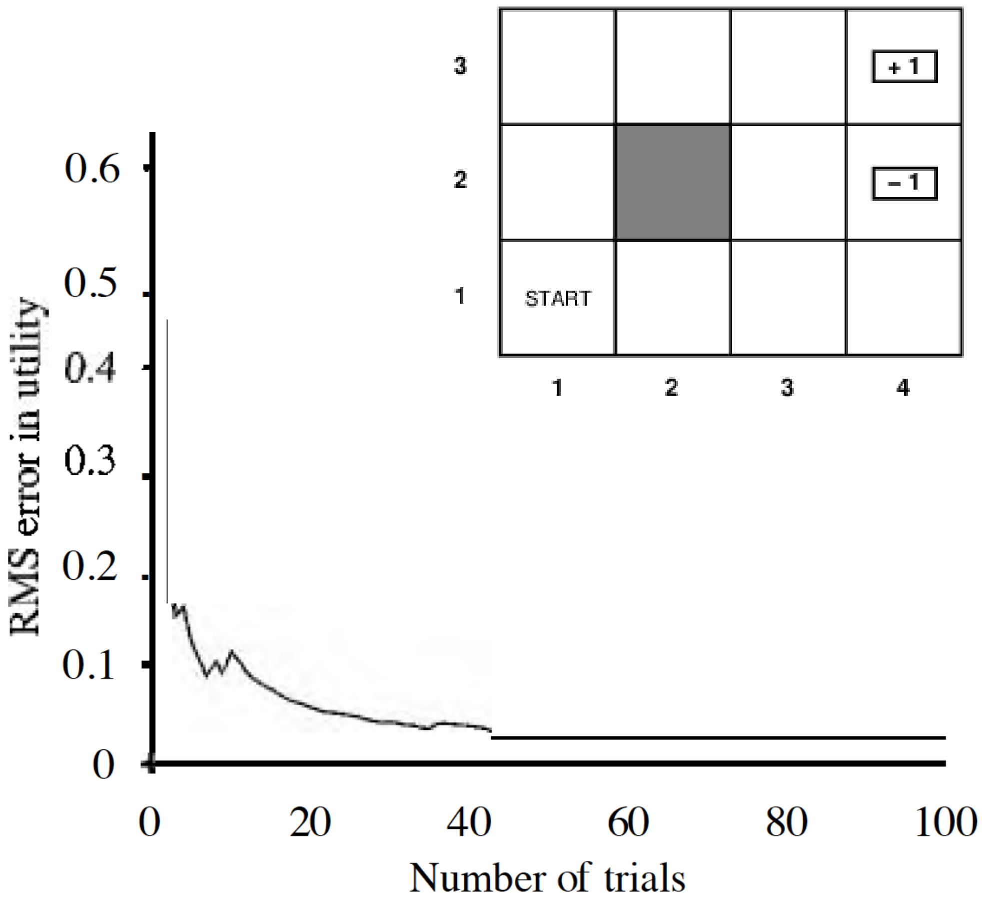
Adaptive Dynamic Programming

function PASSIVE-ADP-AGENT(*percept*) **returns** an action
  inputs: percept, a percept indicating the current state s' and reward signal r'
  static: π, a fixed policy
    mdp, an MDP with model T, rewards R, discount γ
    U, a table of utilities, initially empty
    N_{sa}, a table of frequencies for state-action pairs, initially zero
    $N_{sas'}$, a table of frequencies for state-action-state triples, initially zero
    s, a, the previous state and action, initially null

  if s is new **then do** $U[s] \leftarrow r$; $R[s] \leftarrow r'$
  if s is not null **then do**
    increment $N_{sa}[s,$ **a**] and $N_{sas'}[s, a, s]$
    for each t such that $N_{sas'}[s, a, t]$ is nonzero **do**
      $T[s, a, t] \leftarrow N_{sas'}[s, a, t] \,/\, N_{sa}[s, a]$
  $U \leftarrow$ *POLICY- EVALUATION*(^, U, mdp)
  if TERMINAL?$[s']$ **then** $s, a \leftarrow$ null **else** s, a $\leftarrow s$, $\pi[s']$
  return a

Learning Curve





- Major change at 78^{th} trial: first time terminated in –1 state at (4,2)

Temporal Difference Learning

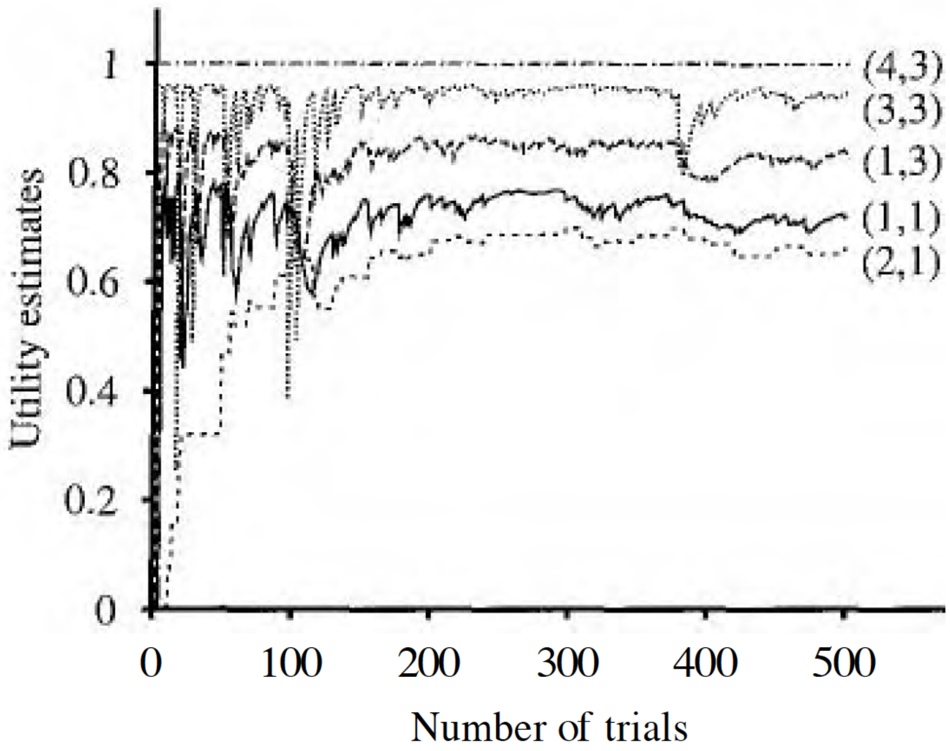
- Idea: do not model $P(s'|s,\pi(s))$, directly adjust utilities $U(s)$ for all visited states
- Estimate of current utility: $U^\pi(s)$
- Estimate of utility after action: $R(s) + \gamma U^\pi(s')$
- Adjust utility of current state $U^\pi(s)$ if they differ

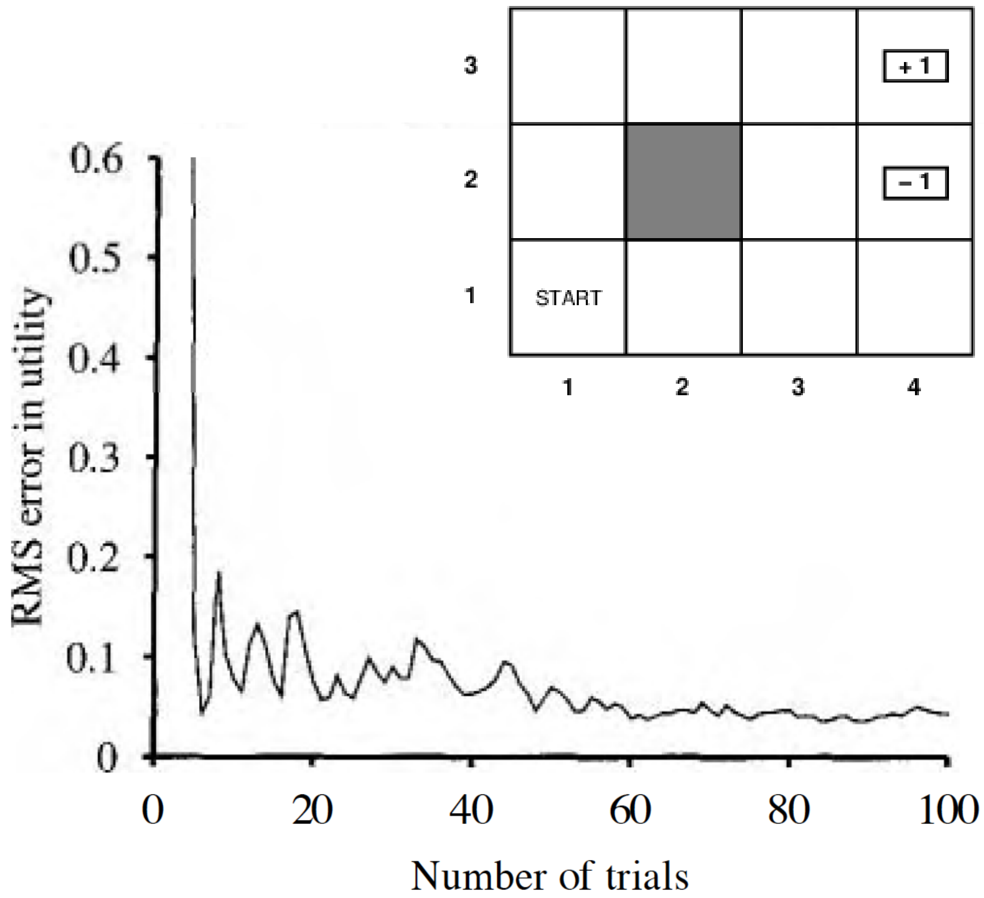
$$\Delta U^\pi(s) = \alpha \left(R(s) + \gamma U^\pi(s') - U^\pi(s) \right)$$

(α = learning rate)

- Learning rate may decrease when state has been visited often

Learning Curve





- Noisier, converging more slowly

Comparison

- Both eventually converge to correct values
- Adaptive dynamic programming (ADP)
  faster than
  Temporal difference learning (TD)
  - both make adjustments to make successors agree
  - but: ADP adjusts all possible successors, TD only observed successor
- ADP computationally more expensive due to policy evaluation algorithm
  (re-computation of all parameters with any new evidence)

active reinforcement learning

Active Reinforcement Learning

- Previously: passive agent follows prescribed policy

- Now: active agent decides which action to take
  - following optimal policy (as currently viewed)
  - exploration

- Goal: optimize rewards for a given time frame

Greedy Agent

1. Start with initial policy

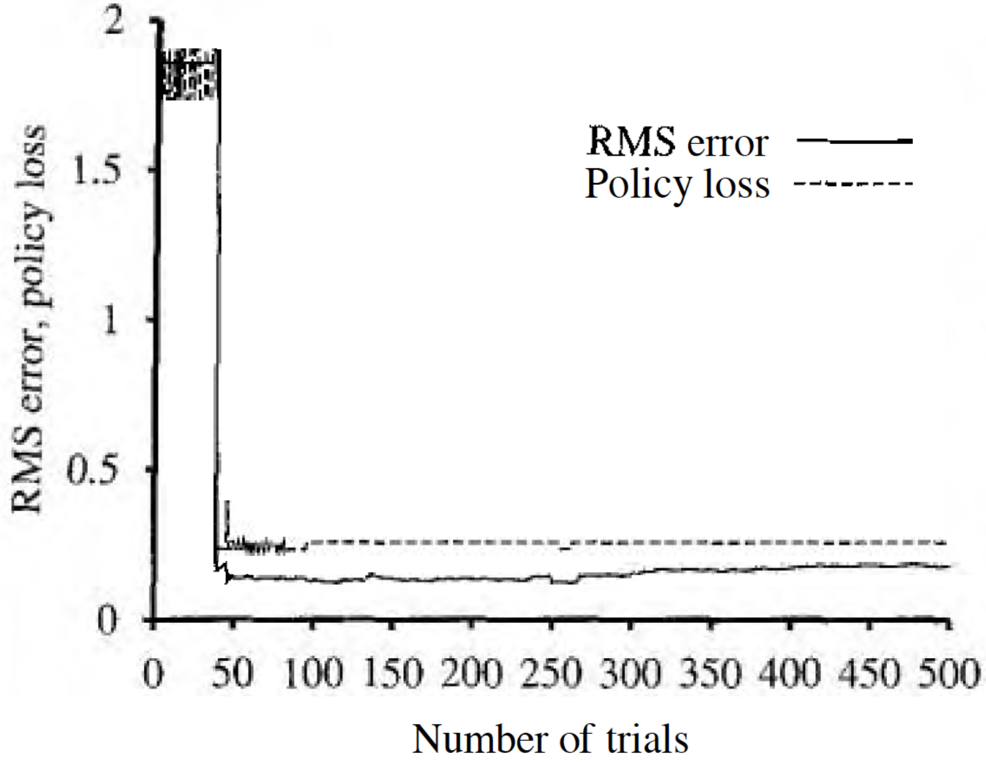
2. Compute utilities (using ADP)

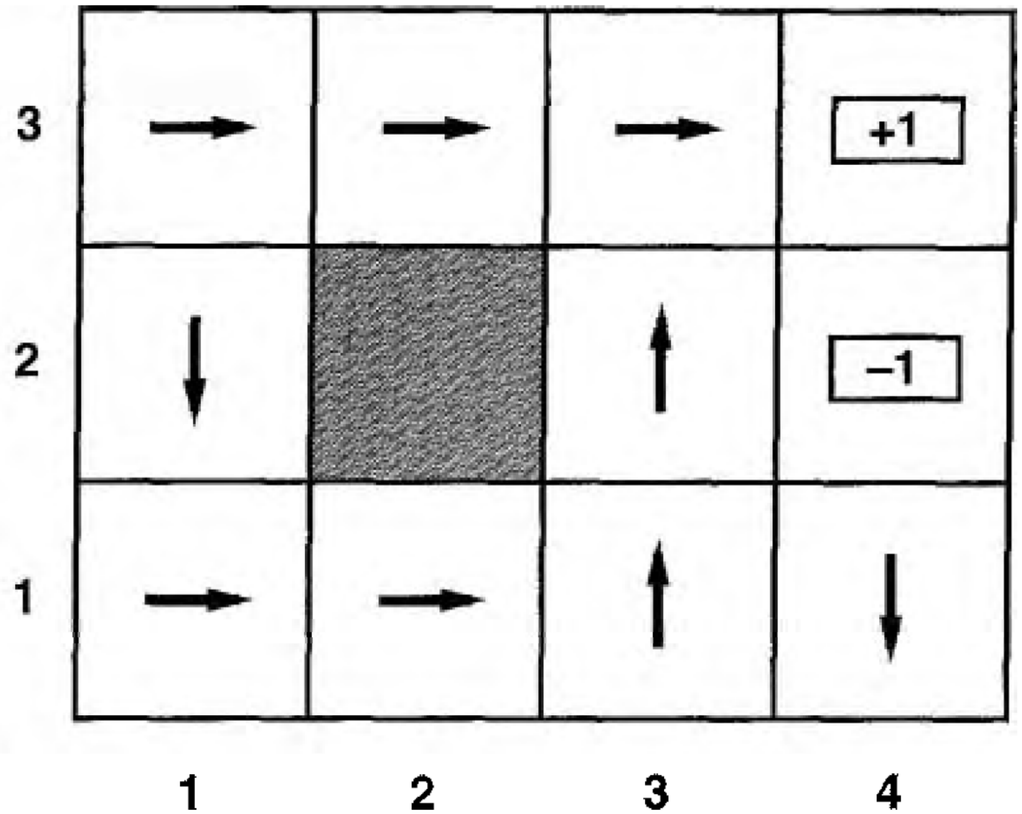
3. Optimize policy

4. Go to Step 2

- This *very seldom* converges to global optimal policy

Learning Curve





- Greedy agent stuck in local optimum

Bandit Problems

- Bandit: slot machine

Bandit Problems

- Bandit: slot machine
- N-armed bandit: n levers
- Each has different probability distribution over payoffs
- Spend coin on
  - presumed optimal payoff
  - exploration (new lever)
- If independent
  - **Gittins index**: formula for solution
  - uses payoff / number of times used

Greedy in the Limit of Infinite Exploration

- Explore any action in any state unbounded number of times

- Eventually has to become greedy
  - carry out optimal policy

  $\Rightarrow$ maximize reward

- Simple strategy
  - with probability $p(1/t)$ take random action
  - initially (t small) focus on exploration
  - later (t big) focus on optimal policy

Extension of Adaptive Dynamic Programming

- Previous definition of utility calculation
  (policy = take best action to maximize utility)

$$U(s) \leftarrow \underbrace{R(s)}_{\text{reward for state}} + \gamma \underbrace{\max_a}_{\text{best action}} \sum_{s'} \underbrace{P(s'|s,a)}_{\text{transition probability}} U(s')$$

- New utility calculation

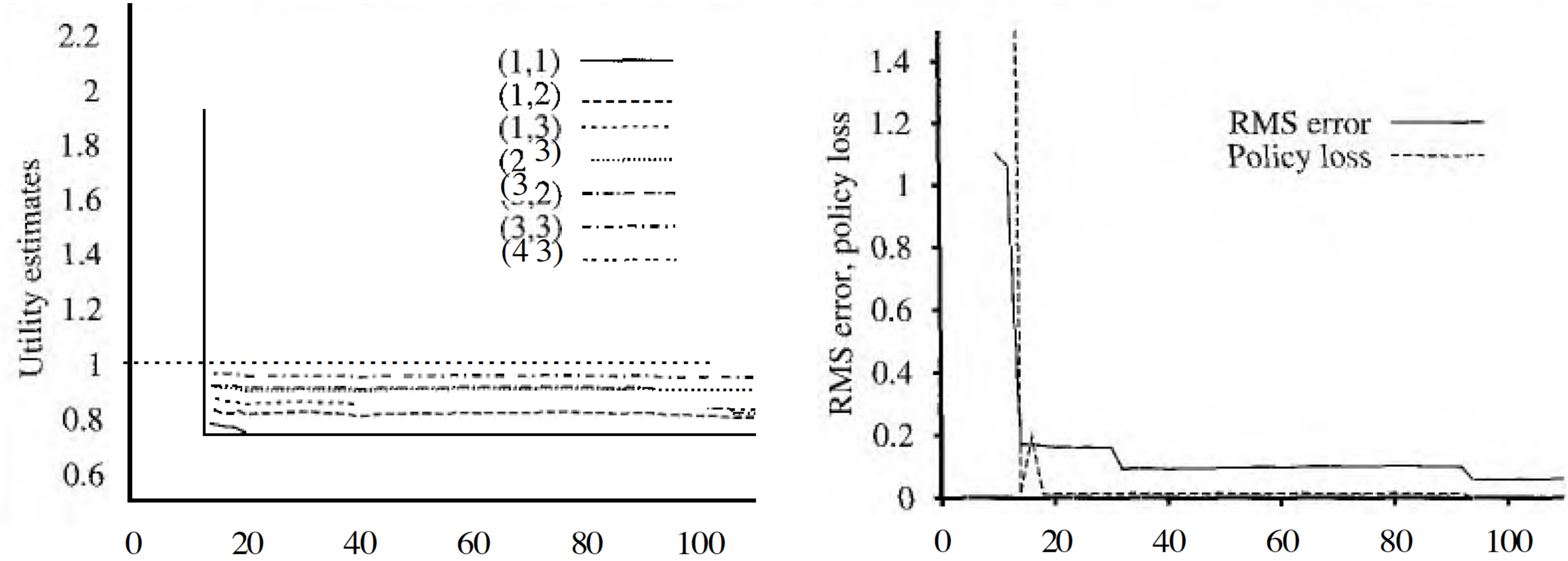
$$U^+(s) \leftarrow R(s) + \gamma \max_a f\left(\sum_{s'} P(s'|s,a)\, U^+(s'), N(s,a)\right)$$

- One possible definition of $f(u,n)$: optimistic reward when visited rarely

$$f(u,n) = \begin{cases} R^+ & \text{if } n < N_c \\ u & \text{otherwise} \end{cases}$$

R^+ is optimistic estimate, best possible award in any state

Learning Curve



- Performance of exploratory ADP agent
- Parameter settings $R^+ = 2$ and $N_e = 5$
- Fairly quick convergence to optimal policy

Q-Learning

- Learning an action utility function $Q(s,a)$
- Allows computation of utilities $U(s) = \text{max}_a Q(s,a)$
- Model-free: no explicit transition model $P(s'|s,a)$
- Theoretically correct Q values

$$Q(s,a) = R(s) + \gamma \sum_{s'} P(s'|s,a) \ \text{max}_{a'} Q(s',a')$$

- Update formula inspired by temporal difference learning (after taking action a to reach state s′)

$$\Delta Q(s,a) = \alpha(R(s,a,s') + \gamma \ \text{max}_{a'} \ Q(s',a') - Q(s,a))$$

- For our example, Q-learning slower, but successful applications (TD-GAMMON)

generalization in reinforcement learning

Large Scale Reinforcement Learning

- Adaptive dynamic programming (ADP) scalable to maybe 10,000 states
  - Backgammon has 10^{20} states
  - Chess has 10^{40} states

- It is not possible to visit all these states multiple times

⇒ Generalization of states needed

Function Approximation

- Define state utility function as linear combination of features

$$\hat{U}_\theta(s) = \theta_1 \ f_1(s) + \theta_2 \ f_2(s) + ... + \theta_n \ f_n(s)$$

- Recall: features to assess Chess state
  - $f_1(s)$ = (number of white pawns) – (number of black pawns)
  - $f_2(s)$ = (number of white rooks) – (number of black rooks)
  - $f_3(s)$ = (number of white queens) – (number of black queens)
  - $f_4(s)$ = king safety
  - $f_5(s)$ = good pawn position
  - etc.

⇒ Reduction from 10^{40} to, say, 20 parameters

- Main benefit: ability to assess unseen states

Learning Feature Weights

- Example: 2 features: x and y

$$\hat{U}_\theta(f_1, f_2) = \theta_0 + \theta_1 f_1 + \theta_2 f_2$$

- Current feature weights $\theta_0, \theta_1, \theta_2 = (0.5, 0.2, 0.1)$

- Model's prediction of utility of specific state, e.g., $\hat{U}_\theta(1,1) = 0.8$

- Sample set of trials, found value $u_\theta(1,1) = 0.4$

- Error $E_\theta = \frac{1}{2}(\hat{U}_\theta(f_1, f_2) - u_\theta(f_1, f_2))^2$

- How do you update the weights θ_i?

Gradient Descent Training

- Compute gradient of error

$$\frac{dE_\theta}{d\theta_i} = (\hat{U}_\theta(f_1, f_2) - u_\theta(f_1, f_2))\ f_i$$

- Update against gradient

$$\Delta\theta_i = -\mu\ \frac{dE_\theta}{d\theta_i}$$

- Our example
  - $\Delta\theta_1 = -\mu(\hat{U}_\theta(f_1, f_2) - u_\theta(f_1, f_2))\ f_i = -\mu(0.8 - 0.4)\ 1 = -0.4\mu$
  - $\Delta\theta_2 = -\mu(\hat{U}_\theta(f_1, f_2) - u_\theta(f_1, f_2))\ f_i = -\mu(0.8 - 0.4)\ 1 = -0.4\mu$

Additional Remarks

- If we know something about the problem
  ⇒ we may want to use more complex features

- Our toy example: utility related to Manhattan distance from goal $(x_{\text{goal}}, y_{\text{goal}})$

$$f_3(s) = (x - x_{\text{goal}}) + (y - y_{\text{goal}})$$

- Gradient descent training can also be used for temporal distance learning

policy search

Policy Search

- Idea: directly optimize policy

- Policy may be parameterized Q functions, hence:

$$\pi(s) = \text{argmax}_a \hat{Q}_\theta(s,a)$$

- Stochastic policy, e.g., given by softmax function

$$\pi_\theta(s,a) = \frac{1}{Z_s} e^{\hat{Q}_\theta(s,a)}$$

- Policy value $\rho(\theta)$: expected reward if π_θ is carried out

Hillclimbing

- Deterministic policy, deterministic environment

  $\Rightarrow$ optimizing policy value $\rho(\theta)$ may be done in closed form

- If $\rho(\theta)$ differentiable

  $\Rightarrow$ gradient descent by following policy gradient

- Make small changes to parameters

  $\Rightarrow$ hillclimb if $\rho(\theta)$ improves

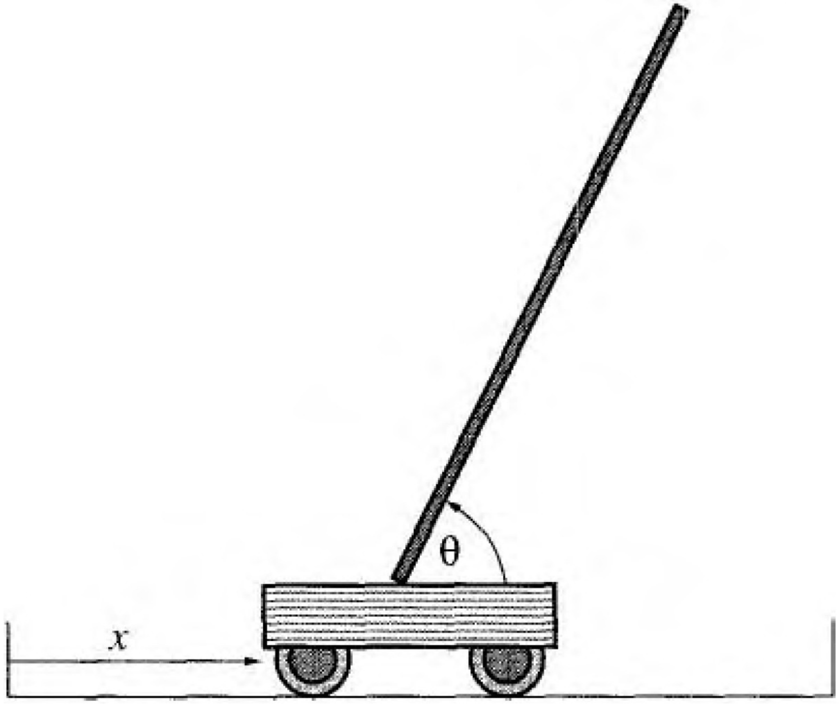
- More complex for stochastic environment

examples

Game Playing

- Backgammon: TD-GAMMON (1992)
- Reward only at end of game
- Training with self-play
- 200,000 training games needed
- Competitive with top human players
- Better positional play, worse end game

Robot Control



- Observe position x, vertical speed $\hat{x}$, angle θ, angle speed $\hat{\theta}$
- Action: jerk left or right
- Reward: time balanced until pole falls, or cart out of bounce
- More complex: multiple stacked poles, helicopter flight, walking

Summary

- Building on Markov decision processes and machine learning
- Passive reinforcement learning
  (fixed policy, partially observable environment, stochastic outcomes of actions)
  - sampling (carrying out trials)
  - adaptive dynamic programming
  - temporal difference learning
- Active reinforcement learning
  - greedy in the limit of infinite exploration
  - following optimal policy vs. exploration
  - exploratory adaptive dynamic programming
- Generalization: representing utility function with small set of parameters
- Policy search