Logical Agents

Philipp Koehn

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The world is everything that is the case.

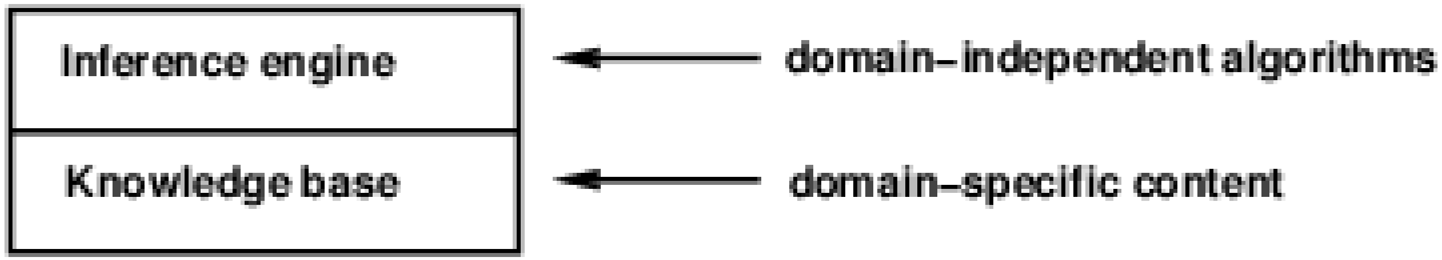
Wittgenstein, Tractatus

Outline

- Knowledge-based agents
- Logic in general—models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - forward chaining
  - backward chaining
  - resolution

knowledge-based agents

Knowledge-Based Agent



- **Knowledge base** = set of **sentences** in a **formal** language
- **Declarative** approach to building an agent (or other system):
  TELL it what it needs to know
- Then it can **Ask** itself what to do—answers should follow from the KB
- Agents can be viewed at the **knowledge level**
  i.e., **what they know**, regardless of how implemented
- Or at the **implementation level**
  i.e., data structures in KB and algorithms that manipulate them

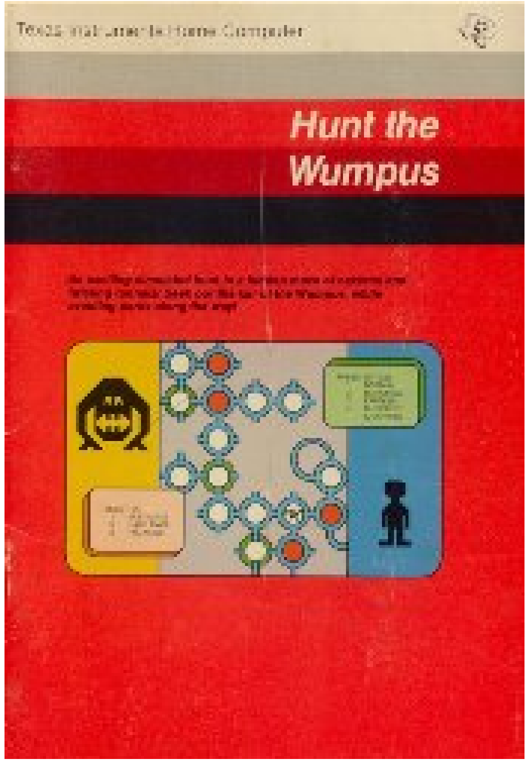
A Simple Knowledge-Based Agent

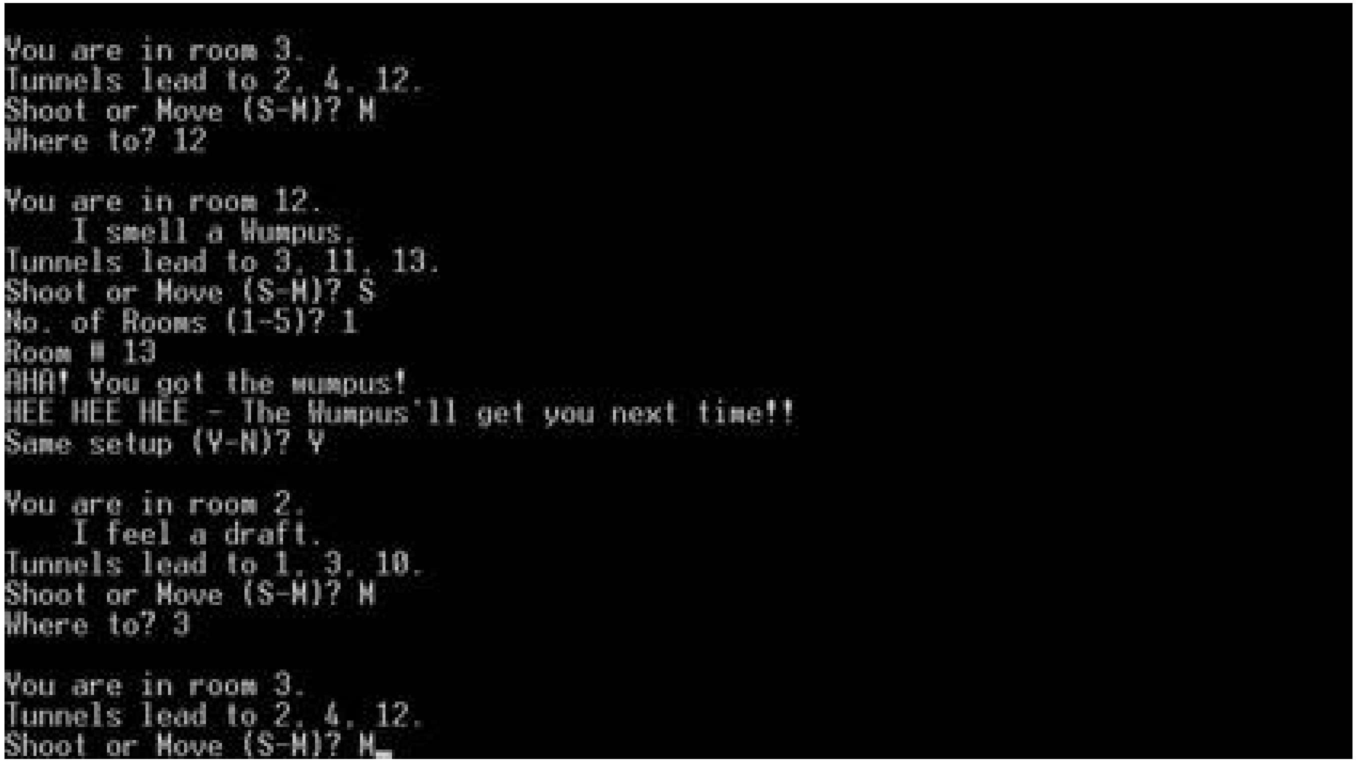
```
function KB-AGENT(percept) returns an action
   static: KB, a knowledge base
           t, a counter, initially 0, indicating time

   TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
   action ← ASK(KB, MAKE-ACTION-QUERY(t))
   TELL(KB, MAKE-ACTION-SENTENCE(action, t))
   t ← t + 1
   return action
```

- The agent must be able to
  - represent states, actions, etc.
  - incorporate new percepts
  - update internal representations of the world
  - deduce hidden properties of the world
  - deduce appropriate actions

example

Hunt the Wumpus

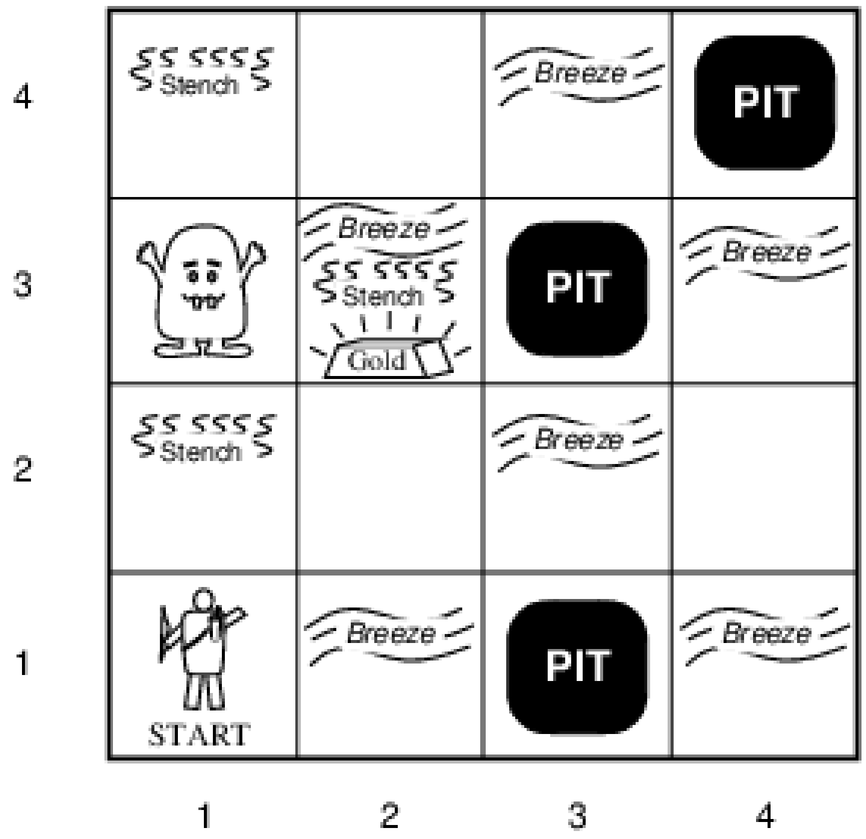




Computer game from 1972

Wumpus World PEAS Description

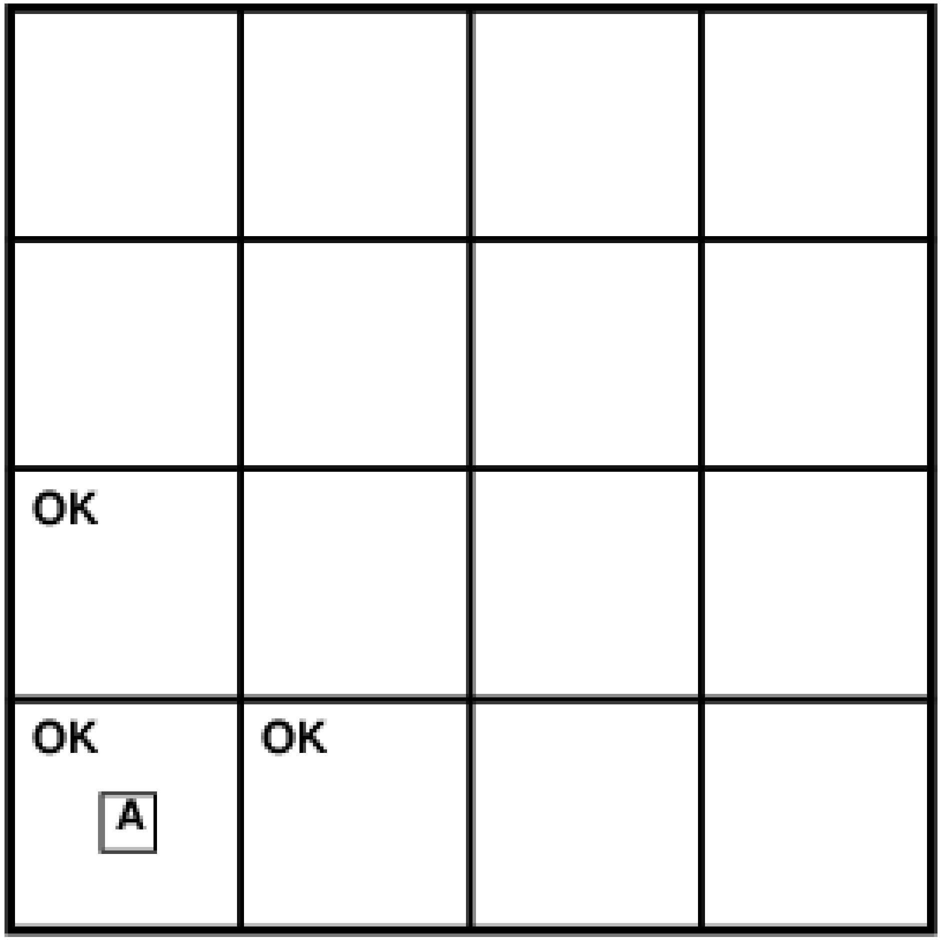
- **Performance measure**
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- **Environment**
  - squares adjacent to wumpus are smelly
  - squares adjacent to pit are breezy
  - glitter iff gold is in the same square
  - shooting kills wumpus if you are facing it
  - shooting uses up the only arrow
  - grabbing picks up gold if in same square
  - releasing drops the gold in same square
- **Actuators** Left turn, Right turn,
  Forward, Grab, Release, Shoot
- **Sensors** Breeze, Glitter, Smell



Wumpus World Characterization

- Observable? No—only **local** perception
- Deterministic? Yes—outcomes exactly specified
- Episodic? No—sequential at the level of actions
- Static? Yes—Wumpus and Pits do not move
- Discrete? Yes
- Single-agent? Yes—Wumpus is essentially a natural feature

Exploring a Wumpus World



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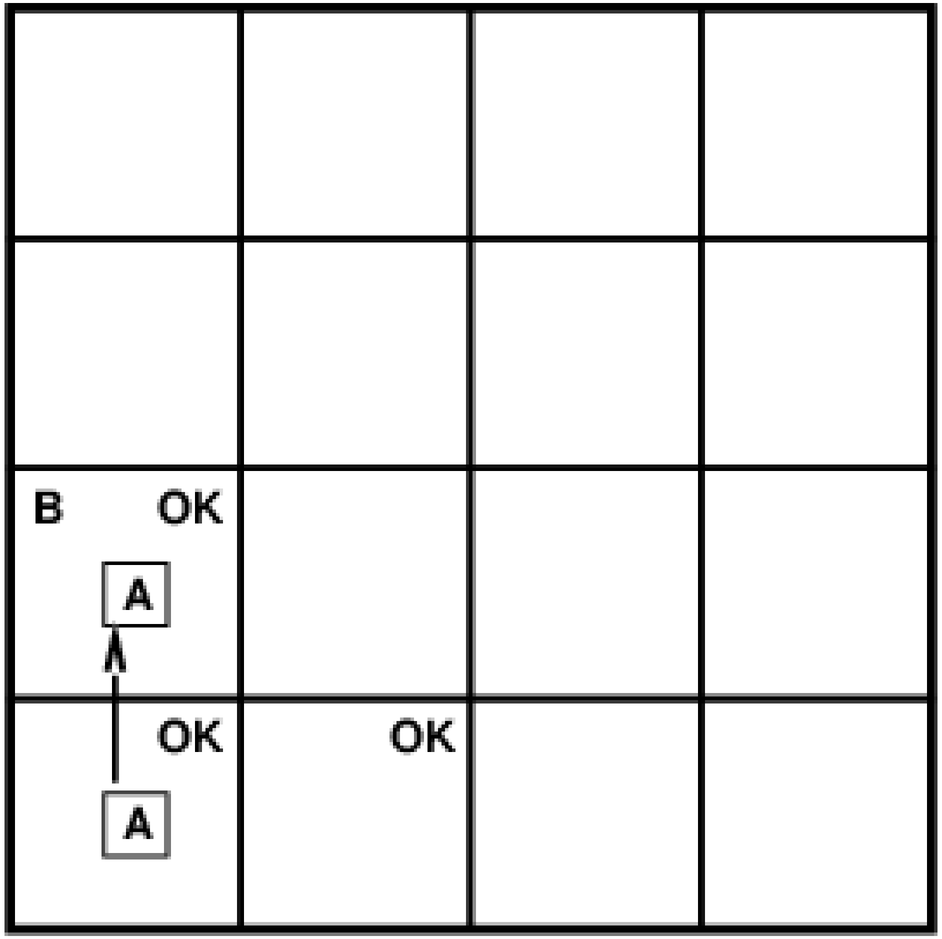

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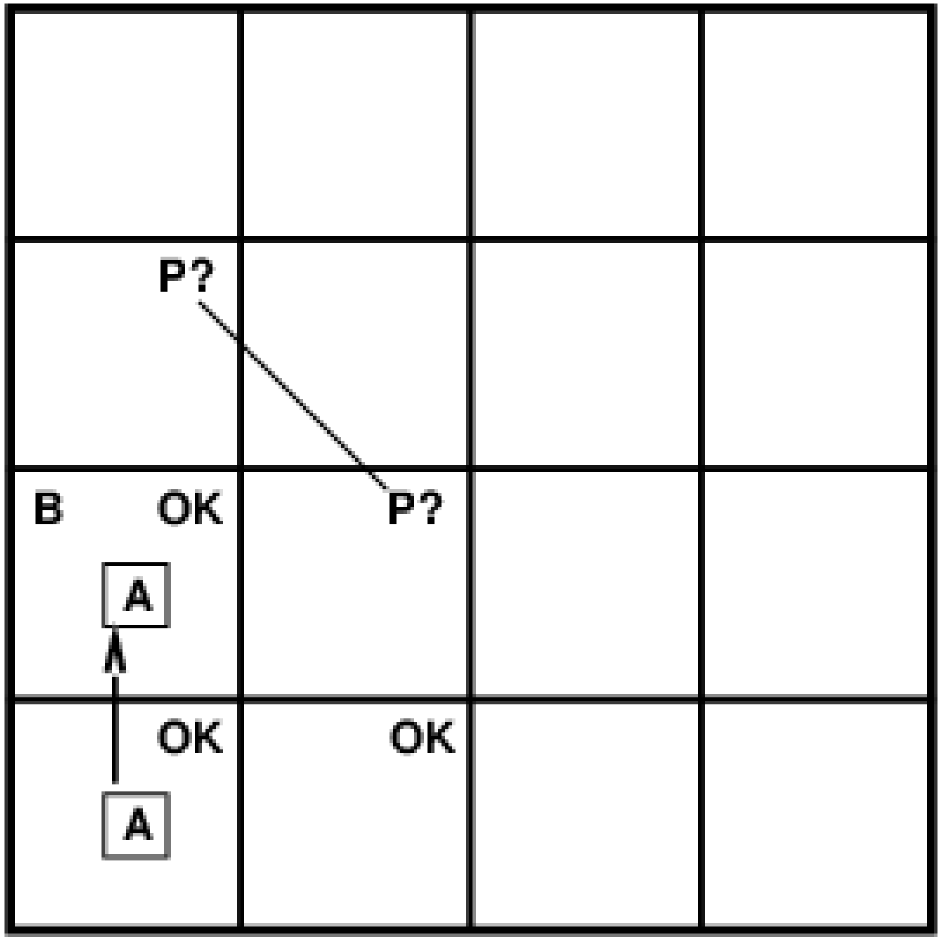

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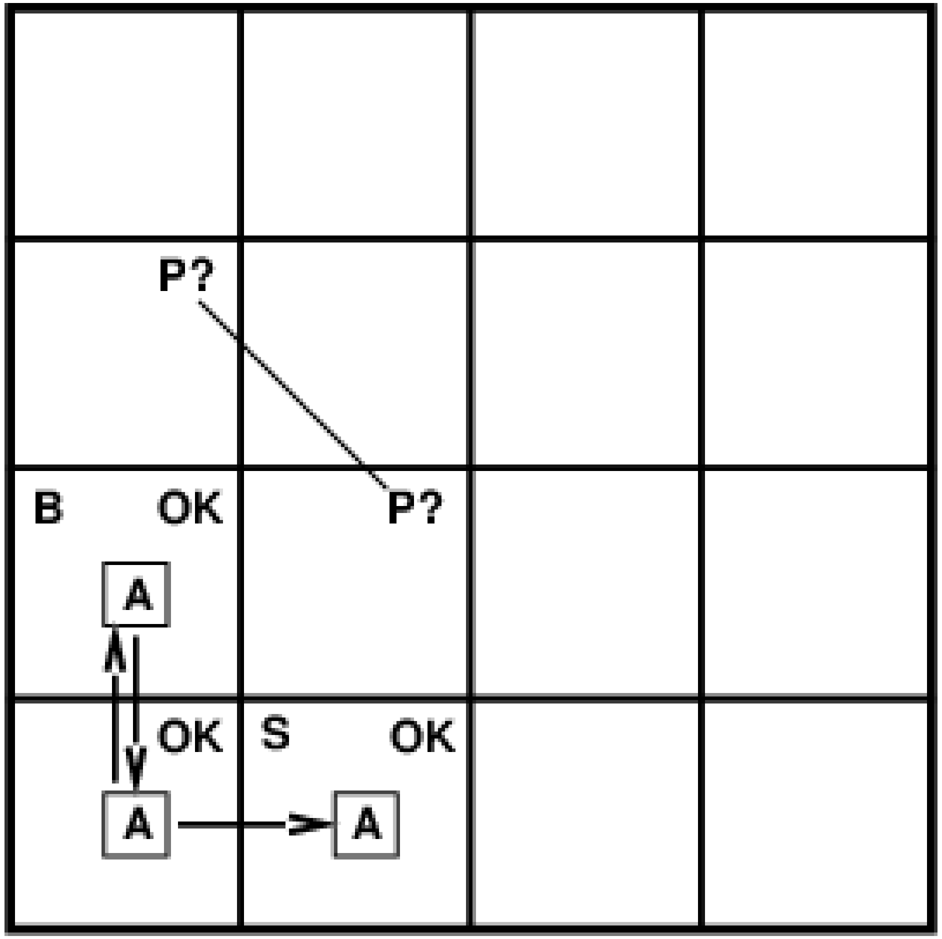

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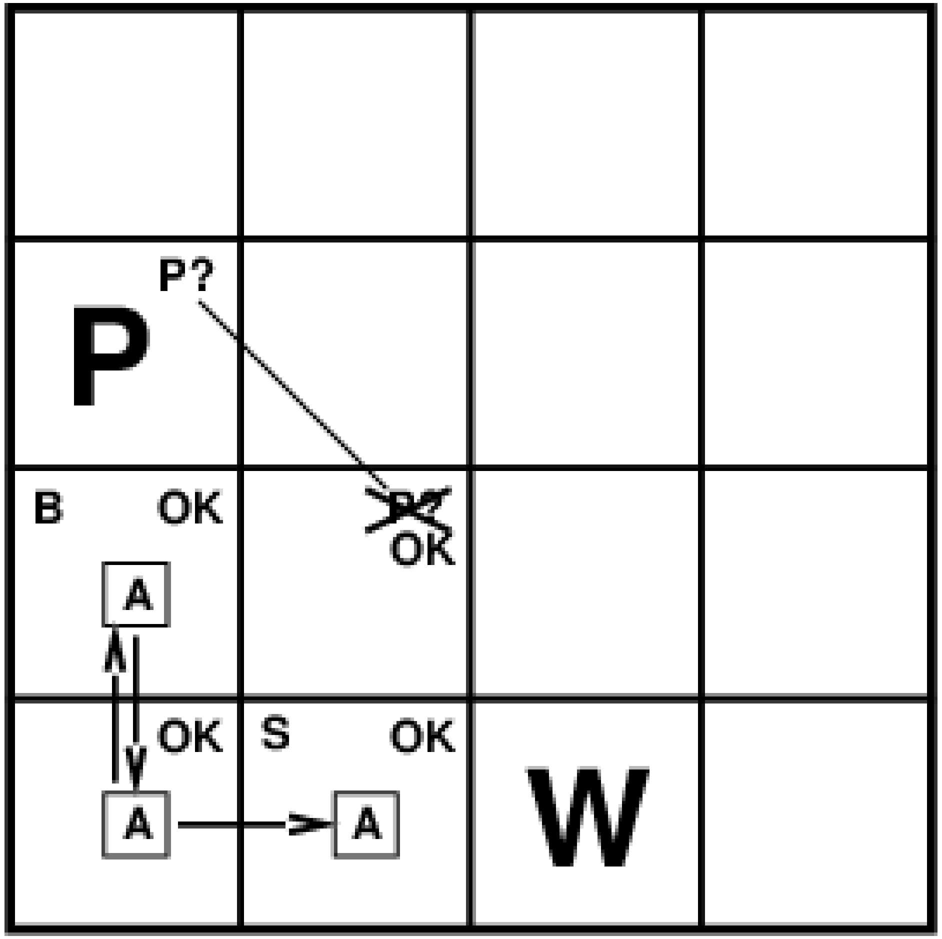

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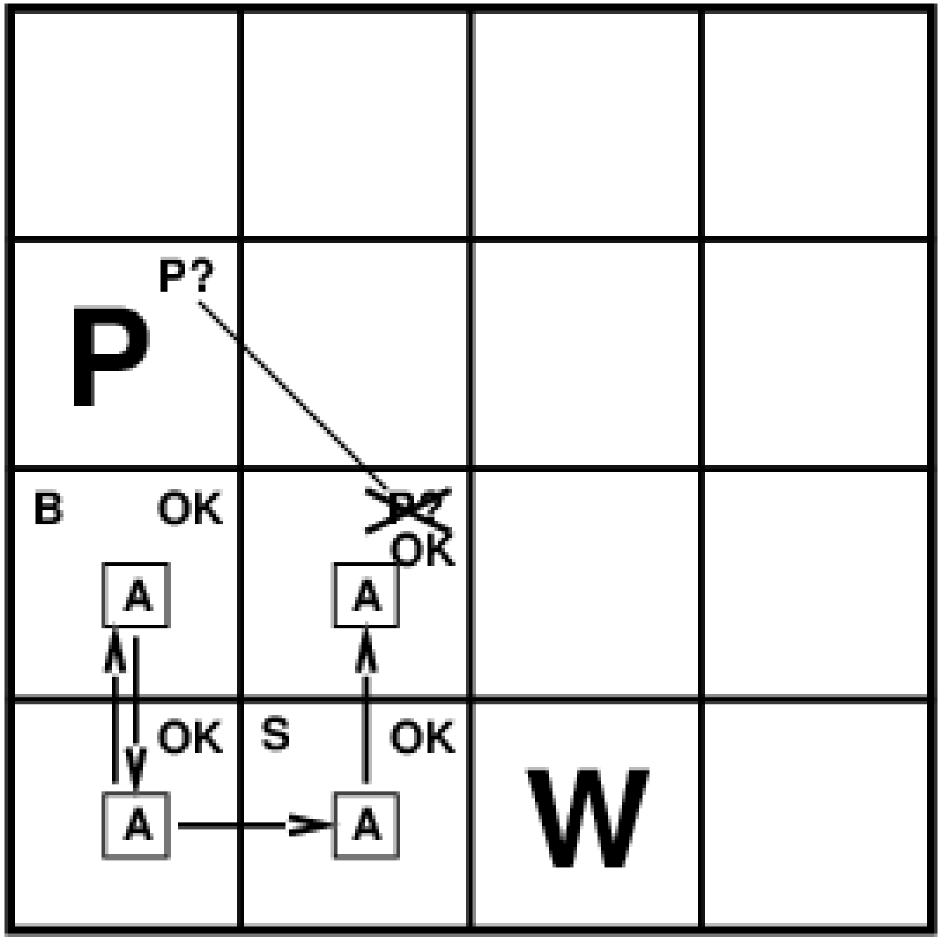

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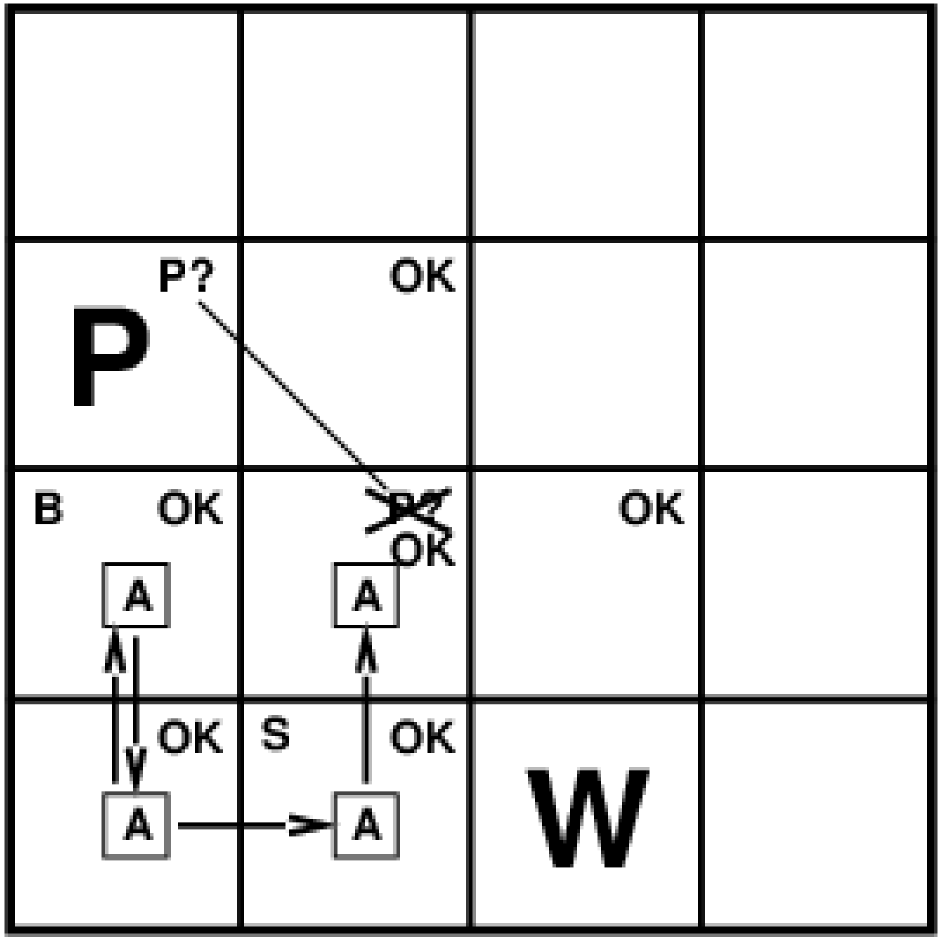
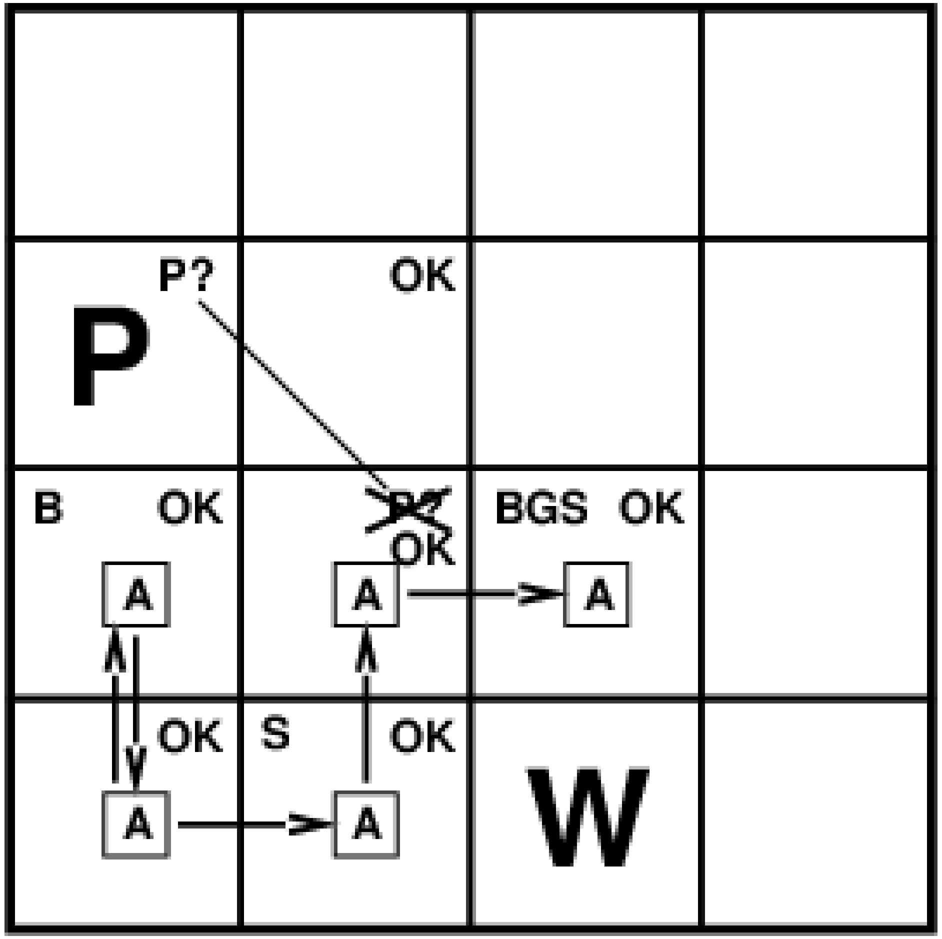
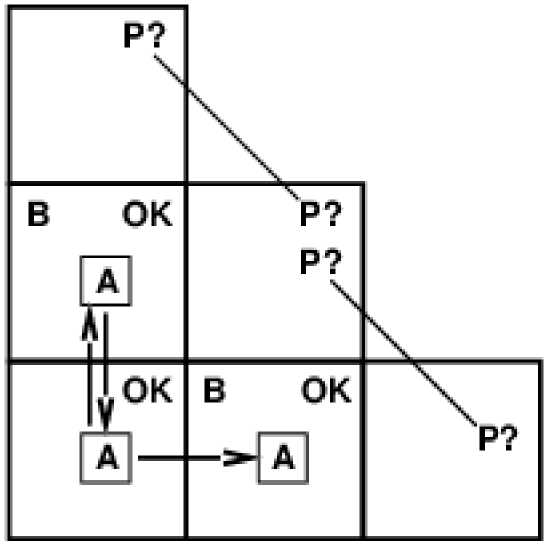


Exploring a Wumpus World



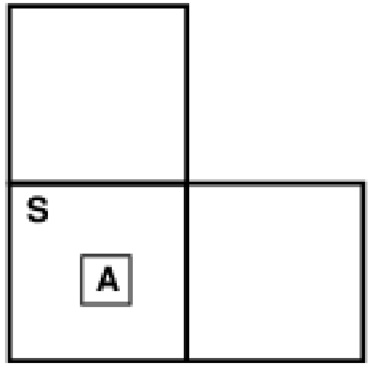
Tight Spot



- Breeze in (1,2) and (2,1)
  $\implies$ no safe actions

- Assuming pits uniformly distributed,
  (2,2) has pit w/ prob 0.86, vs. 0.31

Tight Spot



- Smell in (1,1)
  $\Longrightarrow$ cannot move

- Can use a strategy of **coercion**: shoot straight ahead
  - wumpus was there $\Longrightarrow$ dead $\Longrightarrow$ safe
  - wumpus wasn't there $\Longrightarrow$ safe

logic in general

Logic in General

- **Logics** are formal languages for representing information
  such that conclusions can be drawn
- **Syntax** defines the sentences in the language
- **Semantics** define the "meaning" of sentences;
  i.e., define **truth** of a sentence in a world
- E.g., the language of arithmetic
  - $x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence
  - $x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y
  - $x + 2 \geq y$ is true in a world where $x = 7,\ y = 1$
    $x + 2 \geq y$ is false in a world where $x = 0,\ y = 6$

Entailment

- **Entailment** means that one thing **follows from** another:

$$KB \models \alpha$$

- Knowledge base KB entails sentence α
  if and only if
  α is true in all worlds where KB is true

- E.g., the KB containing "the Ravens won" and "the Jays won" entails "the Ravens won or the Jays won"

- E.g., $x + y = 4$ entails $4 = x + y$

- Entailment is a relationship between sentences (i.e., **syntax**) that is based on **semantics**

Models

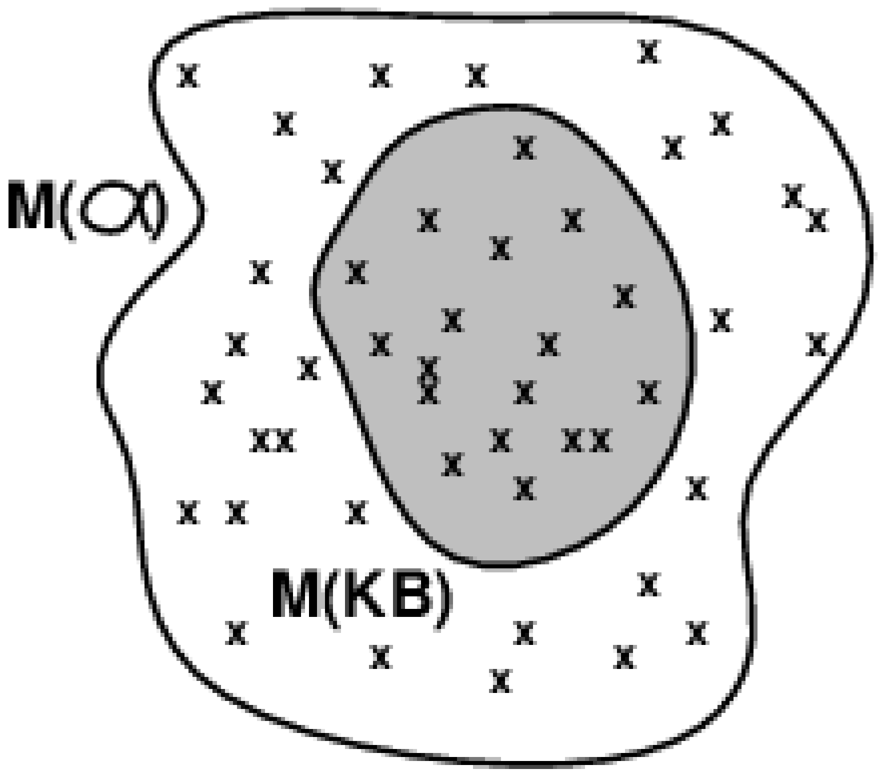
- Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated

- We say m **is a model of** a sentence α if α is true in m

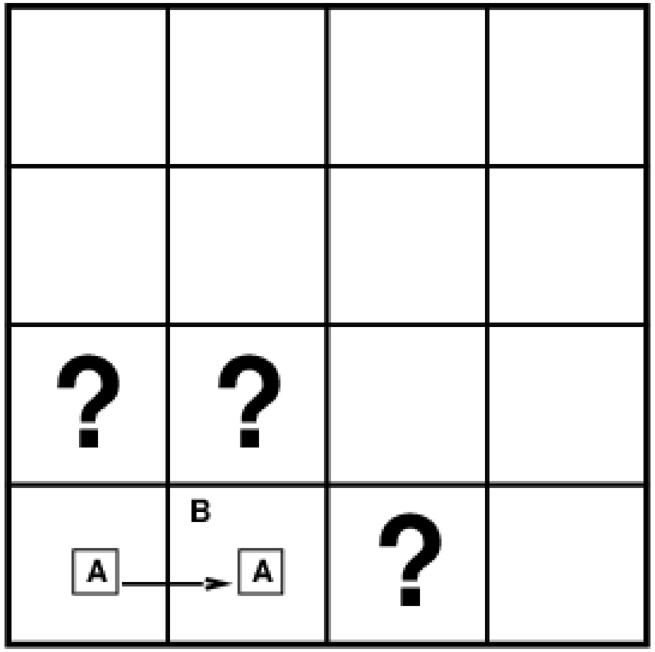
- $M(\alpha)$ is the set of all models of α

⇒ $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

- E.g. KB = Ravens won and Jays won
  α = Ravens won

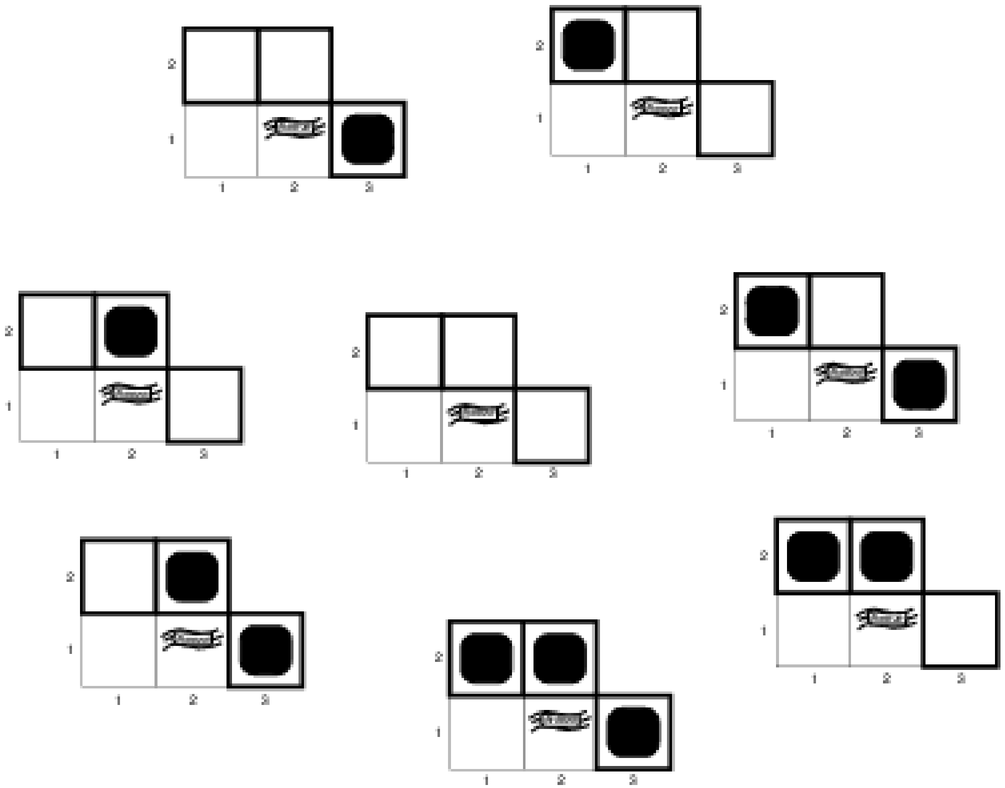


Entailment in the Wumpus World

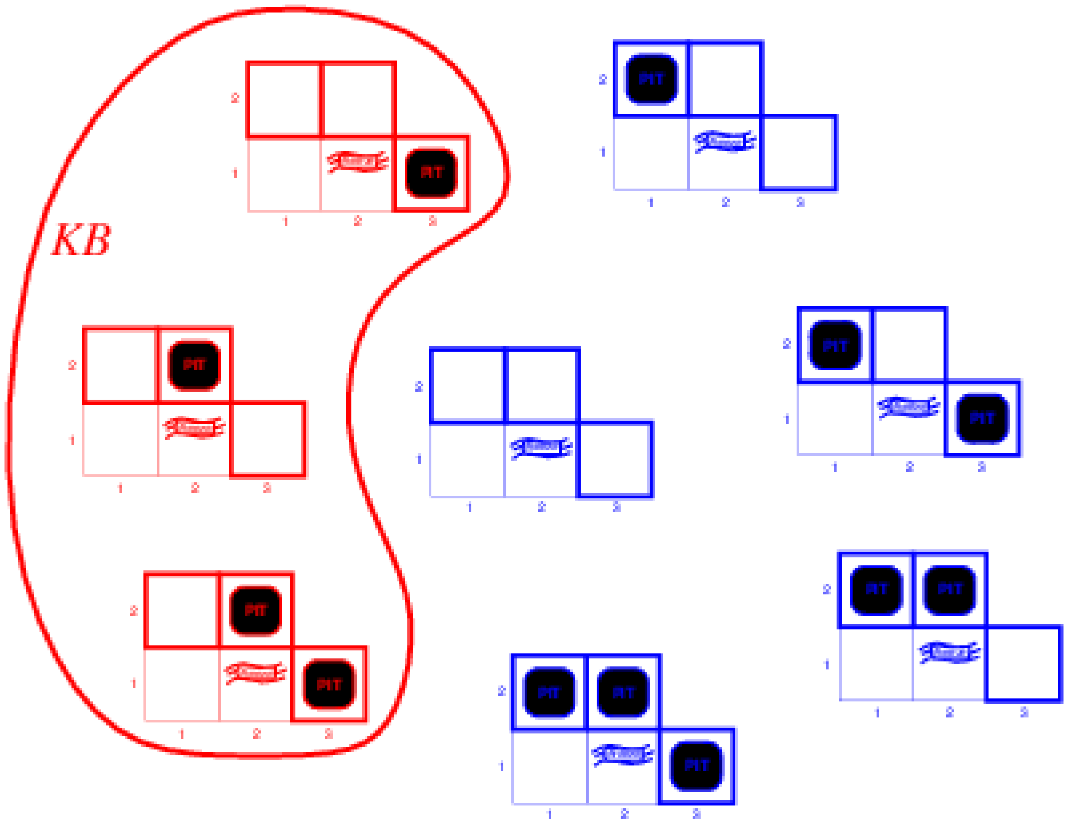


- Situation after detecting nothing in [1,1], moving right, breeze in [2,1]
- Consider possible models for all **?**, assuming only pits
- 3 Boolean choices $\Longrightarrow$ 8 possible models

Possible Wumpus Models

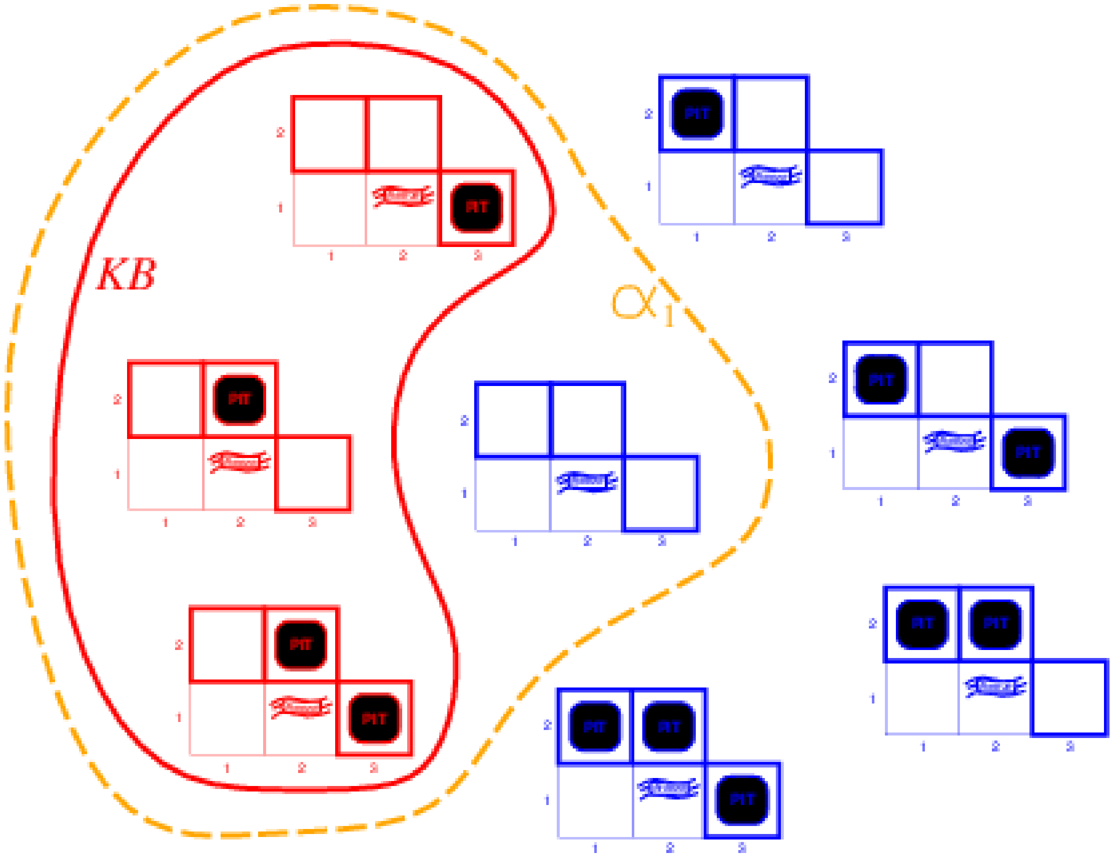


Valid Wumpus Models



KB = wumpus-world rules + observations

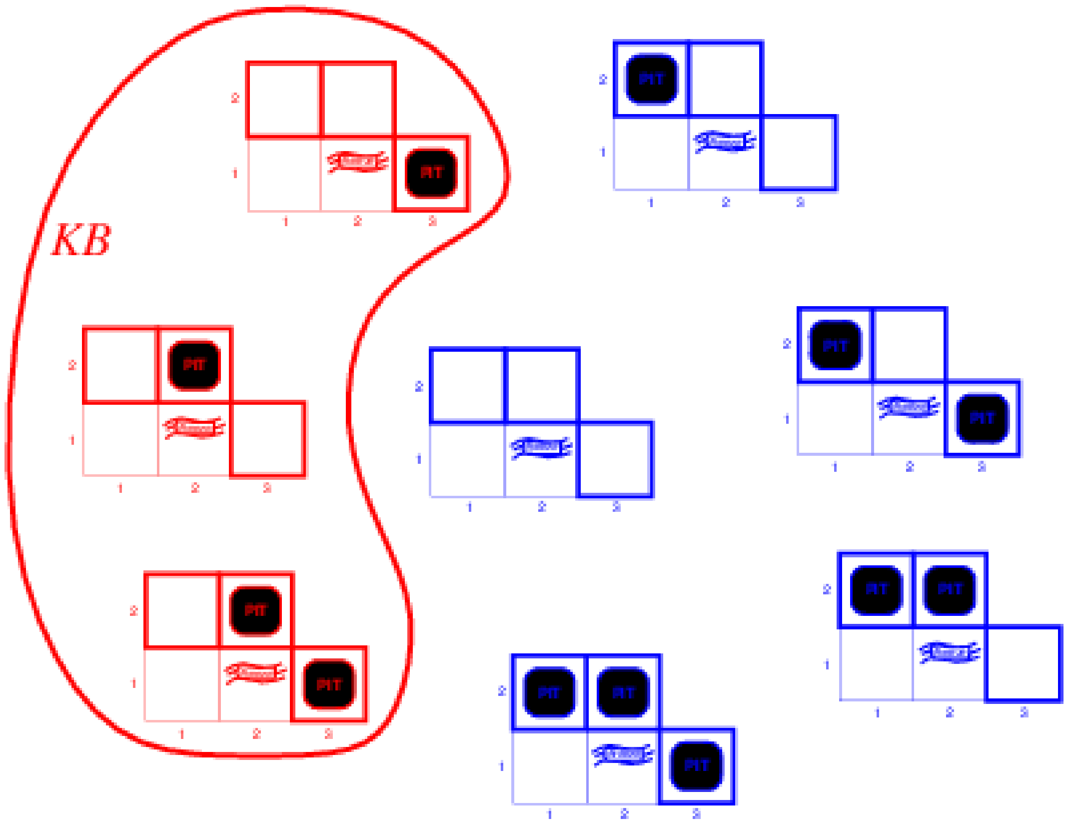
Entailment



KB = wumpus-world rules + observations

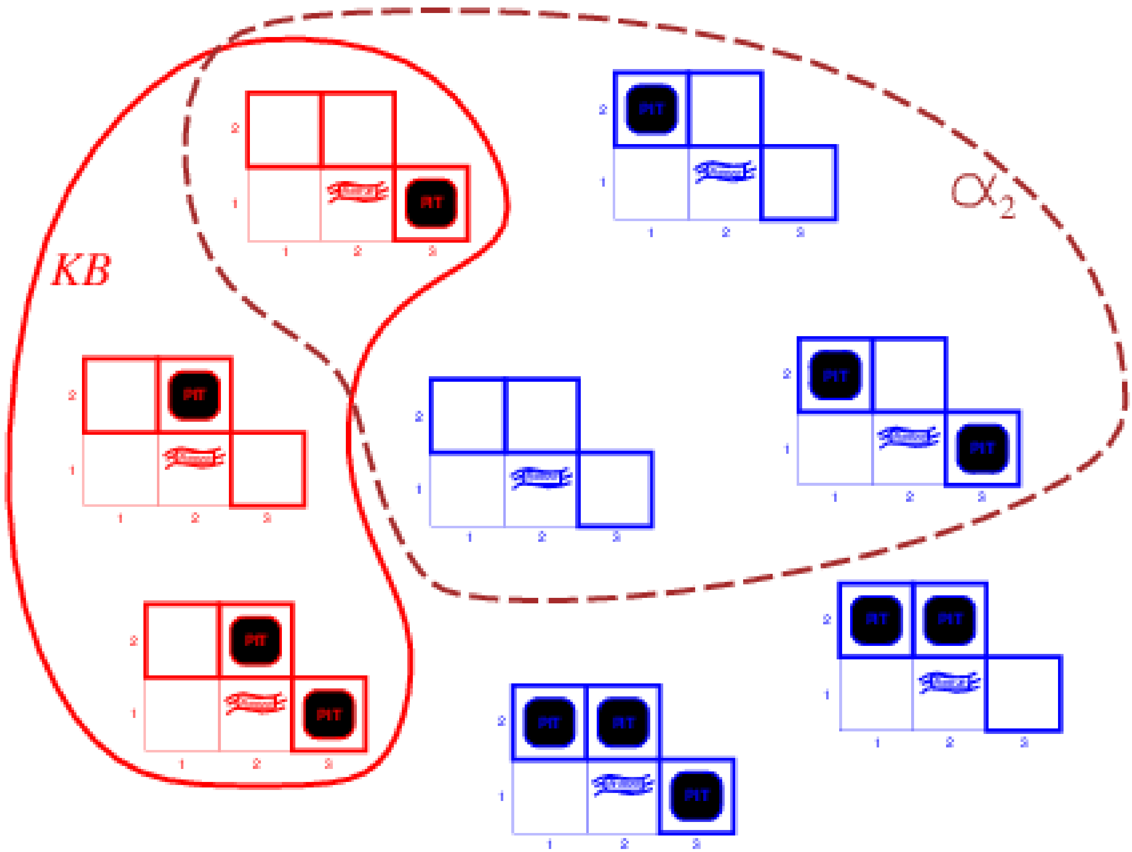
α_1 = "[1,2] is safe", $KB \models \alpha_1$, proved by **model checking**

Valid Wumpus Models



KB = wumpus-world rules + observations

Not Entailed



KB = wumpus-world rules + observations

α_2 = "[2,2] is safe", $KB \not\models \alpha_2$

Inference

- $KB \vdash_i \alpha$ = sentence α can be derived from KB by procedure i

- Consequences of KB are a haystack; α is a needle.
  Entailment = needle in haystack; inference = finding it

- **Soundness**: i is sound if
  whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

- **Completeness**: i is complete if
  whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

- Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

- That is, the procedure will answer any question whose answer follows from what is known by the KB.

propositional logic

Propositional Logic: Syntax

- Propositional logic is the simplest logic—illustrates basic ideas
- The proposition symbols P_1, P_2 etc are sentences
- If P is a sentence, $\neg P$ is a sentence (**negation**)
- If P_1 and P_2 are sentences, $P_1 \wedge P_2$ is a sentence (**conjunction**)
- If P_1 and P_2 are sentences, $P_1 \vee P_2$ is a sentence (**disjunction**)
- If P_1 and P_2 are sentences, $P_1 \implies P_2$ is a sentence (**implication**)
- If P_1 and P_2 are sentences, $P_1 \Leftrightarrow P_2$ is a sentence (**biconditional**)

Propositional Logic: Semantics

- Each model specifies true/false for each proposition symbol

  E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$
  $false$ $true$ $false$

  (with these symbols, 8 possible models, can be enumerated automatically)

- Rules for evaluating truth with respect to a model m:

$\neg P$	is true iff	P	is false		
$P_1 \wedge P_2$	is true iff	P_1	is true **and**	P_2	is true
$P_1 \vee P_2$	is true iff	P_1	is true **or**	P_2	is true
$P_1 \implies P_2$	is true iff	P_1	is false **or**	P_2	is true
i.e.,	is false iff	P_1	is true **and**	P_2	is false
$P_1 \Leftrightarrow P_2$	is true iff	$P_1 \implies P_2$	is true **and**	$P_2 \implies P_1$	is true

- Simple recursive process evaluates an arbitrary sentence, e.g.,
  $\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = true \wedge (false \vee true) = true \wedge true = true$

Truth Tables for Connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Wumpus World Sentences

- Let $P_{i,j}$ be true if there is a pit in $[i, j]$
  - observation $R_1 : \neg P_{1,1}$
- Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.
- "Pits cause breezes in adjacent squares"
  - rule $R_2 : B_{1,1} \;\Leftrightarrow\; (P_{1,2} \vee P_{2,1})$
  - rule $R_3 : B_{2,1} \;\Leftrightarrow\; (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
  - observation $R_4 : \neg B_{1,1}$
  - observation $R_5 : B_{2,1}$
- What can we infer about $P_{1,2}$, $P_{2,1}$, $P_{2,2}$, etc.?

Truth Tables for Inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

- Enumerate rows (different assignments to symbols $P_{i,j}$)
- Check if rules are satisfied (R_i)
- Valid model (KB) if all rules satisfied

Inference by Enumeration

- Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB, α) returns true or false
   inputs: KB, the knowledge base, a sentence in propositional logic
           α, the query, a sentence in propositional logic

   symbols ← a list of the proposition symbols in KB and α
   return TT-CHECK-ALL(KB, α, symbols, [ ])
---------------------------------------------------------------
function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
   if EMPTY?(symbols) then
      if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
      else return true
   else do
      P ← FIRST(symbols); rest ← REST(symbols)
      return TT-CHECK-ALL(KB, α, rest, EXTEND(P, true, model)) and
             TT-CHECK-ALL(KB, α, rest, EXTEND(P, false, model))
```

- $O(2^n)$ for n symbols; problem is **co-NP-complete**

equivalence, validity, satisfiability

Logical Equivalence

- Two sentences are **logically equivalent** iff true in same models:
  $\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

$$
\begin{array}{rcll}
(\alpha \wedge \beta) & \equiv & (\beta \wedge \alpha) & \text{commutativity of } \wedge \\
(\alpha \vee \beta) & \equiv & (\beta \vee \alpha) & \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) & \equiv & (\alpha \wedge (\beta \wedge \gamma)) & \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) & \equiv & (\alpha \vee (\beta \vee \gamma)) & \text{associativity of } \vee \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \implies \beta) & \equiv & (\neg\beta \implies \neg\alpha) & \text{contraposition} \\
(\alpha \implies \beta) & \equiv & (\neg\alpha \vee \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \implies \beta) \wedge (\beta \implies \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) & \equiv & (\neg\alpha \vee \neg\beta) & \text{De Morgan} \\
\neg(\alpha \vee \beta) & \equiv & (\neg\alpha \wedge \neg\beta) & \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) & \equiv & ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) & \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) & \equiv & ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) & \text{distributivity of } \vee \text{ over } \wedge
\end{array}
$$

Validity and Satisfiability

- A sentence is **valid** if it is true in **all** models,
  e.g., $True$, $A \lor \neg A$, $A \implies A$, $(A \land (A \implies B)) \implies B$

- A sentence is **satisfiable** if it is true in **some** model
  e.g., $A \lor B$, C

- A sentence is **unsatisfiable** if it is true in **no** models
  e.g., $A \land \neg A$

- Satisfiability is connected to inference via the following:
  $KB \models \alpha$ if and only if $(KB \land \neg\alpha)$ is unsatisfiable
  i.e., prove α by *reductio ad absurdum*

inference

Proof Methods

- Proof methods divide into (roughly) two kinds

- **Application of inference rules**
  - Legitimate (sound) generation of new sentences from old
  - **Proof** = a sequence of inference rule applications
    Can use inference rules as operators in a standard search alg.
  - Typically require translation of sentences into a **normal form**

- **Model checking**
  - truth table enumeration (always exponential in n)
  - improved backtracking
  - heuristic search in model space (sound but incomplete)
    e.g., min-conflicts-like hill-climbing algorithms

Forward and Backward Chaining

- **Horn Form** (restricted)
  KB = **conjunction** of **Horn clauses**

- Horn clause =
  - proposition symbol; or
  - (conjunction of symbols) $\implies$ symbol

  e.g., C, $B \implies A$, $C \wedge D \implies B$

- **Modus Ponens** (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \cdots \wedge \alpha_n \implies \beta}{\beta}$$

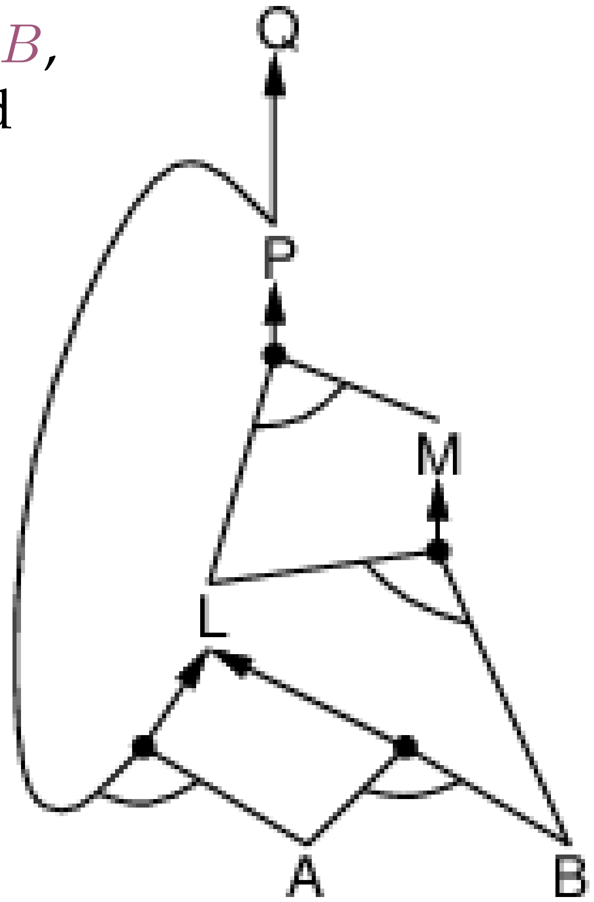
- Can be used with **forward chaining** or **backward chaining**

- These algorithms are very natural and run in **linear** time

Example

- Idea: fire any rule whose premises are satisfied in the KB,
  add its conclusion to the KB, until query is found

$$P \implies Q$$
$$L \wedge M \implies P$$
$$B \wedge L \implies M$$
$$A \wedge P \implies L$$
$$A \wedge B \implies L$$
$$A$$
$$B$$



forward chaining

Forward Chaining

- Start with given proposition symbols (atomic sentence)
  e.g., A and B

- Iteratively try to infer truth of additional proposition symbols
  e.g., $A \wedge B \implies C$, therefor we establish C is true

- Continue until
  - no more inference can be carried out, or
  - goal is reached

Forward Chaining Example

- Given

  $P \implies Q$

  $L \wedge M \implies P$

  $B \wedge L \implies M$

  $A \wedge P \implies L$

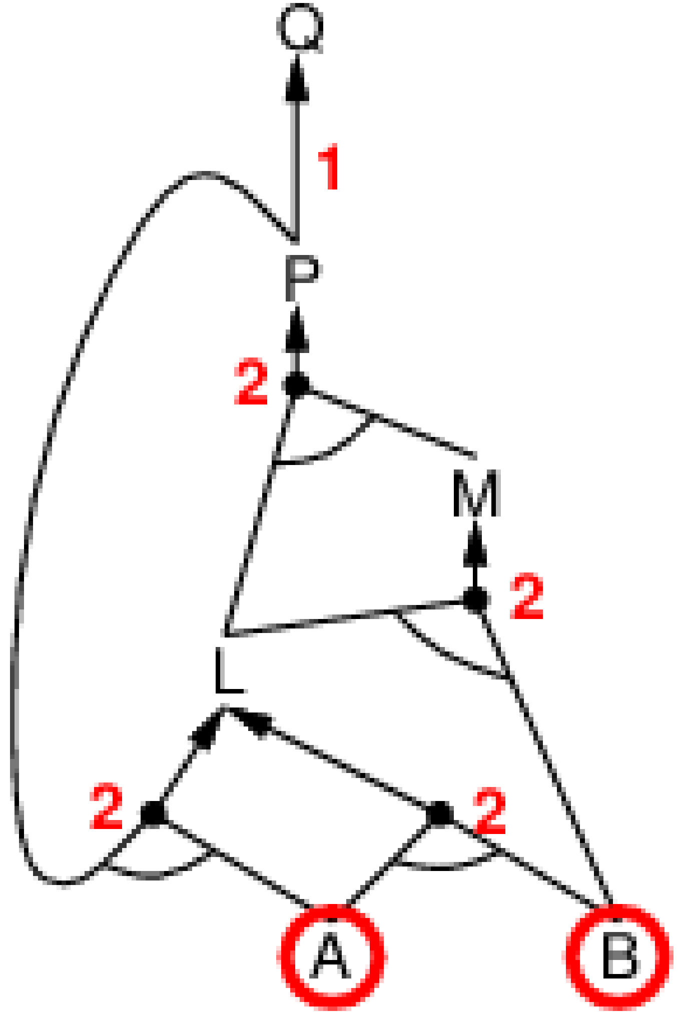
  $A \wedge B \implies L$

  A

  B

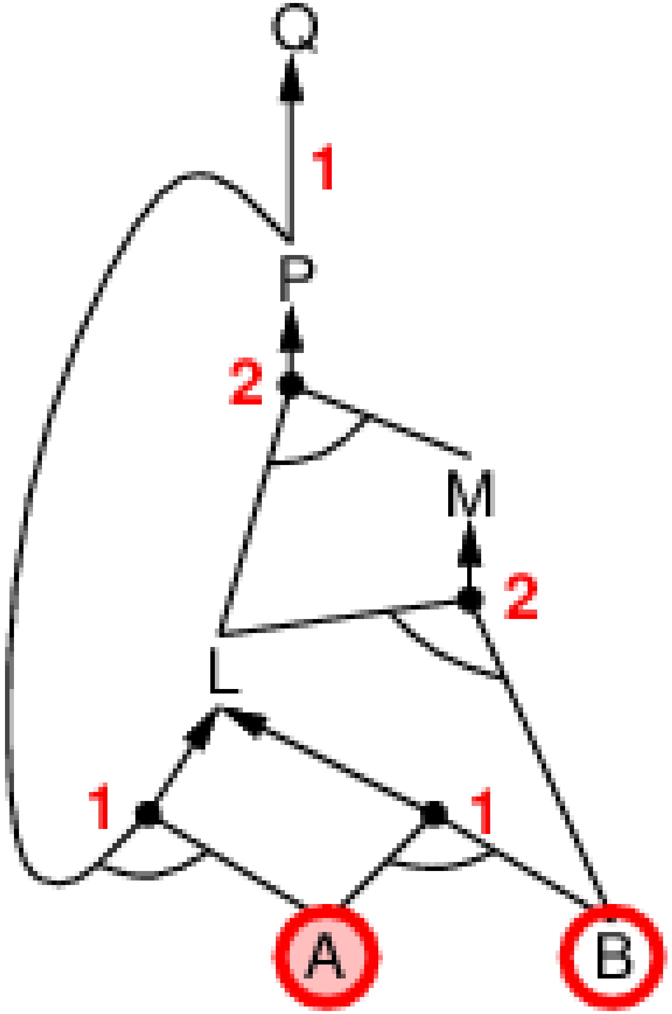
- Agenda: A, B

- Annotate horn clauses with number of premises



Forward Chaining Example

- Process agenda item A
- Decrease count for horn clauses in which A is premise



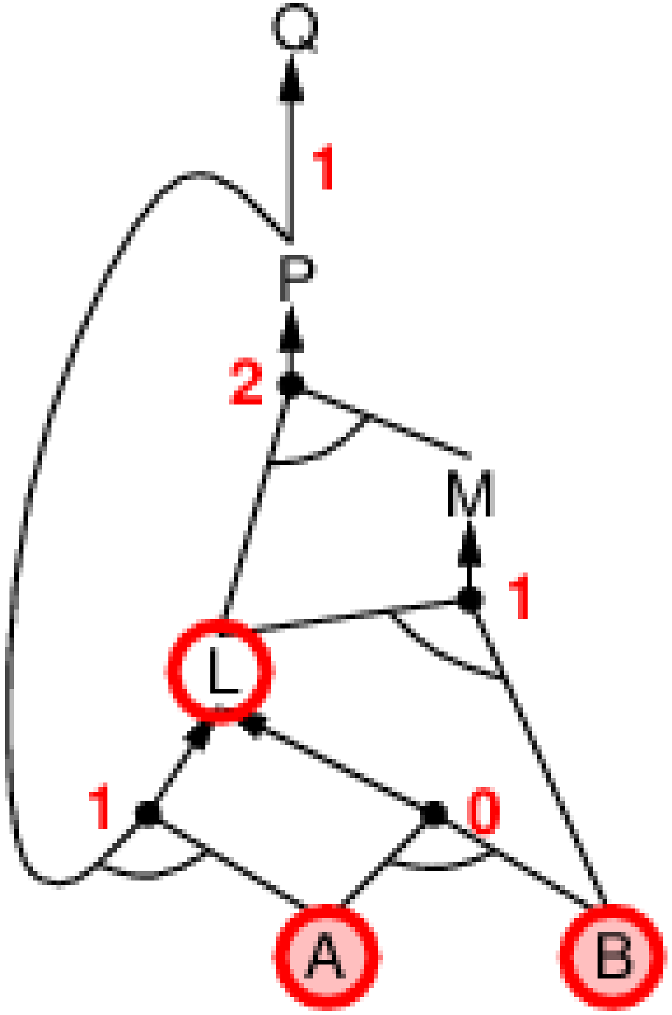
Forward Chaining Example

- Process agenda item B

- Decrease count for horn clauses in which B is premise

- $A \wedge B \implies L$ has now fulfilled premise

- Add L to agenda



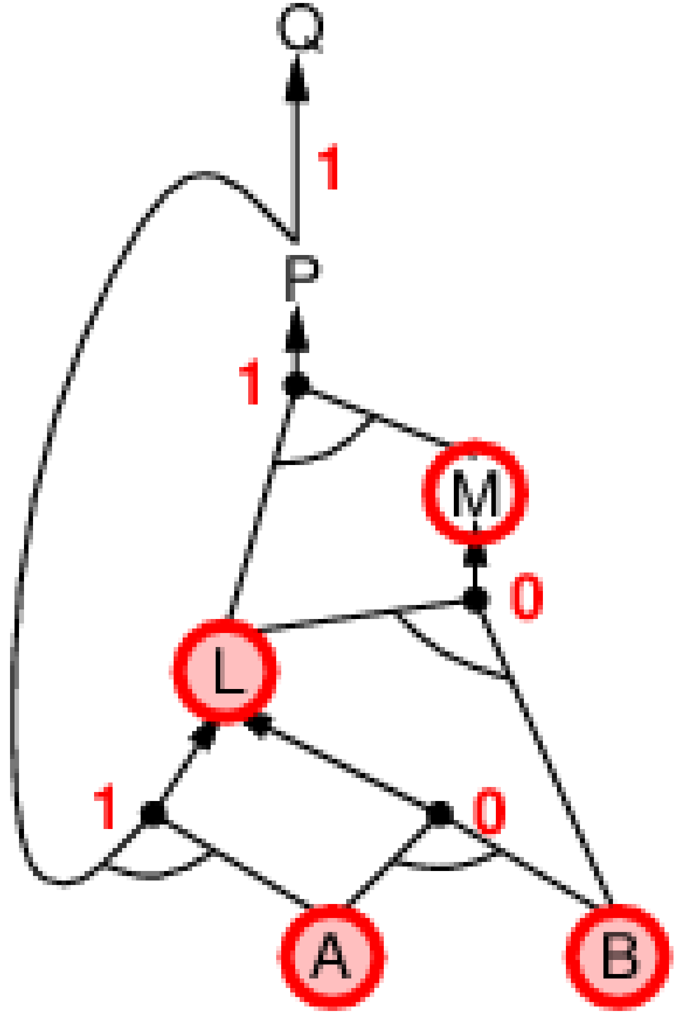
Forward Chaining Example

- Process agenda item L

- Decrease count for horn clauses in which L is premise

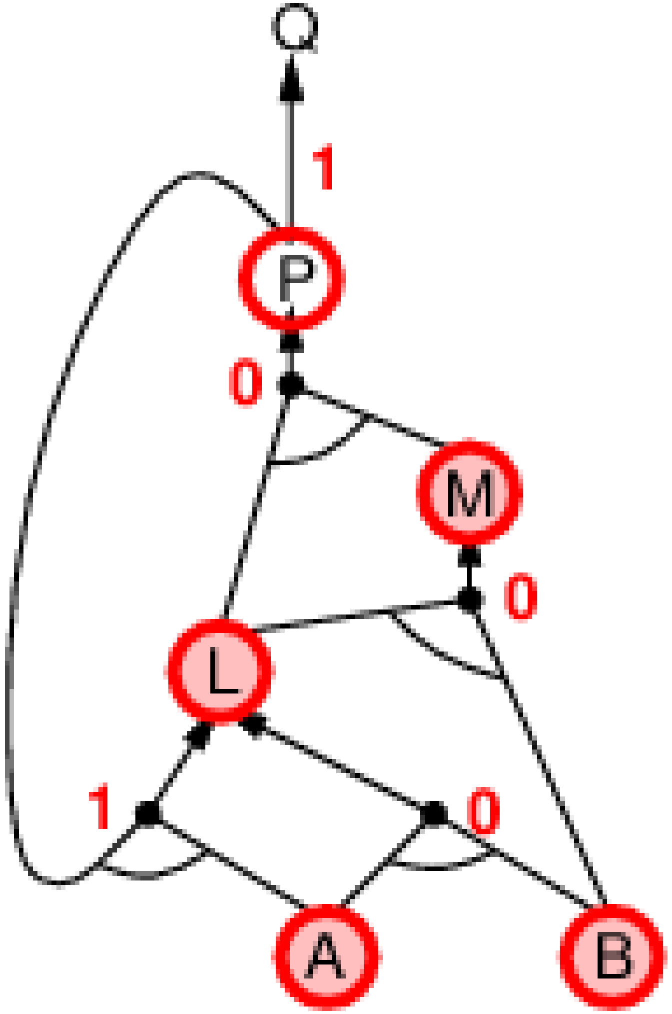
- $B \wedge L \implies M$ has now fulfilled premise

- Add M to agenda



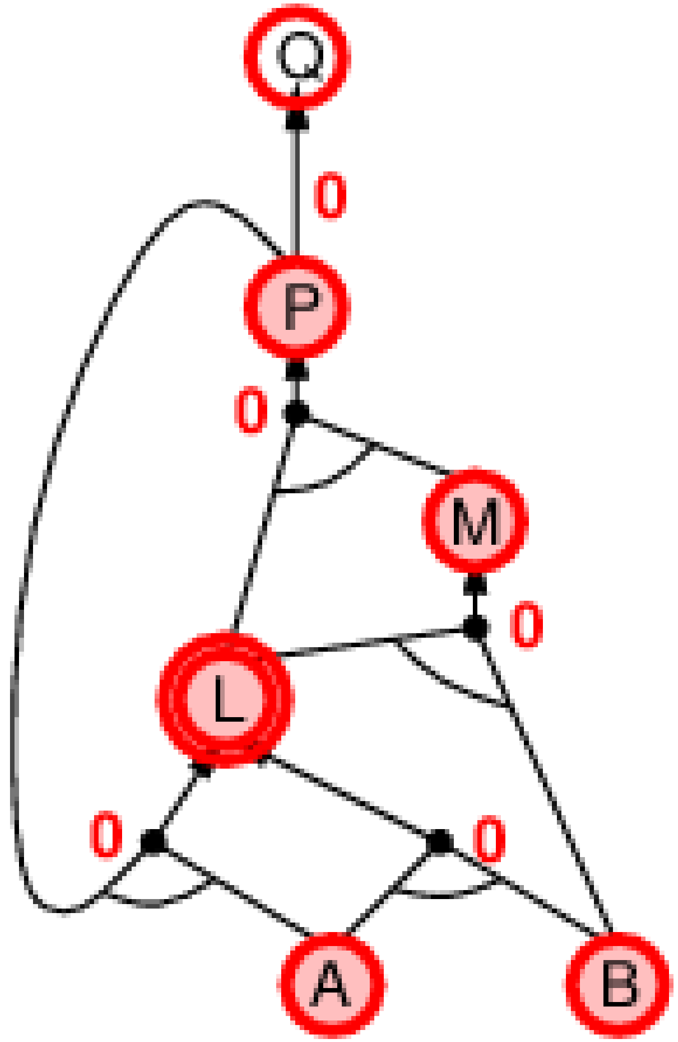
Forward Chaining Example

- Process agenda item M
- Decrease count for horn clauses in which M is premise
- $L \wedge M \implies P$ has now fulfilled premise
- Add P to agenda



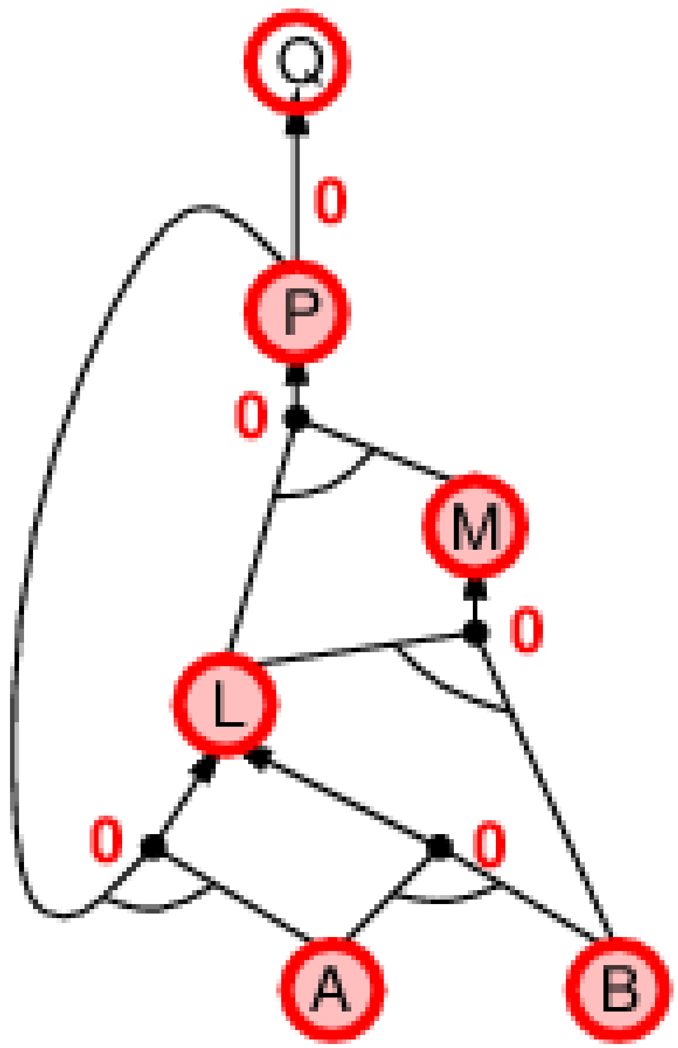
Forward Chaining Example

- Process agenda item P
- Decrease count for horn clauses in which P is premise
- $P \implies Q$ has now fulfilled premise
- Add Q to agenda
- $A \wedge P \implies L$ has now fulfilled premise



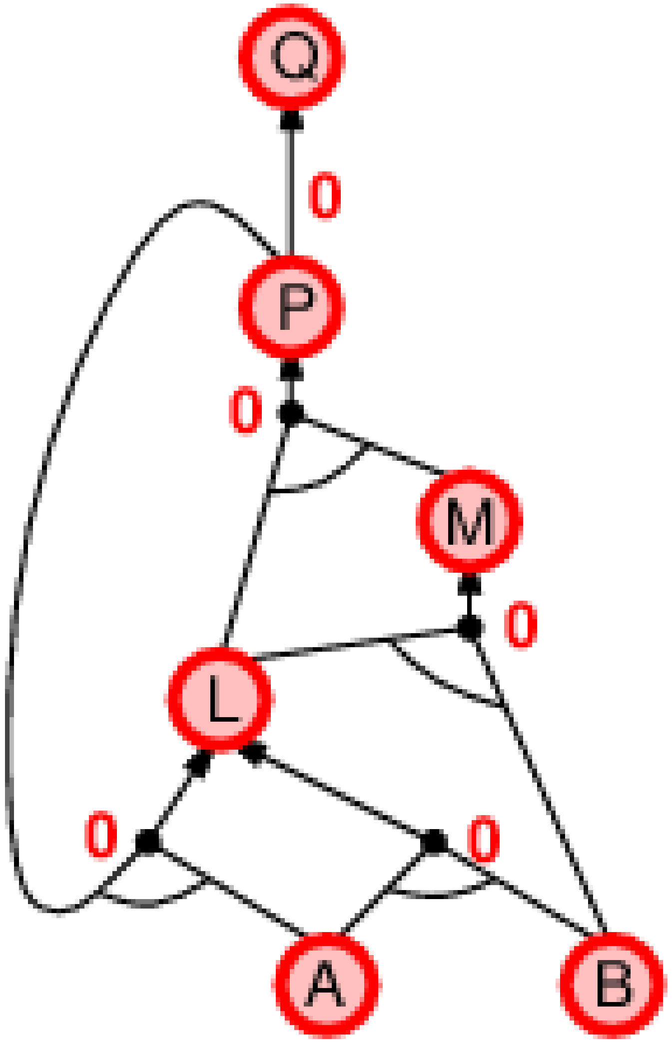
Forward Chaining Example

- Process agenda item P
- Decrease count for horn clauses in which P is premise
- $P \implies Q$ has now fulfilled premise
- Add Q to agenda
- $A \wedge P \implies L$ has now fulfilled premise
- But L is already inferred



Forward Chaining Example

- Process agenda item Q
- Q is inferred
- Done



Forward Chaining Algorithm

```
function PL-FC-ENTAILS?(KB, q) returns true or false
  inputs: KB, the knowledge base, a set of propositional Horn clauses
          q, the query, a proposition symbol
  local variables: count, a table, indexed by clause, init. number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known in KB

  while agenda is not empty do
      p ← POP(agenda)
      unless inferred[p] do
          inferred[p] ← true
          for each Horn clause c in whose premise p appears do
              decrement count[c]
              if count[c] = 0 then do
                  if HEAD[c] = q then return true
                  PUSH(HEAD[c], agenda)
  return false
```

backward chaining

Backward Chaining

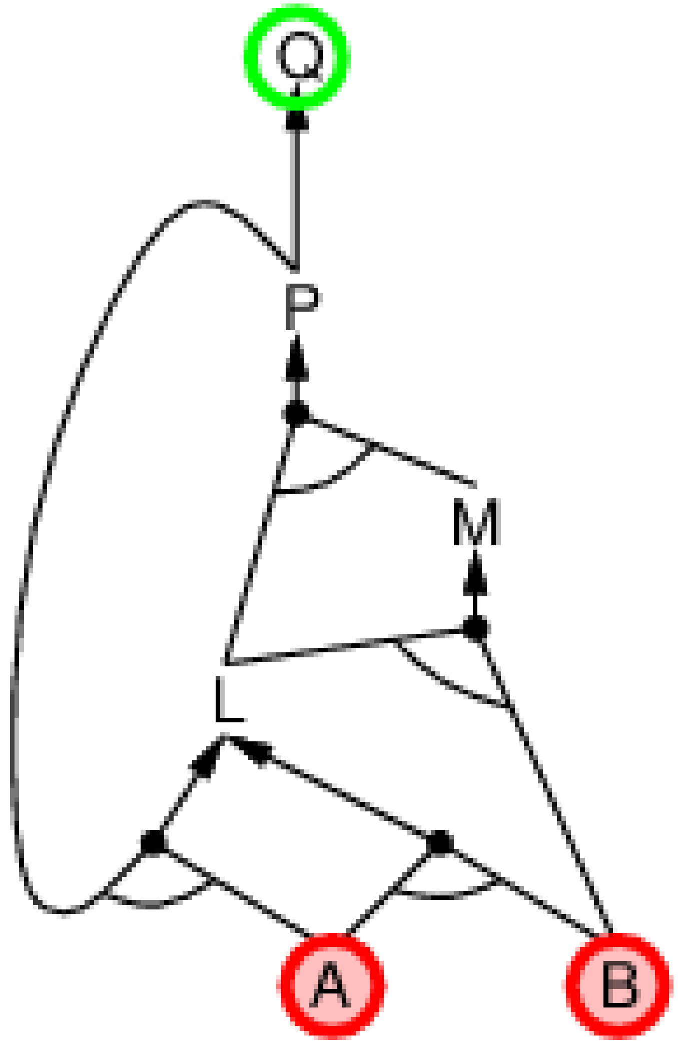
- Idea: work backwards from the query Q:
  to prove Q by BC,
  check if Q is known already, or
  prove by BC all premises of some rule concluding q

- Avoid loops: check if new subgoal is already on the goal stack

- Avoid repeated work: check if new subgoal
  1. has already been proved true, or
  2. has already failed

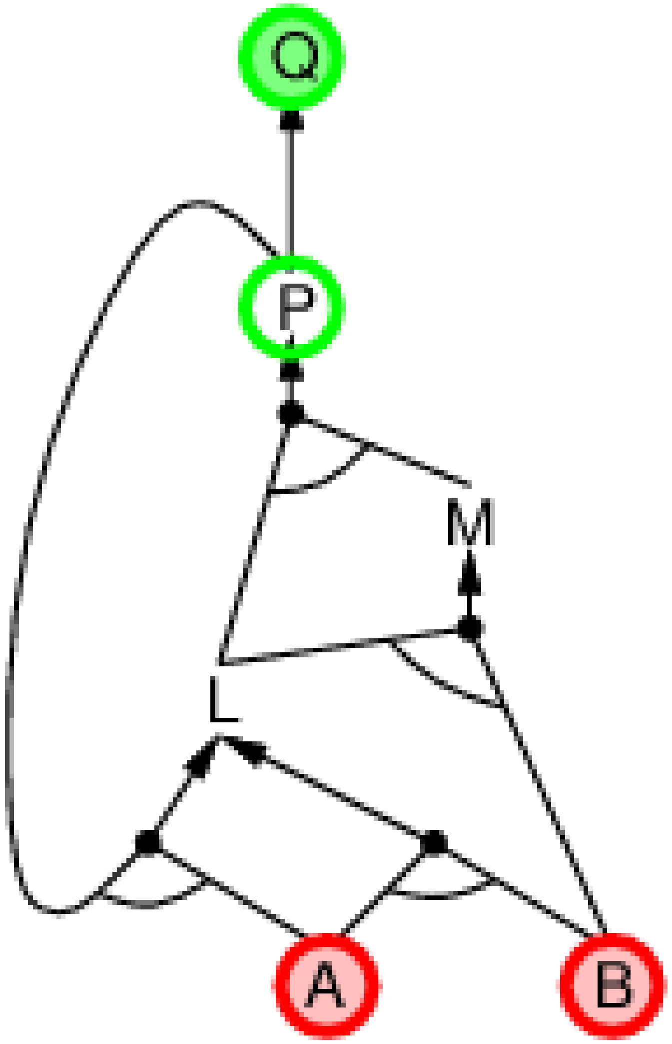
Backward Chaining Example

- A and B are known to be true
- Q needs to be proven



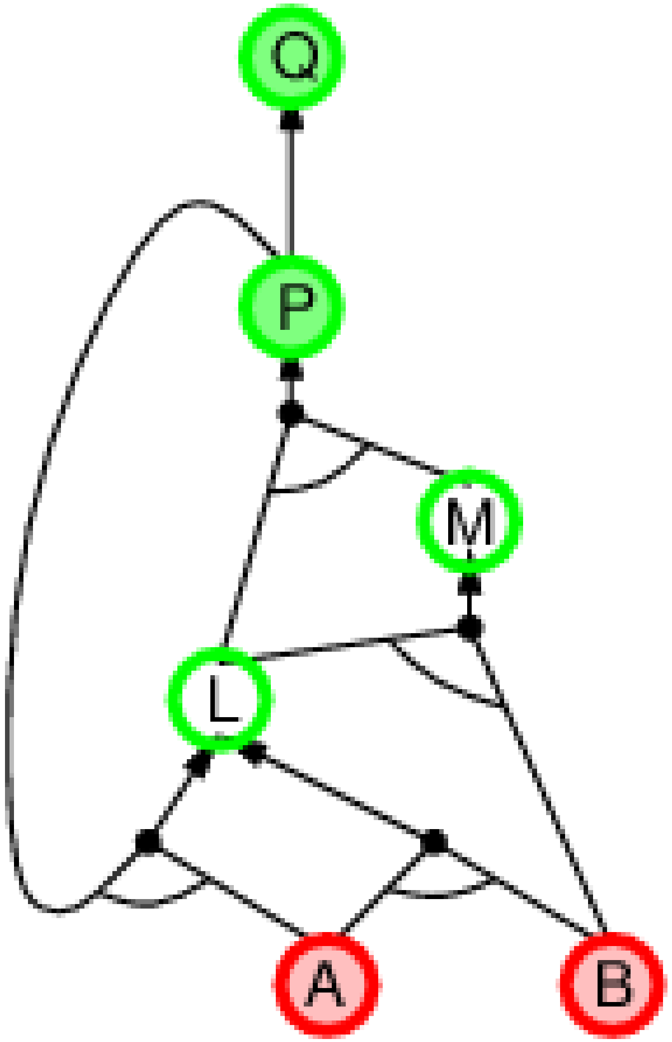
Backward Chaining Example

- Current goal: Q
- Q can be inferred by $P \implies Q$
- P needs to be proven

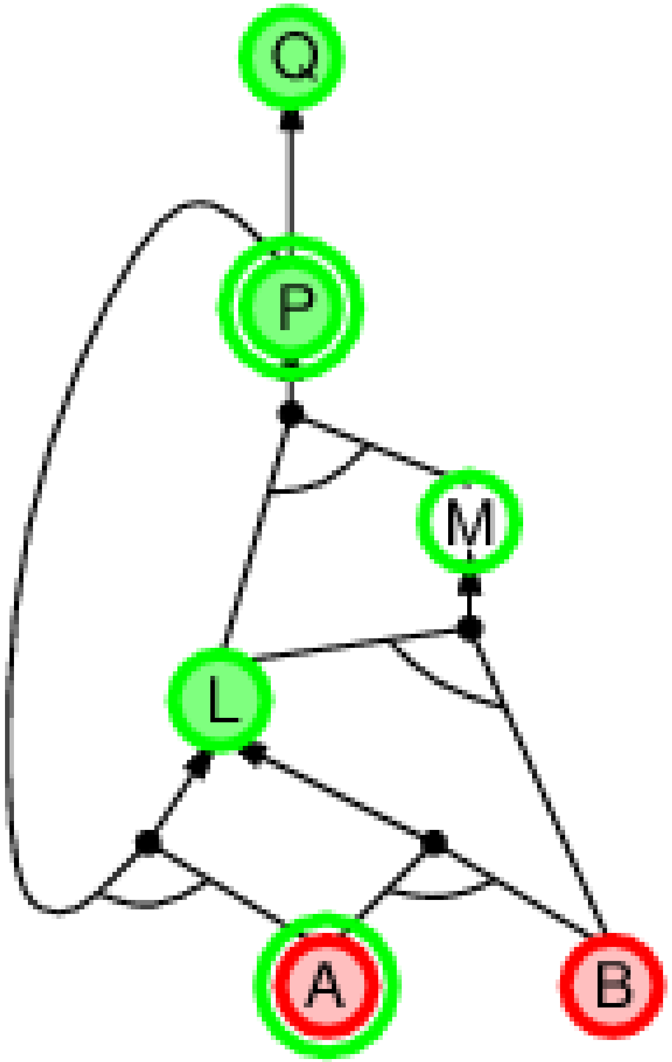


Backward Chaining Example

- Current goal: P
- P can be inferred by $L \wedge M \implies P$
- L and M need to be proven



Backward Chaining Example

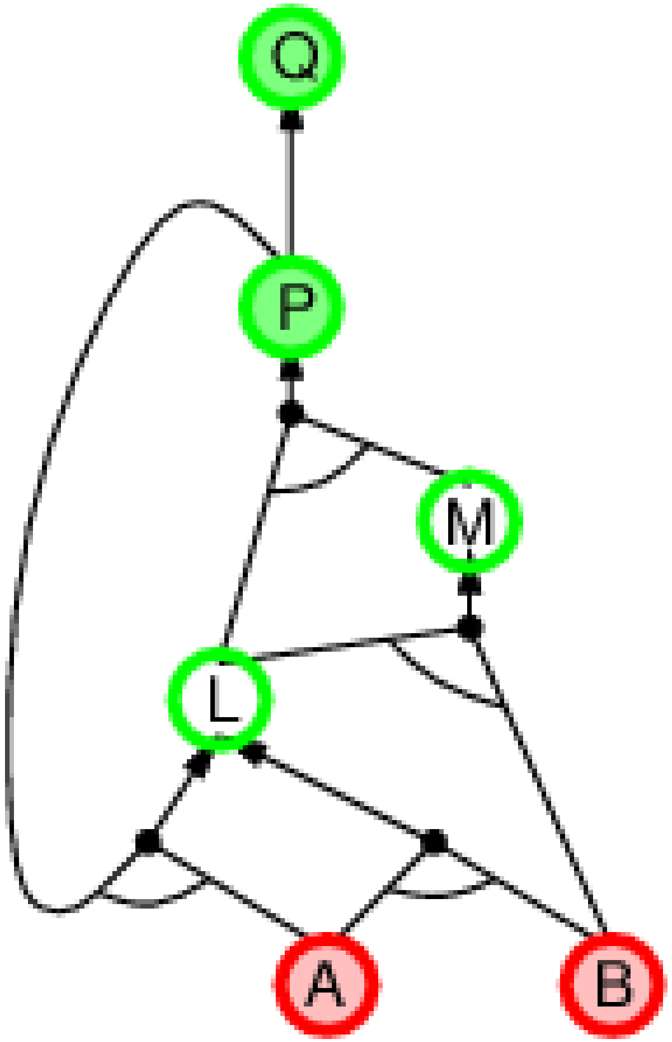


- Current goal: L
- L can be inferred by $A \wedge P \implies L$
- A is already true
- P is already a goal

$\Rightarrow$ repeated subgoal

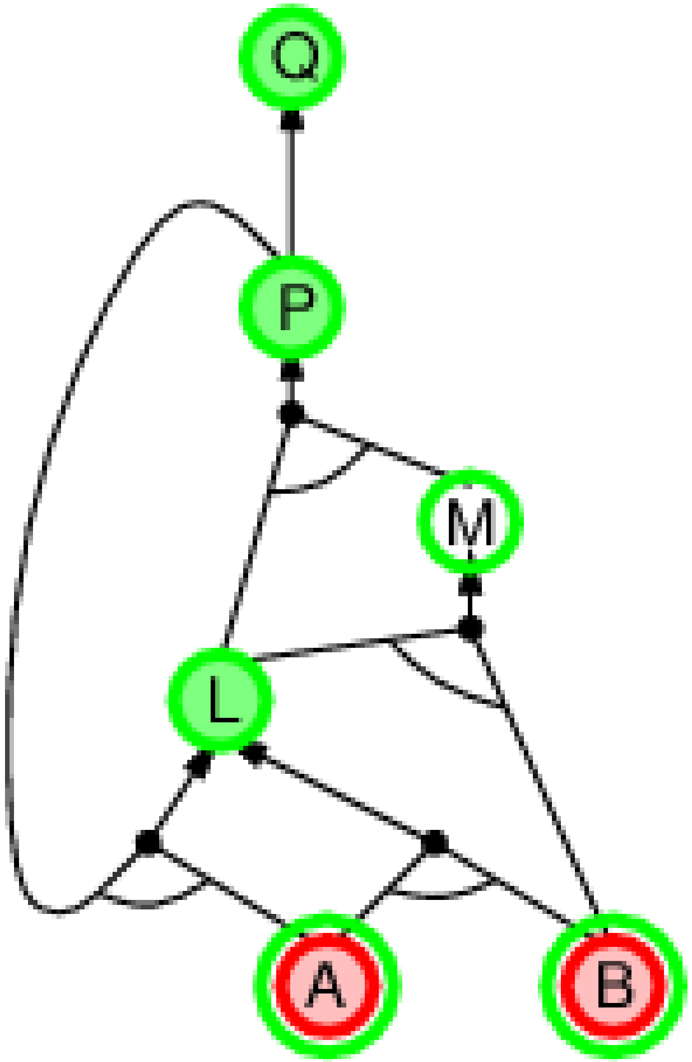
Backward Chaining Example

- Current goal: L



Backward Chaining Example

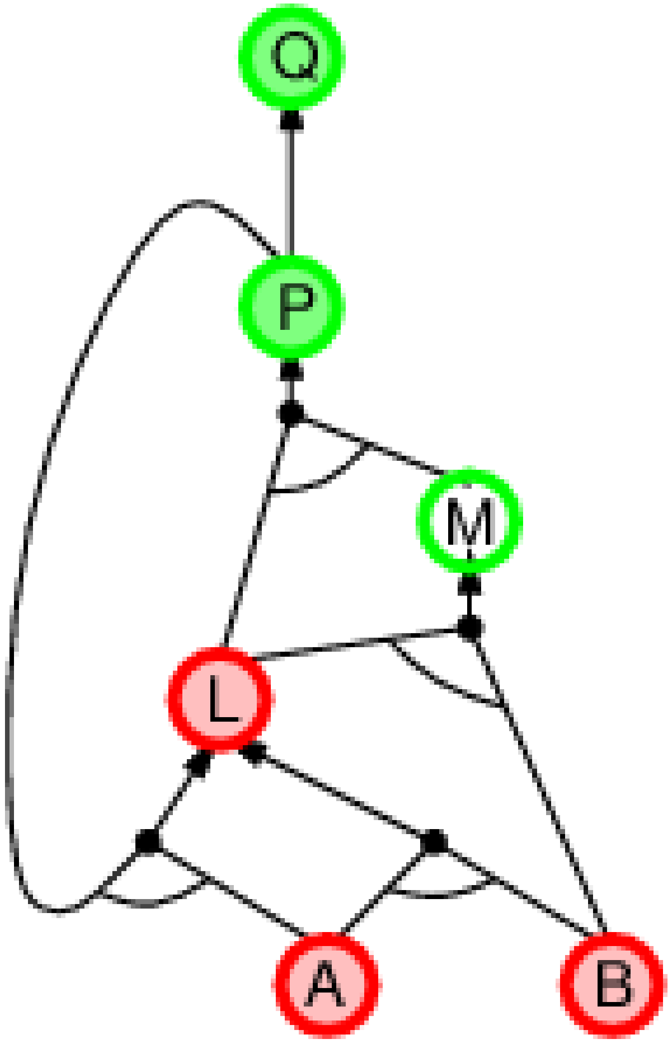
- Current goal: L
- L can be inferred by $A \wedge B \implies L$
- Both are true



Backward Chaining Example

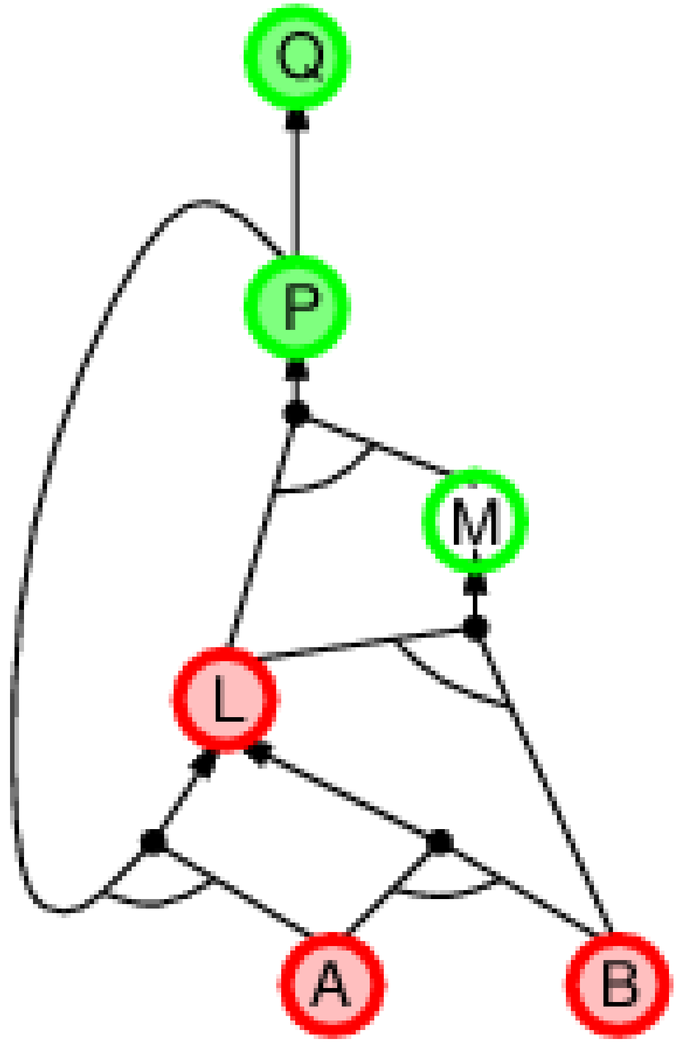
- Current goal: L
- L can be inferred by $A \wedge B \implies L$
- Both are true

$\Rightarrow$ L is true



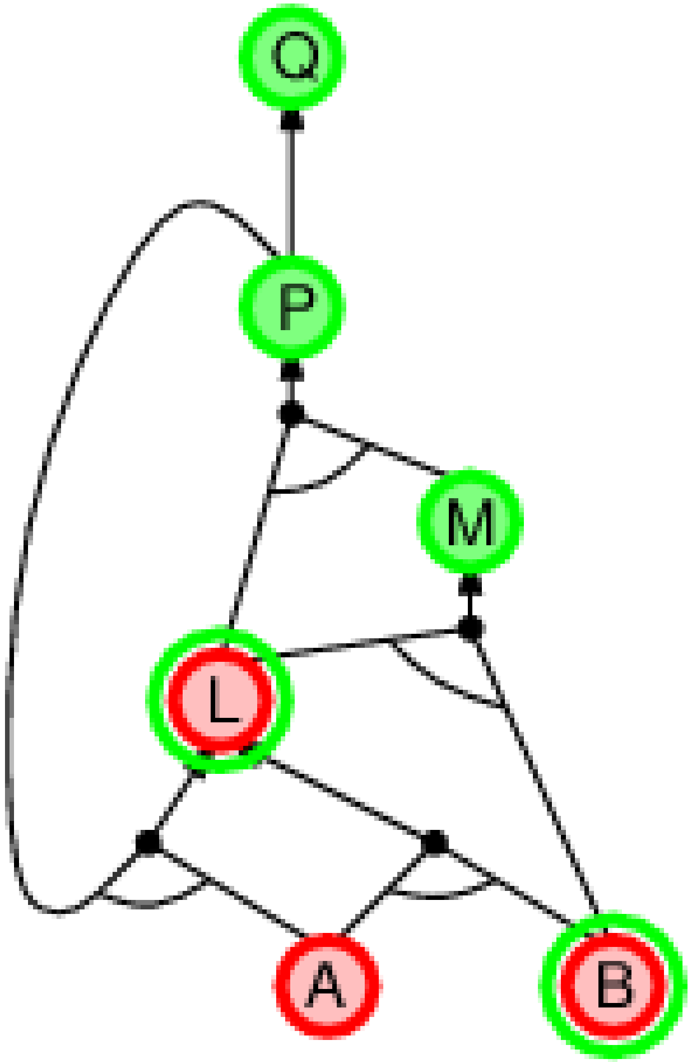
Backward Chaining Example

- Current goal: M



Backward Chaining Example

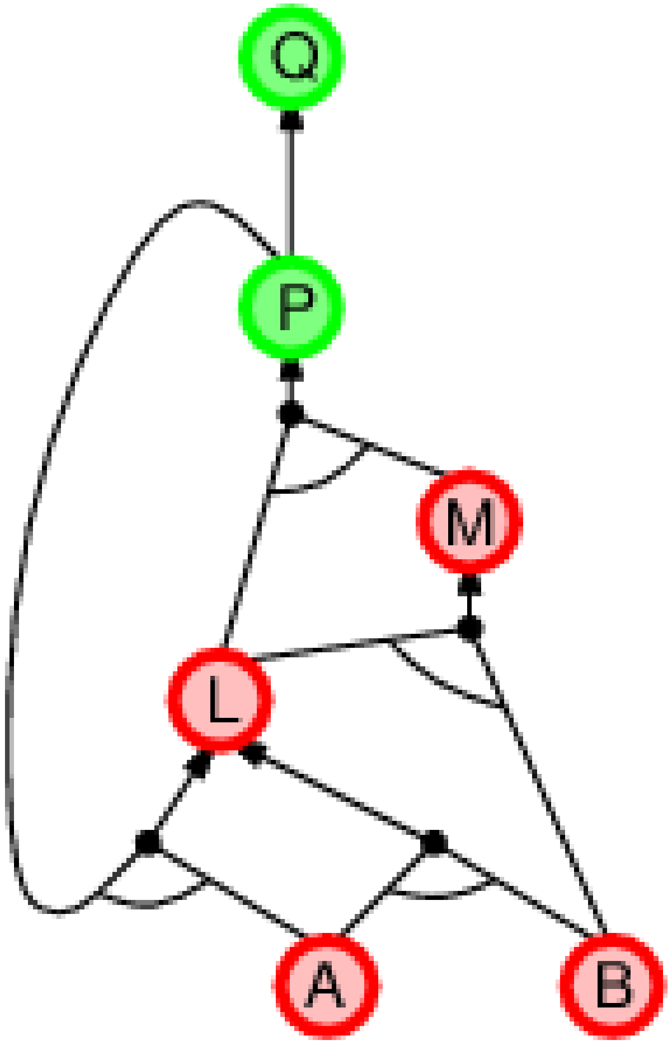
- Current goal: M
- M can be inferred by $B \land L \implies M$



Backward Chaining Example

- Current goal: M
- M can be inferred by $B \wedge L \implies M$
- Both are true

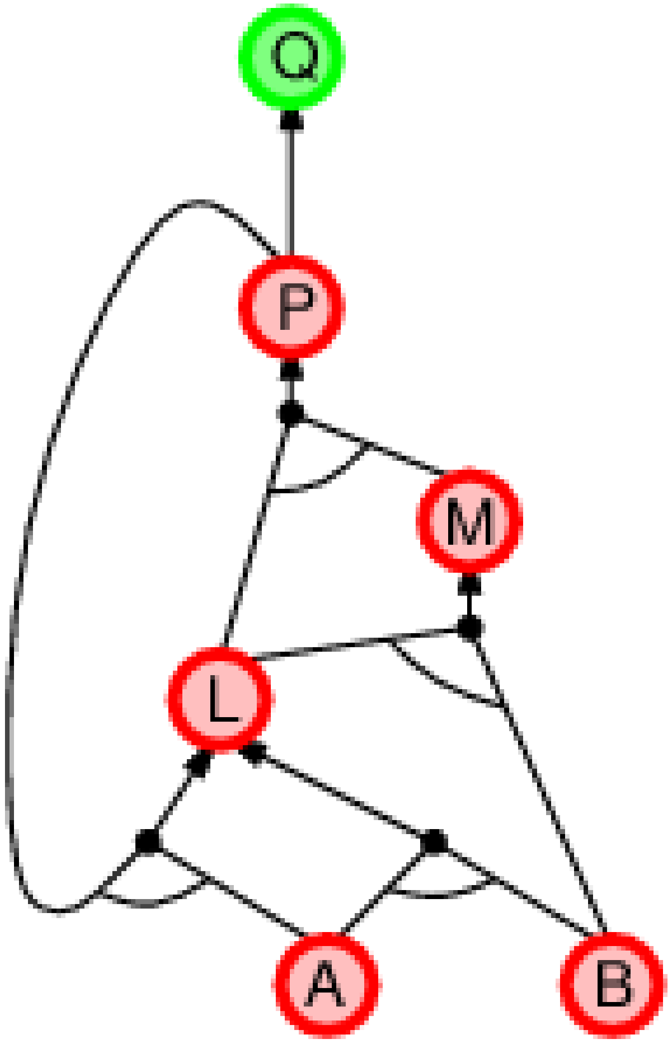
$\Rightarrow$ M is true



Backward Chaining Example

- Current goal: P
- P can be inferred by $L \wedge M \implies P$
- Both are true

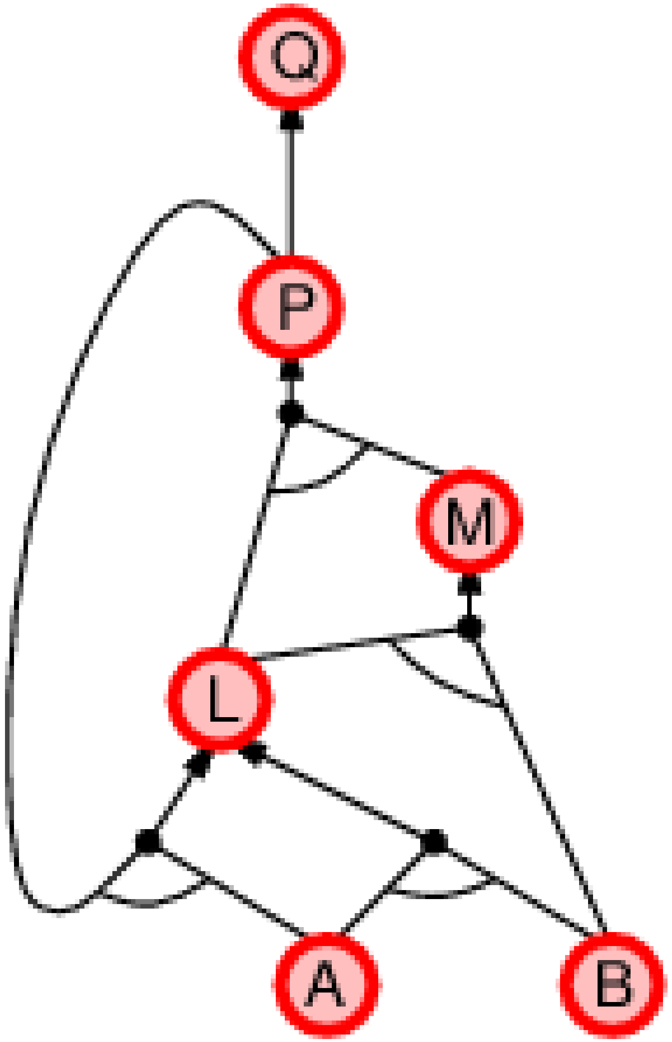
$\Rightarrow$ P is true



Backward Chaining Example

- Current goal: Q
- Q can be inferred by $P \implies Q$
- P is true

$\Rightarrow$ Q is true



Forward vs. Backward Chaining

- FC is **data-driven**, cf. automatic, unconscious processing,
  e.g., object recognition, routine decisions

- May do lots of work that is irrelevant to the goal

- BC is **goal-driven**, appropriate for problem-solving,
  e.g., Where are my keys? How do I get into a PhD program?

- Complexity of BC can be **much less** than linear in size of KB

resolution

Resolution

- **Conjunctive Normal Form** (CNF—universal)

  $\textbf{conjunction} \text{ of } \underbrace{\textbf{disjunctions} \text{ of } \textbf{literals}}_{\textbf{clauses}}$

  E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

- **Resolution** inference rule (for CNF): complete for propositional logic

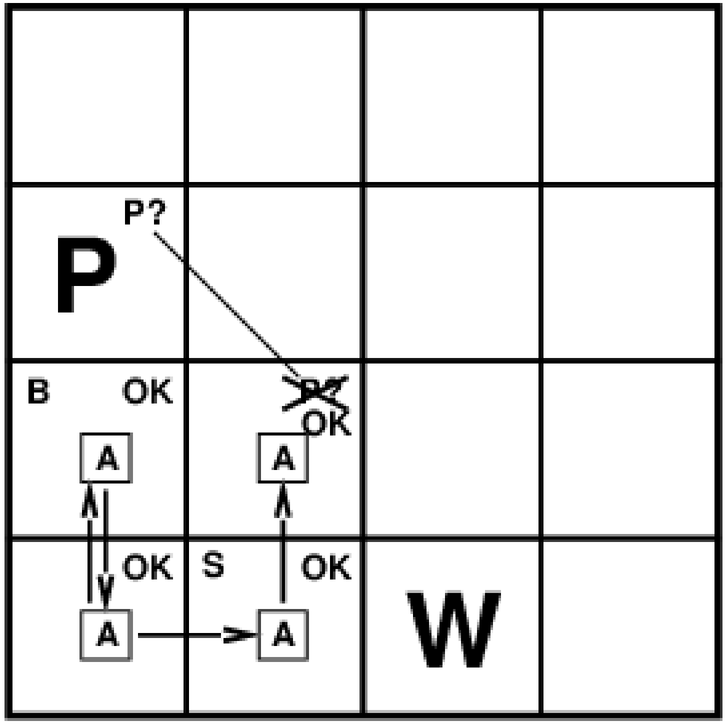
$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

  where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$

- Resolution is sound and complete for propositional logic

Wampus World



- Rules such as: "If breeze, then a pit adjacent."

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

Conversion to CNF

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \implies \beta) \wedge (\beta \implies \alpha)$.

$$(B_{1,1} \implies (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \implies B_{1,1})$$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$$

4. Apply distributivity law ($\vee$ over $\wedge$) and flatten:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

Resolution Example

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1}))$

  reformulated as:

  $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

- Observation: $\neg B_{1,1}$

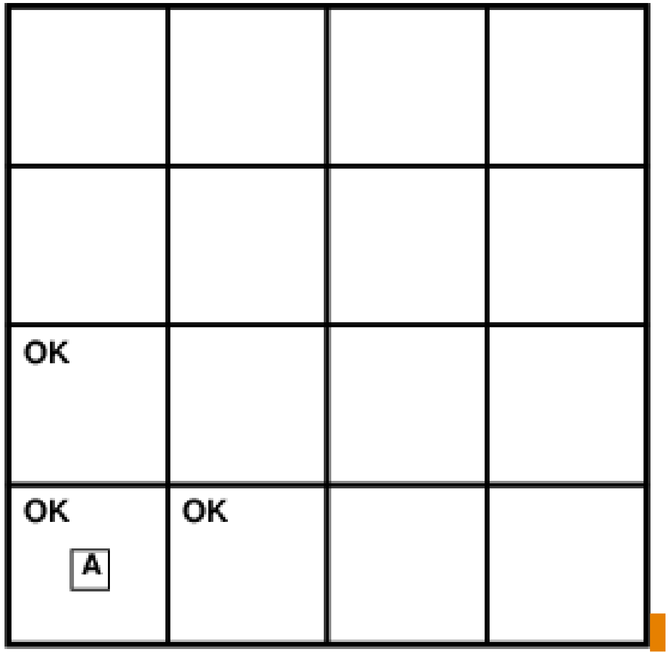
- Goal: disprove: $\alpha = \neg P_{1,2}$
  (we add $P_{1,2}$ to the KB and check for contraction)

- Resolution

$$\frac{\neg P_{1,2} \vee B_{1,1} \qquad \neg B_{1,1}}{\neg P_{1,2}}$$

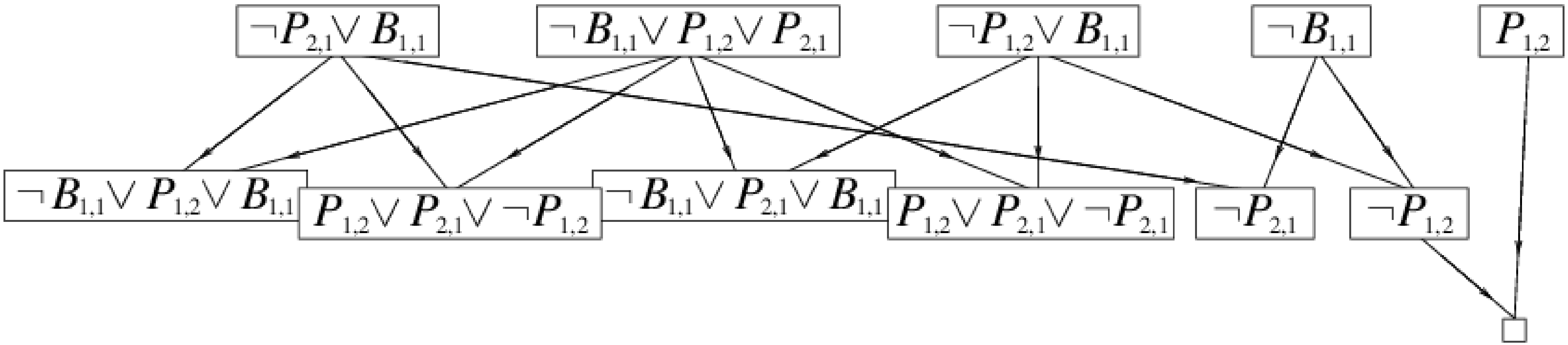
- Resolution

$$\frac{\neg P_{1,2} \qquad P_{1,2}}{\textit{false}}$$



Resolution Example

- In practice: all resolvable pairs of clauses are combined



Resolution Algorithm

- Proof by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable

function PL-RESOLUTION(*KB*, α) **returns** *true* or *false*
    inputs: *KB*, the knowledge base, a sentence in propositional logic
            α, the query, a sentence in propositional logic

    clauses ← the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
    new ← { }
    loop do
        for each C_i, C_j **in** *clauses* **do**
            resolvents ← PL-RESOLVE(C_i, C_j)
            if *resolvents* contains the empty clause **then return** *true*
            new ← *new* ∪ *resolvents*
        if *new* ⊆ *clauses* **then return** *false*
        clauses ← *clauses* ∪ *new*

Logical Agent

- Logical agent for Wumpus world explores actions
  - observe glitter → done
  - unexplored safe spot → plan route to it
  - if Wampus in possible spot → shoot arrow
  - take a risk to go possibly risky spot

- Propositional logic to infer state of the world

- Heuristic search to decide which action to take

Summary

- Logical agents apply **inference** to a **knowledge base**
  to derive new information and make decisions

- Basic concepts of logic:
  - **syntax**: formal structure of **sentences**
  - **semantics**: **truth** of sentences wrt **models**
  - **entailment**: necessary truth of one sentence given another
  - **inference**: deriving sentences from other sentences
  - **soundess**: derivations produce only entailed sentences
  - **completeness**: derivations can produce all entailed sentences

- Wumpus world requires the ability to represent partial and negated information, inference to determine state of the world, etc.

- Forward, backward chaining are linear-time, complete for Horn clauses

- Resolution is complete for propositional logic