Game Playing

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18 February 2025

Outline

- Games
- Perfect play
  - minimax decisions
  - α–β pruning
- Resource limits and approximate evaluation
- Games of chance
- Games of imperfect information

games

Games vs. Search Problems

- "Unpredictable" opponent ⇒ solution is a strategy specifying a move for every possible opponent reply

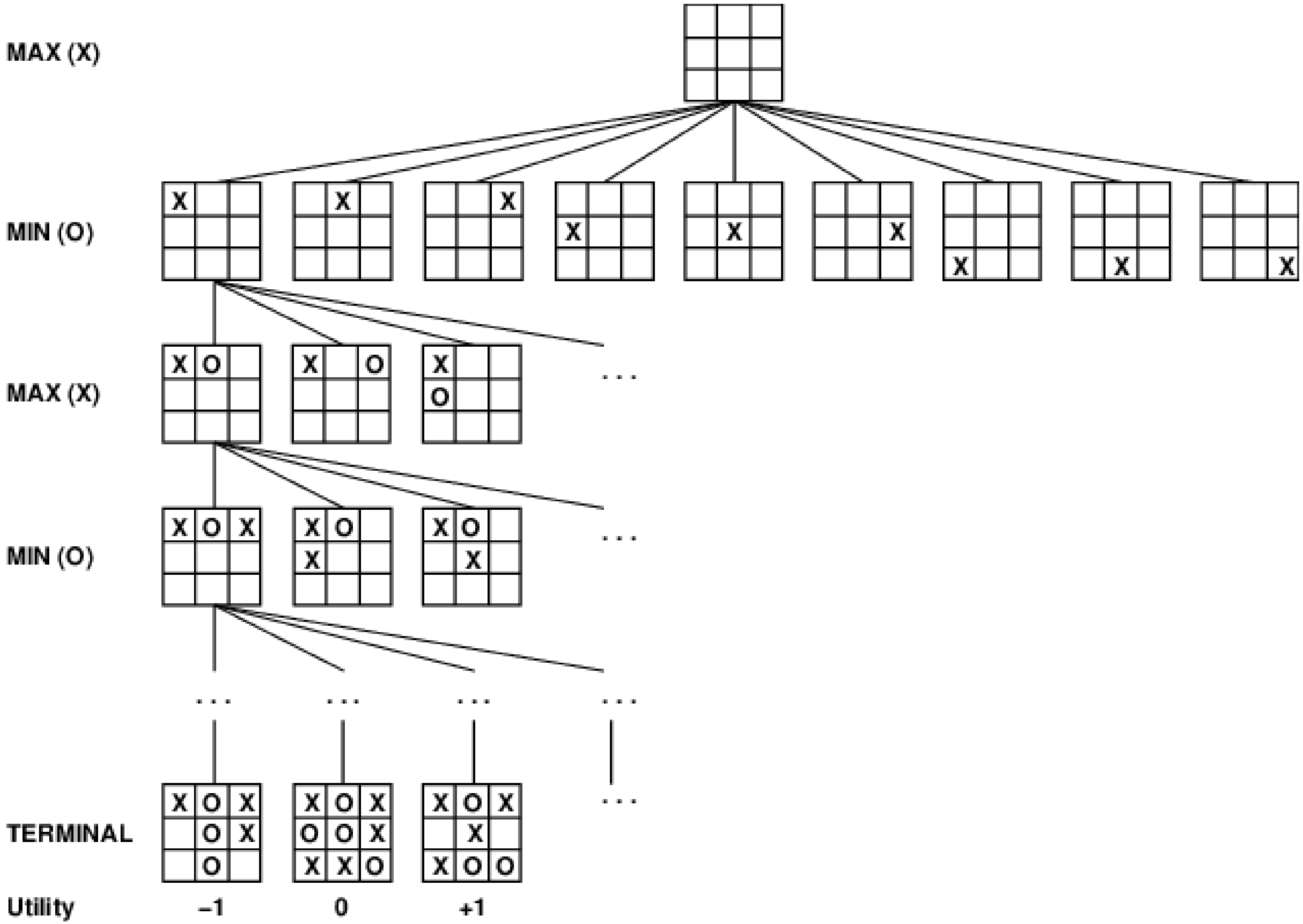
- Time limits ⇒ unlikely to find goal, must approximate

- Plan of attack:
  - computer considers possible lines of play (Babbage, 1846)
  - algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
  - finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
  - first Chess program (Turing, 1951)
  - machine learning to improve evaluation accuracy (Samuel, 1952–57)
  - pruning to allow deeper search (McCarthy, 1956)

Types of Games

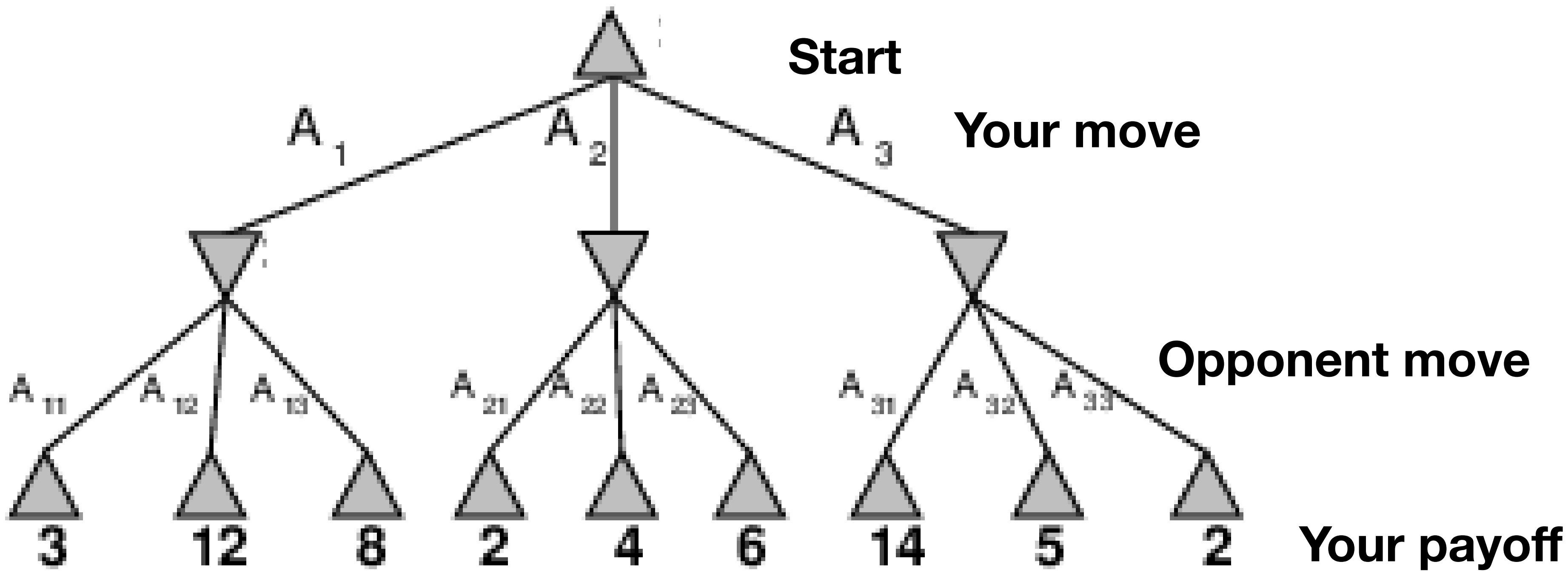
	deterministic	**chance**
perfect information	Chess Checkers Go Othello	Backgammon Monopoly
imperfect information	battleships Blind Tic Tac Toe	Bridge Poker Scrabble

Game Tree (2-player, Deterministic, Turns)



Simple Game Tree

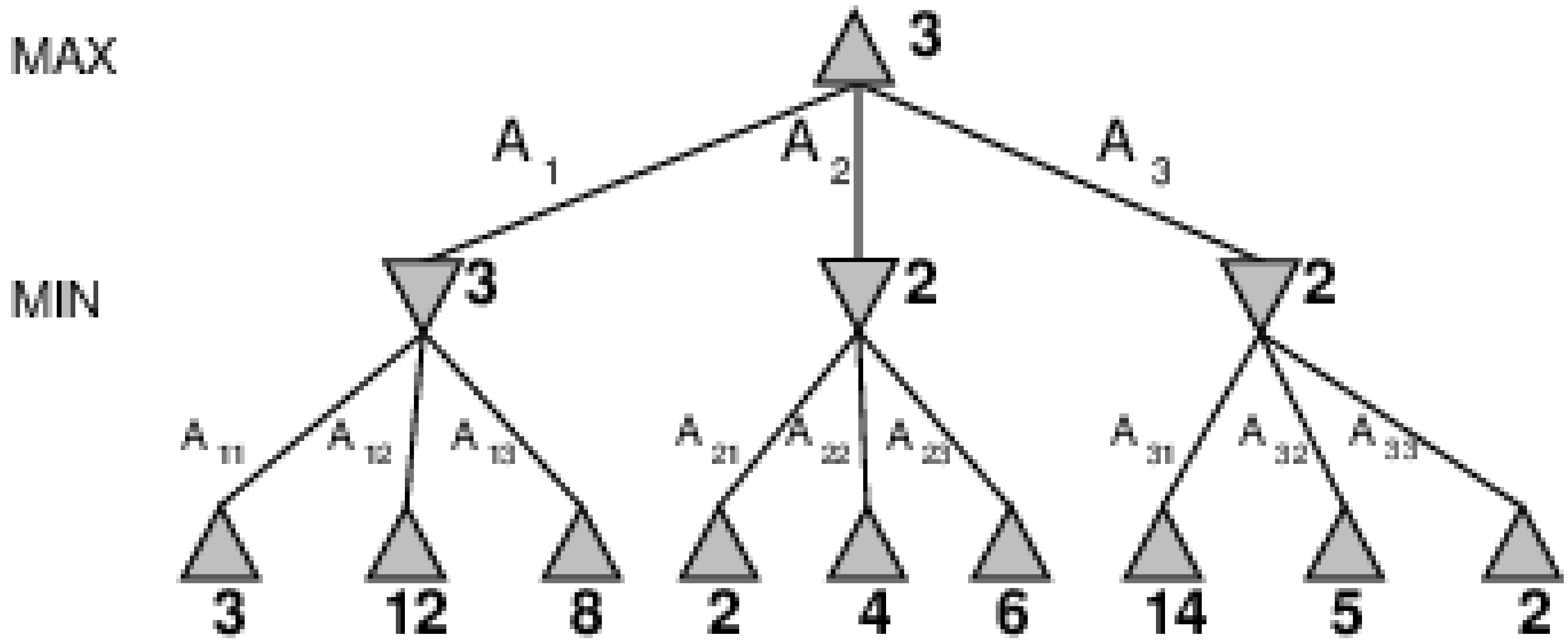
- 2 player game
- Each player has one move
- You move first
- Goal: optimize your payoff (utility)



minimax

Minimax

- Perfect play for deterministic, perfect-information games
- Idea: choose move to position with highest minimax value
  = best achievable payoff against best play
- E.g., 2-player game, one move each:



Minimax Algorithm

```
function MINIMAX-DECISION(state) returns an action
   inputs: state, current state in game

   return the a in ACTIONS(state) maximizing MIN-VALUE(RESULT(a, state))

function MAX-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for a, s in SUCCESSORS(state) do v ← MAX(v, MIN-VALUE(s))
   return v

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```
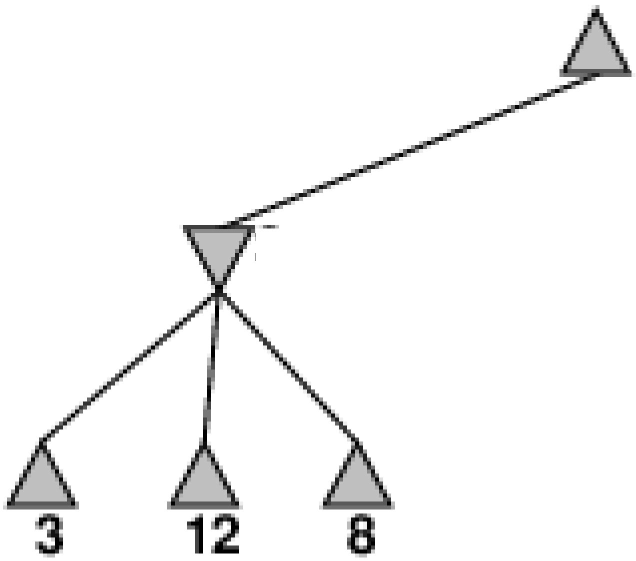
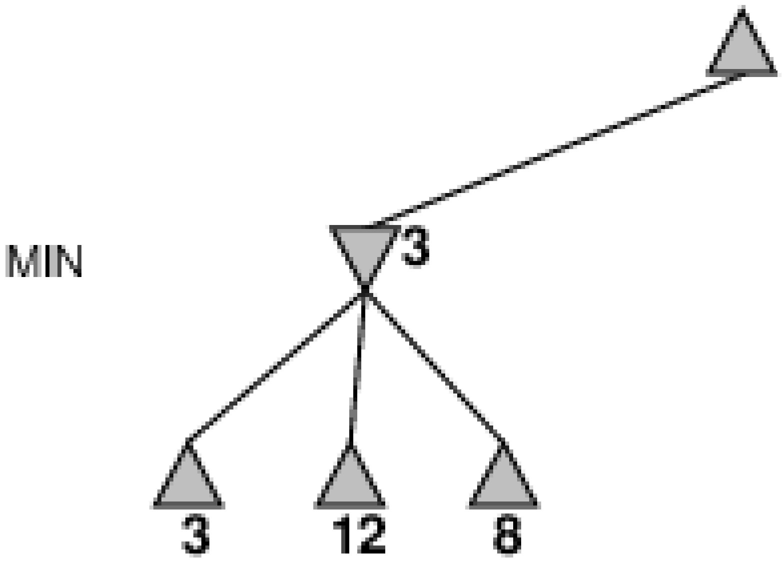

Properties of Minimax

- Complete? Yes, if tree is finite
- Optimal? Yes, against an optimal opponent. Otherwise?
- Time complexity? $O(b^m)$
- Space complexity? $O(bm)$ (depth-first exploration)
- For Chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
  $\Rightarrow$ exact solution completely infeasible
- But do we need to explore every path?

α–β Pruning Example



α–β Pruning Example



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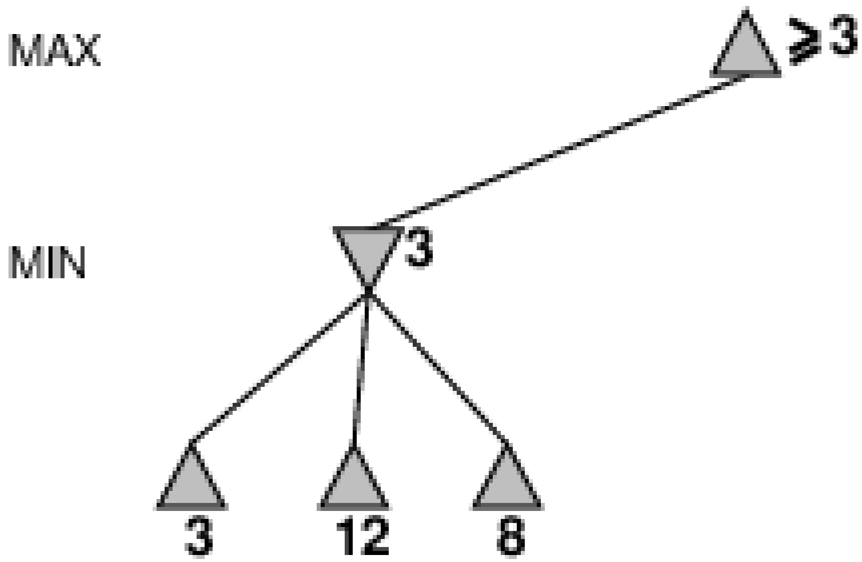

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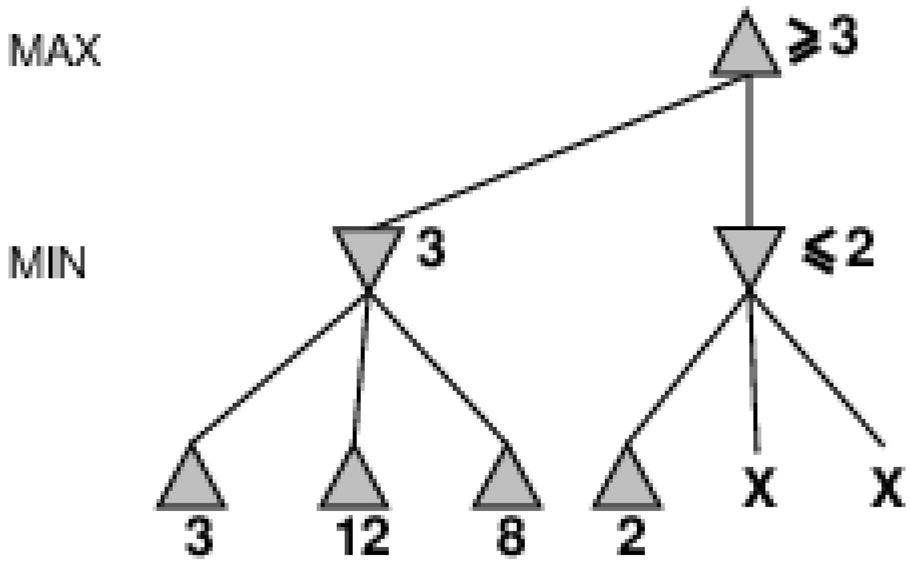

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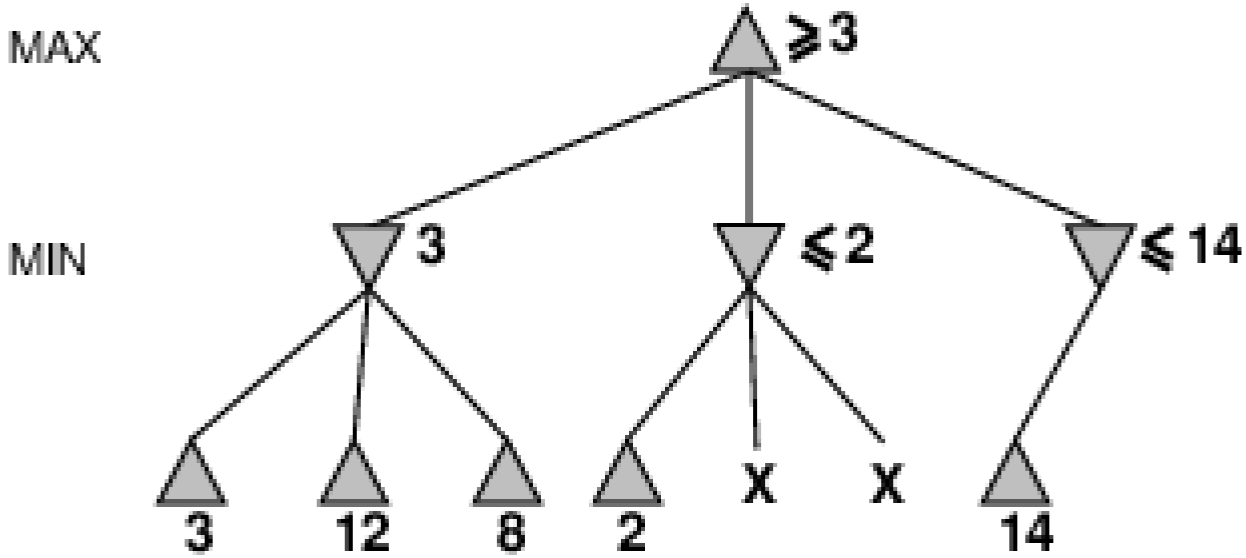

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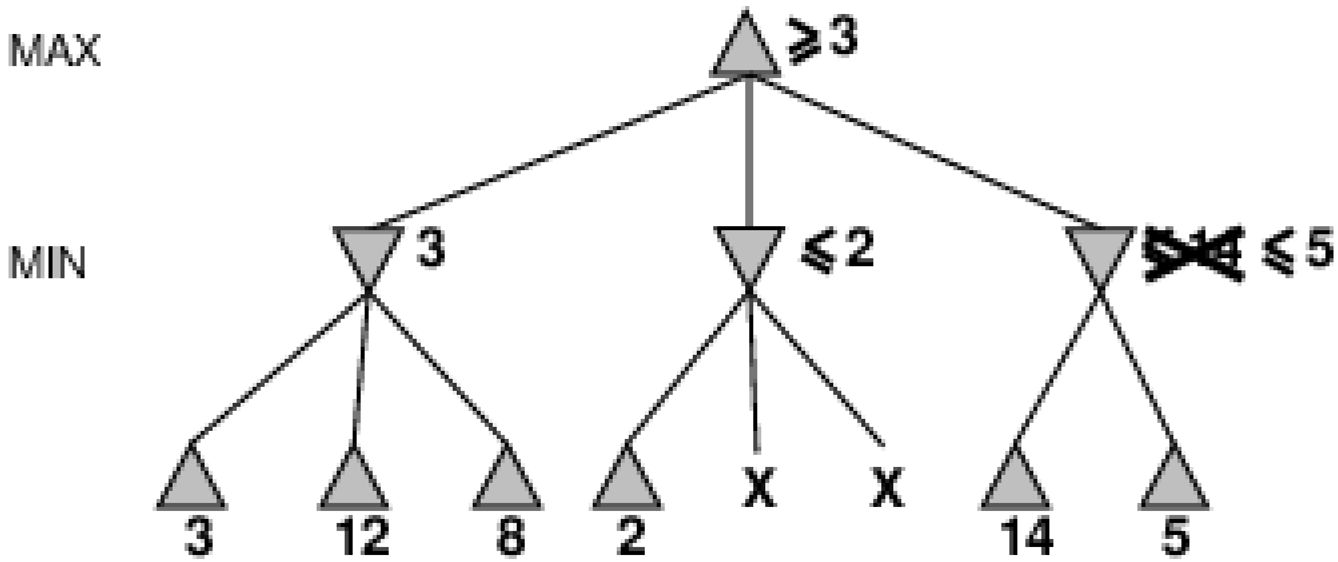

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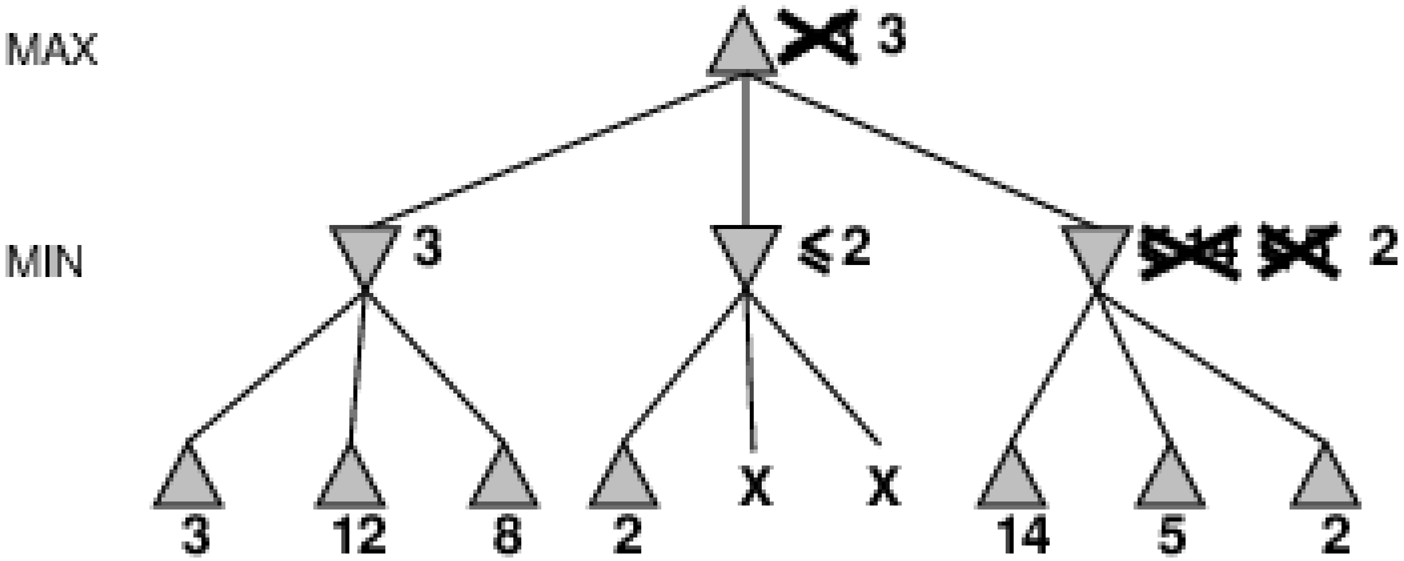
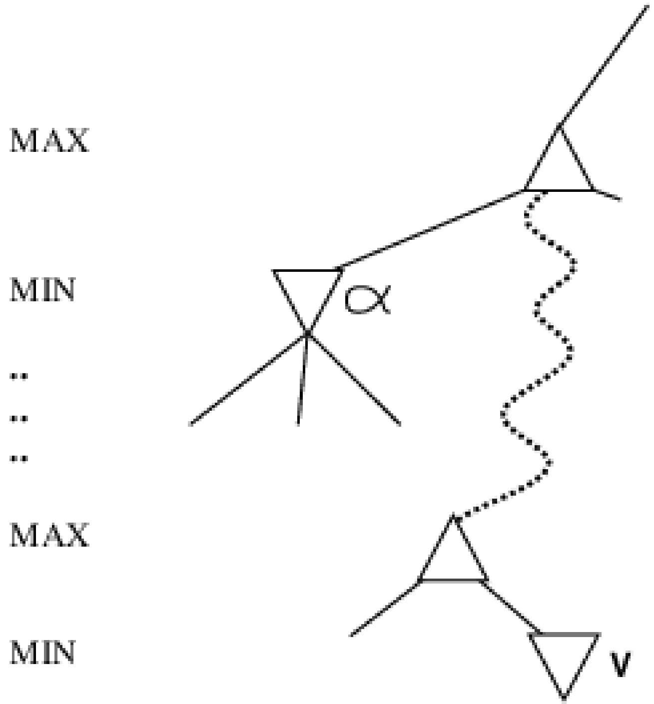


Why is it Called α–β?



- α is the best value (to MAX) found so far off the current path
- If V is worse than α, MAX will avoid it $\Rightarrow$ prune that branch
- Define β similarly for MIN

Properties of α–β

- Safe: Pruning **does not** affect final result

- Good move ordering improves effectiveness of pruning

- With "perfect ordering," time complexity = $O(b^{m/2})$
  $\Rightarrow$ **doubles** solvable depth

- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)

- Unfortunately, 35^{50} is still impossible!

Solved Games

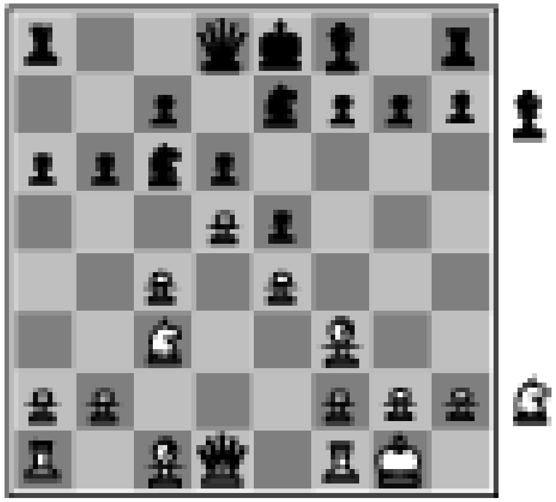
- A game is solved if optimal strategy can be computed
- Tic Tac Toe can be trivially solved
- Biggest solved game: Checkers
  - proof by Schaeffer in 2007
  - both players can force at least a draw
- Most games (Chess, Go, etc.) too complex to be solved

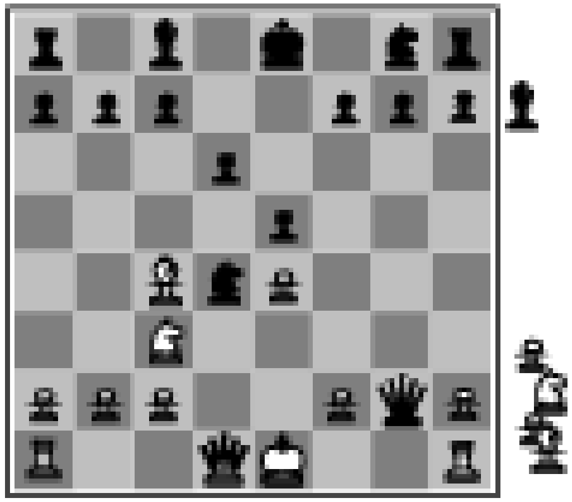
resource limits

Resource Limits

- Standard approach:
  - Use CUTOFF-TEST instead of TERMINAL-TEST
    e.g., depth limit
  - Use EVAL instead of UTILITY
    i.e., evaluation function that estimates desirability of position
- Suppose we have 100 seconds, explore 10^4 nodes/second
  $\Rightarrow$ 10^6 nodes per move $\approx 35^{8/2}$
  $\Rightarrow$ α–β reaches depth 8 $\Rightarrow$ pretty good Chess program

Evaluation Functions





- For Chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

e.g., $f_1(s)$ = (number of white queens) – (number of black queens)

Evaluation Function for Chess

- Long experience of playing Chess

⇒ Evaluation of positions included in Chess strategy books

  - bishop is worth 3 pawns
  - knight is worth 3 pawns
  - rook is worth 5 pawns
  - good pawn position is worth 0.5 pawns
  - king safety is worth 0.5 pawns
  - etc.

- Pawn count → weight for features

Learning Evaluation Functions

- Designing good evaluation functions requires a lot of expertise

- Machine learning approach
  - collect a large database of games play
  - note for each game who won
  - try to predict game outcome from features of position

  $\Rightarrow$ learned weights

- May also learn evaluation functions from self-play

Some Concerns

- Quiescence
  - position evaluation not reliable if board is unstable
  - e.g., Chess: queen will be lost in next move
  - → deeper search of game-changing moves required

- Horizon Effect
  - adverse move can be delayed, but not avoided
  - search may prefer to delay, even if costly

Forward Pruning

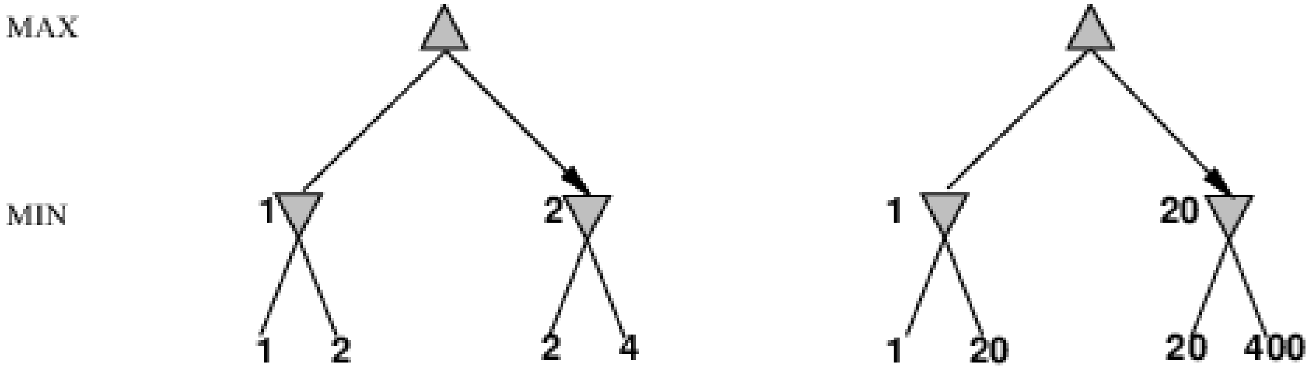
- Idea: avoid computation on clearly bad moves
- Cut off searches with bad positions before they reach max-depth
- Risky: initially inferior positions may lead to better positions
- Beam search: explore fixed number of promising moves deeper

Lookup instead of Search

- Library of opening moves
  - even expert Chess players use standard opening moves
  - these can be memorized and followed until divergence

- End game
  - if only few pieces left, optimal final moves may be computed
  - Chess end game with 6 pieces left solved in 2006
  - can be used instead of evaluation function

Digression: Exact Values do not Matter



- Behaviour is preserved under any **monotonic** transformation of EVAL
- Only the order matters:
  payoff in deterministic games acts as an ordinal utility function

Deterministic Games in Practice

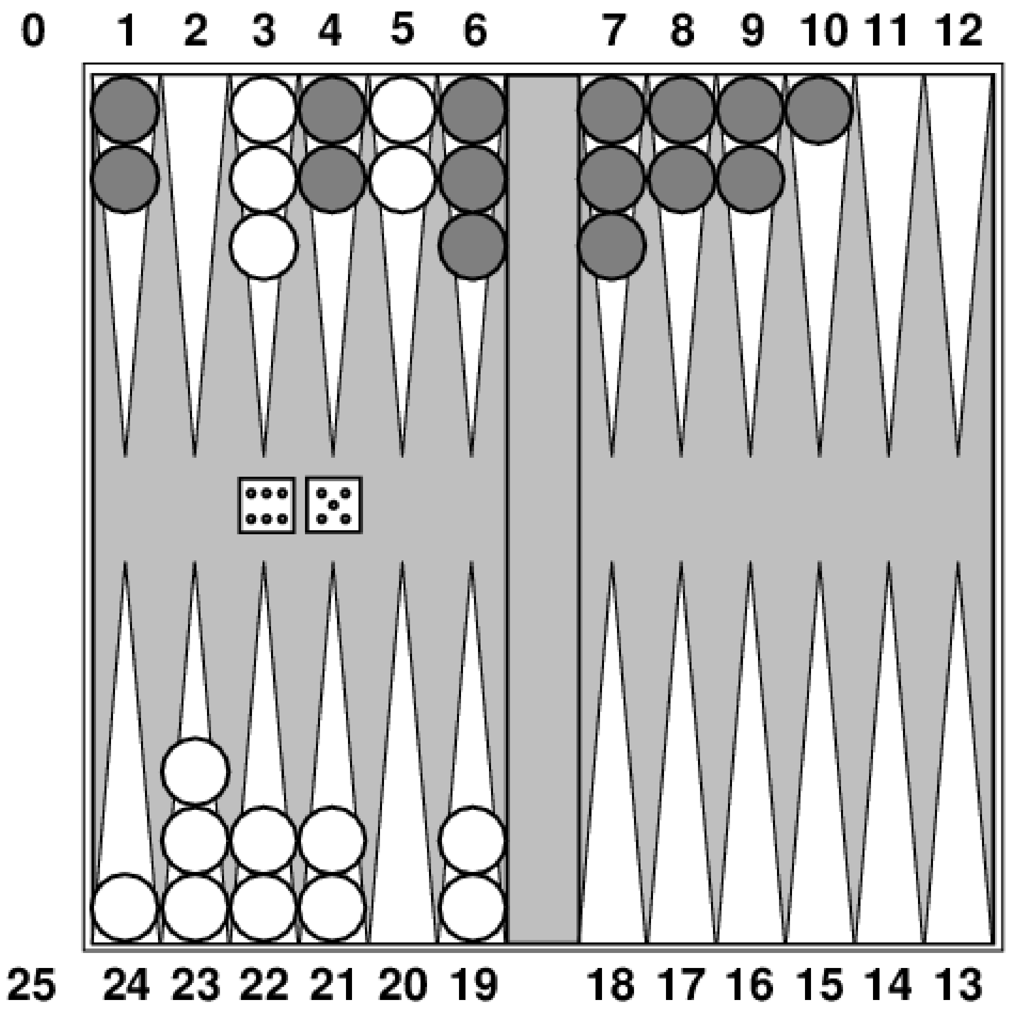
- **Checkers:** Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions. Weakly solved in 2007 by Schaeffer (guaranteed draw).

- **Chess:** Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

- **Go:** In 2016, computer using a neural network for the board evaluation function, was able to beat the human Go champion for the first time. Given the huge branching factor ($b > 300$), Go was long considered too difficult for machines. **We will have a dedicated lecture about this towards the end of the course**

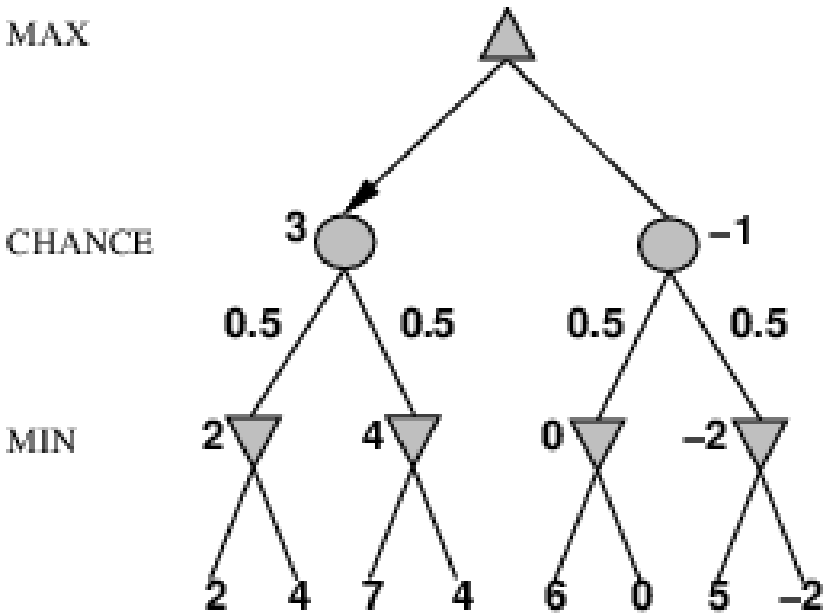
games of chance

Nondeterministic Games: Backgammon



Nondeterministic Games in General

- In nondeterministic games, chance introduced by dice, card-shuffling
- Simplified example with coin-flipping:



Algorithm for Nondeterministic Games

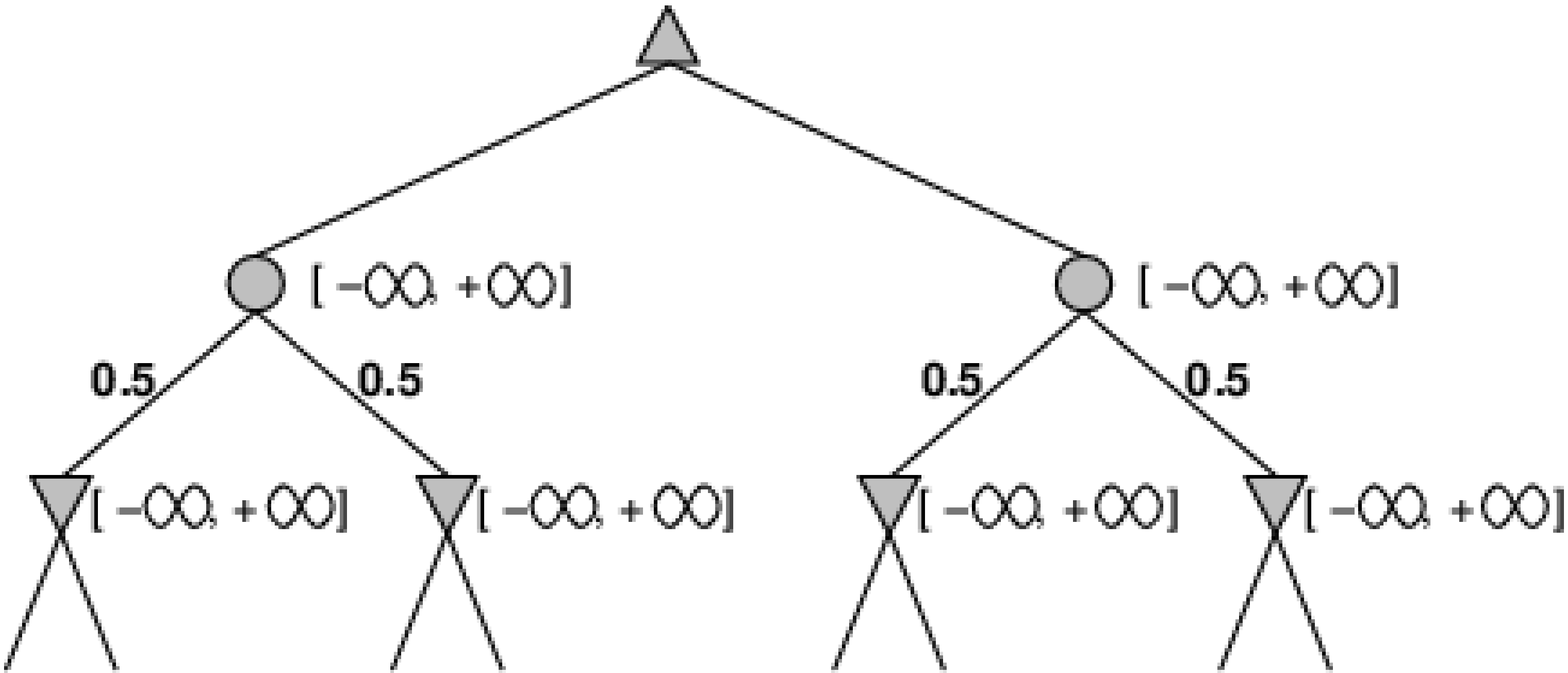
- EXPECTIMINIMAX gives perfect play

- Just like MINIMAX, except we must also handle chance nodes:

...
if *state* is a MAX node **then**
    return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a MIN node **then**
    return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a chance node **then**
    return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
...

Pruning in Nondeterministic Game Trees

A version of α-β pruning is possible:



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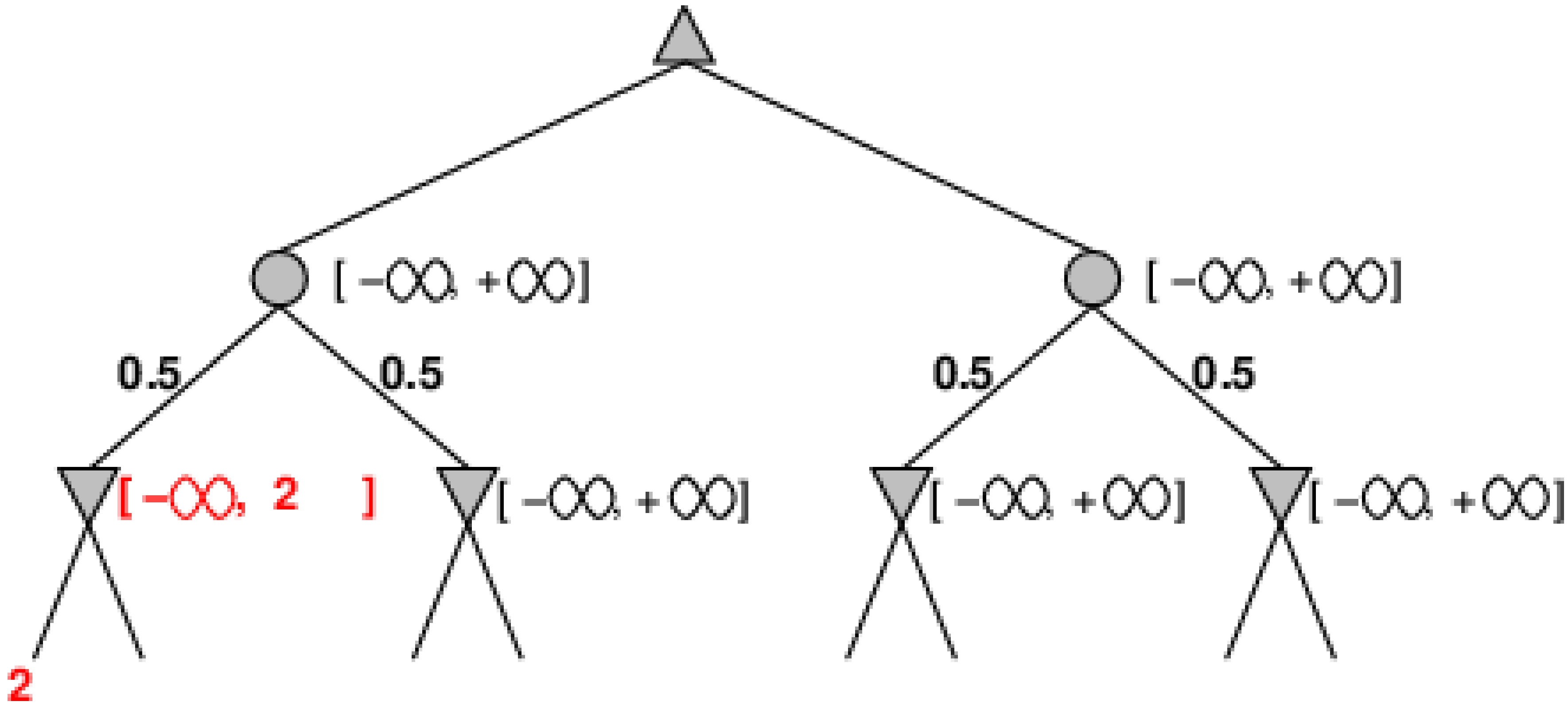

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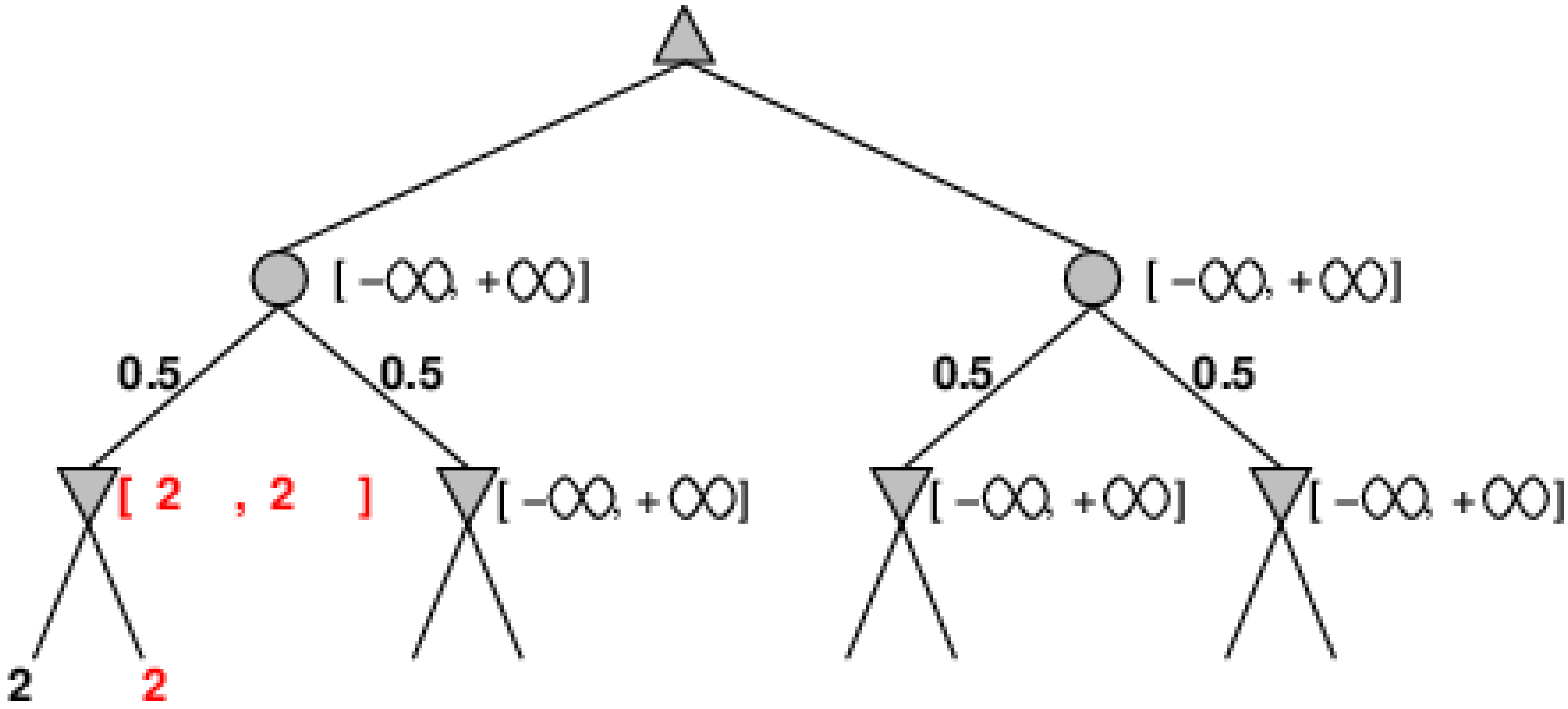

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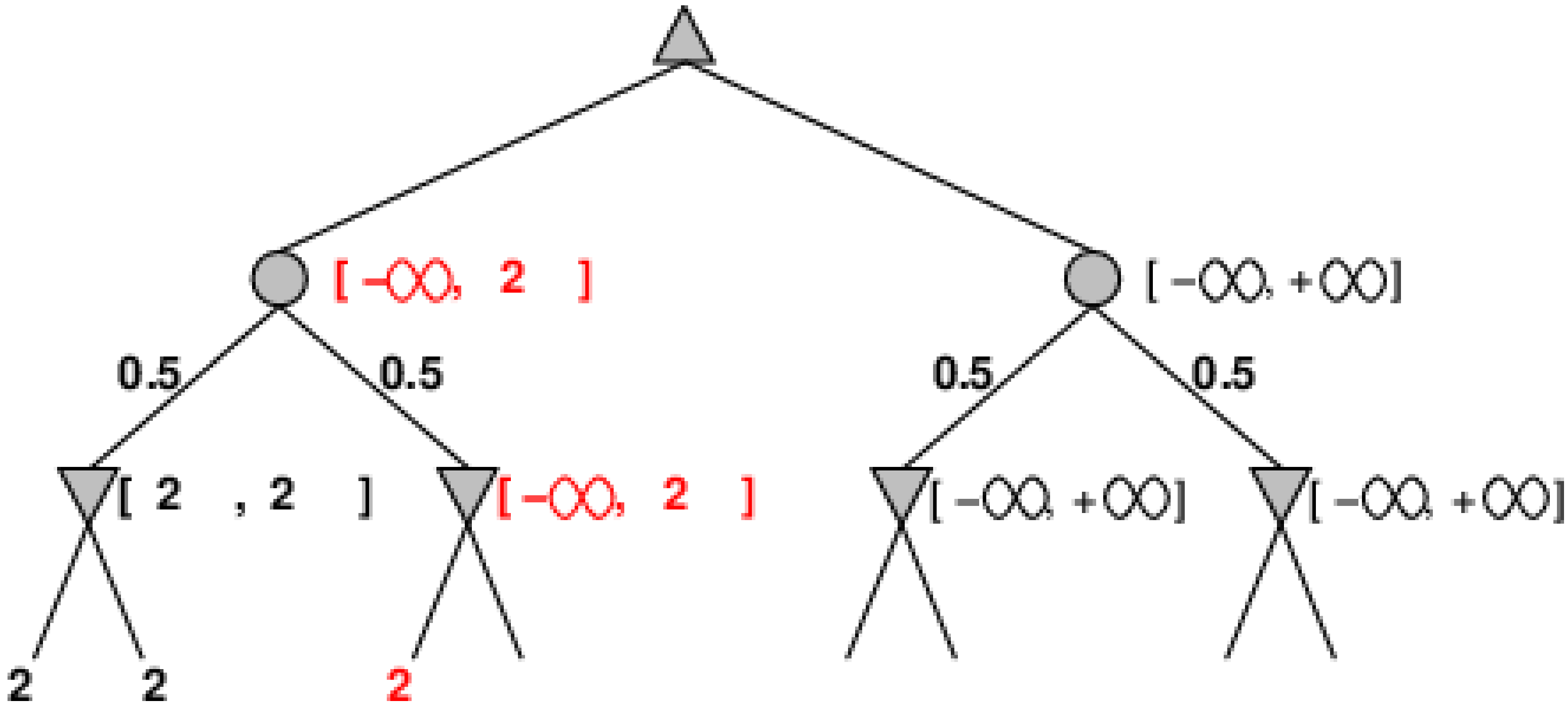

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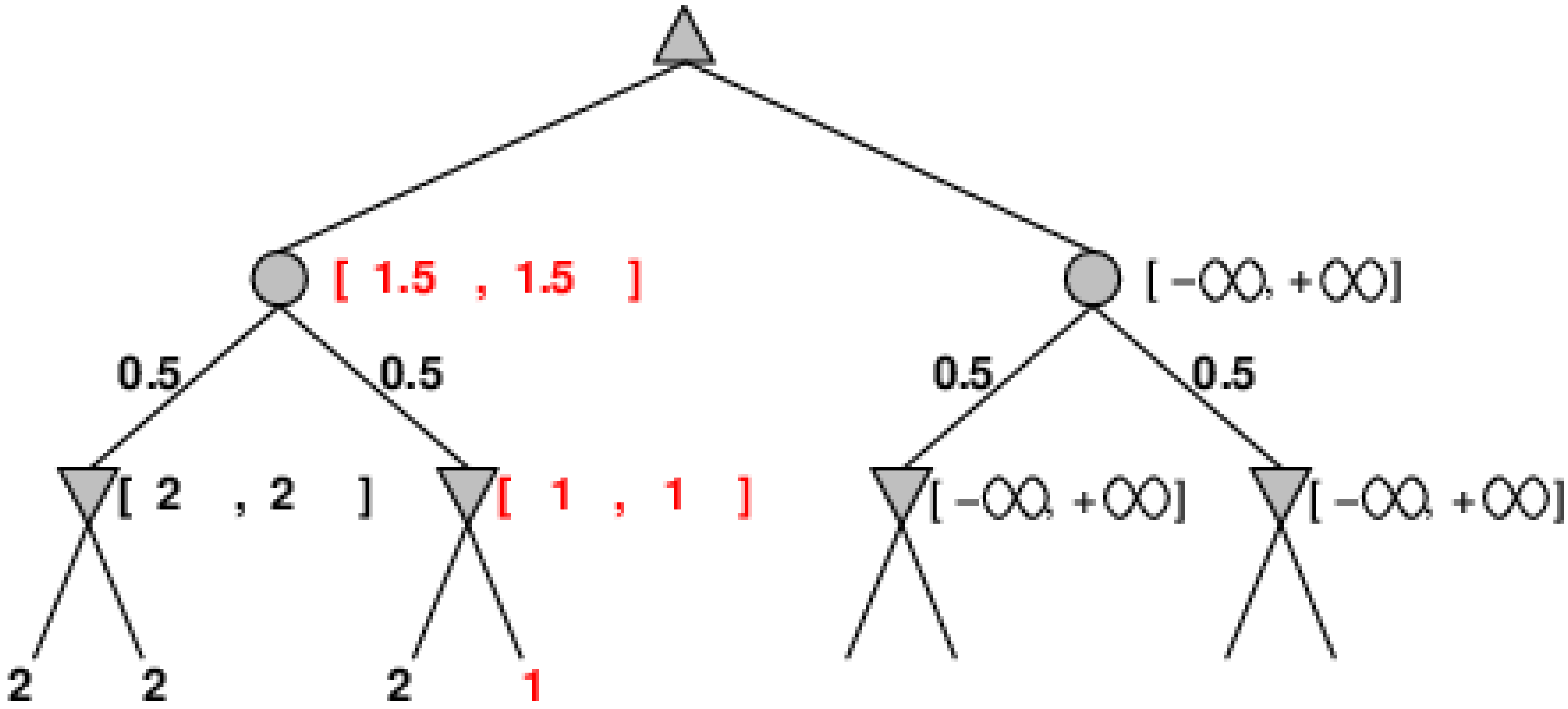

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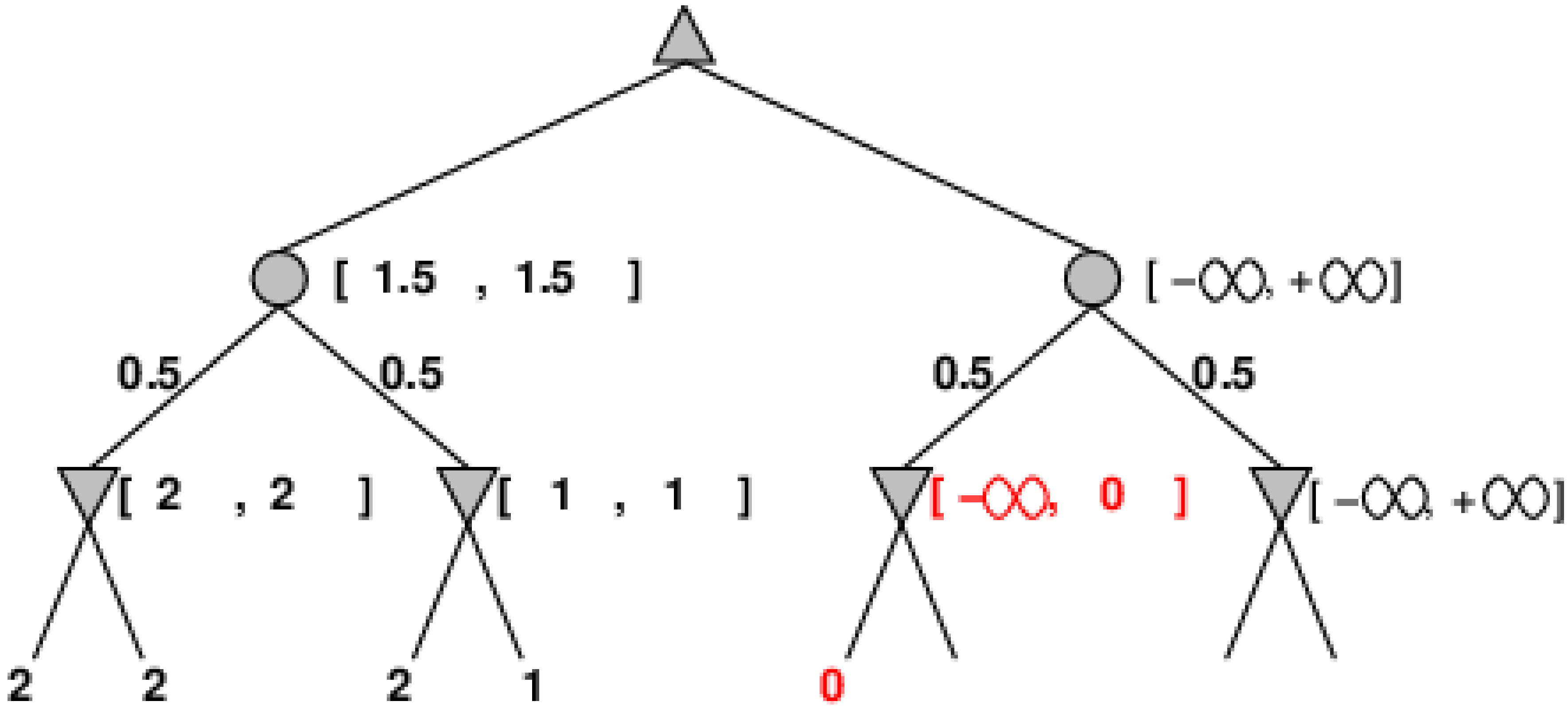

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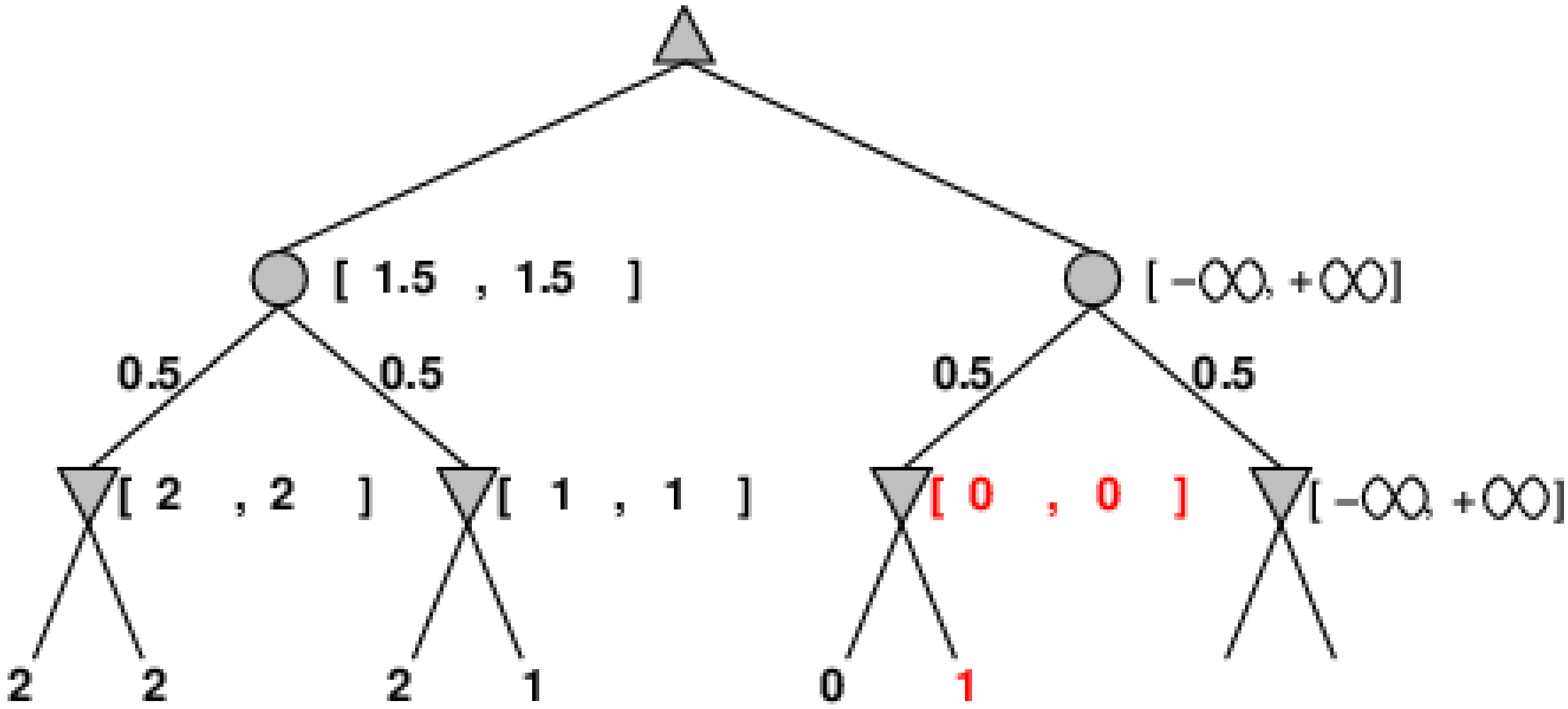
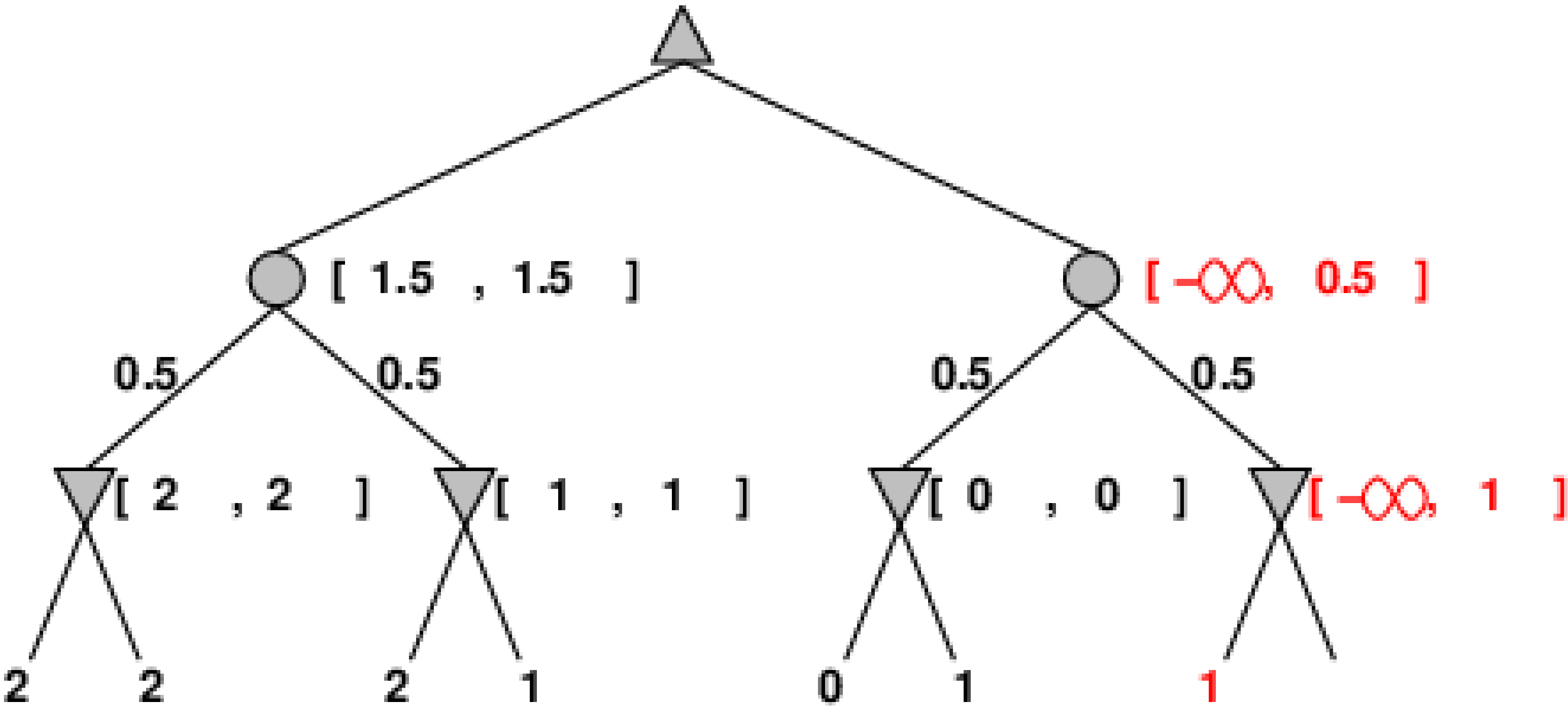


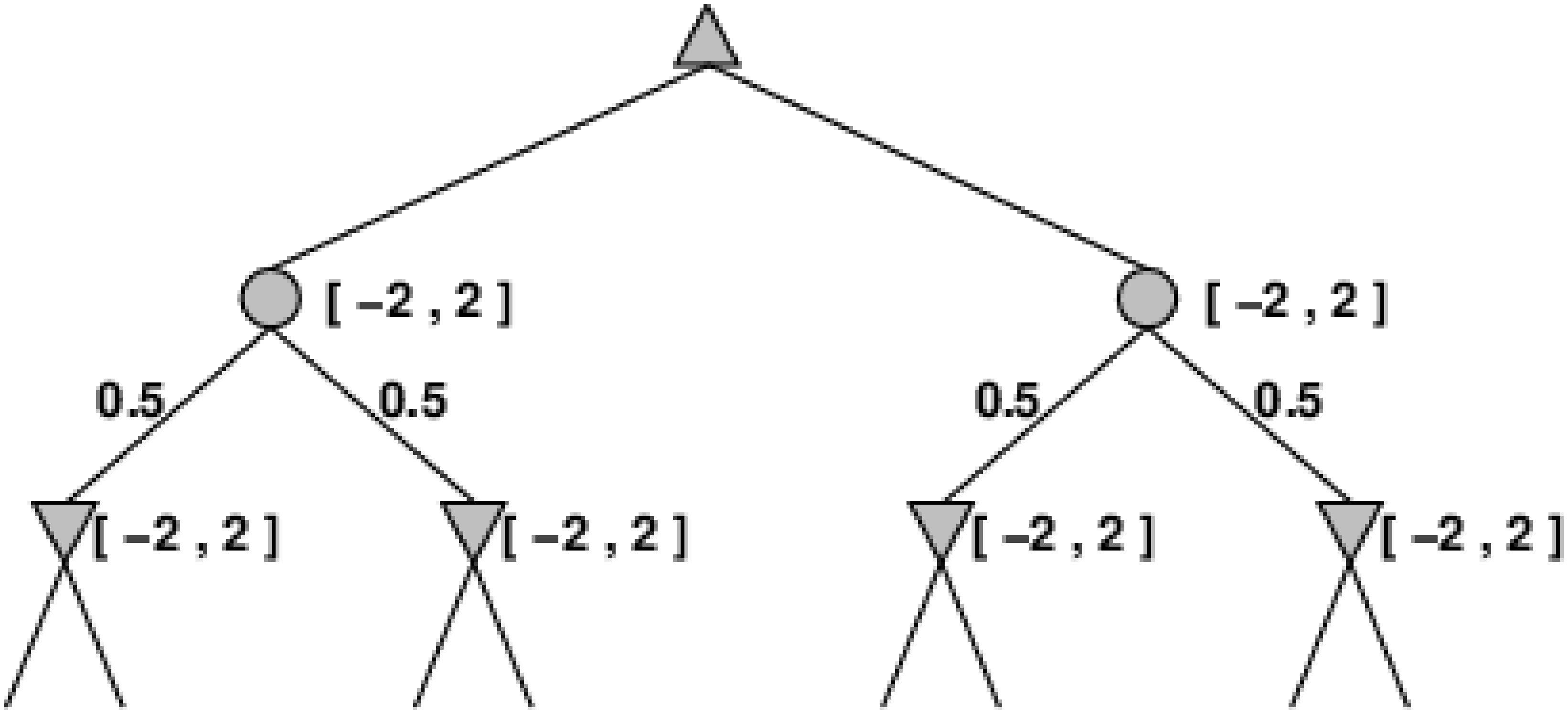
Pruning in Nondeterministic Game Trees

Terminate, since left path will be worth more on average.



Pruning with Bounds

More pruning occurs if we can bound the leaf values (0,1,2)



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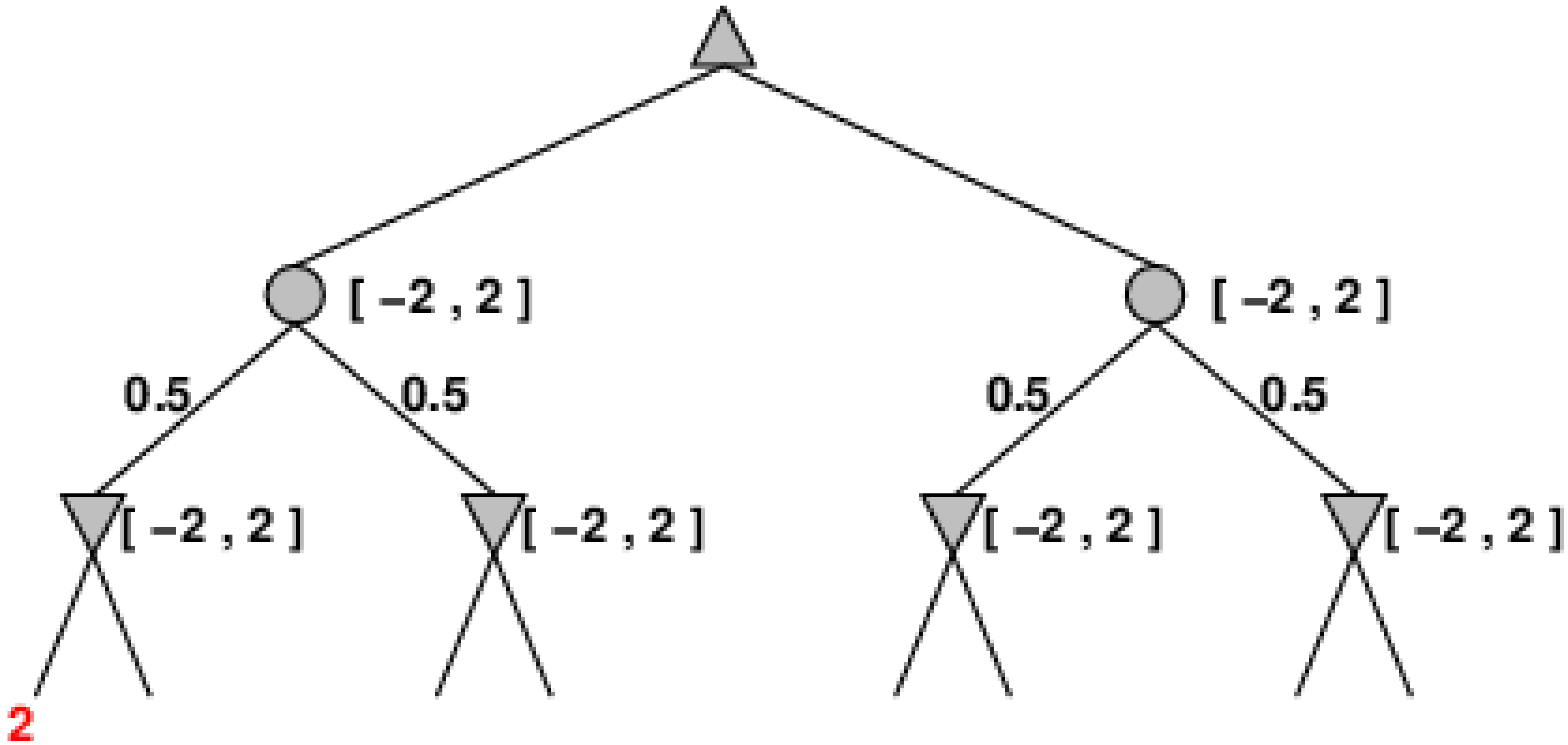

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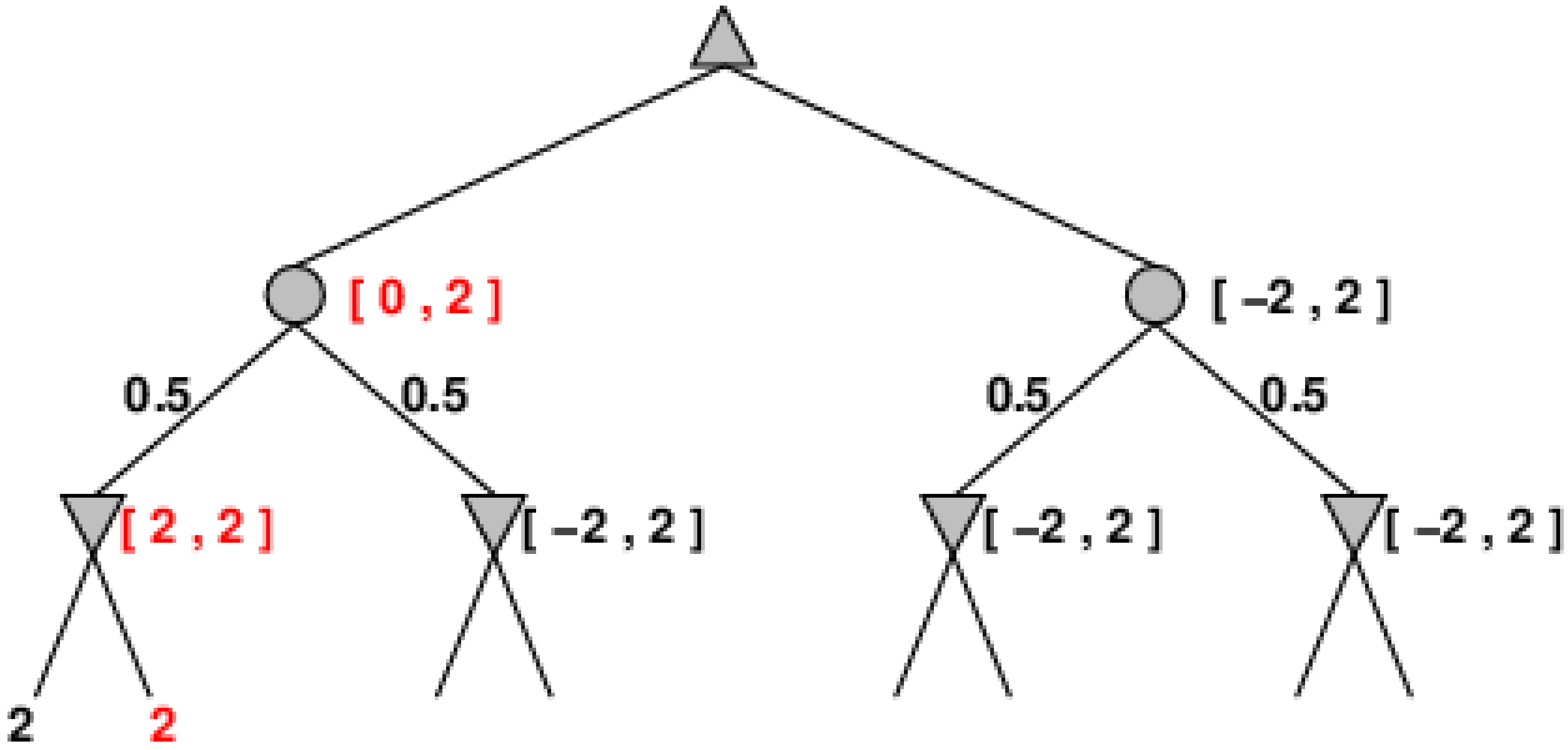

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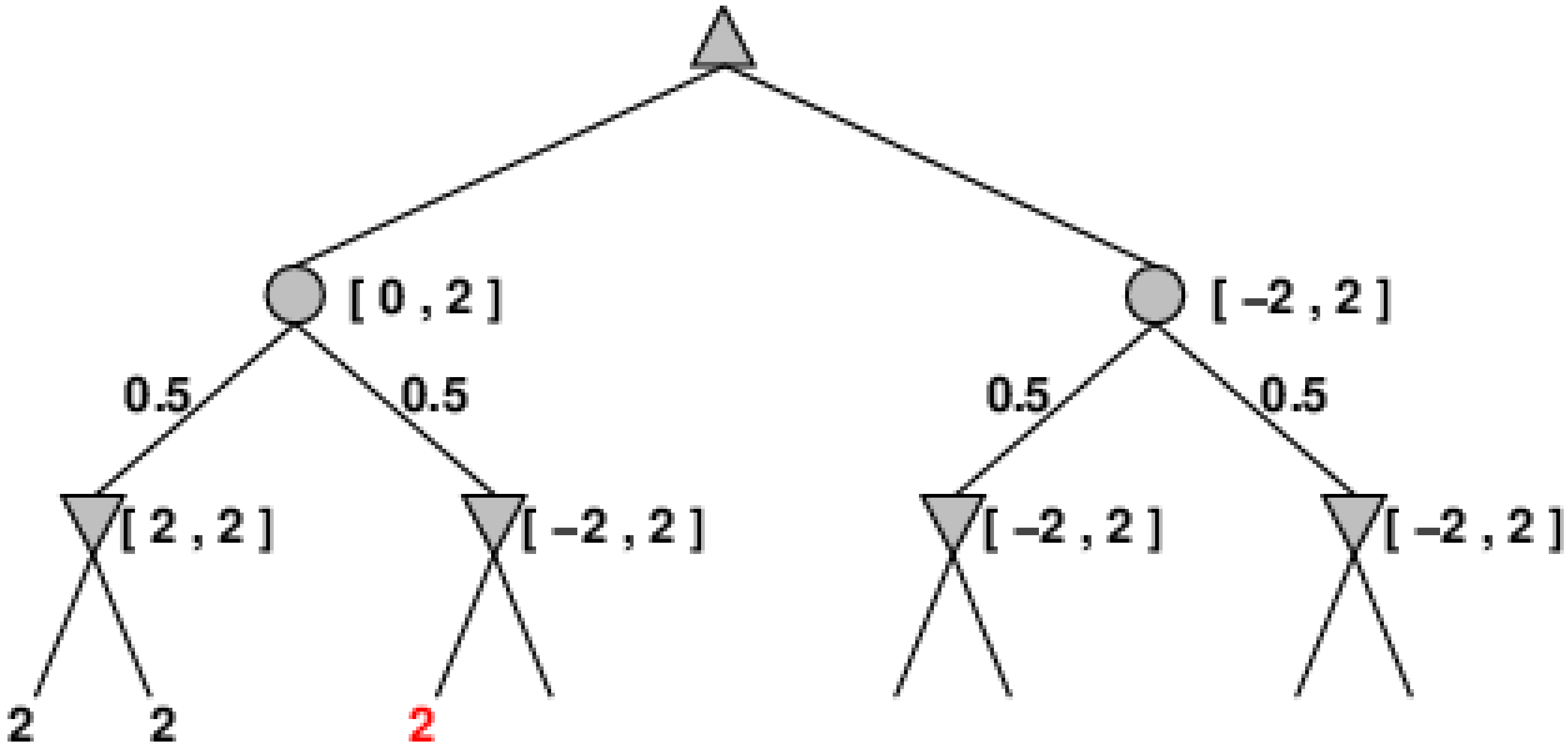

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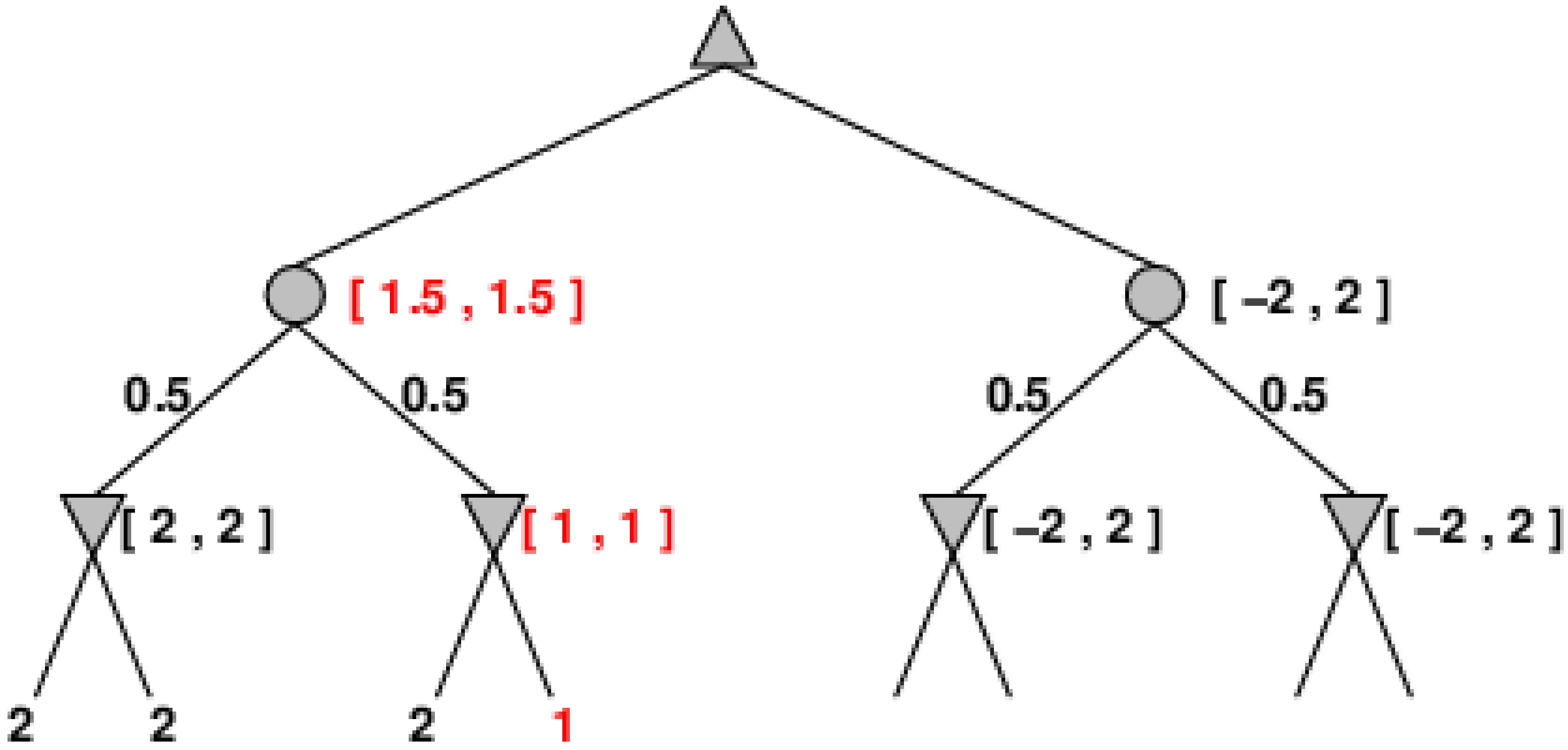
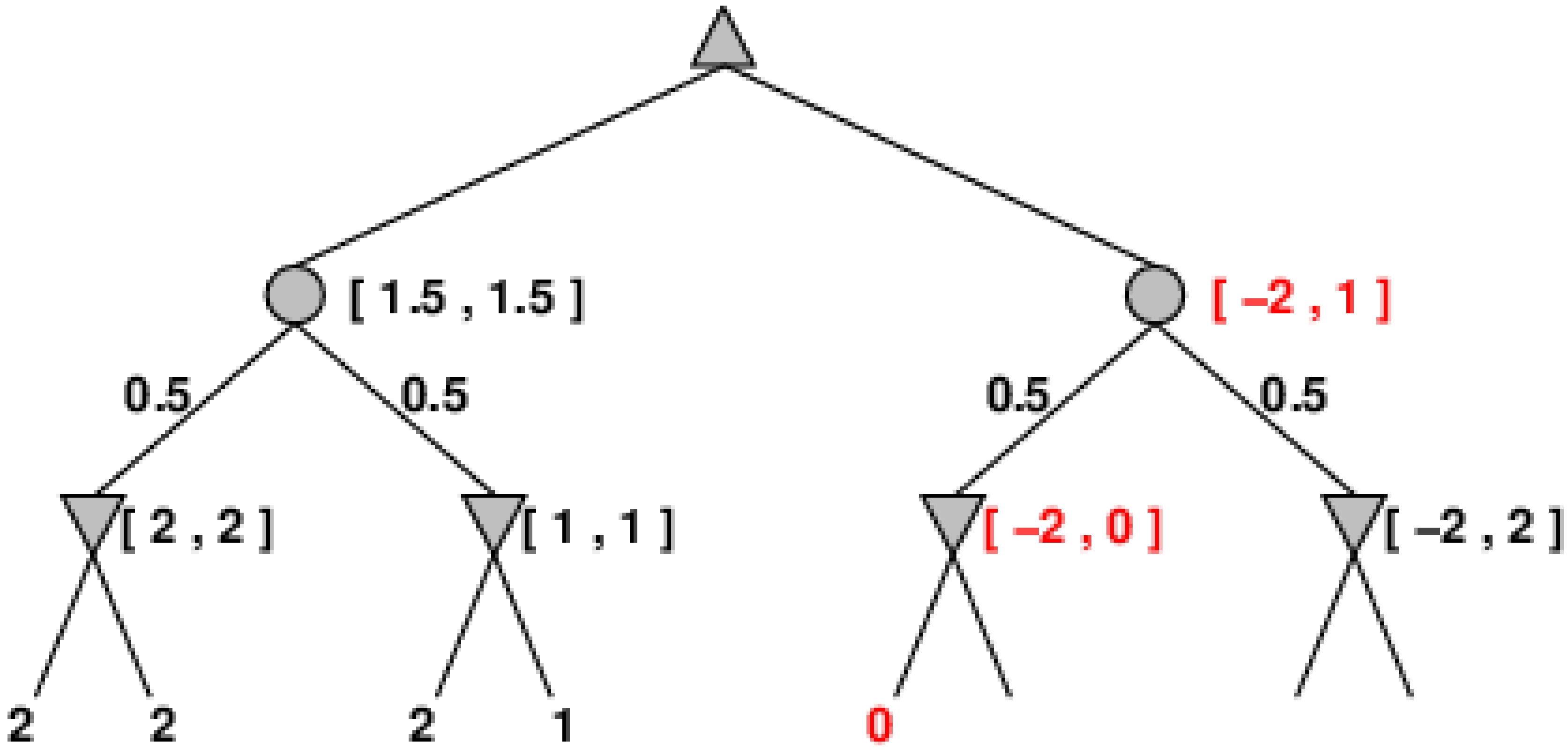


Pruning with Bounds

Can already stop search: best expected value for right branch is 1



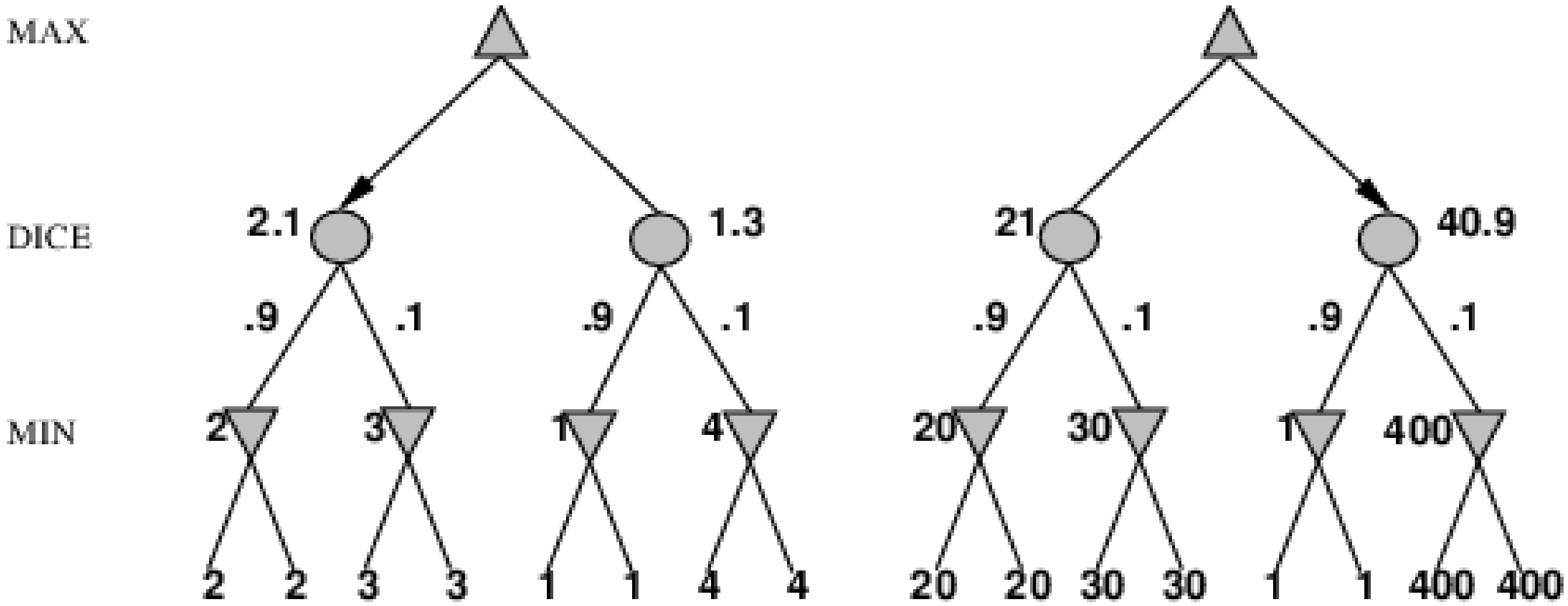
Nondeterministic Games in Practice

- Dice rolls increase b: 21 possible rolls with 2 dice
- Backgammon ≈ 20 legal moves (can be 6,000 with 1-1 roll)

$$\text{depth } 4 = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$$

- As depth increases, probability of reaching a given node shrinks
  ⇒ value of lookahead is diminished
- α–β pruning is much less effective
- TDGAMMON uses depth-2 search + very good EVAL
  ≈ world-champion level

Digression: Exact Values Do Matter



- Behavior is preserved only by positive linear transformation of EVAL
- Hence EVAL should be proportional to the expected payoff

imperfect information

Games of Imperfect Information

- E.g., card games, where opponent's initial cards are unknown
- Typically we can calculate a probability for each possible deal
- Seems just like having one big dice roll at the beginning of the game
- Idea: compute the minimax value of each action in each deal,
  then choose the action with highest expected value over all deals
- Special case: if an action is optimal for all deals, it's optimal.

Commonsense Counter-Example

- Road A leads to a small heap of gold pieces ($)
  Road B leads to a fork:
  take the left fork and you'll find a mound of jewels ($$$);
  take the right fork and you'll be run over by a bus.

- Road A leads to a small heap of gold pieces ($)
  Road B leads to a fork:
  take the left fork and you'll be run over by a bus;
  take the right fork and you'll find a mound of jewels ($$$);.

⇒ *does not matter if jewels are left or right on road B, it's always better choice*

- Road A leads to a small heap of gold pieces ($);
  Road B leads to a fork:
  guess correctly and you'll find a mound of jewels ($$$);
  guess incorrectly and you'll be run over by a bus.

⇒ *it does matter if we know where forks on road B lead to*

Proper Analysis

- Intuition that the value of an action is the average of its values in all actual states is **WRONG**
- With partial observability, value of an action depends on the information state or belief state the agent is in
- Can generate and search a tree of information states
- Leads to rational behaviors such as
  - acting to obtain information
  - signalling to one's partner
  - acting randomly to minimize information disclosure

Computer Poker

- Hard game
  - imperfect information — including bluffing and trapping
  - stochastic outcomes — cards drawn at random
  - partially observable — may never see other players hand when they fold
  - non-cooperative multi-player — possibility for coalitions

- Few moves (fold, call, raise), but large number of stochastic states

- Relative balance of deception plays very important
  also: when to bluff

⇒ There is no single best move

- Need to model other players (style, collusion, patterns)

- Hard to evaluate (not just win/loss, different types of opponents)

Summary

- Games are fun to work on

- They illustrate several important points about AI
  - perfection is unattainable ⇒ must approximate
  - good idea to think about what to think about
  - uncertainty constrains the assignment of values to states
  - optimal decisions depend on information state, not real state

- Games are to AI as grand prix racing is to automobile design