Geometry Processing (601.458/658)

Misha Kazhdan

Course Content

Processing Signals on Triangle Meshes

Theory

- Linear algebra
- Calculus
- Finite elements

Application

- Smoothing and Sharpening
- Geodesics in Heat
- Spectral Representation, PDEs, and Unconditional Stability
- Heat Kernel Signature
- Shape Deformation

What I Expect From You

Homework

- A code-base is provided. (Assignment 1 posted and due 2/07/26.)
- Assignments will focus on implementation.

~~Presentations~~

~~Exams~~

Readings

- There is no text-book.
- Class notes will be posted.
- Supplementary readings will be suggested.

https://www.cs.jhu.edu/~misha/Spring26/

Motivating Problem

Given a 2D surface $\mathcal{M} \subset \mathbb{R}^3$, and a function $\phi: \mathcal{M} \to \mathbb{R}$,

Evolve ϕ so that it gets progressively smoother over time.

Motivating Problem

Newton's law of cooling:

"The rate of change in the temperature of a body is directly proportional to the temperature difference between the body and the surroundings"

For a signal $\phi: \mathcal{M} \to \mathbb{R}$

- The *rate of change* is $\partial\phi/\partial t$
- The *difference between the body and the surroundings* is $\Delta\phi$

This gives the PDE:

$$\frac{\partial \phi}{\partial t} = \lambda \cdot \Delta \phi$$

Signal Smoothing Visualization

Motivating Problem

$$\frac{\partial \phi}{\partial t} = \lambda \cdot \Delta \phi$$

Goal:

- Study the *linear algebra* and *calculus* involved in formulating the problem and use it to obtain a solution…
- To a whole class of problems requiring similar construction:
  - Smoothing
  - Wave equation
  - Stitching
  - Deformation
  - Shape matching

Wave Simulation Visualization

General Note

Many techniques designed for processing signals on surfaces are trivially applied to the surface itself by processing the x-, y-, and z-coordinates.

Example:

Instead of smoothing a signal defined on the surface, we can smooth the geometry.

Geometry Smoothing Visualization

Course Thesis

In geometry processing we regularly work with matrices.

- ✓Enables numerical computation
  - ⇒ Linear solvers, eigen-decomposition, etc.
- ✗Lose track of the spaces we are mapping from/to.
  - ⇒ End up saying nonsensical things

In this course:

Revisit geometry processing from the perspective of tracking what the underlying operators (not matrices) are.

Assignment 1

Naïve Implementation:

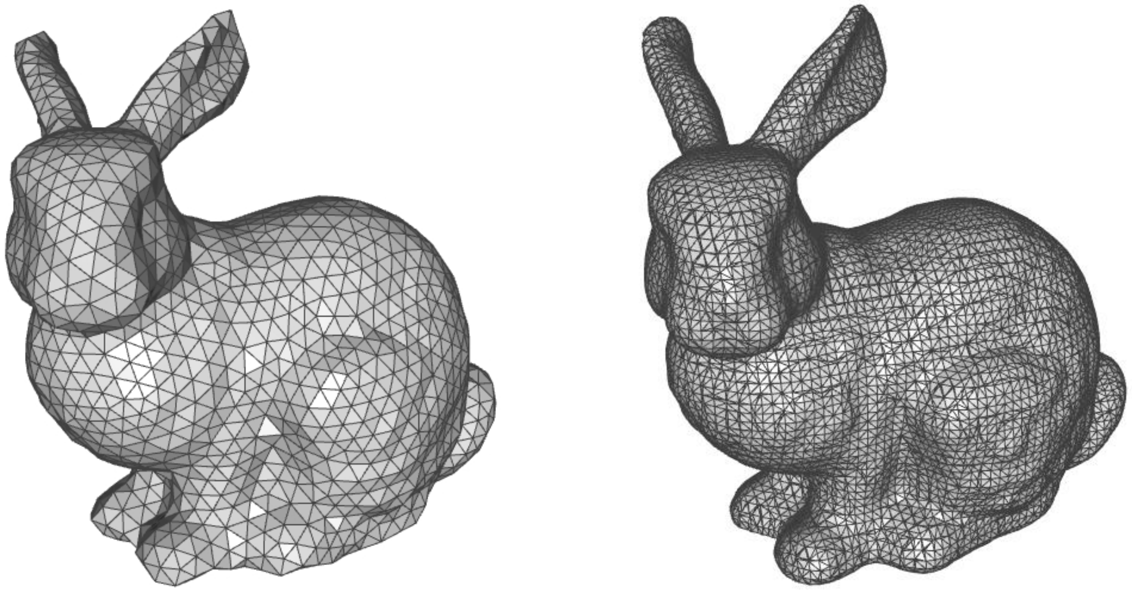
Iteratively smooth a signal described by values at vertices of a triangle mesh.

⇒ At each time-step, set the new value at a vertex to be the weighted average of the values at the vertex and its neighbors.

Assignment 1: Surface Representation

Surface are represented as triangle meshes:

- A vector of vertex positions in 3D
- A vector of triplets of indices into the vertex list



Include/Mesh.h

```
struct Mesh
{
    using Real = double;
    static const unsigned int K = 2;
    static const unsigned int Dim = 3;

    std::vector< Point< Real , Dim > > vertices;

    std::vector< SimplexIndex< K > > triangles;

    ...
}
```

Assignment 1: Surface Representation

Surface are represented as triangle meshes:

- A vector of vertex positions in 3D
- A vector of triplets of indices into the vertex list

The Point<Real,Dim> class is an array of size Dim storing Real values.*

Include/Mesh.h

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struct Mesh
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    using Real = double;
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    std::vector< Point< Real , Dim > > vertices;

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    ...
}
```

*Defined in ThirdParty/Include/Misha/Geometry.h

Assignment 1: Surface Representation

Surface are represented as triangle meshes:

- A vector of vertex positions in 3D
- A vector of triplets of indices into the vertex list

The SimplexIndex<K> class is an array of size K+1 storing unsigned int values.*

Include/Mesh.h

```
struct Mesh
{
    using Real = double;
    static const unsigned int K = 2;
    static const unsigned int Dim = 3;

    std::vector< Point< Real , Dim > > vertices;

    std::vector< SimplexIndex< K > > triangles;

    ...
}
```

*Defined in ThirdParty/Include/Misha/Geometry.h

Assignment 1: Signal Representation

Signals are represented as a vector of Real values, in one-to-one correspondence with vertices.

Include/Mesh.h

```
struct Mesh
{
    …

    std::vector< Real > values;

    …
}
```

Assignment 1: Invocation

1. To smooth the signal, invoke:

   `OneRingSmoothing --in <input file>*`

   If the mesh has colors/values, those will be smoothed.
   Otherwise, you will need to provide heat sources/sinks.

2. To smooth the geometry, invoke:

   `OneRingSmoothing --in <input file>* --geometry`

* The input file can be in either `.ply` or `.obj` format.

Assignment 1: Interaction

You can visualization the animation:

- [space]: pause/continue
- ‘+’: advance one time-step

You can move around the animation:

- [left mouse]: rotate
- [right mouse]: zoom
- [left mouse]+[ctrl]: pan

Assignment 1: Interaction

Selection

‘s’: enter/exit selection mode

In selection mode:

[left mouse] (hold): add source

[right mouse] (hold): add sink

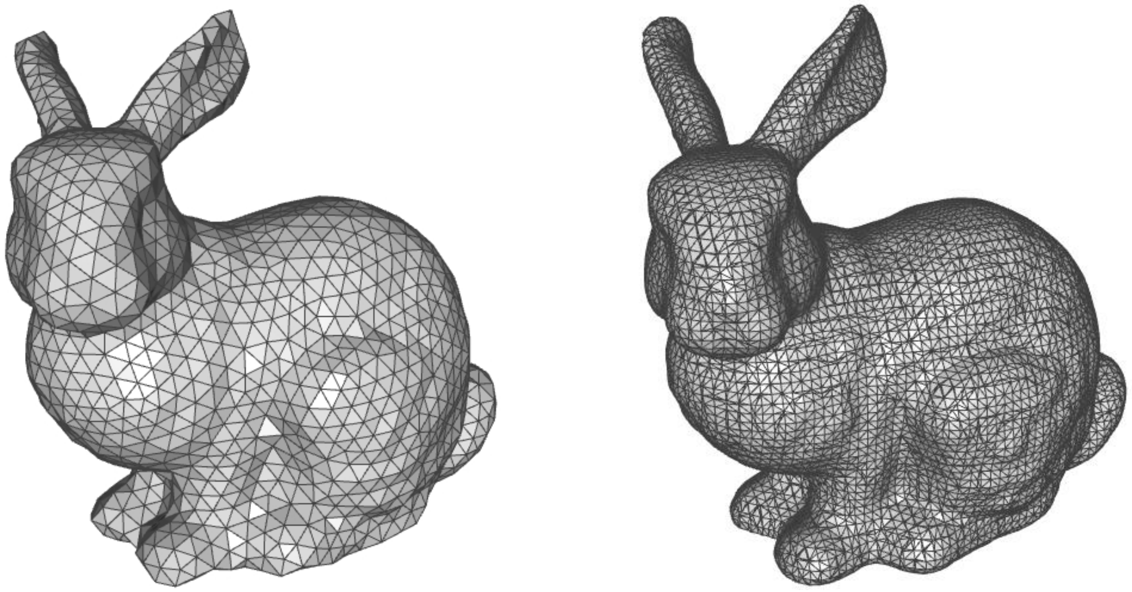
Adding Smoothing Sources/Sinks Visualization

Assignment 1: Thoughts

Q: Why do the two bunnies smooth differently?

Q: How do the results depend on the tessellation?

Q: How do we get geometry into the picture?



Miscellany

Contacts:

Professor:

Misha Kazhdan

misha@cs.jhu.edu

TA:

Hongyi Liu

liuhongyi@jhu.edu

Piazza:

https://piazza.com/jhu/spring2026/en601458658

I will not respond to queries on Piazza unless I'm notified about them. (Don't hesitate to notify me if you want me to respond.)

Resources:

Consider downloading https://www.meshlab.net/ to visualize, convert, process triangle meshes