Shape Matching

Michael Kazhdan

(601.457/657)

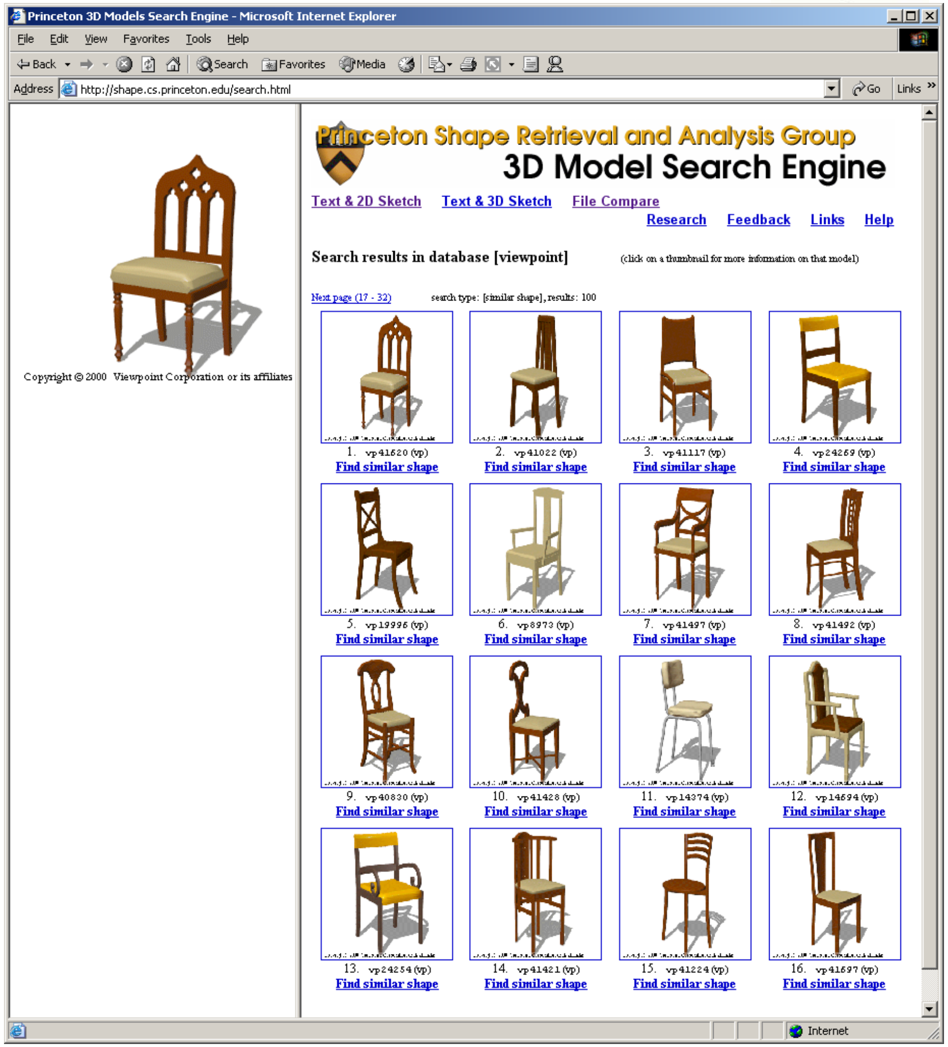
Overview

- Intro
- General Approach
- Minimum SSD Descriptor

Goal

Given:
- 3D model database
- query shape

Find:
- The database models most similar to the query.

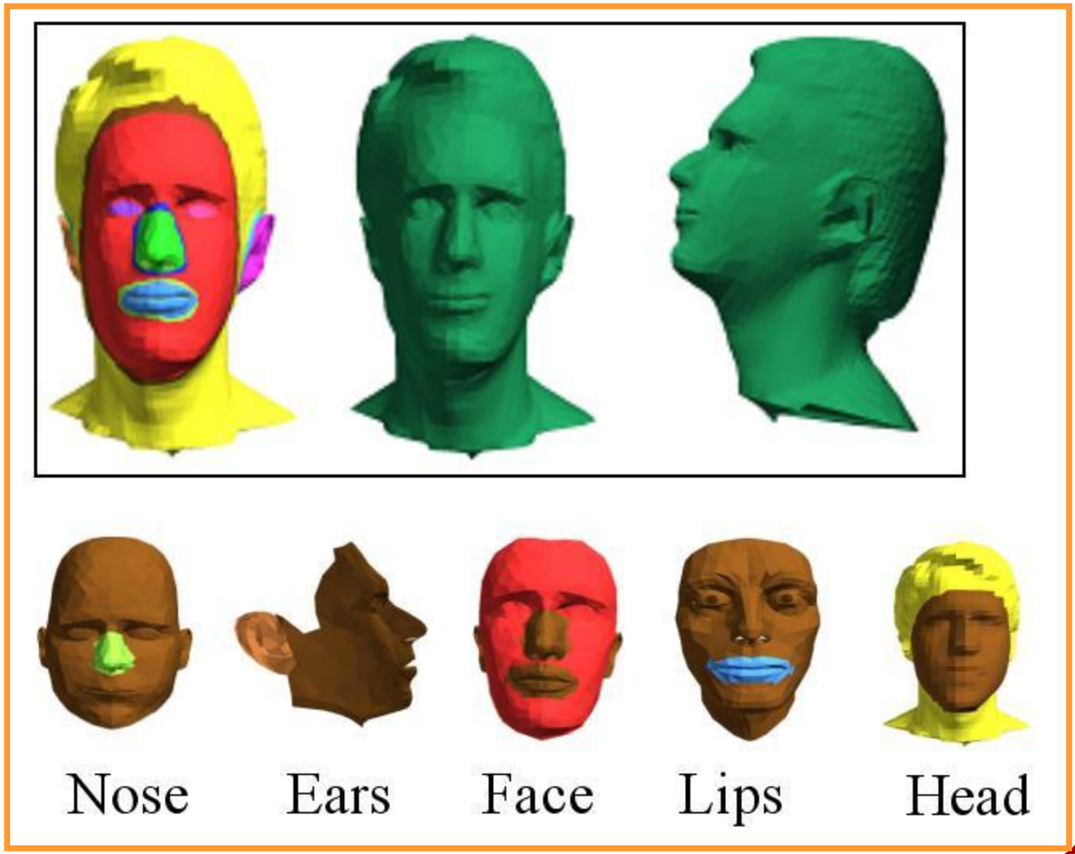


Applications

- Entertainment
- Medicine
- Chemistry/Biology
- Archaeology

Applications

- Entertainment
  - Model generation
- Medicine
- Chemistry/Biology
- Archaeology



Applications

- Entertainment
- Medicine
  - Automated diagnosis
- Chemistry/Biology
- Archaeology

Images courtesy of NLM

Applications

- Entertainment
- Medicine
- Chemistry/Biology
  - Docking and binding
- Archaeology

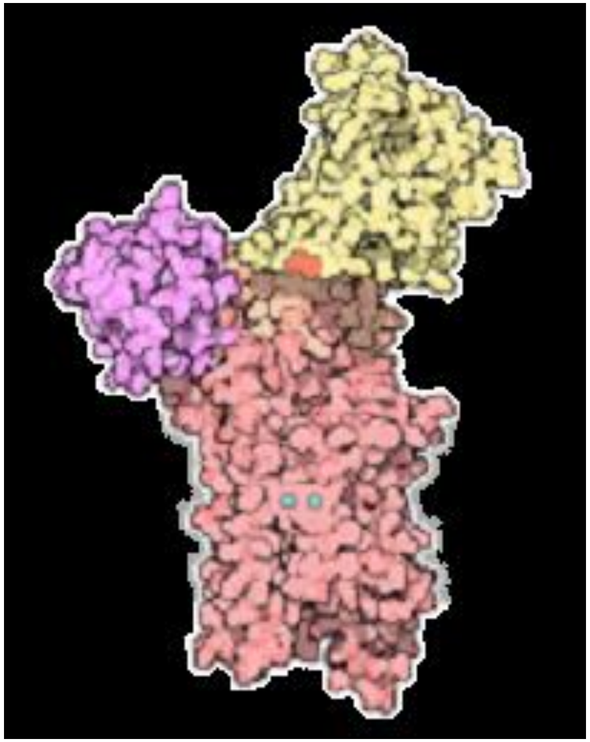


Image Courtesy of PDB

Applications

- Entertainment
- Medicine
- Chemistry/Biology
- Archaeology
  - Reconstruction

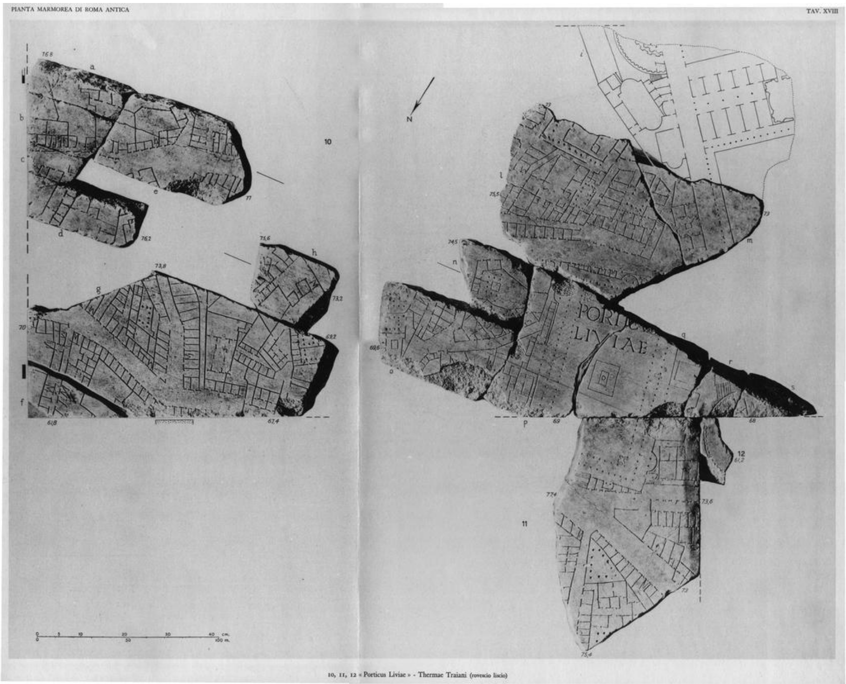


Image Courtesy of Stanford

Overview

- Motivation
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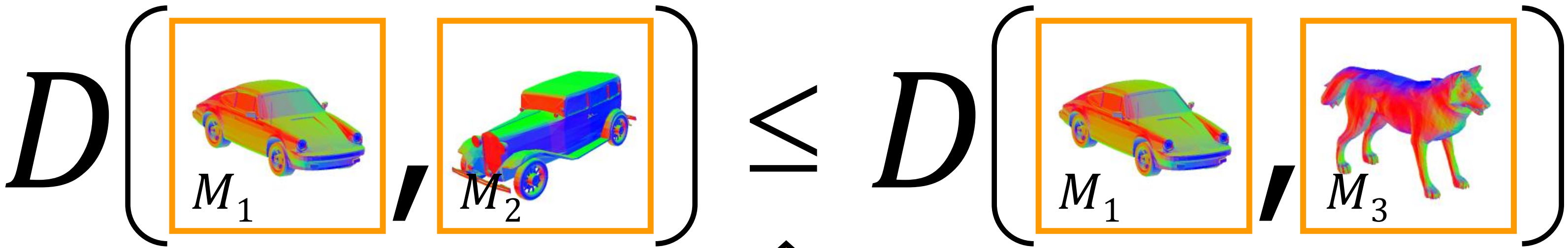
- Minimum SSD Descriptor

Shape Matching

General approach:

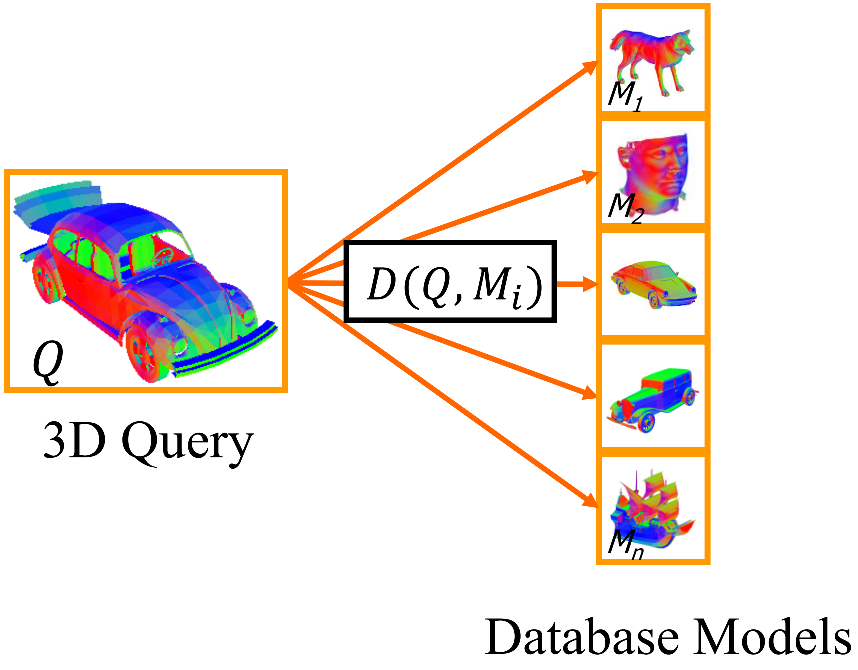
Define a function that takes in two models and returns a measure of their proximity.



M_1 is closer to M_2 than it is to M_3

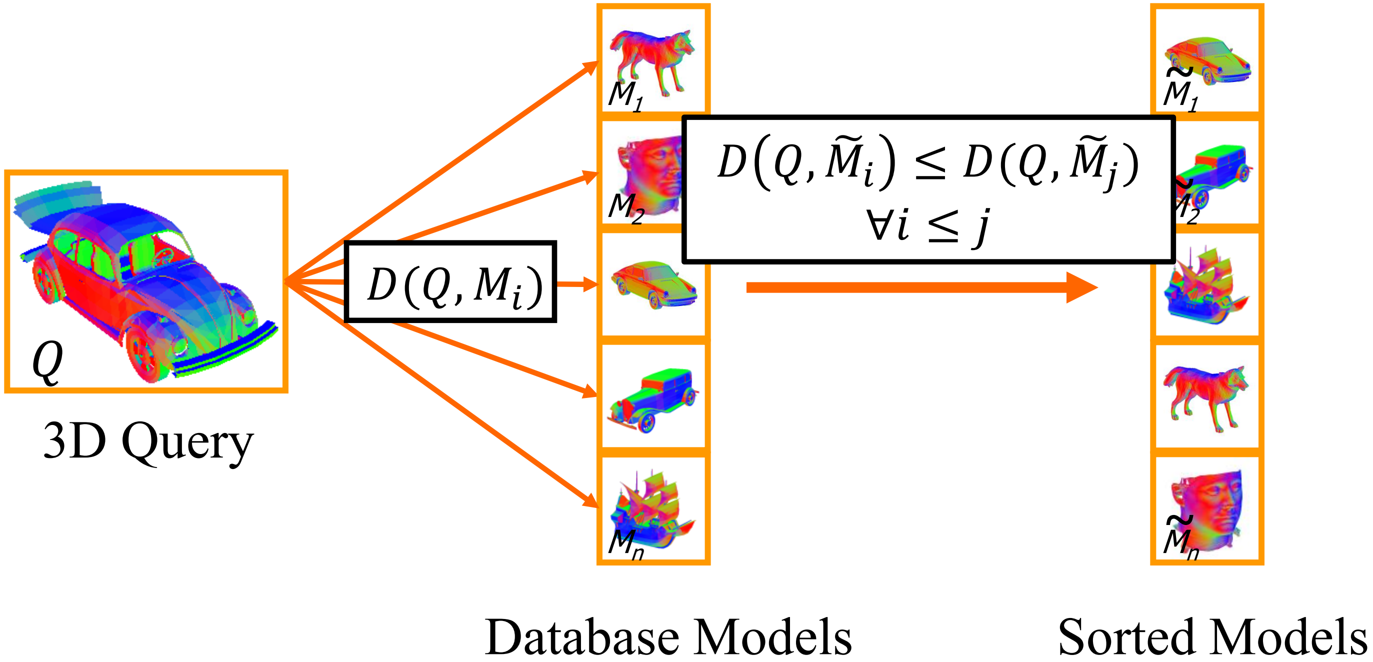
Database Retrieval

- Compute the distance from the query to each database model



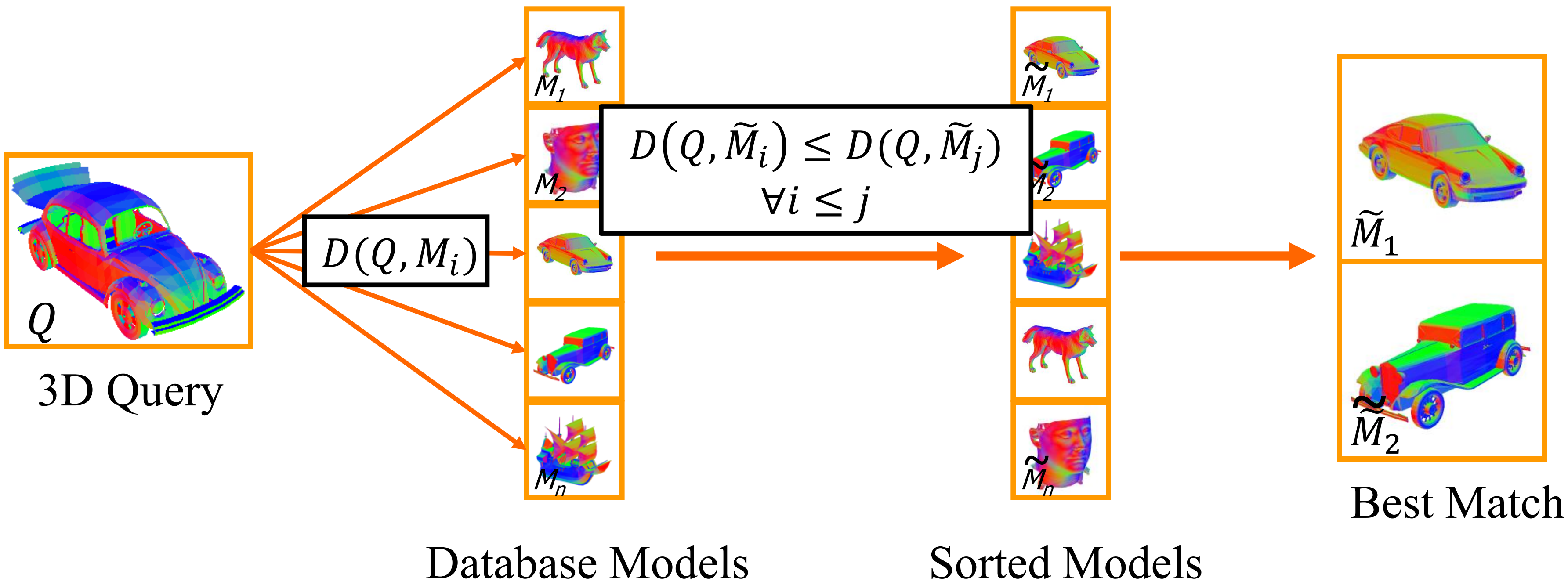
Database Retrieval

- Sort the database models by proximity



Database Retrieval

- Return the closest matches



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- General Approach
  - Shape Descriptors
- Minimum SSD Descriptor

Shape Matching

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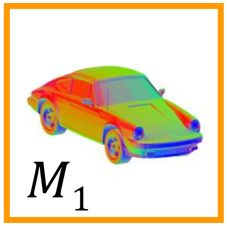
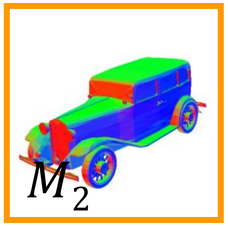
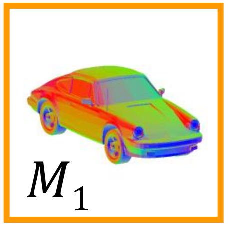
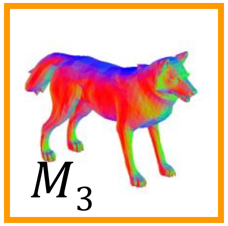
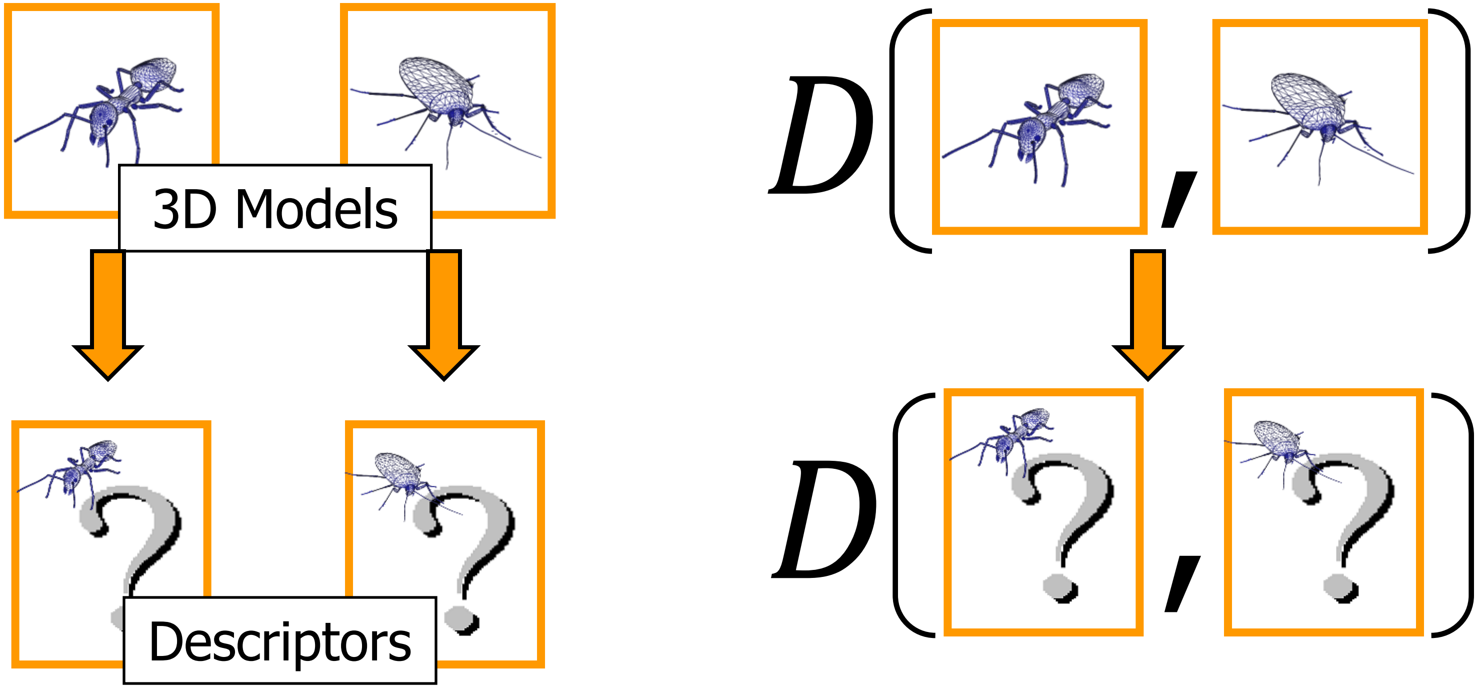
$D($  M_1 ,  M_2 $) \leq D($  M_1 ,  M_3 $)$

M_1 is closer to M_2 than it is to M_3

Shape Descriptors

<u>Shape Descriptor</u>:

A structured abstraction of a 3D model that is well suited to the challenges of shape matching

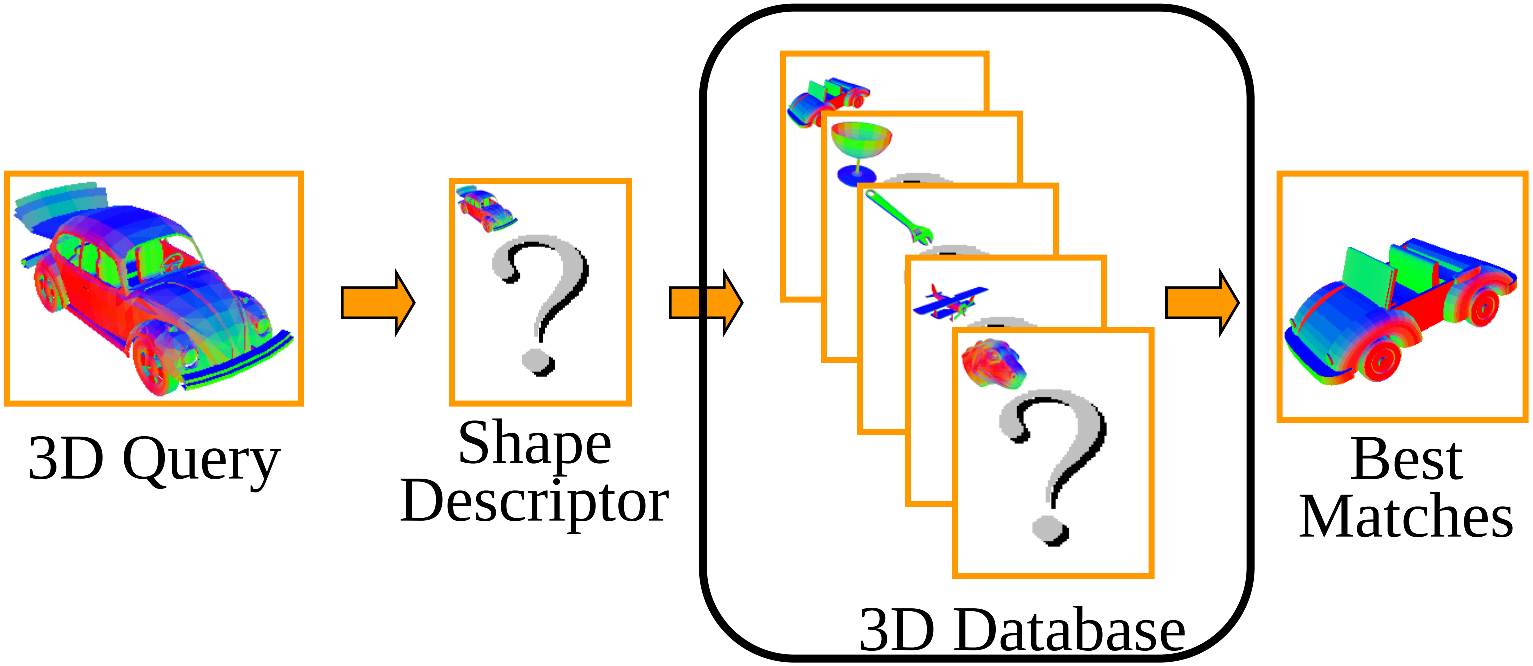


Matching with Descriptors

Preprocessing

- Compute database descriptors

Run-Time



Matching with Descriptors

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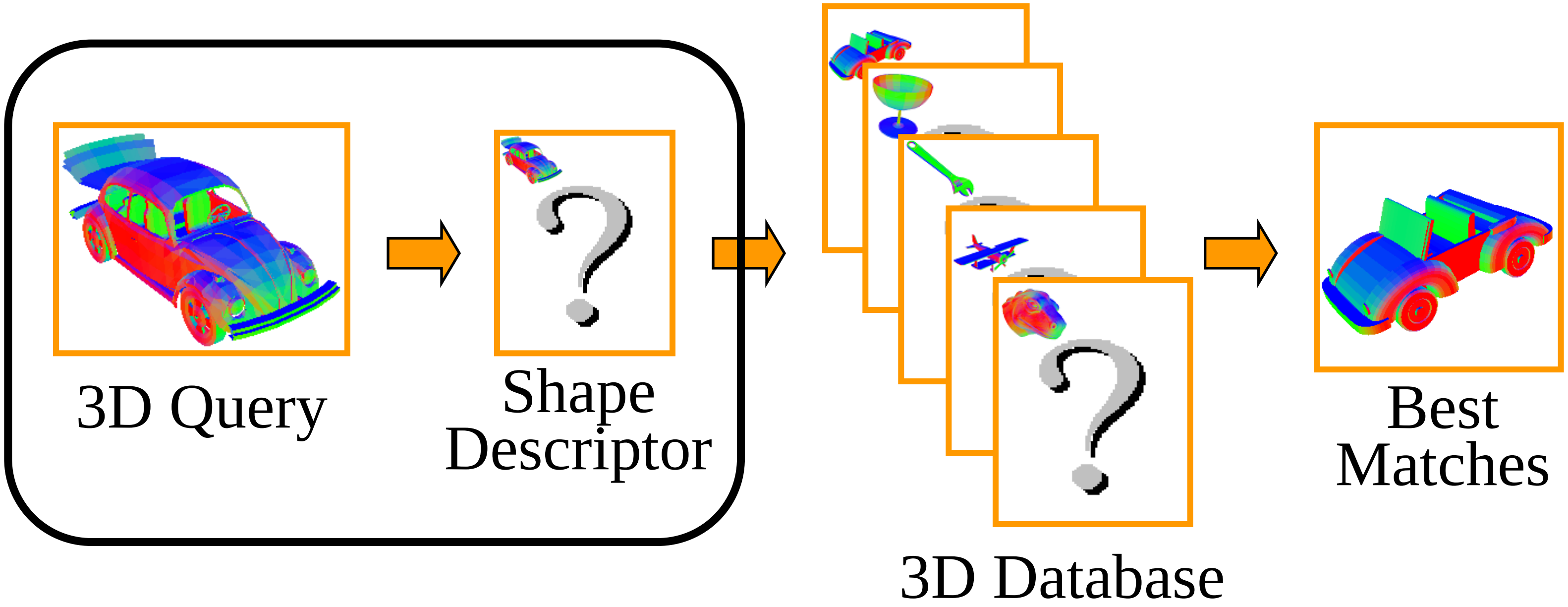
Run-Time

- Compute query descriptor



Matching with Descriptors

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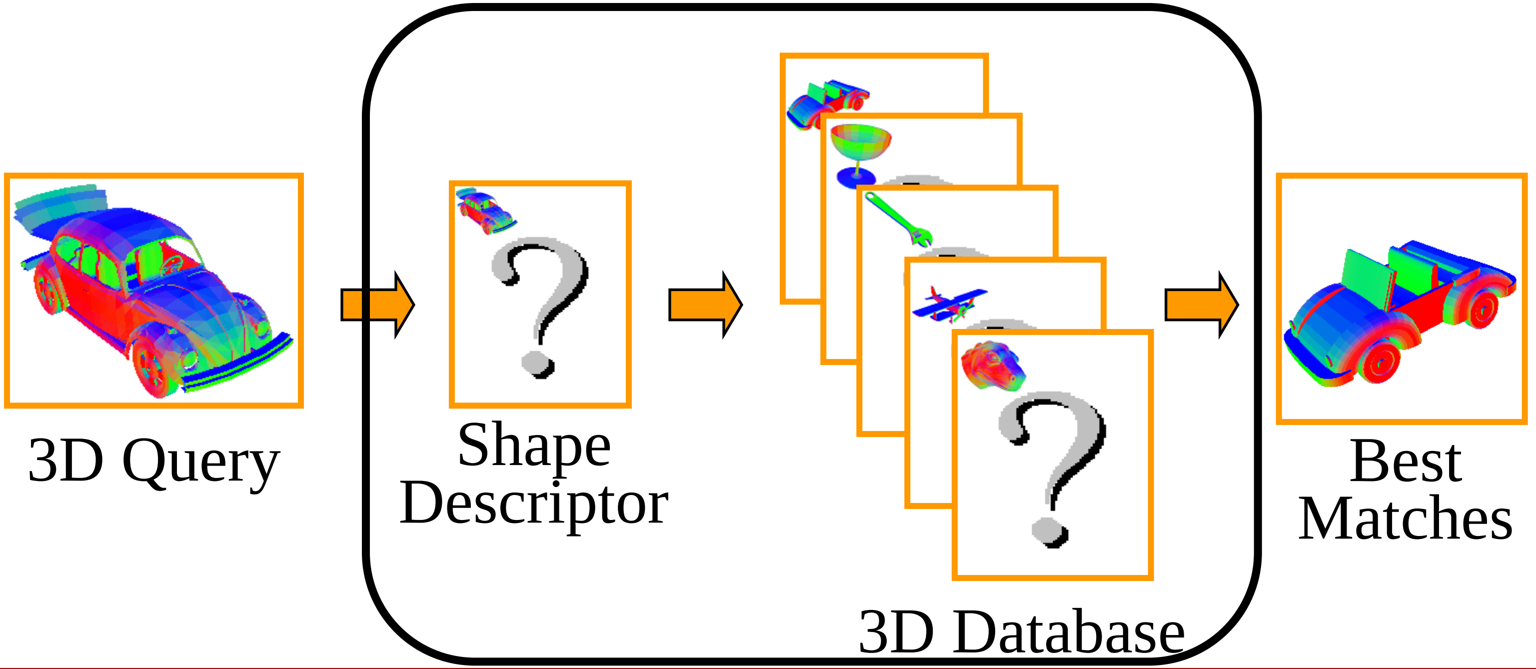
Run-Time

- Compute query descriptor
- Compare query descriptor to database descriptors



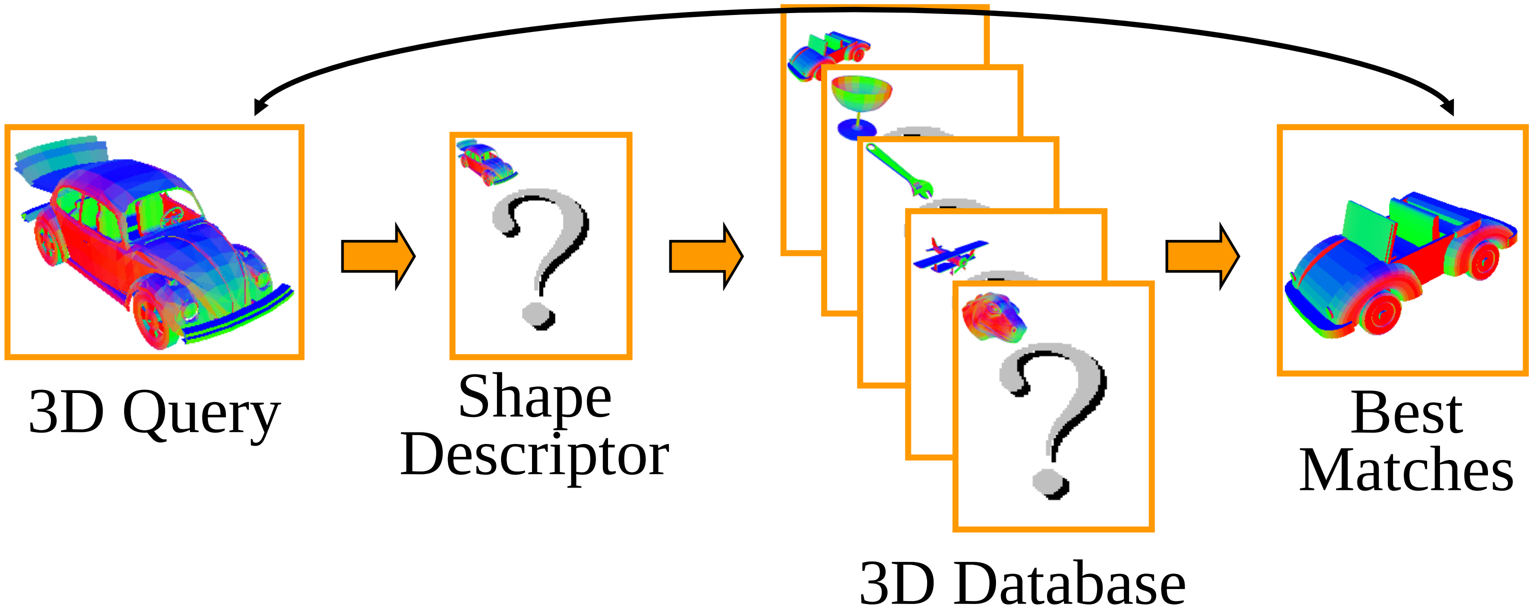
Matching with Descriptors

Preprocessing

- Compute database descriptors

Run-Time

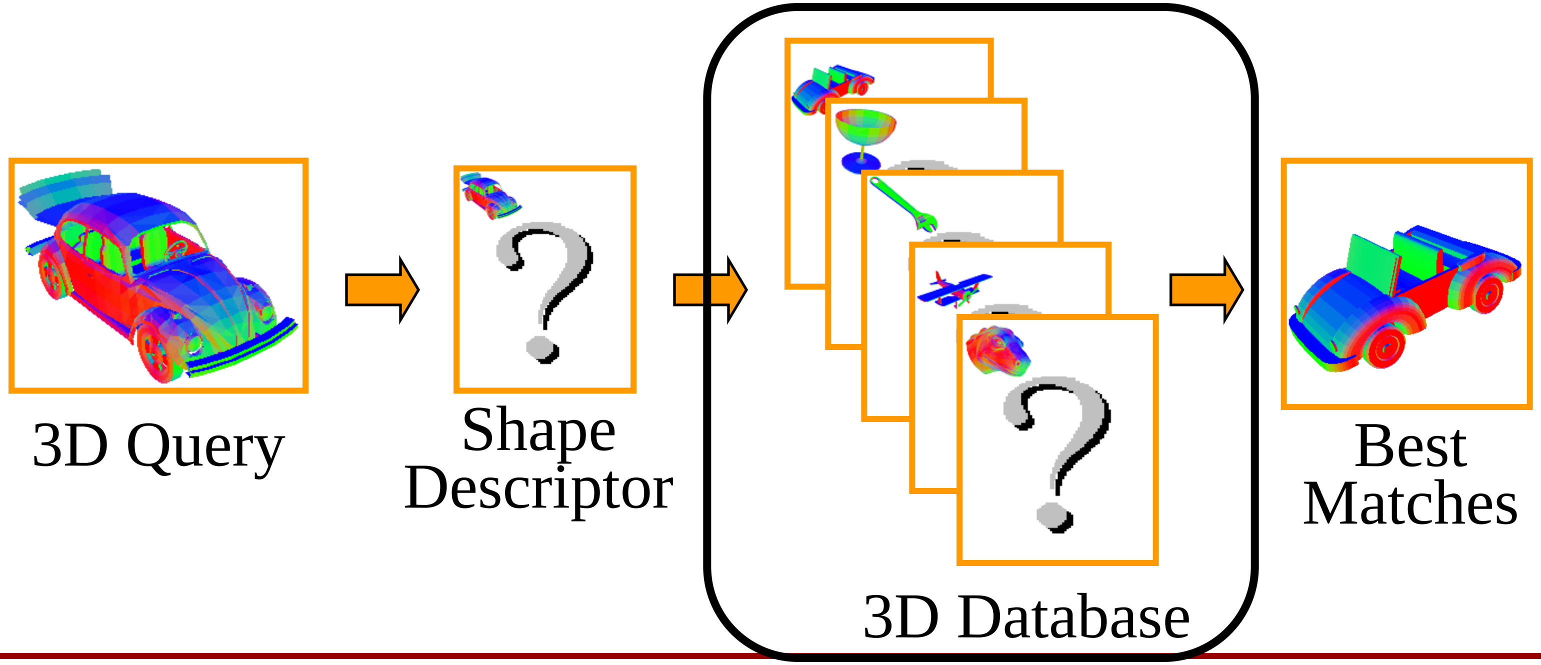
- Compute query descriptor
- Compare query descriptor to database descriptors
- Return best Match(es)



Shape Matching Challenge

Need shape descriptor that is:

- Concise to store
- Quick to compute
- Efficient to match
- Discriminating



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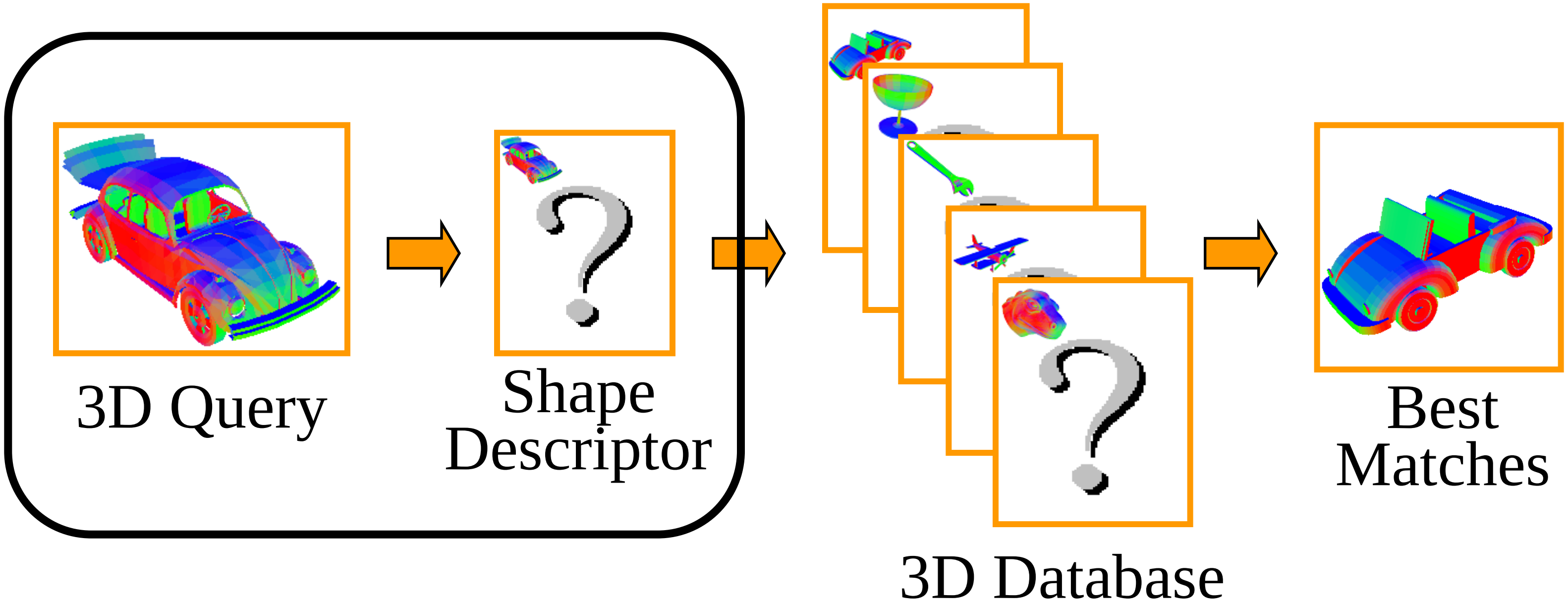

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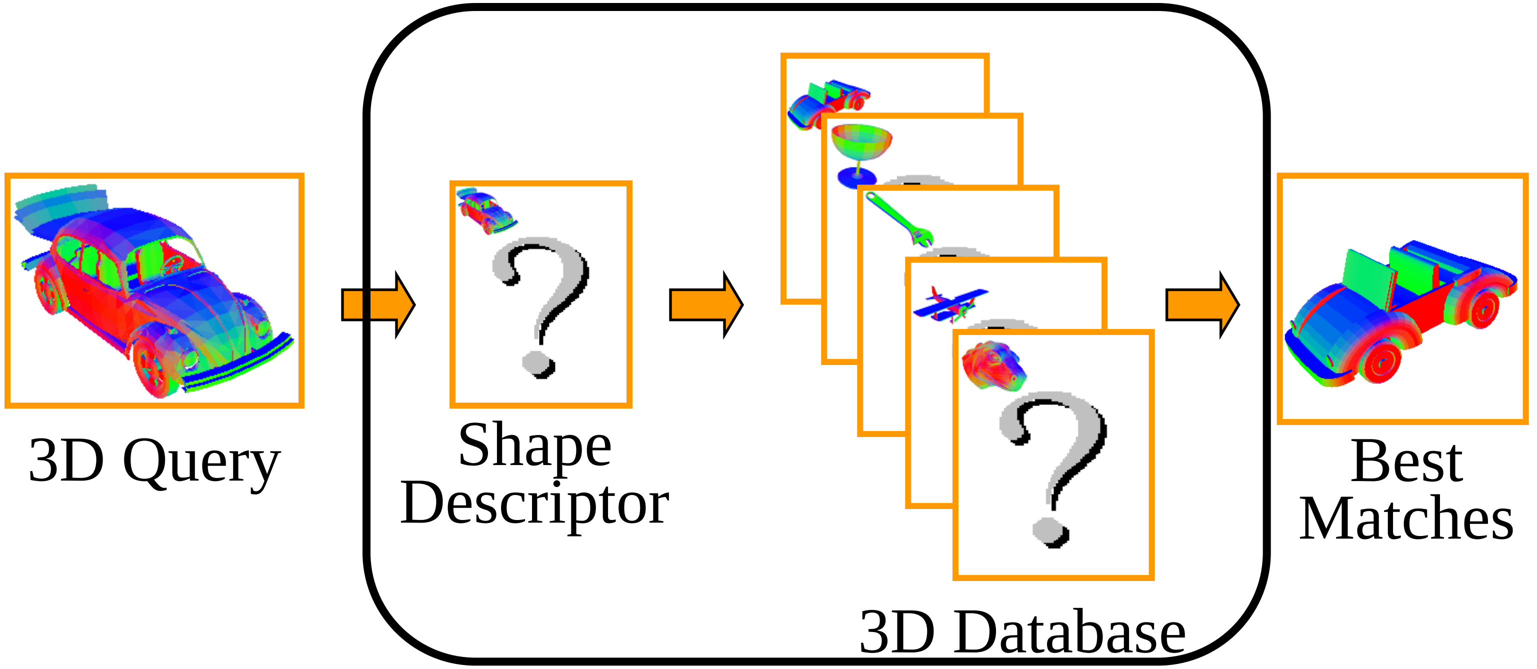

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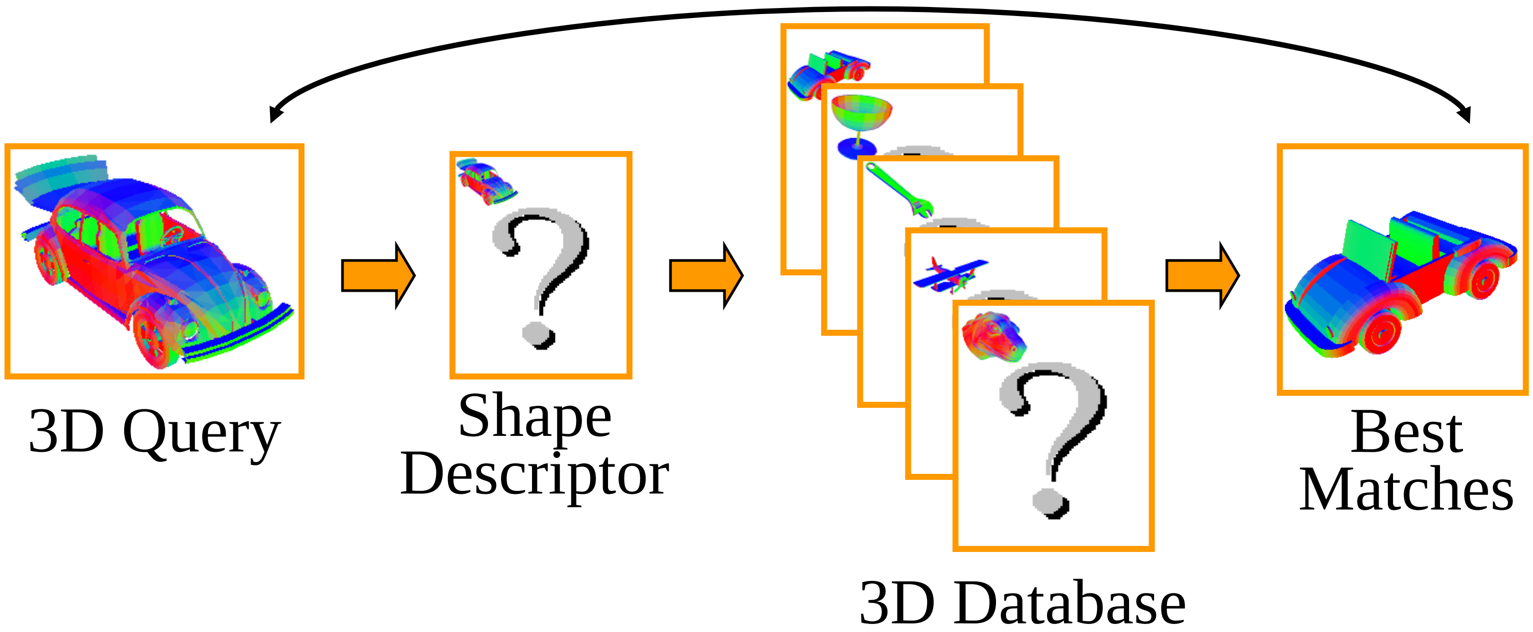

Shape Matching Challenge

Need shape descriptor that is:

- Concise to store
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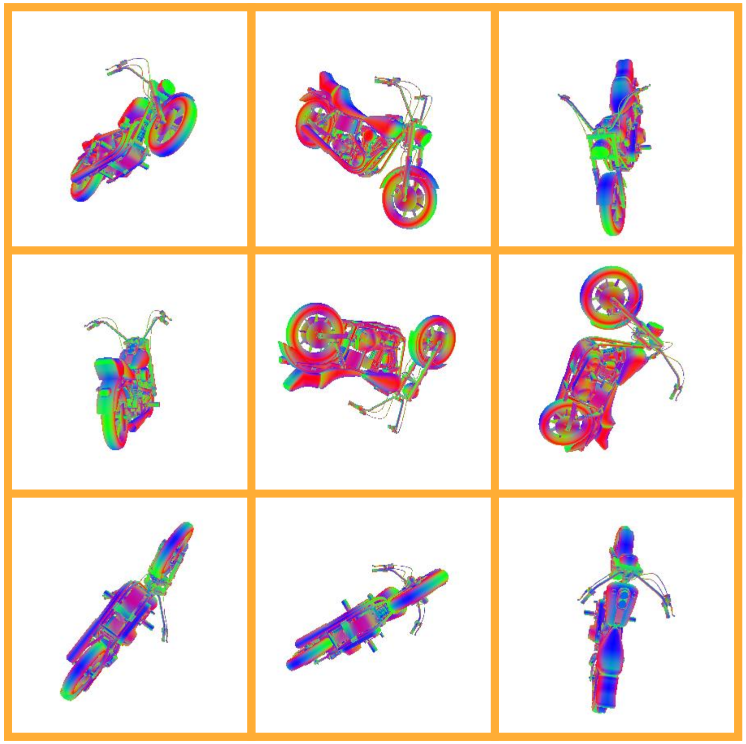
- ➢ Invariant to transformations
- Invariant to deformations
- Insensitive to noise
- Insensitive to topology



Different Transformations
(translation, scale, rotation, mirror)

Shape Matching Challenge

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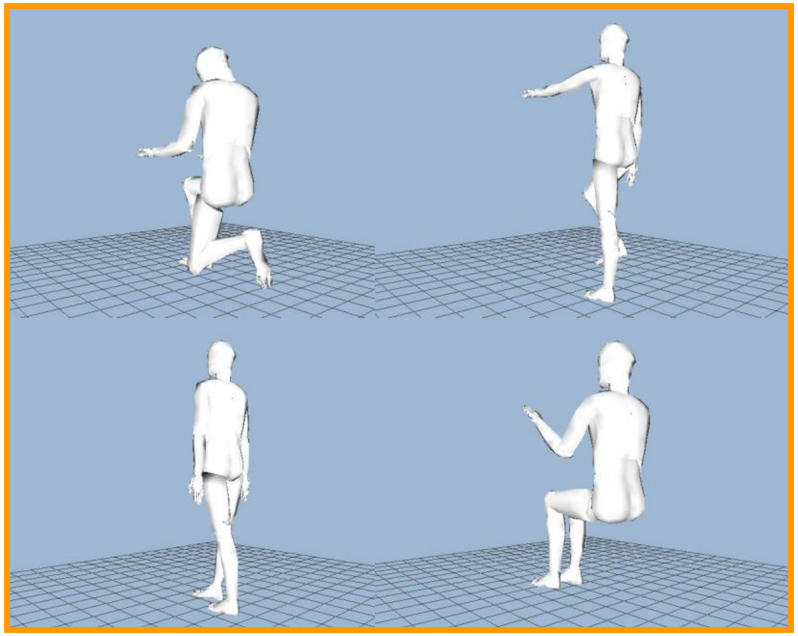

Different Articulated Poses

Shape Matching Challenge

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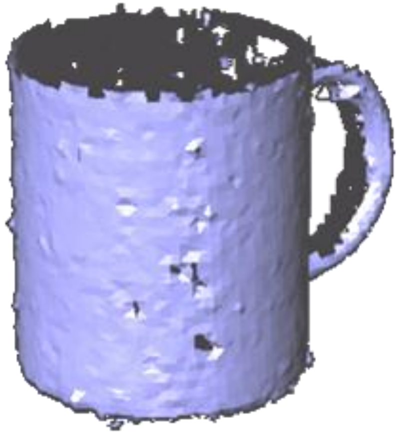

Scanned Surface

Image courtesy of Ramamoorthi et al.

Shape Matching Challenge

Need shape descriptor that is:

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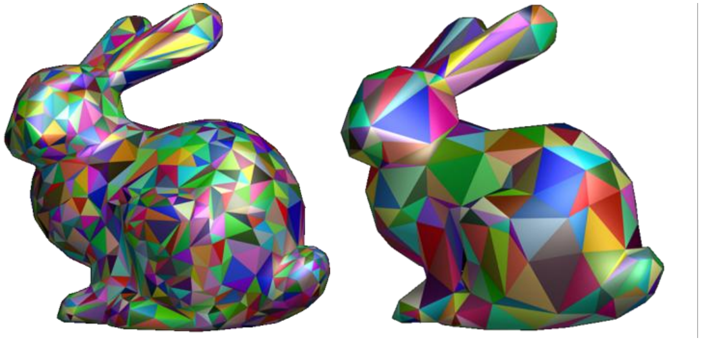
Different Genus



Different Tessellations

Images courtesy of
Viewpoint & Stanford

Overview

- Applications
- General Approach
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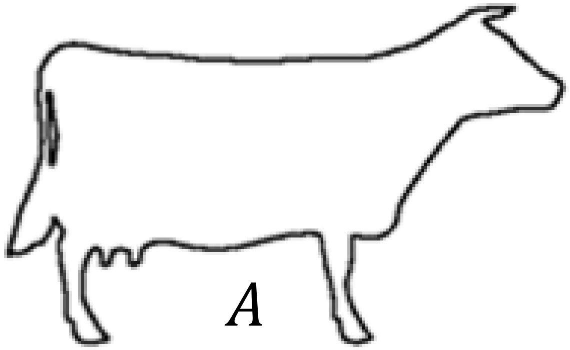
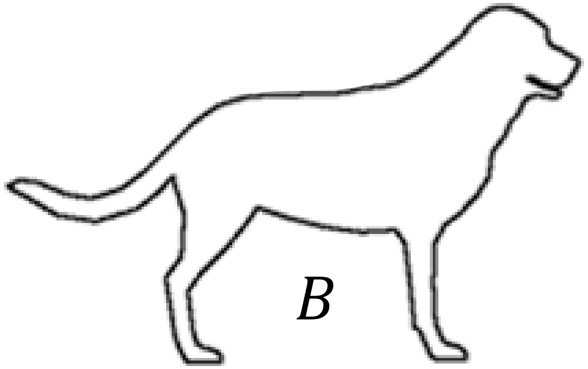
Shape Matching Approach

Q: How should we measure the similarity between two shapes?



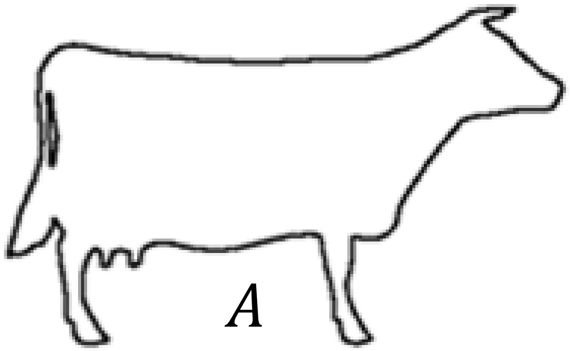
A



B

Shape Matching Approach

A: Define shape (dis)similarity as the sum of squared distances from points on one surface to the closest points on the other.



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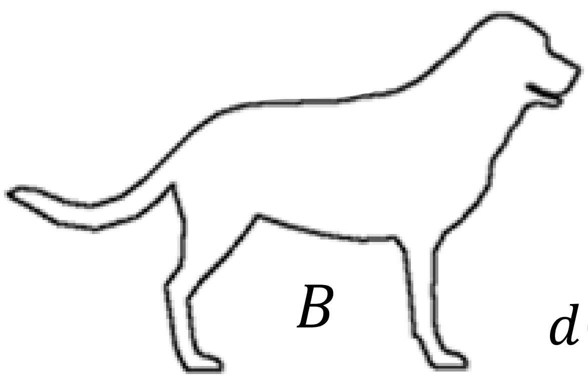

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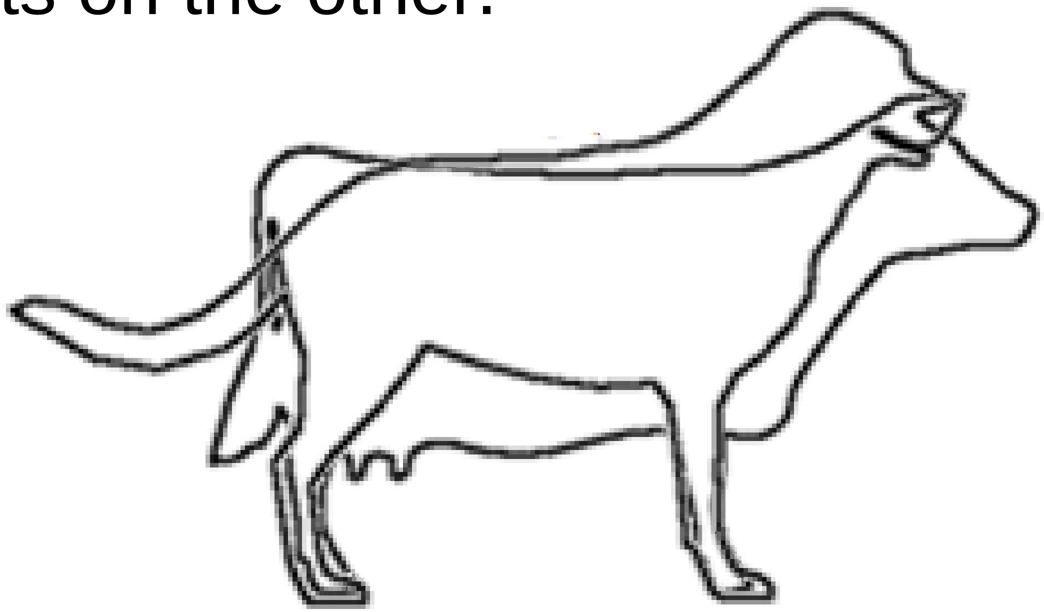
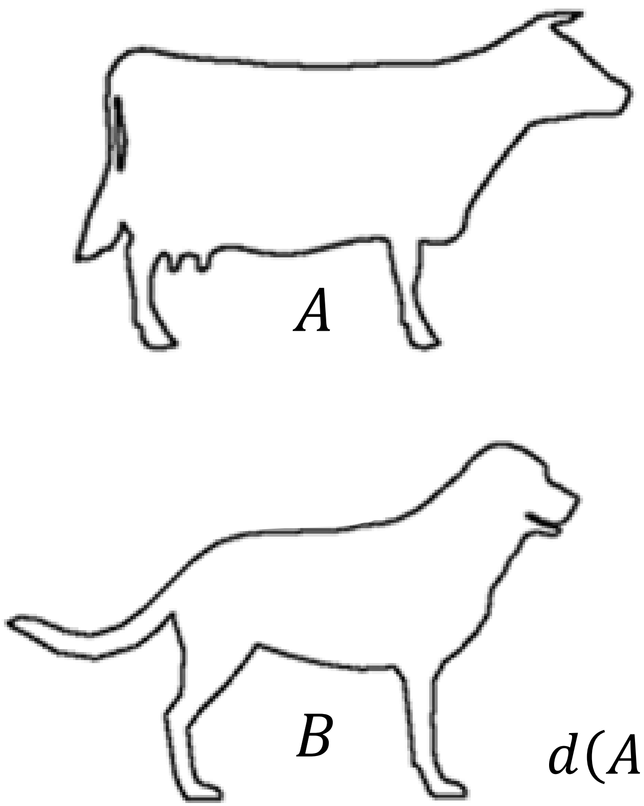
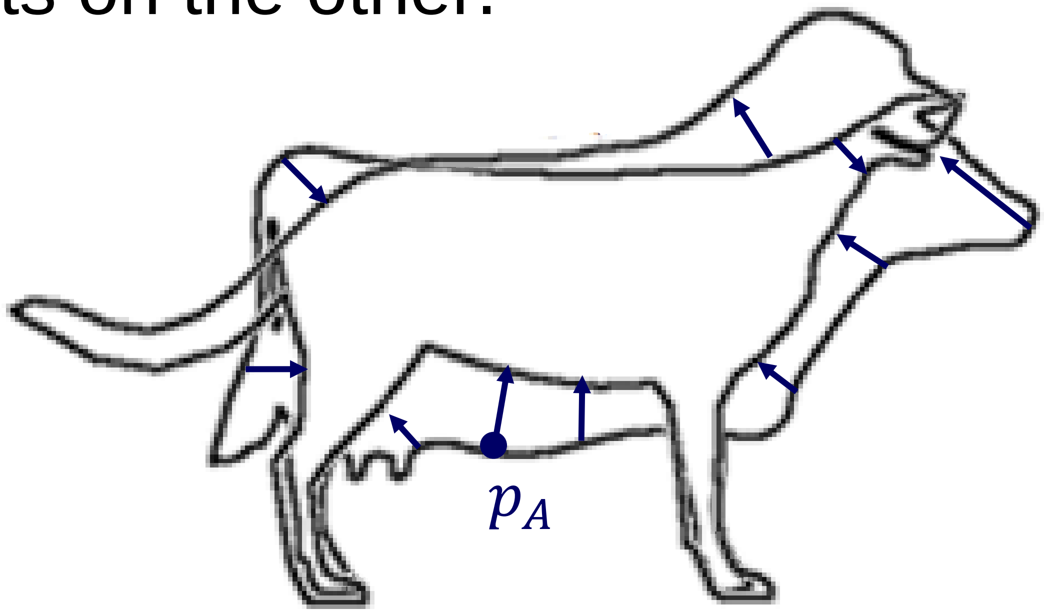


$$d(A,B) = \int_A dist(p_A, B)^2 dp_A + \int_B dist(p_B, A)^2 dp_B$$

Shape Matching Approach

A: Define shape (dis)similarity as the sum of squared distances from points on one surface to the closest points on the other.





$$d(A,B)=\boxed{\int_A dist(p_A,B)^2\,dp_A}+\int_B dist(p_B,A)^2\,dp_B$$

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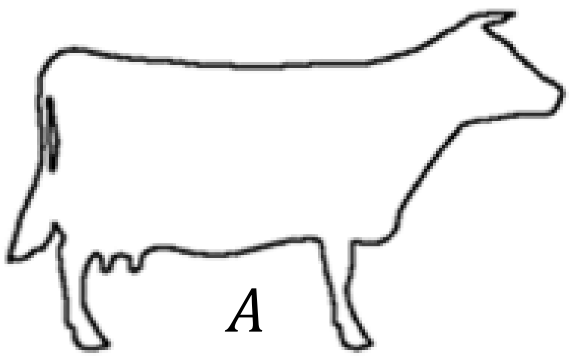

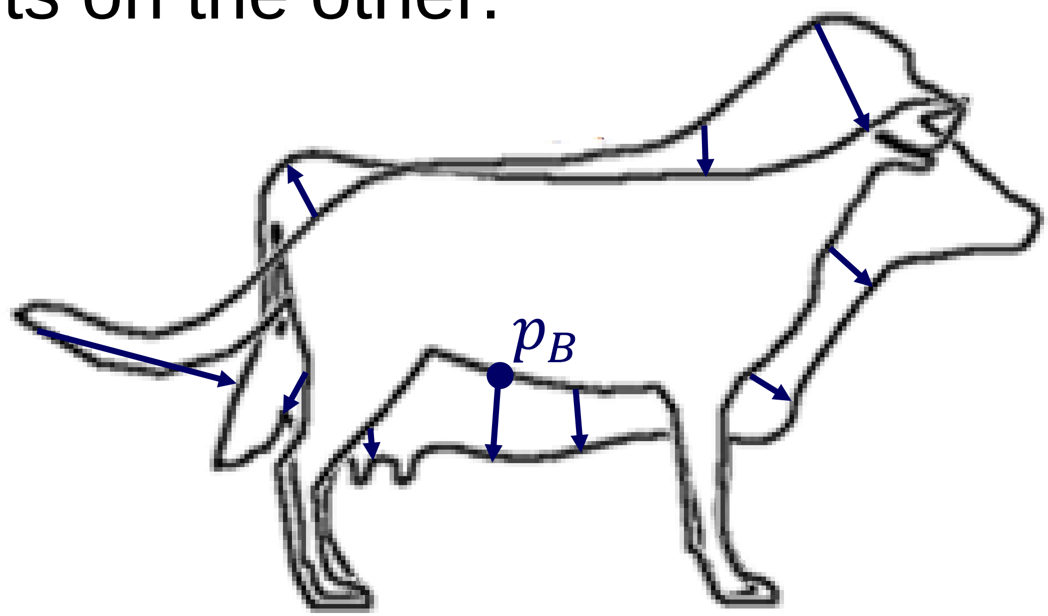

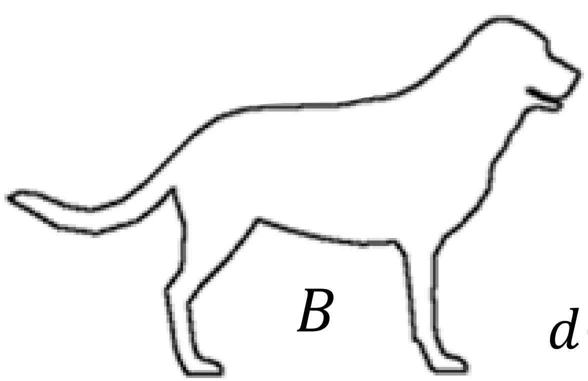

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Overview

- Applications
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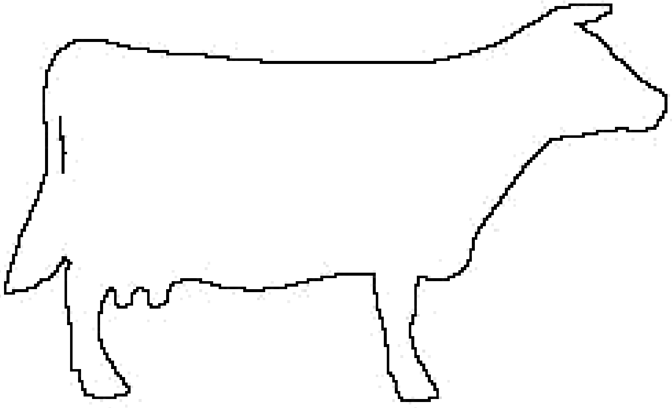
  - (Euclidean) Distance Transform

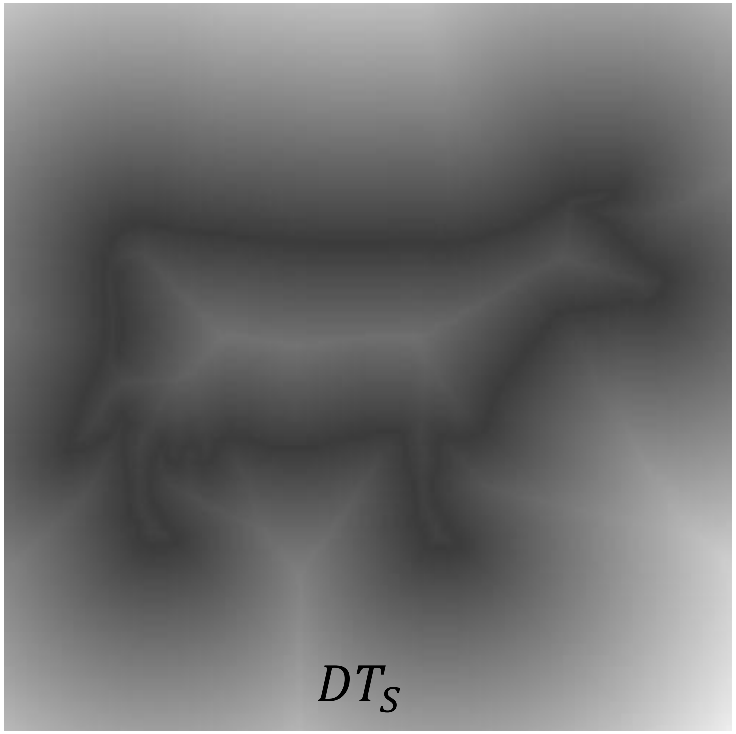
(Euclidean) Distance Transform

The (Euclidean) Distance Transform (DT) of a surface is a function (defined in 3D) returning the distance to the nearest surface point.

$$\boxed{DT_S(p) = \min_{q \in S} \|p - q\|}$$



S

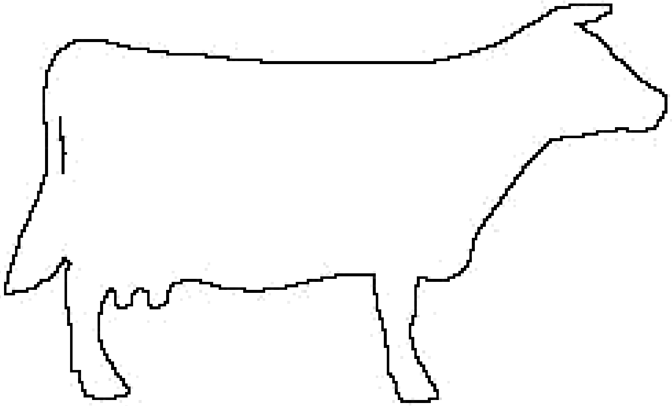


DT_S

(Euclidean) Distance Transform

Grass-Fire Algorithm:

- Think of space as a field of dry grass.
- Set fire to the boundary and measure the amount of time for the fire to reach each point.



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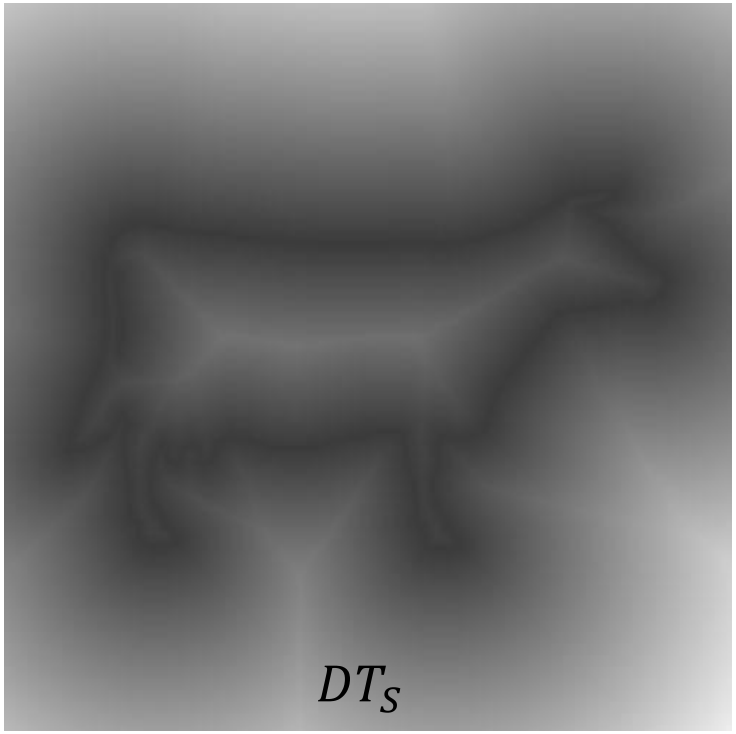
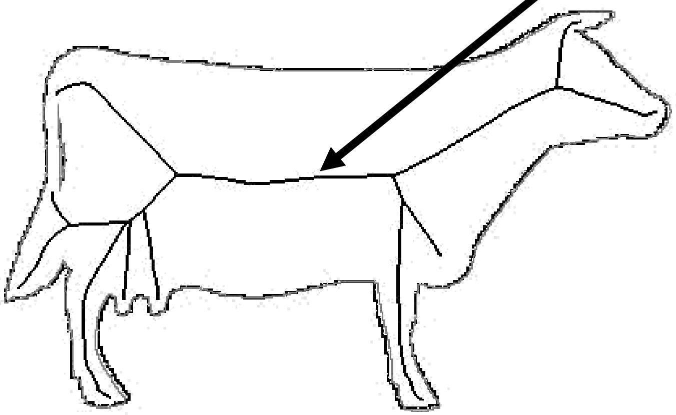


(Euclidean) Distance Transform

Grass-Fire Algorithm:

- Think of space as a field of dry grass.
- Set fi ure the
  amou
  fire to

The points where the fire gets quenched define the <u>skeleton</u> of the shape.



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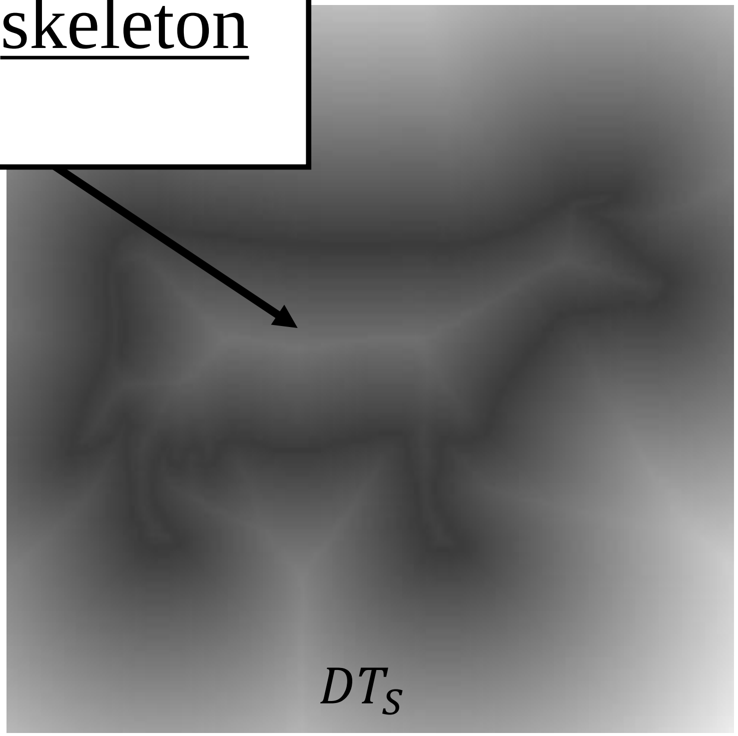


DT_S

Computing DT_S

Brute Force:

Compute the distance to each surface point and store the minimum.

If there are m surface points and we want the values on a grid of resolution R, the overall complexity becomes:

- $O(R^2 m) \approx O(R^2 \cdot R)$ for a 2D grid
- $O(R^3 m) \approx O(R^3 \cdot R^2)$ for a 3D grid

Computing DT_S

Graphics Hardware (2D):

1. For each surface point (x, y), draw a 3D right-cone with apex at $(x, y, 0)$ and axis aligned with the positive z-axis.

2. Render with orthographic projection, looking down the positive the z-axis.

3. Read the values of the depth-buffer to get the values of DT_S.

Computing DT_S

General Problem:

Given a set of points, $P = \{p_1, \ldots, p_n\} \subset \mathbb{R}^2$ and given a point $p \in \mathbb{R}^2$ we would like to compute the distance to the closest point in P:

$$d(p, P) = \min_i \|p - p_i\|$$

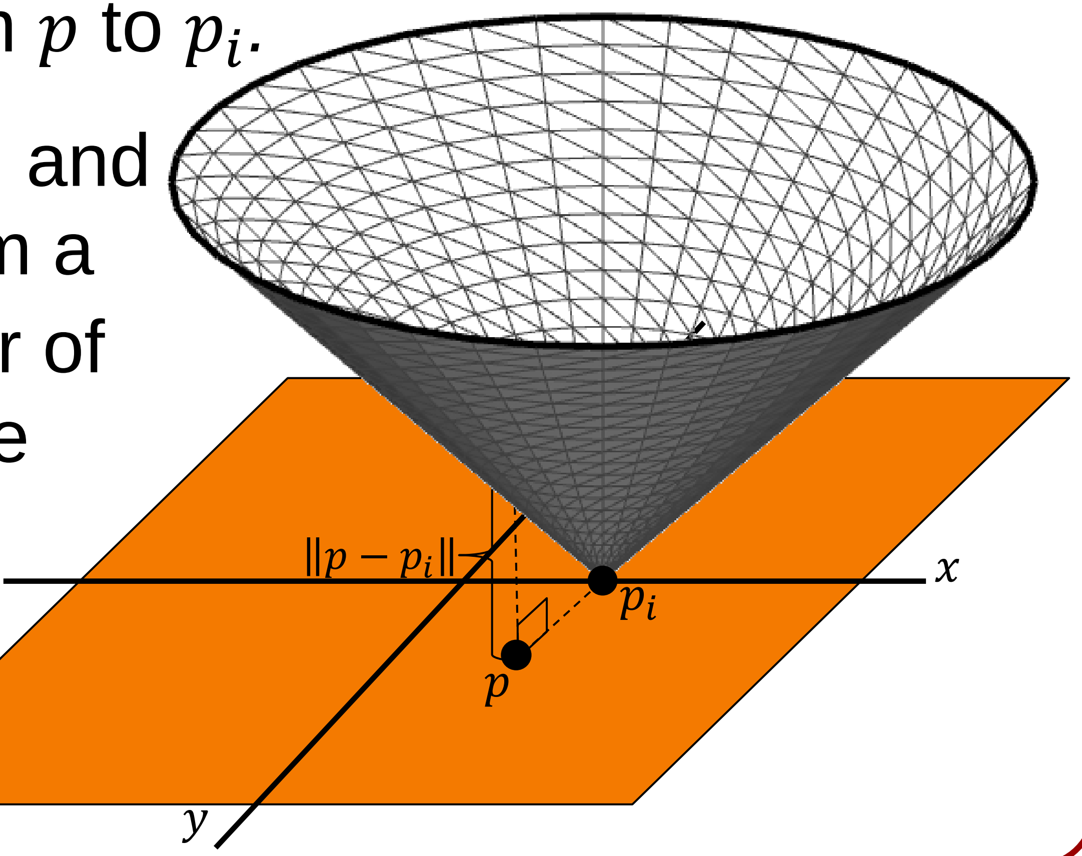
Start by considering how we can compute the distance from the point p to a single point p_i.

Computing DT_S

Graphics Hardware (2D):

At p, the height of a **right**-cone with apex at p_i is the distance from p to p_i.

Given two points p_i and p_j the distance from a point p to the closer of the two points is the minimum of the two heights.



Computing DT_S

Graphics Hardware (2D):

At p, the height of a **right**-cone with apex at p_0 is the distance from p to p_0.

Given a collection of points in the xy-plane:

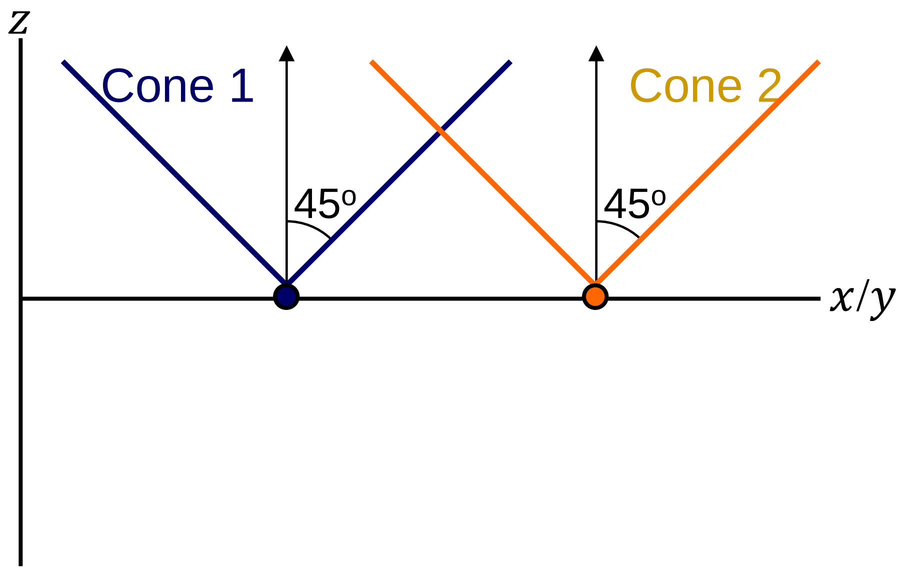
Computing DT_S

<u>Graphics Hardware (2D)</u>:

At p, the height of a **right**-cone with apex at p_0 is the distance from p to p_0.

Given a collection of points in the xy-plane:

- Draw right-cones at each point



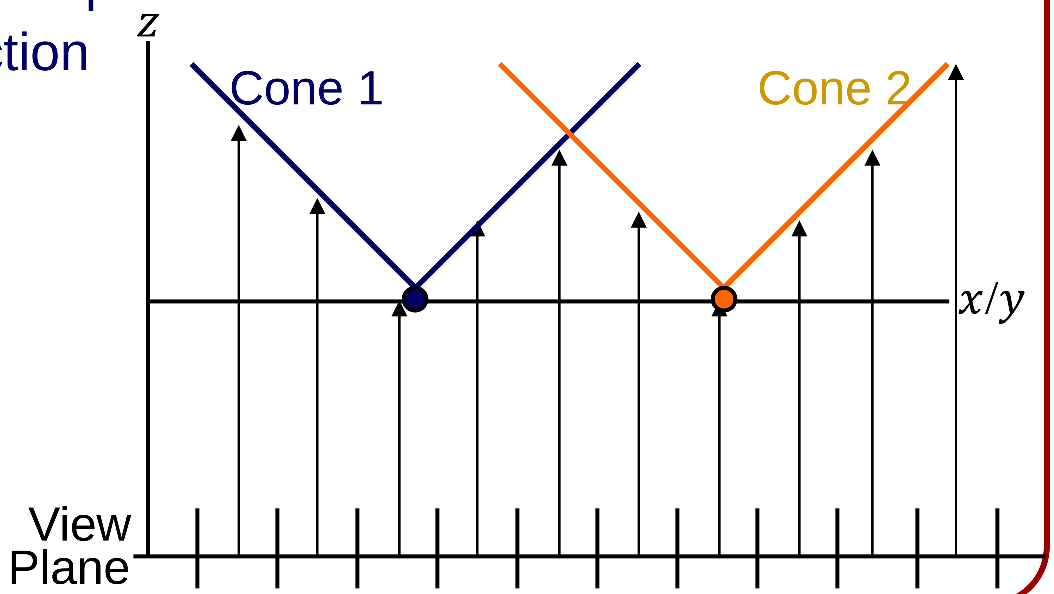
Computing DT_S

<u>Graphics Hardware (2D)</u>:

At p, the height of a **right**-cone with apex at p_0 is the distance from p to p_0.

Given a collection of points in the xy-plane :

- Draw right-cones at each point
- View along the z-direction



Computing DT_S

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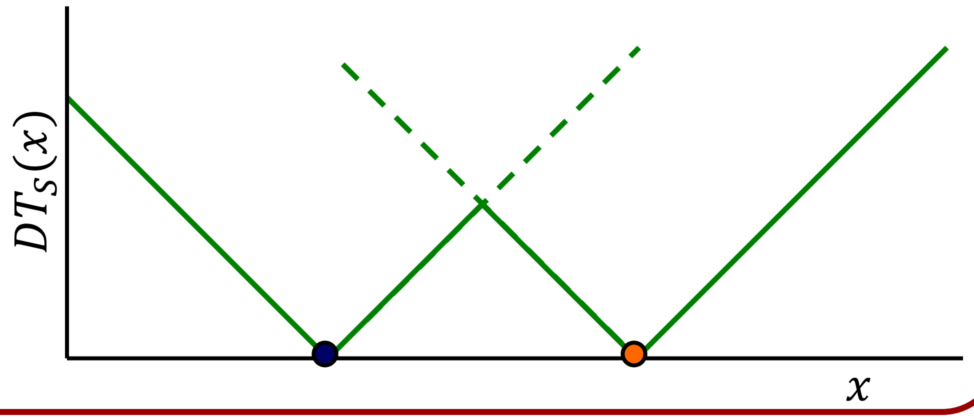

Computing DT_S

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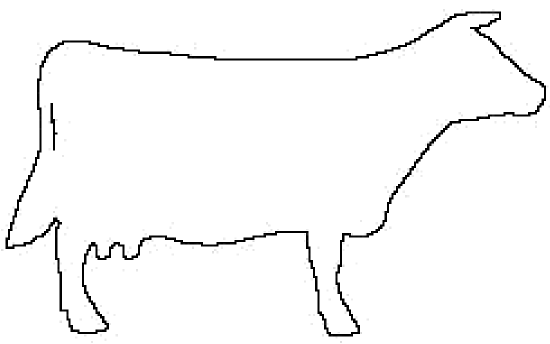
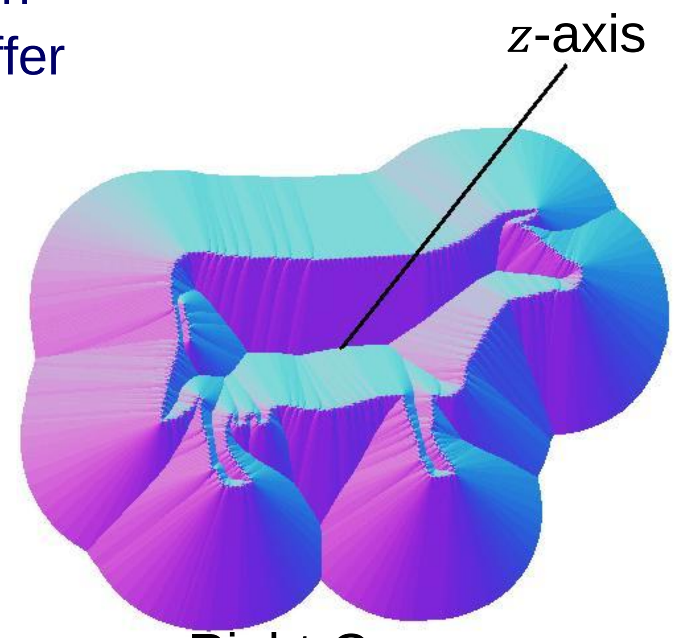


Surface



Right-Cones

$\boxed{\text{Visualization}}$

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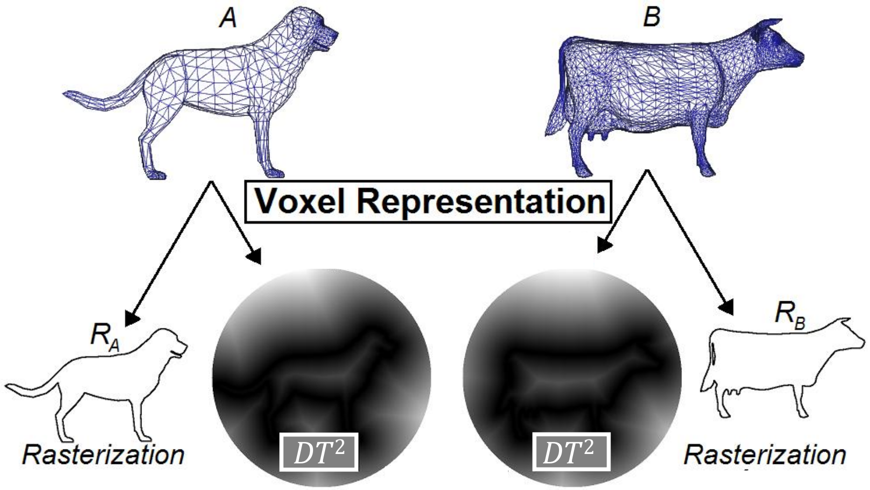
Shape Matching Implementation

Preprocessing:

Compute **rasterization** and **squared distance transforms**



- The value of the rasterization at a 3D point (voxel) is:

$$R_A(p) = \begin{cases} 1 & \text{if} \quad p \in A \\ 0 & \text{otherwise} \end{cases}$$

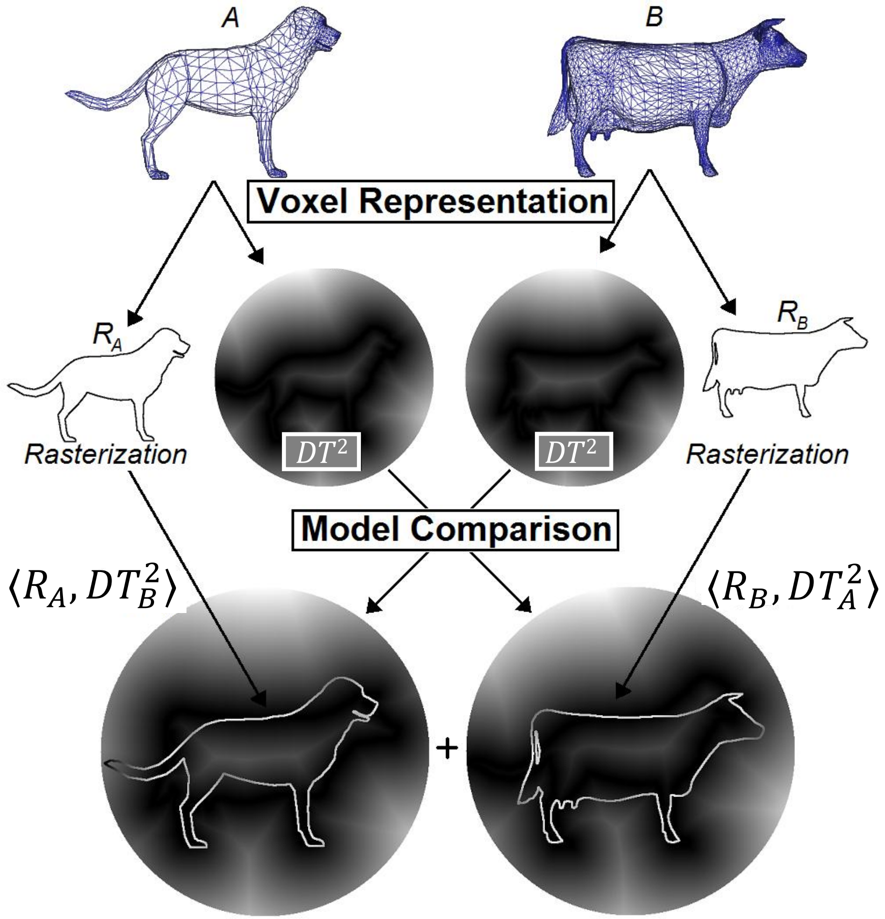
- The value of the distance transform at a 3D point is:

$$DT_A^2(p) = \min_{q \in A} \|p - q\|^2$$

Shape Matching Implementation

Run-Time:

Compute mesh similarity with two dot-products/integrals

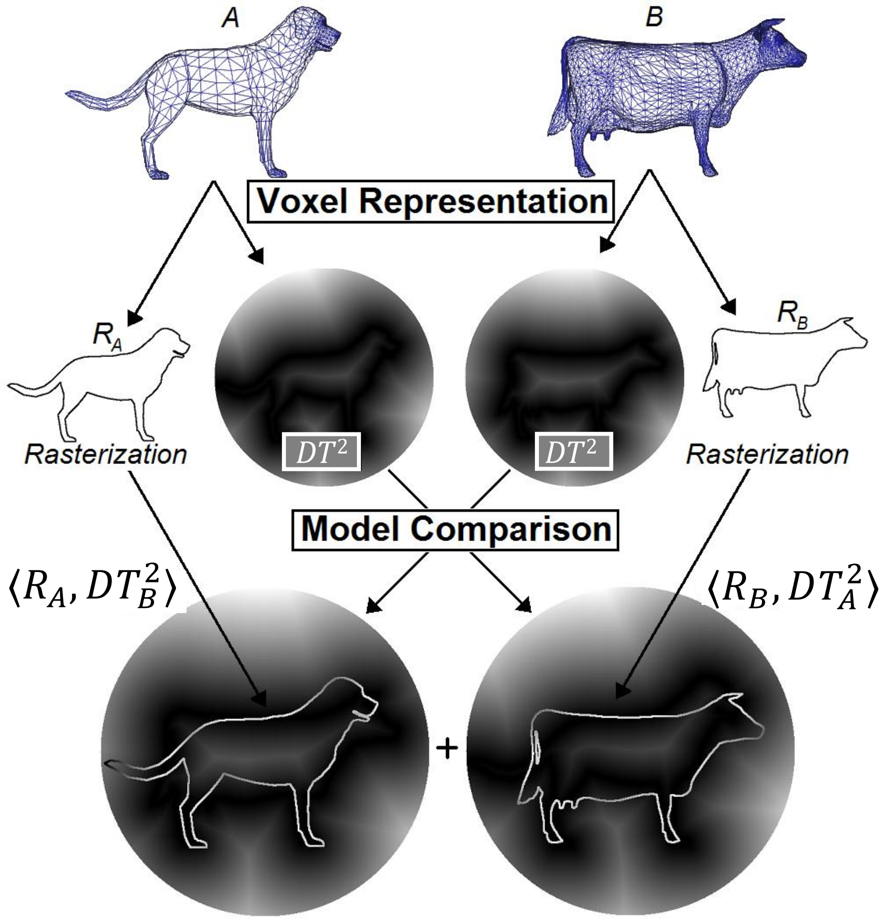


$$d(A,B) = \langle R_A, DT_B^2 \rangle + \langle DT_A^2, R_B \rangle$$

Shape Matching Implementation

Run-Time:

Compute mesh similarity with two dot-products/integrals



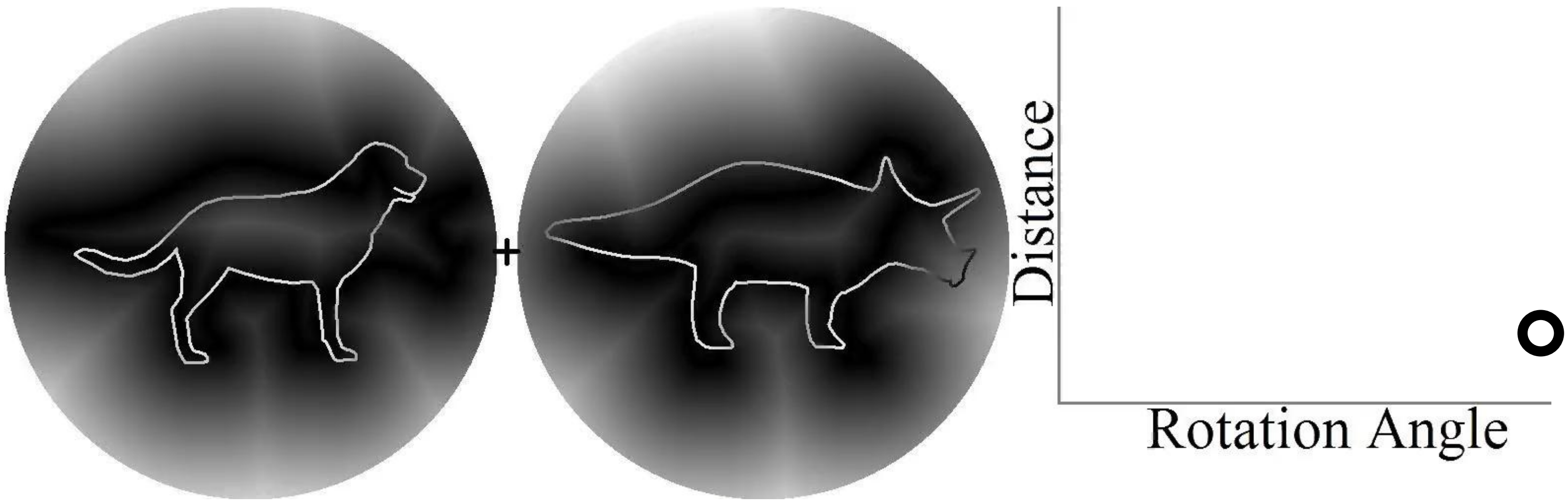
The dot product of R_A with DT_B^2 is the sum of the product of the two functions:

$$\begin{aligned}\langle R_A, DT_B^2 \rangle &\equiv \int_{\mathbb{R}^3} R_A(p) \cdot DT_B^2(p)dp \\ &= \int_A DT_B^2(p)dp \\ &= \int_A \min_{q \in B} \|p-q\|^2 dp\end{aligned}$$

because the rasterization R_A is equal to zero off of A and is equal to one on it.

Shape Matching Implementation

- Advantages:
  - Squared EDT is quick to compute
  - Match surfaces without correspondences
  - Can use compression techniques to reduce storage.
  - Can solve for the optimal rigid-body alignment using fast signal processing techniques.



Summary

Minimum sum of squared distances descriptor:

- Advantages:
  - Compact
  - Discriminating
  - Quick to compute
  - Allows for matching over rigid body transformations

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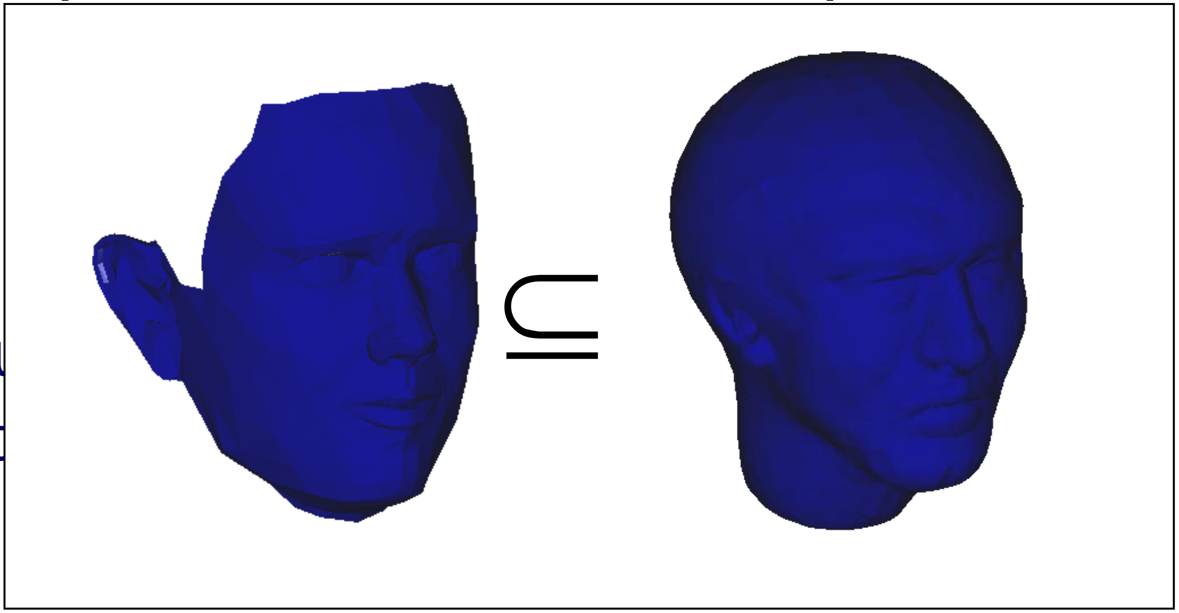
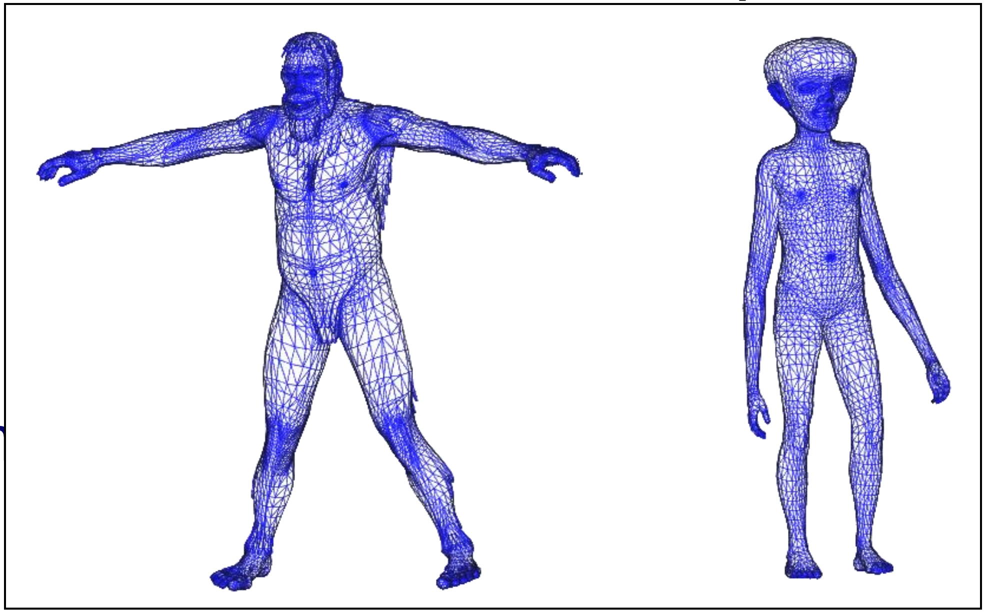


- Limitations:
  - Difficult to use for partial object matching

Summary

Minimum sum of squared distances descriptor:

- Advantages:
  - Compact
  - Discriminating
  - Quick to compute
  - Allows for matchin



- Limitations:
  - Difficult to use for partial object matching
  - Difficult to use for articulated figures

Midterm 2 Review

Michael Kazhdan

(601.457/657)

Midterm

Content:

Everything that we have covered since the first midterm:

- Radiosity
- Subdivision Surfaces
- Spline Curves/Surfaces
- Procedural Models
- Solid Models
- 3D Scanning
- Surface Reconstruction
- Animation
- Image Stitching
- Shape Matching

Midterm

Format:

- Short answer questions only
- No essays
- No True/False
- No multiple choice