Surface Reconstruction

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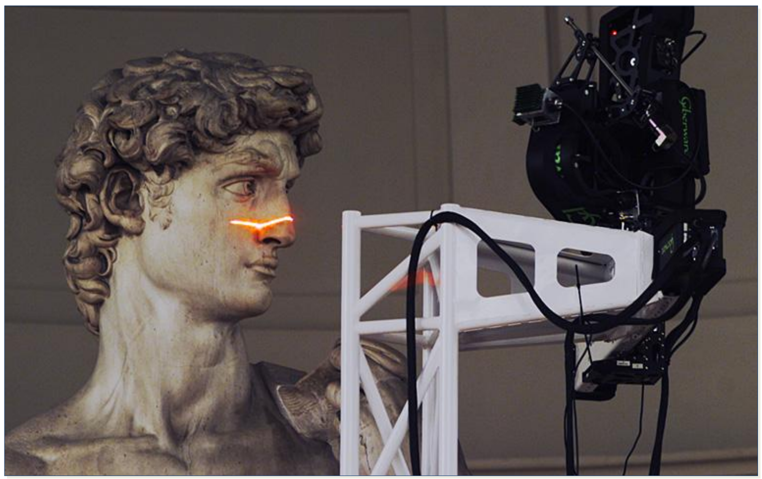
(601.457/657)

Motivation

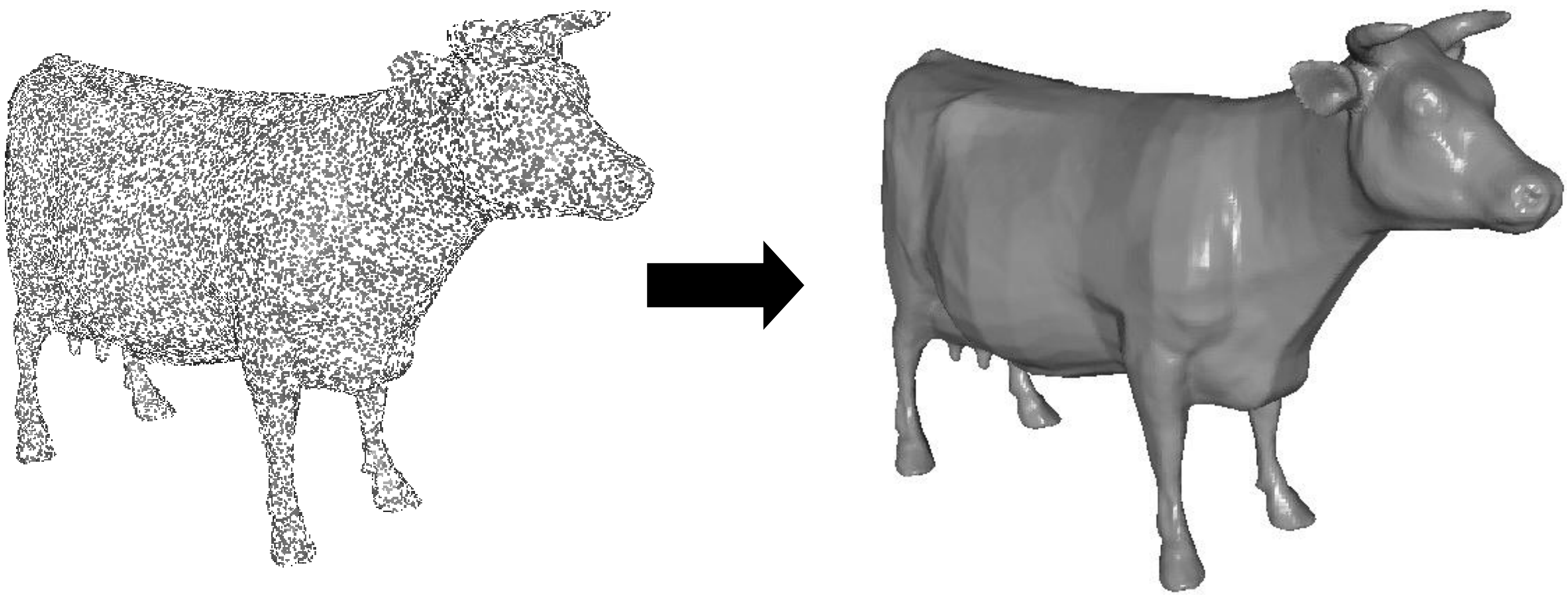
3D Scanners are ubiquitous (and cheap)



[Images courtesy of Rusinkiewicz, Strecha, createdigitalmotion.com, and NextEngine]

Motivation

Merged scans typically consist of un/semi-structured sets of points that need to be connected into a single (water-tight) model.



Related Work

[1] GVU Center Georgia Tech, Graphics Research Grupo, Variational Implicit Surfaces Web site: http://www.cc.gatech.edu/gvu/geometry/implicit/. [6] T. Gentils R. Smith A. Hilton, D. Beresford and W. Sun. Virtual people: Capturing human models to populate virtual worlds. In Proc. Computer Animation, page 174185, Geneva, Switzerland, 1999. IEEE Press. [7] Anders Adamson and Marc Alexa. Approximating and intersecting surfaces from points. In Proceedings of the Eurographics/ACM SIG- GRAPH Symposium on Geometry Processing 2003, pages 230{239. ACM Press, Jun 2003. [8] Anders Adamson and Marc Alexa. Approximating bounded, nonorientable surfaces from points. In SMI '04: Proceedings of Shape Modeling Applications 2004, pages 243{252, 2004. 153 [9] U. Adamy, J. Giesen, and M. John. Surface reconstruction using umbrella ¿ters. Computational Geometry, 21(1-2):63{86, 2002. [10] G. J. Agin and T. O.Binford. Computer description of curved objects. In Proceedings of the International Joint Conference on Artìlial In- telligence, pages 629{640, 1973. [11] Irene Albrecht, J?org Haber, and Hans-Peter Seidel. Construction and Animation of Anatomically Based Human Hand Models. In Proceedings ACM SIGGRAPH/Eurographics Symposium on Computer Animation (SCA '03), pages 98{108, July 2003. [12] M. Alexa, J. Behr, D. Cohen-Or, S. Fleishman, D. Levin, and C. T. Silva. Point set surfaces. In IEEE Visualization 2001, pages 21{28, October 2001. [13] Marc Alexa and Anders Adamson. On normals and projection operators for surfaces de¿ed by point sets. In Marc Alexa, Markus Gross, Hanspeter P¿ter, and Szymon Rusinkiewicz, editors, Proceedings of Eurographics Symposium on Point-based Graphics, pages 149{156. Eurographics, 2004. [14] Marc Alexa, Johannes Behr, Daniel Cohen-Or, Shachar Fleishman, David Levin, and Claudio T. Silva. Computing and rendering point set surfaces. IEEE Transactions on Computer Graphics and Visualization, 9:3{15, 2003. [15] International 2003, pages 49{58, Seoul, Korea, May 2003. [17] H. Alt, B. Behrends, and J. Blomer. Approximate matching of polygonal shapes. Annals of Mathematics and Artìlial Intelligence, pages 251{265, 1995. 154 [18] N. Amenta, M. Bern, and M. Kamvysselis. A new Voronoi-based Surface Reconstruction Algorithm. In Proceedings of ACM SIGGRAPH' 98, pp.415-421, 1998. [19] N. Amenta, M. Bern, and M. Kamvysselis. A new Voronoi-based surface reconstruction algorithm. In Proceedings of ACM SIGGRAPH 1998, pages 415{421, 1998. [20] N. Amenta, S. Choi, T. K. Dey, and N. Leekha. A simple algorithm for homeomorphic surface reconstruction. In Proc. 16th Annu. ACM Sympos. Comput. Geom., pages 213{222, 2000. [21] N. Amenta, S. Choi, and R. Kolluri. The power crust. In Proceedings of 6th ACM Symposium on Solid Modeling, pages 249{260, 2001. [22] N. Amenta, S. Choi, and R. Kolluri. The power crust. In ACM Solid Modeling, 2001. [23] N. Amenta, S. Choi, and R. K. Kolluri. The power crust, unions of balls, and the medial axis transform. Comput. Geom. Theory Appl., 19:127{153, 2001. [24] N. Amenta and Y. Kil. De¿ing point-set surfaces. Acm Transactions on Graphics, 23, Aug 2004. [25] N. Amenta and Y. Kil. The domain of a point set surface. In Symposium on Point-Based Graphics 2004, 2004. [26] N. Amenta, S.Choi, t. K. Dey, and N. Leekha. A Simple Algorithm for Homeomorphic Surface Reconstruction, International Journal of Computational Geometry and Applications, vol.12 n.1-2, pp.125-141, 2002. [27] Nina Amenta and Marshall Bern. Surface reconstruction by Voronoi ¿tering. Discrete Comput. Geom., 22(4):481{504, 1999. [28] P. Anandan. A computational framework and an algorithm for the measurement of visual motion. Int. Journal of Computer Vision, 2:283{ 310, 1989. [29] Anonymous. The Anthropometry Source Book, volume I & II. NASA Reference Publication 1024. 155 [30] Anonymous. Nasa man-systems integration manual. Technical Report NASA-STD-3000. [31] H.J. Antonisse. Image segmentation in pyramids. Computer Vision, Graphics and Image Processing, 19(4):367{383, 1982. [32] A. Asundi. Computer aided moire methods. Optical Laser Engineering, 17:107{116, 1993. [33] D. Attali. r-regular Shape Reconstruction from Unorganized Points. In Proceedings of the ACM Symposium on Computational Geometry, pp.248-253, 1997. [34] D. Attali and J. O. Lachaud. Delaunay conforming iso-surface; skeleton extraction and noise removal, Computational Geometry: Theory and Applications. 19(2-3): 175-189, 2001. [35] D. Attali and J.-O. Lachaud. Constructing Iso-Surfaces Satisfying the Delaunay Constraint; Application to the Skeleton Computation. In Proc. 10th International Conference on Image Analysis and Processing (ICIAP'99), Venice, Italy, September 27-29, pages 382-387, 1999. [36] Dominique Attali and Jean-Daniel Boissonnat. Complexity of the Delaunay triangulation of points on polyhedral surfaces. In Proc. 7th ACM Symposium on Solid Modeling and Applications, 2002. [37] M. Attene, B. Falcidino, M. Spagnuolo, and J. Rossignac. Sharpen & bend: Recovering curved edges in triangle meshes produced by featureinsensitive sampling. Technical report, Georgia Institute of Technology, 2003. GVU-GATECH 34/2003. [38] M. Attene and M. Spagnuolo. Automatic Surface Reconstruction from point sets in space. In EUROGRAPHICS 2000 Proceedings, pp. 457- 465, Vol.19 n.3, 2000. [39] M. Attene and M. Spagnuolo. Automatic surface reconstruction from point sets in space. Computer Graphics Forum, 19(3):457{465, 2000. Proceedings of EUROGRAPHICS 2000. [40] Marco Attene, Bianca Falcidieno, Jarek Rossignac, and Michela Spagnuolo. Edge-sharpener: Recovering sharp features in triangulations of non-adaptively re-meshed surfaces. 156 [41] Marco Attene and Michela Spagnuolo. Automatic surface reconstruction from point sets in space. Computer Graphics Forum, 19(3):457{ 465, 2000. [42] C.K. Au and M.M.F. Yuen. Feature-based reverse engineering of mannequin for garment design. Computer-Aided Design 31:751-759, 1999. [43] S. Ayer and H. Sawhney. Layered representation of motion video using robust maximum-likelihood estimation of mixture models and mdl encoding. International Conference on Computer Vision, pages 777{784, 1995. [44] Z. Popovic B. Allen, B. Curless. Articulated body deformation from range scan data. In Proceedings SIGGRAPH 02, page 612619, San Antonio, TX, USA, 2002. Addison-Wesley. [45] Z. Popovic B. Allen, B. Curless. The space of all body shapes: reconstruction and parameterization from range scans. In Proceedings SIGGRAPH 03, page 587594, San Diego, CA, USA, 2003. Addison- Wesley. [46] ed B. M. ter haar Romeny. Geometry-Driven Di¯sion in Computer Vision. Kluwer Academic Pubs., 1994. [47] A. Bab-Hadiashar and D. Suter. Robust optic ow estimation using least median of squares. Proceedings ICIP-96, September 1996. Switzerland. [48] C. Bajaj, Fausto Bernardini, and Guoliang Xu. Automatic reconstruction of surfaces and scalar Îlds from 3d scans. In International Confer- ence on Computer Graphics and Interactive Techniques, pages 109{118, 1995. [49] C .L. Bajaj, F. Bernardini, J. Chen, and D. Schikore. utomatic Reconstruction of 3D Cad Models. In Proceedings of Theory and Practice of Geometric Modelling, 1996. [50] C.L. Bajaj, E.J. Coyle, and K.N. Lin. Arbitrary topology shape reconstruction from planar cross sections. Graphical.Models.and Image Processing., 58:524{543, 1996. 157 [51] G. Barequet, M.T. Goodrich, A. Levi-Steiner, and D. Steiner. Contour interpolation by straight skeletons. Graphical.models., 66:245{260, 2004. [52] G. Barequet, D. Shapiro, and A. Tal. Multilevel sensitive reconstruction of polyhedral surfaces from parallel slices. The Visual Computer, 16(2):116{133, 2000. [53] G. Barequet and M. Sharir. Piecewise-linear interpolation between polygonal slices. Computer Vision and Image Understanding, 63(2):251{272, 1996. [54] G. Barequet and M. Sharir. Partial surface and Beraldin. Practical considerations for a design of a high precision 3d laser scanner system. Proceedings of SPIE, 959:225{246, 1988. [74] B. Blanz and T. Vetter. A morphable model for the synthesis of 3d faces. In Proceedings SIGGRAPH 99, page 187194, Los Angeles, CA, USA, 1999. Addison-Wesley. [75] Volker Blanz, Curzio Basso, Tomaso Poggio, and Thomas Vetter. Reanimating Faces in Images and Video. In Pere Brunet and Dieter Fellner, editors, Computer Graphics Forum (Proceedings of Eurographics 2003), volume 22, pages 641{650, September 2003. [76] Volker Blanz and Thomas Vetter. A Morphable Model for the Synthesis of 3D Faces. In Alyn Rockwood, editor, Computer Graphics (SIGGRAPH '99 Conference Proceedings), pages 187{194. ACM SIGGRAPH, August 1999. [77] J. F. Blinn. A generalization of algebraic surface drawing. ACM Trans- actions on Graphics, 1(3):235{256, July 1982. [78] J. Bloomenthal, editor. Introduction to Implicit Surfaces. Morgan Kaufmann, 1997. [79] J. D. Boissonnat. Geometric Structures for Three-dimensional Shape Representation. ACM Transaction on Graphics, pp.266-286, Vol.3, 1984. [80] J.-D. Boissonnat. Geometric structures for three-dimensional shape representation. ACM Transactions on Graphics, 3(4):266{286, October 1984. [81] J.D. Boissonnat. Shape reconstruction from planar cross sections… [403] M.J. Zyda, A.R. Jones, and P.G. Hogan. Surface construction from planar contours. Computers.and Graphics, 11:393{408, 1987.

…/… [403]

[List courtesy of Alliez]

Related Work

<u>Classification</u>:

- Approach:
  - Computational Geometry
  - Implicit Surfaces
- Input:
  - Oriented vs. Unoriented
  - Structured vs. Unstructured
- Output:
  - Water-tight vs. Surface with Boundary

Related Work

Classification:

- Computational Geometry (Unoriented Points)
  - Use input to partition space
  - Use a subset of the partition to define the shape
- Implicit Surfaces (Oriented Points)
  - Fit implicit function to the input
  - Extract iso-surface

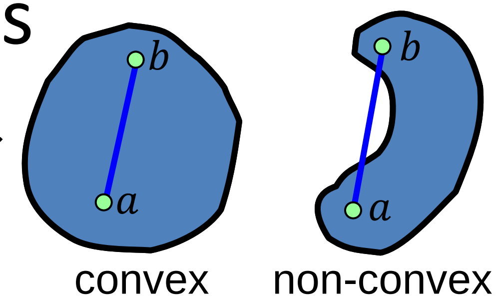
Outline

- Introduction
- Preliminaries
  - Convex Hulls
  - Delaunay Triangulations
  - Voronoi Diagrams
  - Medial Axes
- A sampling of methods
- Why is reconstruction hard?

Computational Geometry

<u>Convex Hulls</u>:

A set S is *convex* if for any two points $a, b \in S$, the line segment between a and b is also in S.



[Notes courtesy of Alliez]

Computational Geometry

Convex Hulls:

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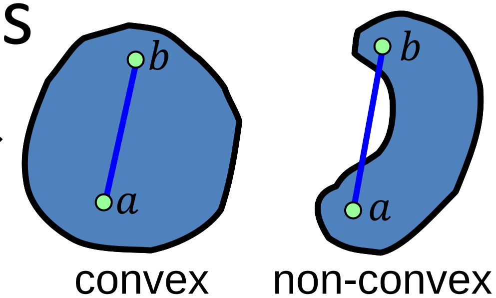
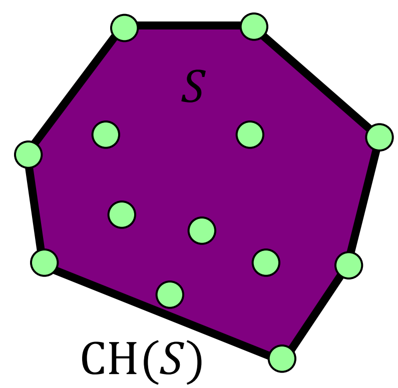


The *convex hull* of a set of points is the smallest convex set containing S.



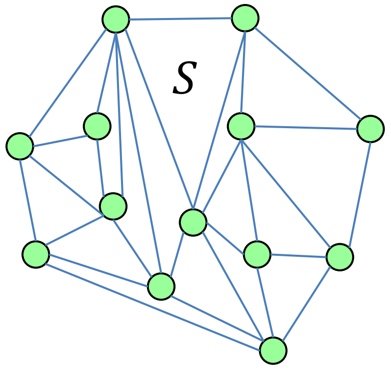
[Notes courtesy of Alliez]

Computational Geometry

<u>Triangulation</u>:

A *triangulation* of a set of sites/points S a decomposition of the convex hull of the points into triangles, whose vertex set is the set of sites/points.

- There are many ways to triangulate the set S.
- Not all are equally "good" (e.g. can have skinny triangles with small angles)

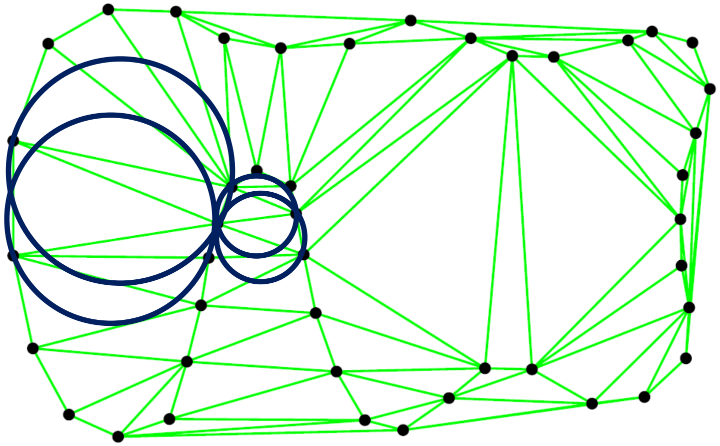


[Notes courtesy of Alliez]

Computational Geometry

Delaunay Triangulation:

A *Delaunay Triangulation* of a set of sites S is a triangulation of S such that the circumscribing circle of any triangle contains no other site in S*.



Compactness Property:

This triangulation maximizes the minimum angle.

[*Assuming general position]

[Notes courtesy of Alliez]

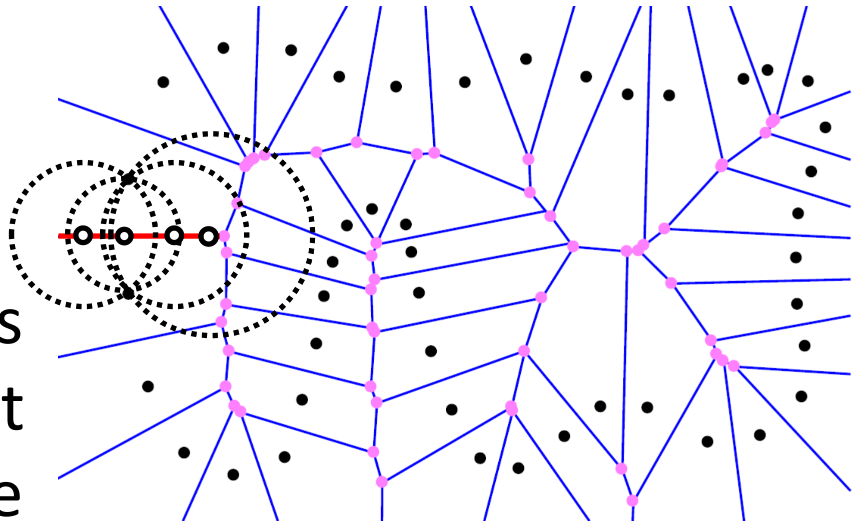
Computational Geometry

<u>Voronoi Diagrams</u>:

The *Voronoi Diagram* of S is a partition of space into regions $\mathrm{VD}(s)$, with $s \in S$, s.t. all points in $\mathrm{VD}(s)$ are closer to s than to any other site.

– Edges are equidistant from the two sites in the incident cells.

– For each edge point there is an empty circle, centered at the point, only touching the sites in the two incident cells.



[Notes courtesy of Alliez]

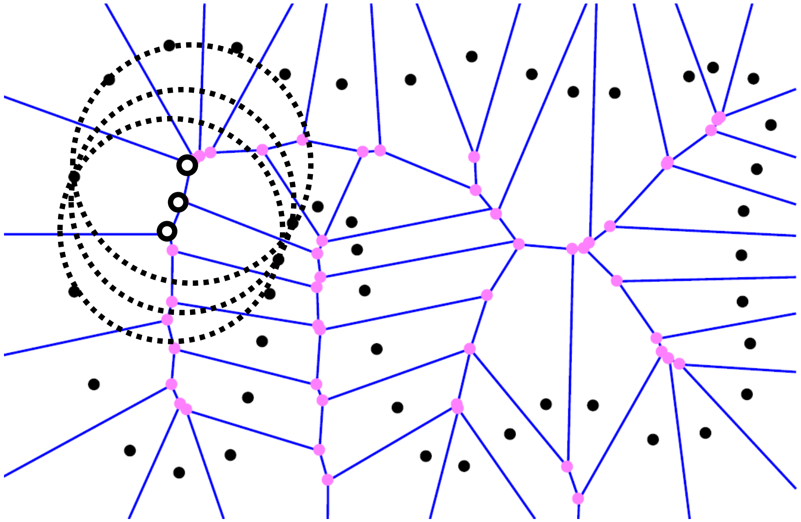
Computational Geometry

<u>Voronoi Diagrams</u>:

The *Voronoi Diagram* of S is a partition of space into regions $\mathrm{VD}(s)$, with $s \in S$, s.t. all points in $\mathrm{VD}(s)$ are closer to s than to any other site.

– Vertices are equidistant from three (or more) sites in the incident cells.

– For a vertex, we can draw an empty circle, centered at the vertex, that just touches the sites in the three (or more) incident cells.



[Notes courtesy of Alliez]

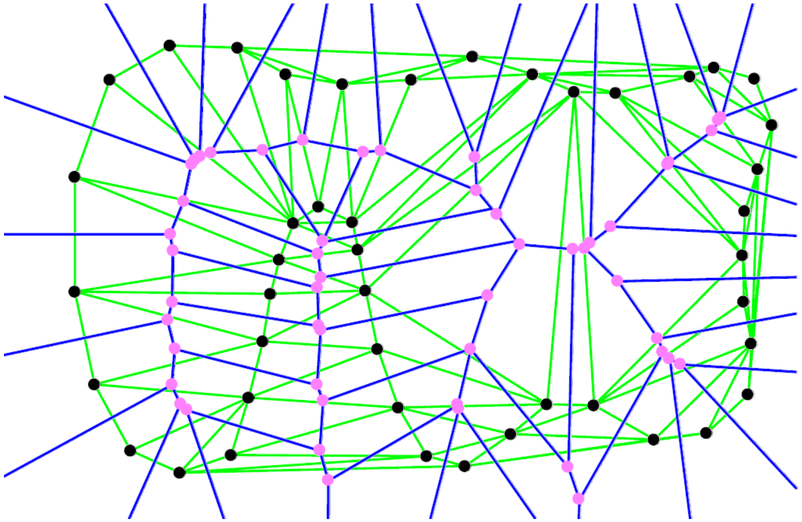
Computational Geometry

Voronoi Diagrams:

The *Voronoi Diagram* of S is a partition of space into regions $\mathrm{VD}(s)$, with $s \in S$, s.t. all points in $\mathrm{VD}(s)$ are closer to s than to any other site.

Duality:

Each Voronoi vertex is in one-to-one correspondence with a Delaunay triangle.

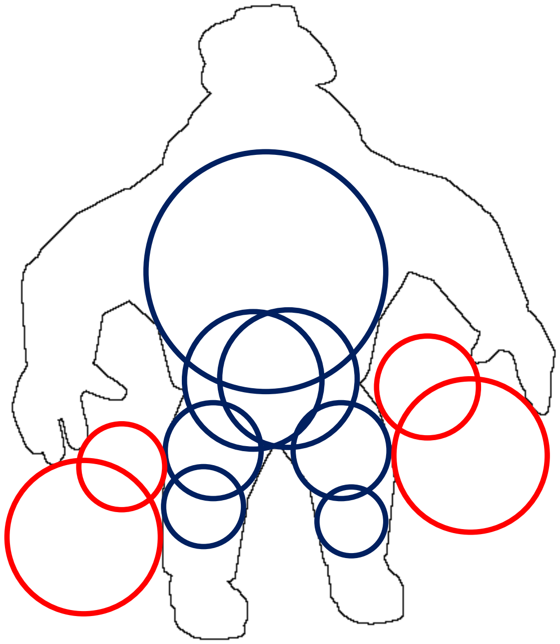


[Notes courtesy of Alliez]

Computational Geometry

Medial Axis:

For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.

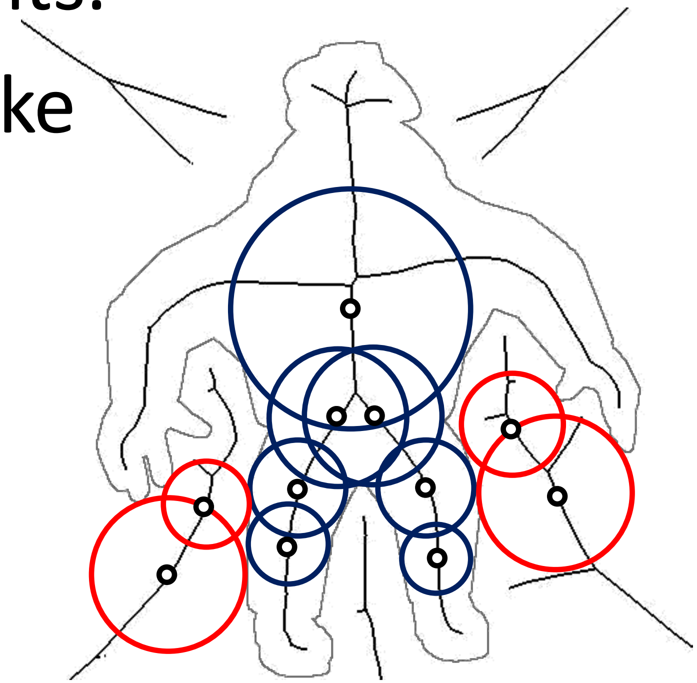


Computational Geometry

<u>Medial Axis</u>:

For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.

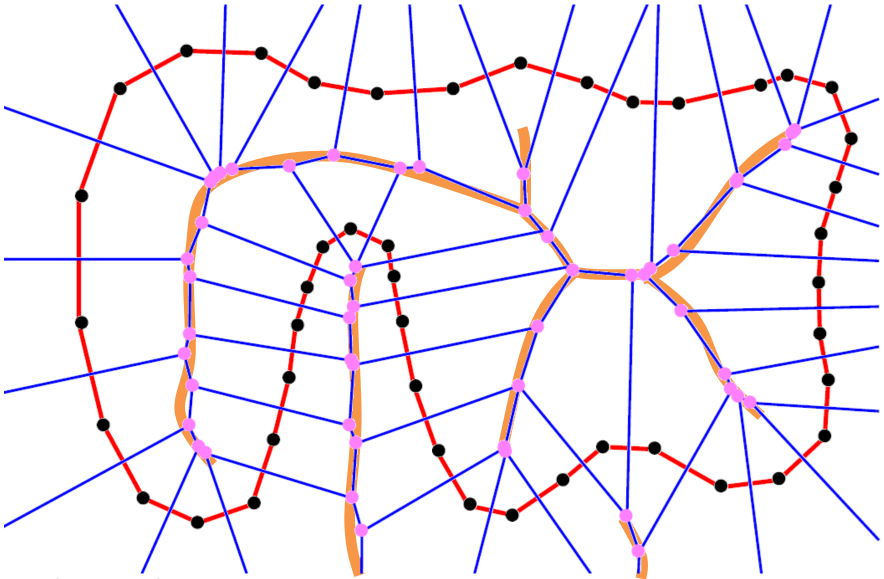
The centers of all such balls make up the *medial axis/skeleton*.



Computational Geometry

<u>Observation in 2D*</u>:

For a reasonable point sample, the medial axis is well-sampled by the Voronoi vertices.



*In 3D, this is only true for a subset of the Voronoi vertices – the *poles*.

Outline

- Introduction
- Preliminaries
- A sampling of methods
  - **Space Partitioning**
  - Crust
  - … from Unorganized Points
  - Poisson Reconstruction
- Why is reconstruction hard?

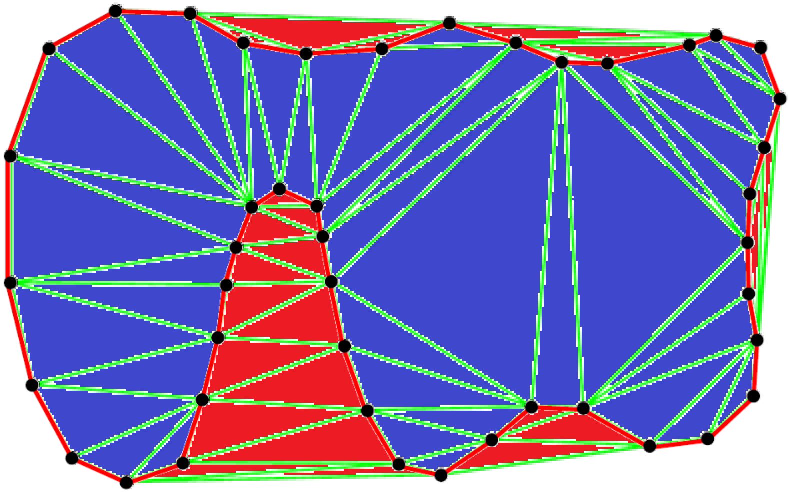
Computational Geometry

Implicit Surfaces

Space Partitioning

Given a set of points, we can construct the Delaunay triangulation.

If we could label each triangle as inside/outside, then the surface of interest is the set of edges that lie between inside and outside triangles.

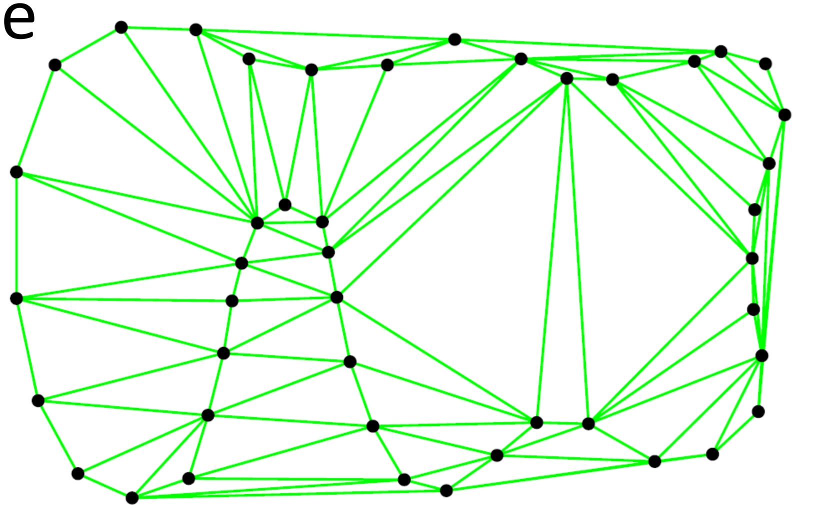


Space Partitioning

Q: How should we assign labels?

A: Spectral Partitioning [Kolluri et al. 2004]

1. **Local**: Assign a weight to each (interior) edge indicating if the two triangles should have the same label.
2. **Global**: Evenly partition the triangles, minimizing the sum of the weights along partitioning edges.



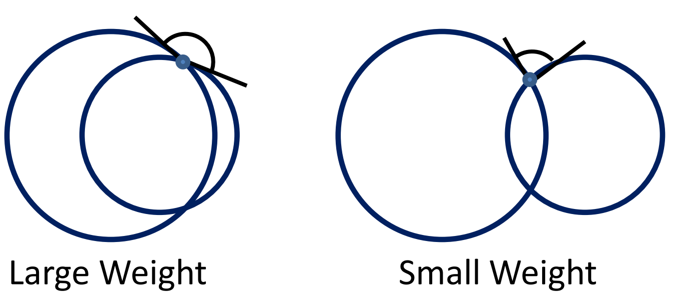
Space Partitioning

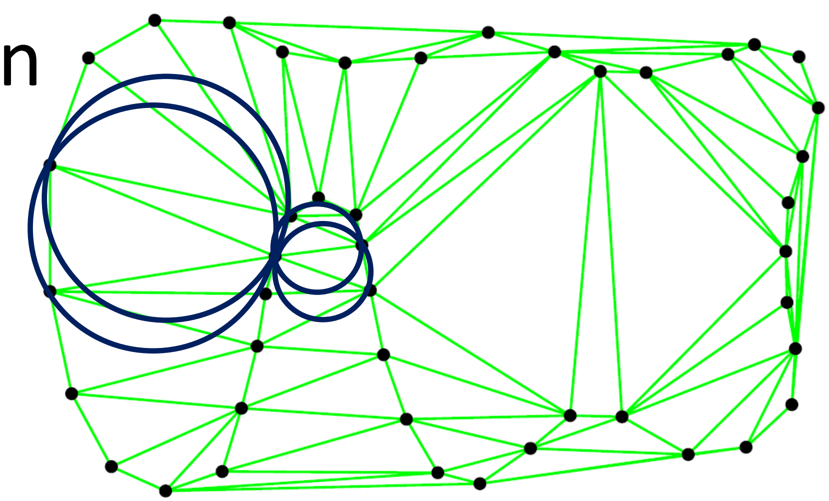
[Local] Assigning Edge Weights:

Q: When are triangles on opposite sides of an edge likely to have the same label?

A: If the triangles are on the same side, their circumscribing circles intersect deeply.

Use the angle of intersection to set the weight.





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Computational Geometry (Space Partitioning, Crust)

Implicit Surfaces (… from Unorganized Points, Poisson Reconstruction)

Crust [Amenta et al. 1998]

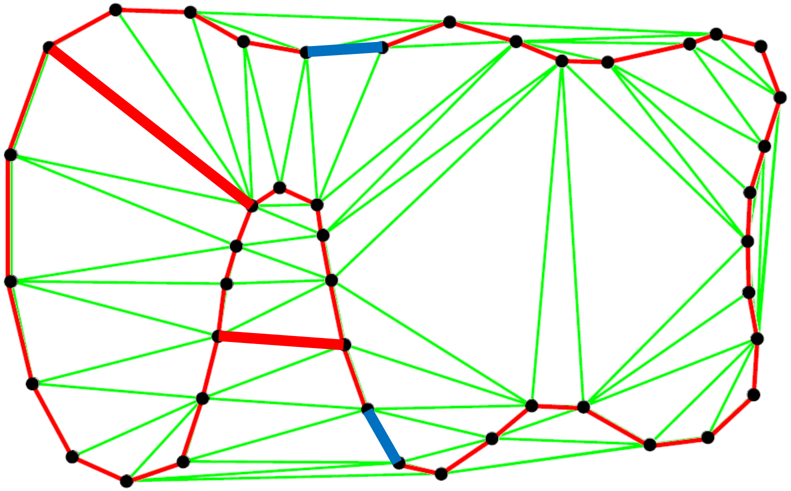
If we consider the Delaunay Triangulation of a point set sampling a curve, the curve should be (approximately) a subset of the Delaunay edges.

Q: How do we determine which edges to keep?

A: Two types of edges:

1. Those connecting adjacent points on the curve
2. Those traversing.

Discard those that traverse.

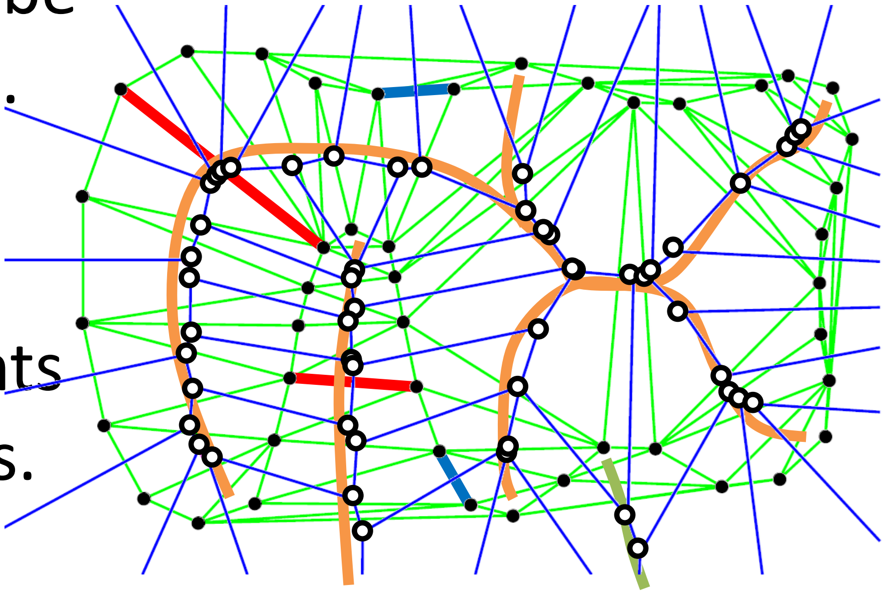


Crust [Amenta et al. 1998]

Observation:

Edges that traverse cross the medial axis.

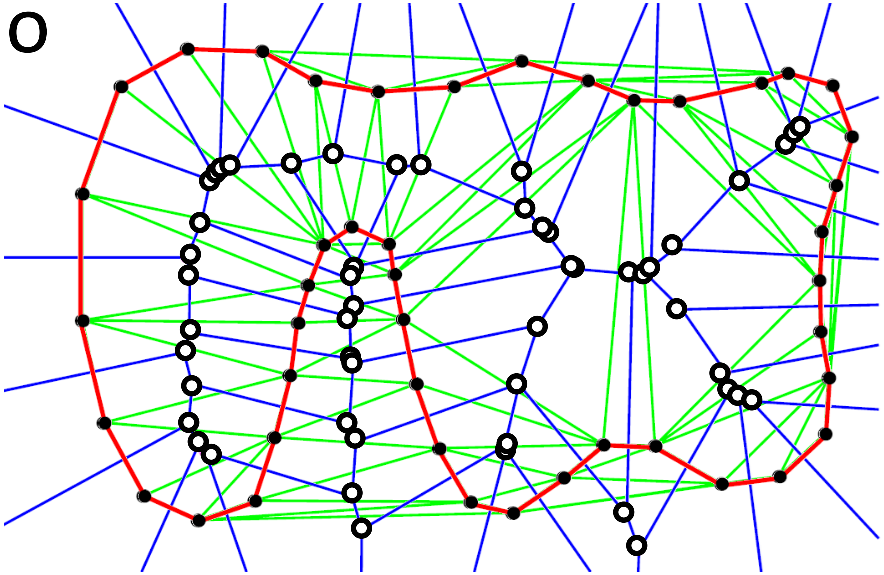
- Although we don't know the medial axis, we can sample it with the Voronoi vertices.
- Edges that traverse must be near the Voronoi vertices.
- We say an edge does not traverse if we can draw a circle through its endpoints empty of Voronoi vertices.



Crust [Amenta et al. 1998]

Algorithm:

1. Compute the Delaunay triangulation.
2. Compute the Voronoi vertices
3. Keep all edges for which there is a circle that contains the edge but no Voronoi vertices.



Note:
As opposed to the previous method, it is not obvious that this will generate a closed, manifold curve/surface.

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Computational Geometry (Space Partitioning, Crust)

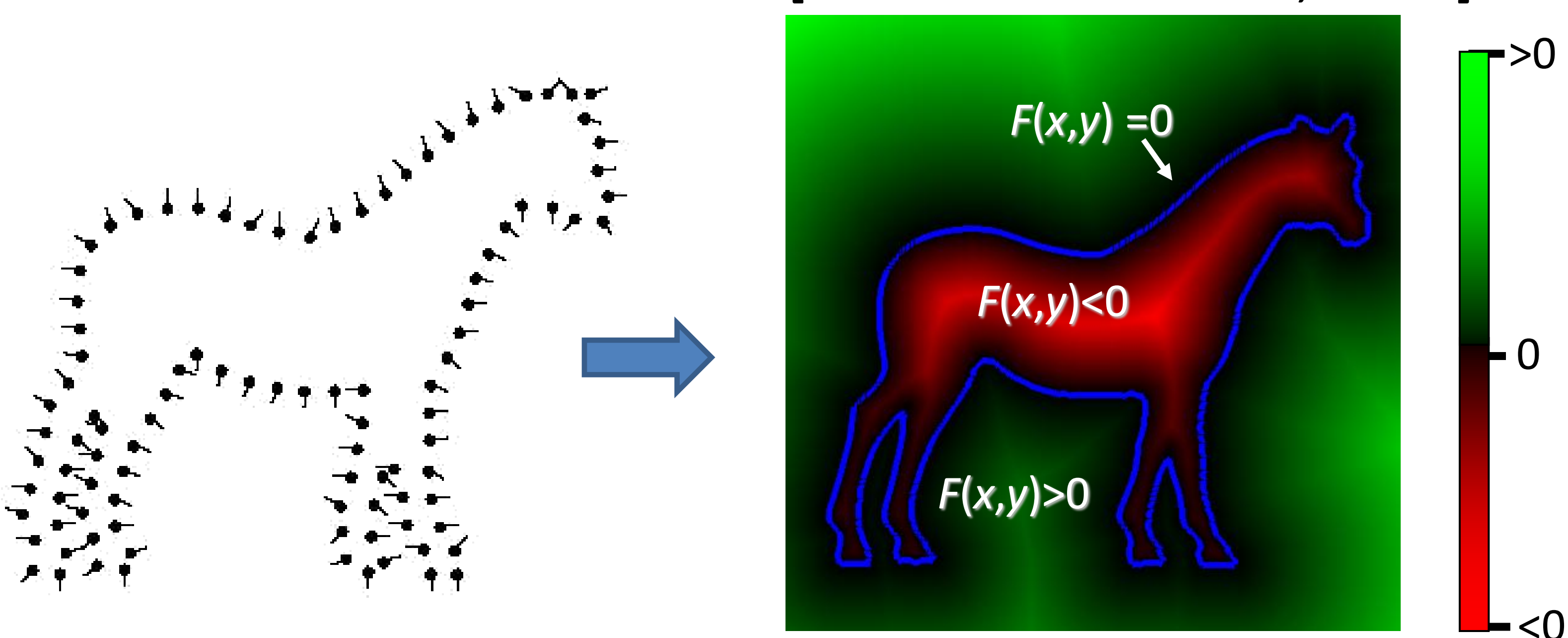
Implicit Surfaces (… from Unorganized Points, Poisson Reconstruction)

Implicit Surface Reconstruction

Key Idea:

– Use the point samples to define a function whose value at each sample positions is zero.

– Extract the zero level set. [Lorensen and Cline, 1987]

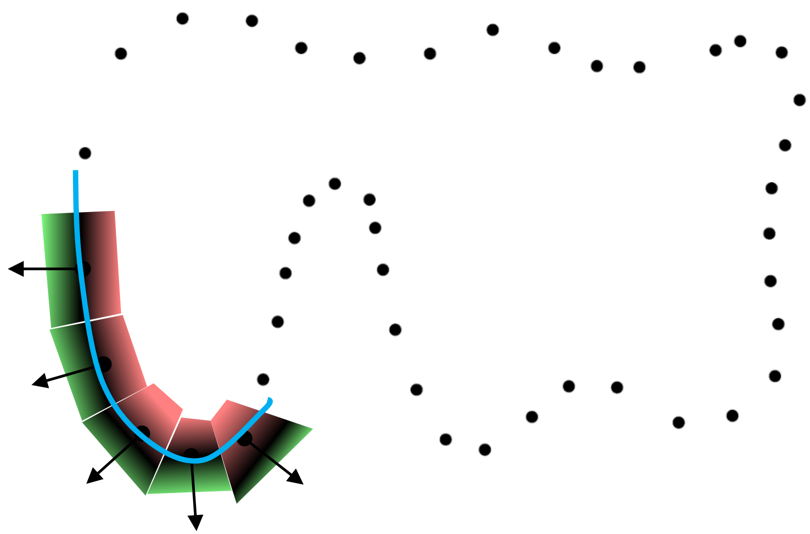


Sample Points

$F(x,y) = 0$

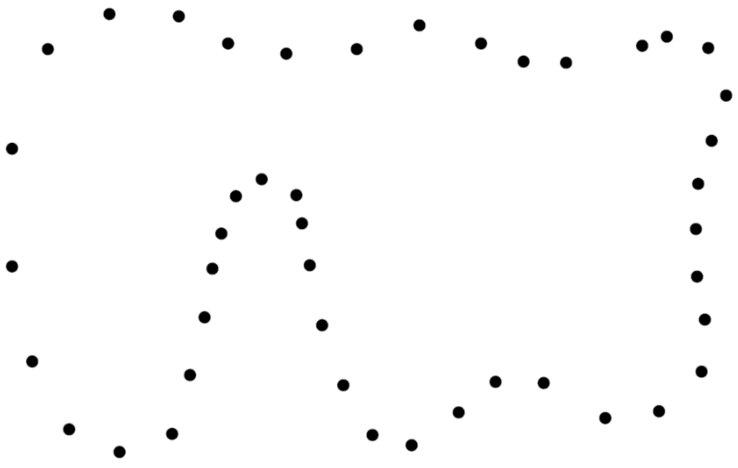
… Unorganized Points [Hoppe et al. 1992]

- Compute a local *signed distance function* by using the sample normals to define a **local** linear approximation to the function.
- Blend the linear approximations.
- Extract the zero level (where defined).



… Unorganized Points [Hoppe et al. 1992]

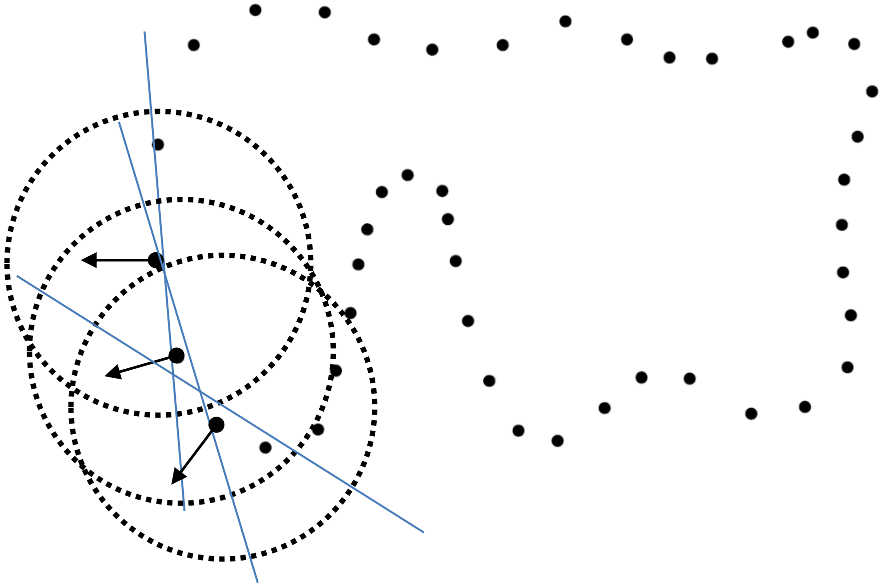
Q: How do we get the normals?



… Unorganized Points [Hoppe et al. 1992]

Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.



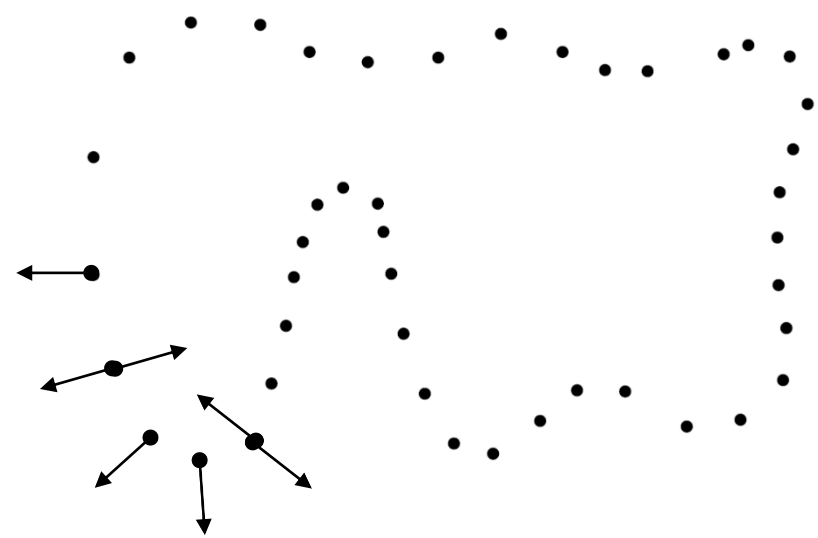
… Unorganized Points [Hoppe et al. 1992]

Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.

This doesn't guarantee a consistent orientation!

For the orientation to be consistent, neighboring points should point in the same direction.

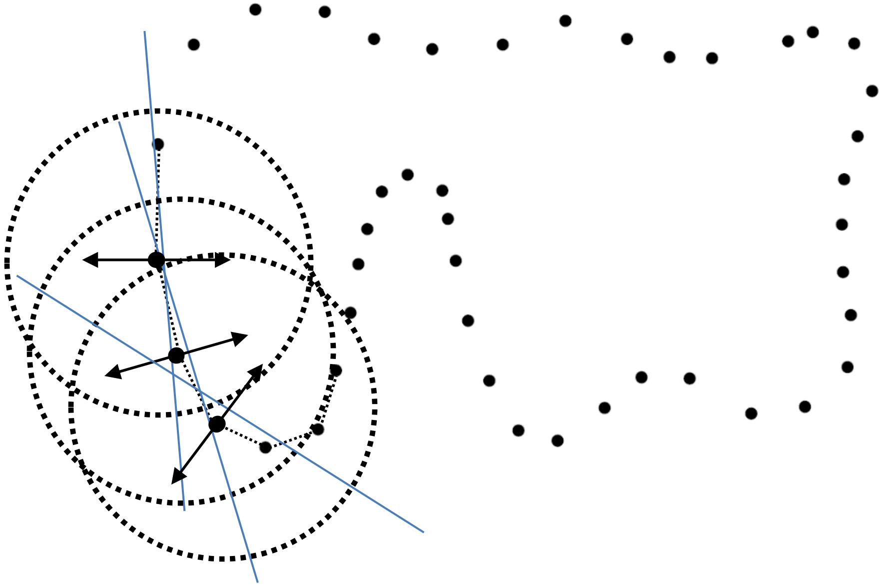


… Unorganized Points [Hoppe et al. 1992]

Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.

A2: Build a (Euclidian) minimal spanning tree and propagate the orientation from a root.



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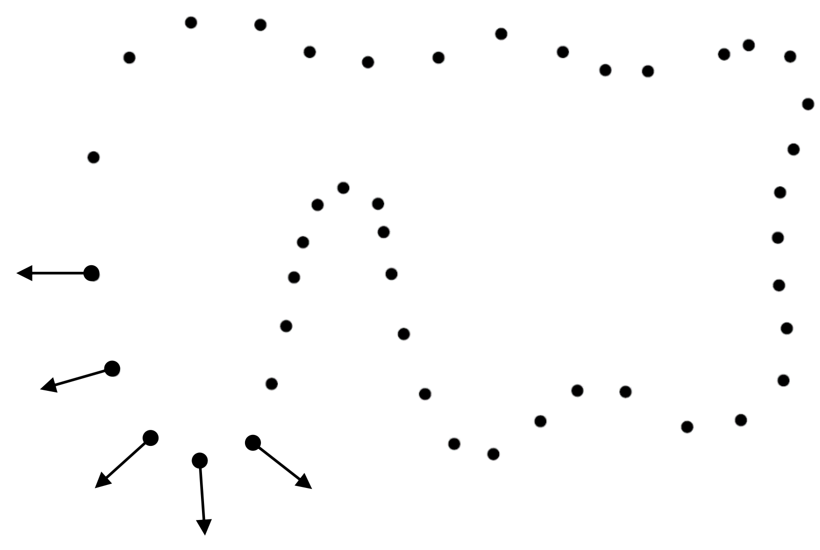

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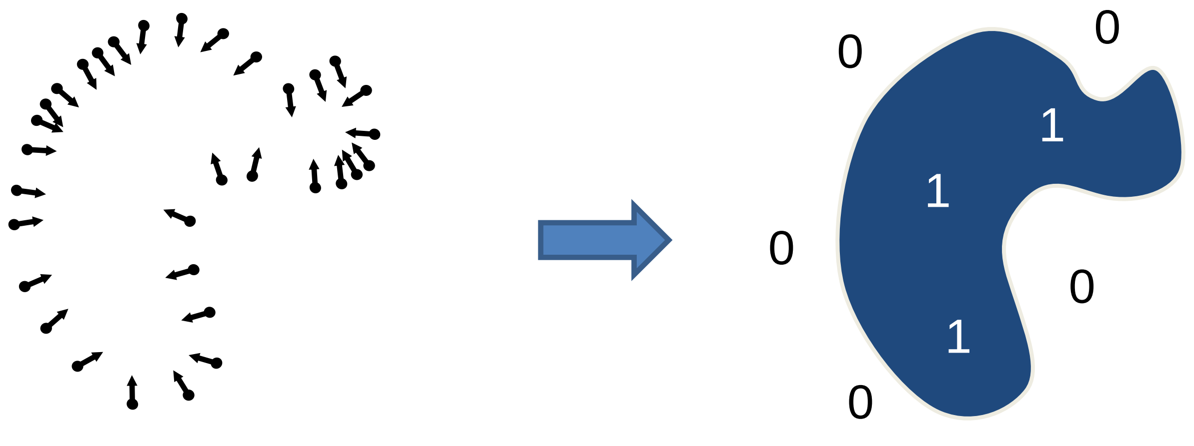
Computational Geometry (Space Partitioning, Crust)

Implicit Surfaces (… from Unorganized Points, Poisson Reconstruction)

Poisson Reconstruction [Kazhdan et al. 2006]

Reconstruct the *indicator function* of the surface and then extract the boundary.

Q: How to fit the function to the samples?



Oriented points

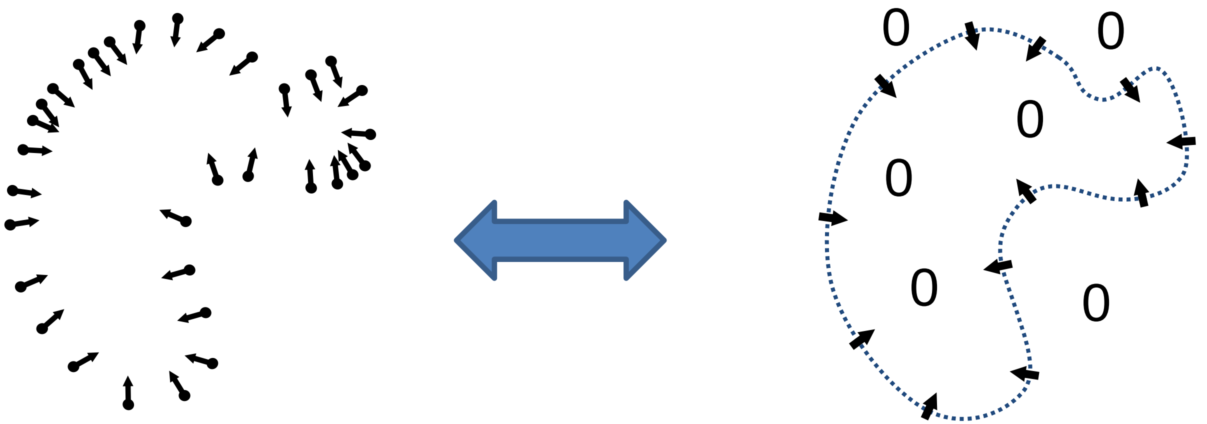
Indicator function

Poisson Reconstruction [Kazhdan et al. 2006]

Reconstruct the *indicator function* of the surface and then extract the boundary.

Q: How to fit the function to the samples?

A: Normals are samples of function's gradients.



Oriented points

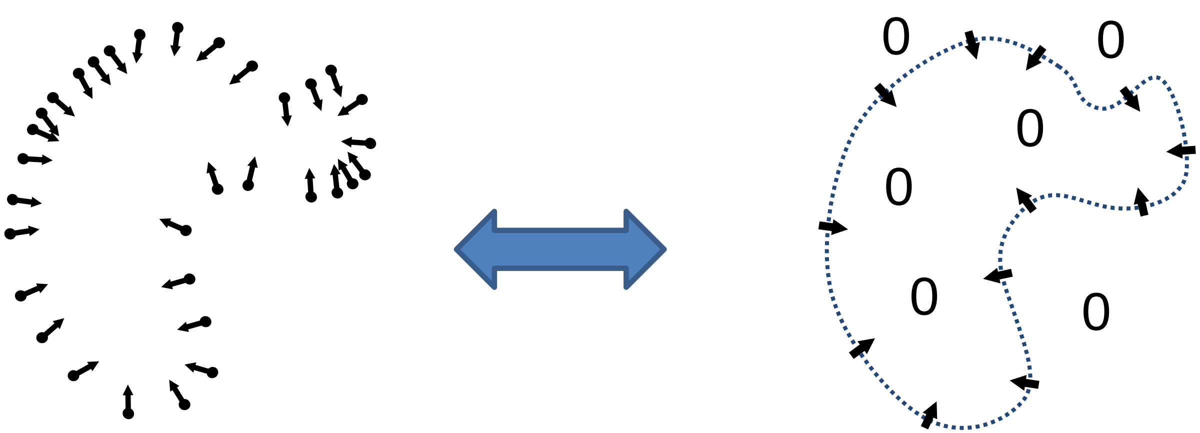
Indicator gradient

Poisson Reconstruction [Kazhdan et al. 2006]

To fit a scalar field F to the gradients $\vec{V}$ solve:

$$\nabla F = \vec{V}$$

- This is an over-constrained problem, so there is (usually) no solution.



Oriented points

Indicator gradient

Poisson Reconstruction [Kazhdan et al. 2006]

To fit a scalar field F to the gradients $\vec{V}$ solve:

$$\nabla F = \vec{V}$$

✘ This is an over-constrained problem, so there is (usually) no solution.

✓ Solve for the best (least-squares) solution:

$$\arg\min_F \left\|\nabla F - \vec{V}\right\|^2$$

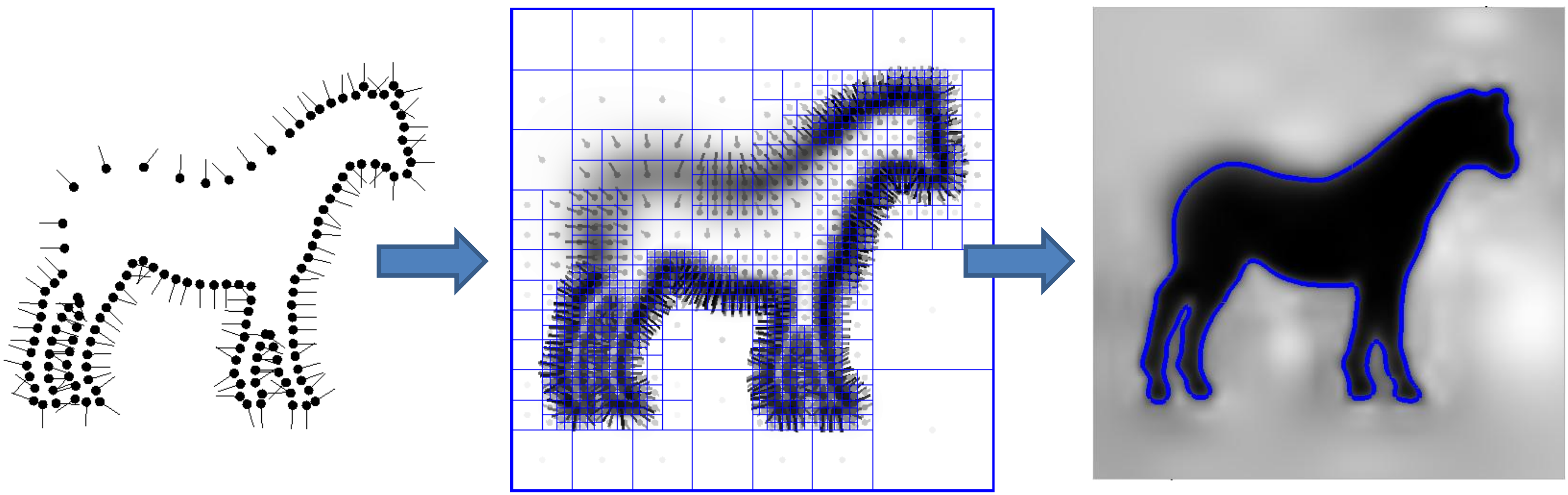
⇒ Taking the divergence, this becomes:

$$\nabla \cdot \left(\nabla F - \vec{V}\right) = 0 \quad \Leftrightarrow \quad \Delta F = \nabla \cdot \vec{V}$$

Poisson Reconstruction [Kazhdan et al. 2006]

Algorithm:

1. Transform samples into a vector field.
2. Fit a scalar-field to the gradients.
3. Extract the isosurface.



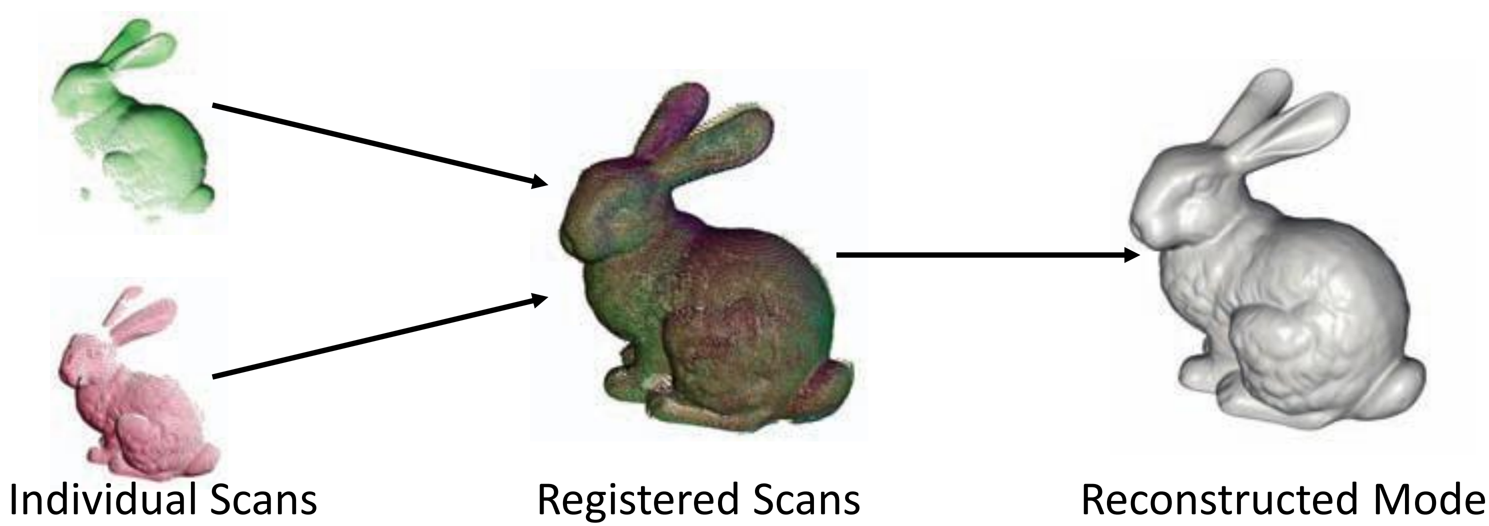
Outline

- Introduction
- Preliminaries
- A sampling of methods
- Why is reconstruction hard?

Why is Reconstruction Hard?

The point-set is often the result of:

- Scanning
- Registering
- Etc.

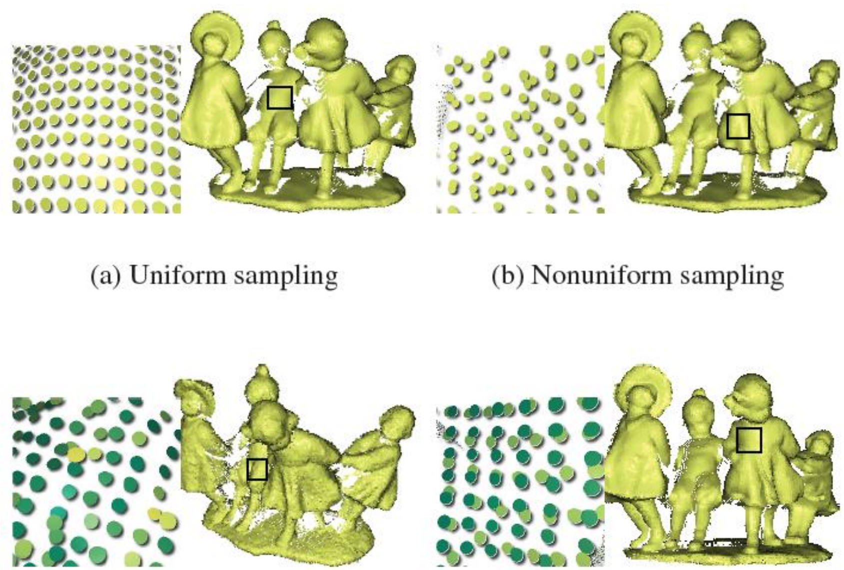


[Image courtesy of Bolitho]

Why is Reconstruction Hard?

Susceptible to:

- Scanning
  - Nonuniform sampling
  - Grazing angles
  - Scanner noise
  - Imprecise estimates
- Registering
  - Misalignment
  - Non-linear camera model



(a) Uniform sampling

(b) Nonuniform sampling

(c) Noisy data

(d) Misaligned scans

[Image courtesy of Berger *et al*.]

Practical Concerns

- Performance in the presence of bad data
- Interpolating vs. approximating
- Efficiency of reconstruction
- Quality guarantees
- Manifold / water-tight
- Incorporation of prior knowledge