# Radiosity

Michael Kazhdan

(601.457/657)

# Overview

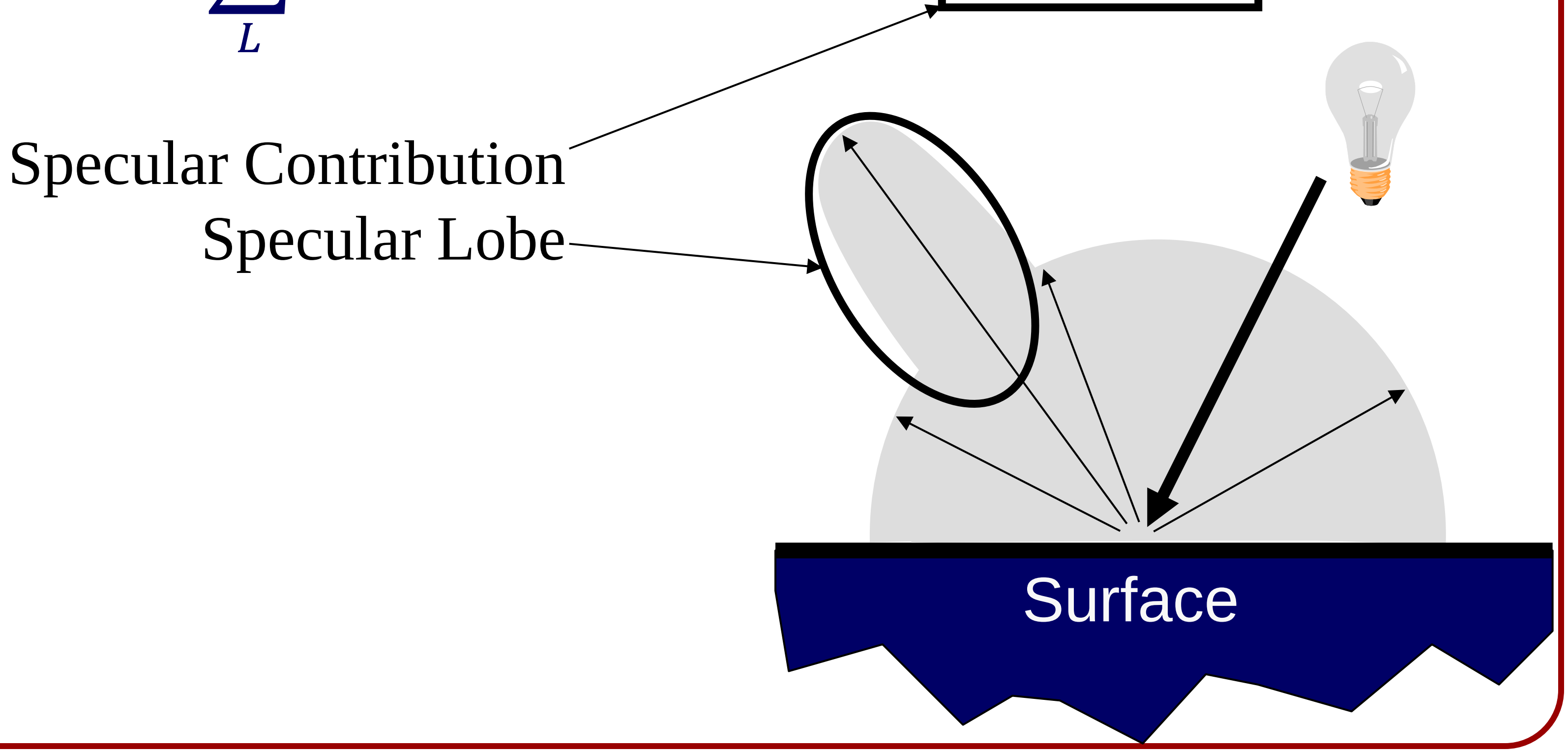
- Ray Tracing Revisited
- Radiosity

# Ray Casting

Ray tracing is based on the Phong lighting model:

- A surface reflects light non-uniformly, with stronger reflection in the specular direction:

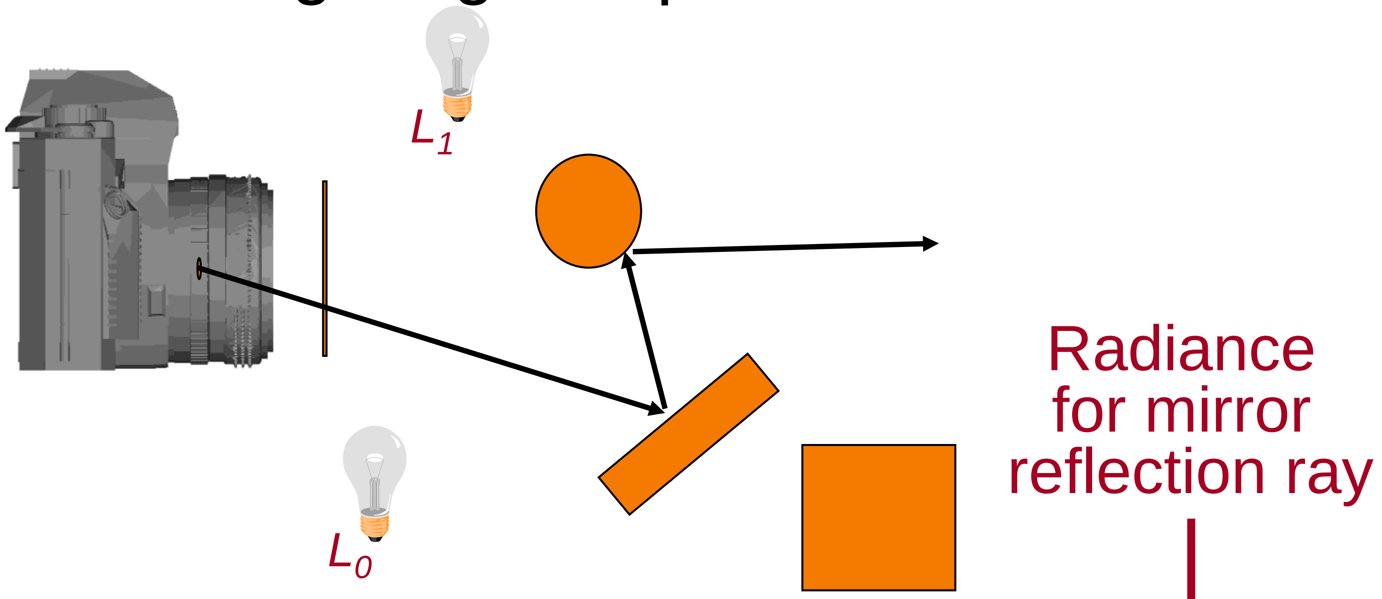
$$I = I_E + \sum_L \left[ K_A \cdot I_L^A + \left( K_D \cdot \langle \vec{N}, \vec{L} \rangle + \boxed{K_S \cdot \langle \vec{V}, \vec{R} \rangle^n} \right) \cdot I_L \cdot S_L \right]$$



# Ray Tracing

Ray tracing is based on the Phong lighting model:

For the same reason, we only cast secondary rays in the reflected direction – to maximize the contribution to the lighting computation.

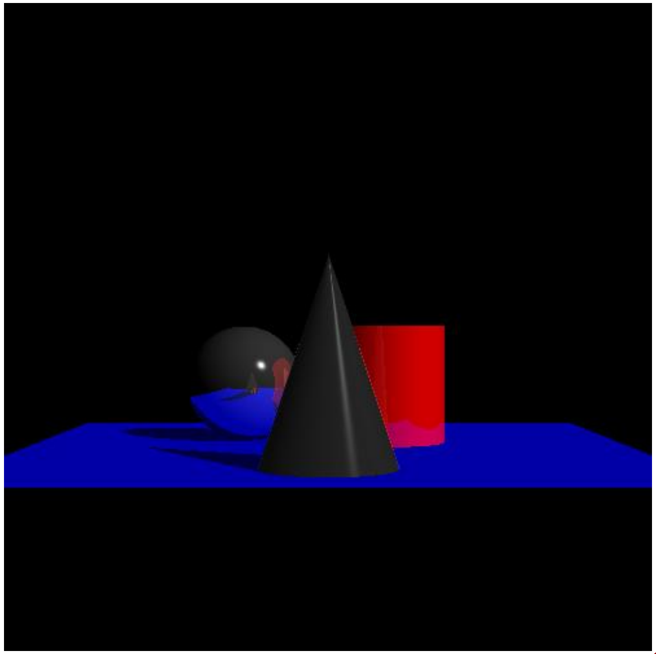



$$I = I_E + \sum_L \left[ K_A \cdot I_L^A + \left( K_D \cdot \langle \vec{N}, \vec{L} \rangle + K_S \cdot \langle \vec{V}, \vec{R} \rangle^n \right) \cdot I_L \cdot S_L \right] + K_s \cdot I_R$$

# Ray Tracing

Advantage:

- Good at capturing the specular properties of materials



# Ray Tracing

Advantage:

- Good at capturing the specular properties of materials

Disadvantages:

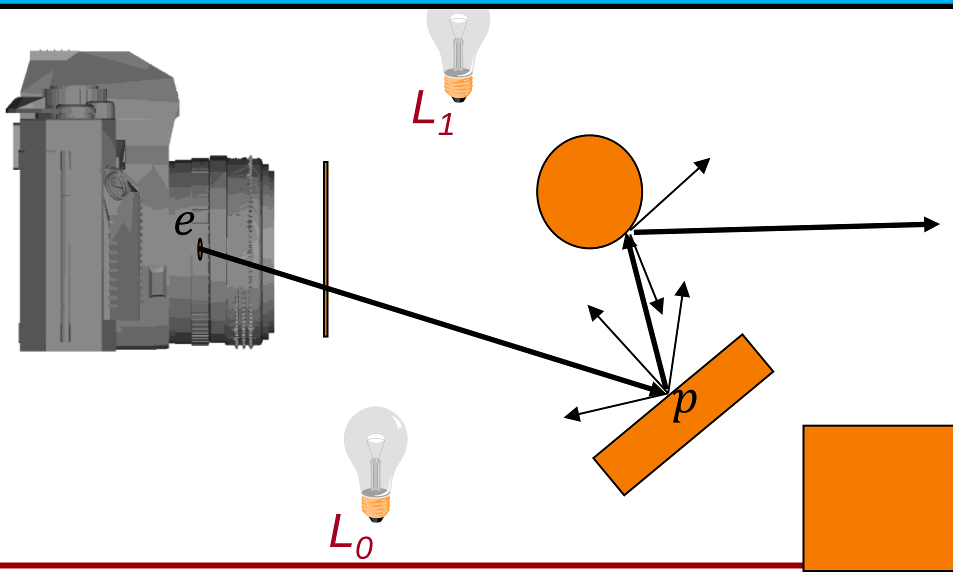
- Difficult to support soft shadows from area lights
- Difficult to support caustics
- Need the ambient term as a hack for the global illumination

# Lighting

What do we really want to compute?

The accumulation of light coming in from **all** directions, **modulated** by how much the light is reflected in/from that direction.

In practice, use Monte-Carlo integration with importance sampling to generate more reflected rays in directions that contribute more strongly.

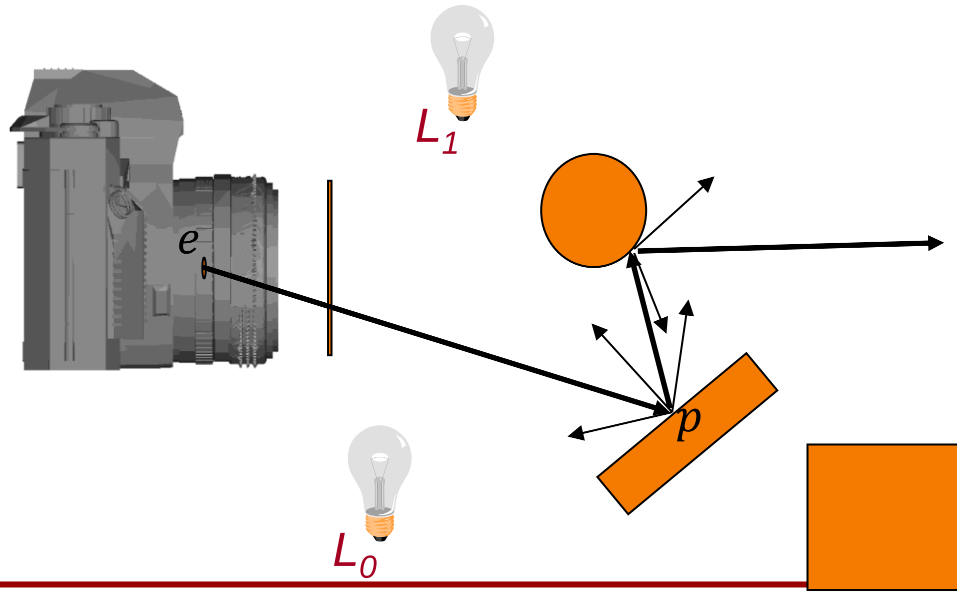


# Lighting

What do we really want to compute?

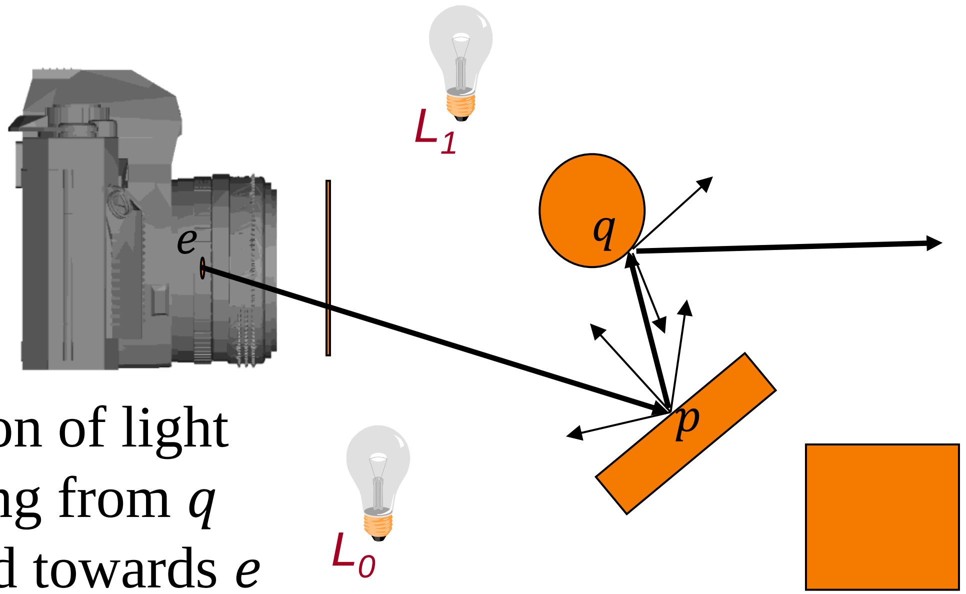
The brightness of the light reaching the camera eye, $e$, from a point, $p$, in the scene is the sum of:

1. The light emitted from $p$, to $e$, and
2. The light emanating from all points in the scene scaled by the extent to which it is reflected through $p$ to $e$.



# Lighting

What do we really want to compute?




Amount of light from $p$ going towards $e$

Amount of light emitted from $p$ going towards $e$

Fraction of light arriving from $q$ reflected towards $e$

Amount of light from $q$ going towards $p$

$$B(p \to e) = E(p \to e) + \int_{\Omega} F_r(q \to p \to e) \cdot B(q \to p) dq$$

# Lighting

Challenge:

- The integral needs to be estimated precisely to capture discontinuities.
- The function is recursive since the amount of light entering a point depends on the amount of light leaving it.


*Jensen*

$$B(p \to e) = E(p \to e) + \int_{\Omega} F_r(q \to p \to e) \cdot B(q \to p)\, dq$$

# Ray-Tracing

Specular assumption:

- The surface only reflects lights from the reflected ray direction:

$$F_r(q \to p \to e) = \begin{cases} K_s(p) & q = I\big(p, \text{Ref}(p \to e)\big) \\ 0 & \text{otherwise} \end{cases}$$

$$B(p \to e) = E(p \to e) + \int_\Omega F_r(q \to p \to e) \cdot B(q \to p) dq$$

$$B(p \to e) = E(p \to e) + K_s(p) \cdot B\Big(I\big(p, \text{Ref}(p \to e)\big) \to p\Big)$$

$I\big(p, \text{Ref}(p \to e)\big)$ is the first intersection of the ray in the reflected view direction.

# Radiosity

Lambertian assumption:

- The apparent brightness a patch of surface is constant (i.e. independent of the view direction).

$$B(p \to e) = E(p \to e) + \int_\Omega F_r(q \to p \to e) \cdot B(q \to p) dq$$

$$B(p) = E(p) + \int_\Omega F_r(q \to p) \cdot B(q) dq$$

# Radiosity

<u>Lambertian assumption</u>:

- The apparent brightness a patch of surface is constant (i.e. independent of the view direction).
  - » **Emitters appear equally bright from all directions**

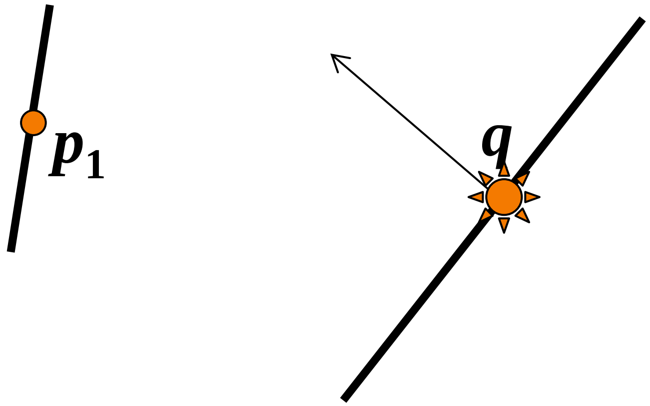
Emission $\neq$ 0

$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q)\,dq$$

# Lambertian Emitters

Given an emitter at point $q$, the apparent brightness point $p$ is independent of its orientation w.r.t. to $q$.

By assumption, the apparent brightness of $q$ is independent of the view direction of $p$.




$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q) dq$$

# Lambertian Emitters

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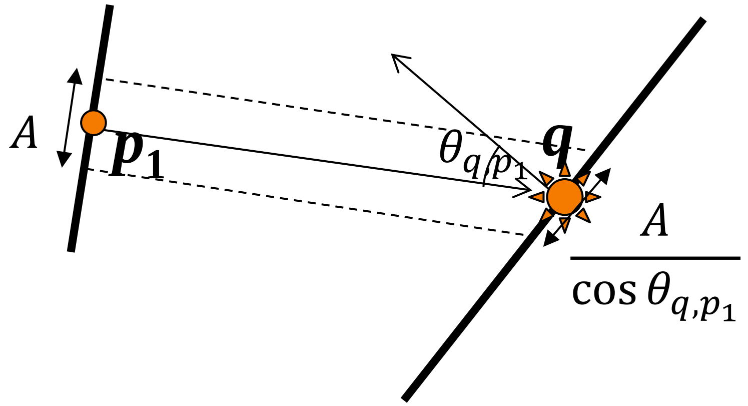


By assumption, the apparent brightness of $q$ is independent of the view direction of $p$.

But a patch about $p$ receives more of $q$'s surface as angle $\theta_{q,p}$ is more grazing.

$$B(p) = E(p) + \int_\Omega F_r(q \to p) \cdot B(q)dq$$

# Lambertian Emitters

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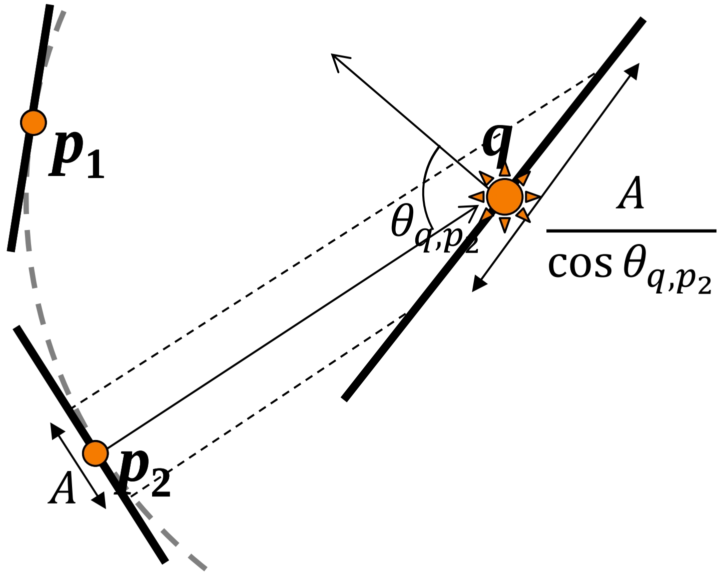



By assumption, the apparent brightness of $q$ is independent of the view direction of $p$.

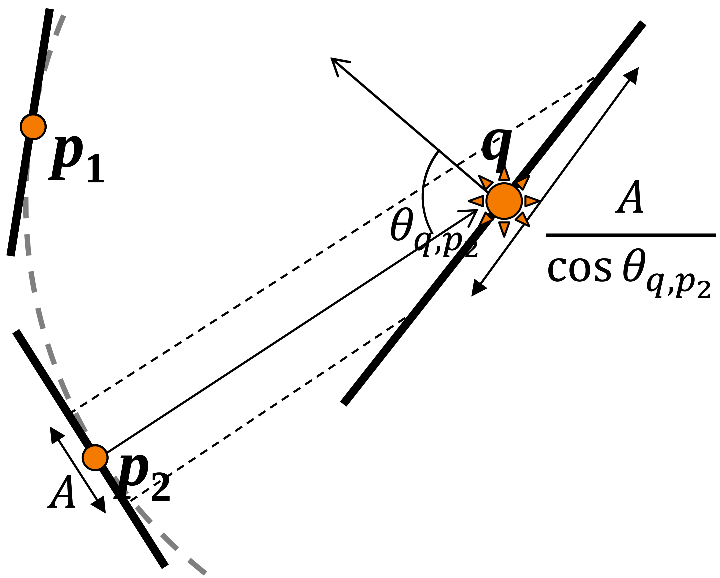
But a patch about $p$ receives more of $q$'s surface as angle $\theta_{q,p}$ is more grazing.

$\Rightarrow$ A patch of size $A$ about $p$ receives a patch of size $A/\cos\theta_{q,p}$ about $q$.

$$B(p) = E(p) + \int_\Omega F_r(q \to p) \cdot B(q)dq$$

# Lambertian Emitters

Given an emitter at point $q$, the apparent brightness point $p$ is independent of its orientation w.r.t. to $q$.




By assumption, the apparent brightness of $q$ is independent of the view direction of $p$.

But a patch about $p$ receives more of $q$'s surface as angle $\theta_{q,p}$ is more grazing.

$\Rightarrow$ A patch of size $A$ about $p$ receives a patch of size $A/\cos\theta_{q,p}$ about $q$.

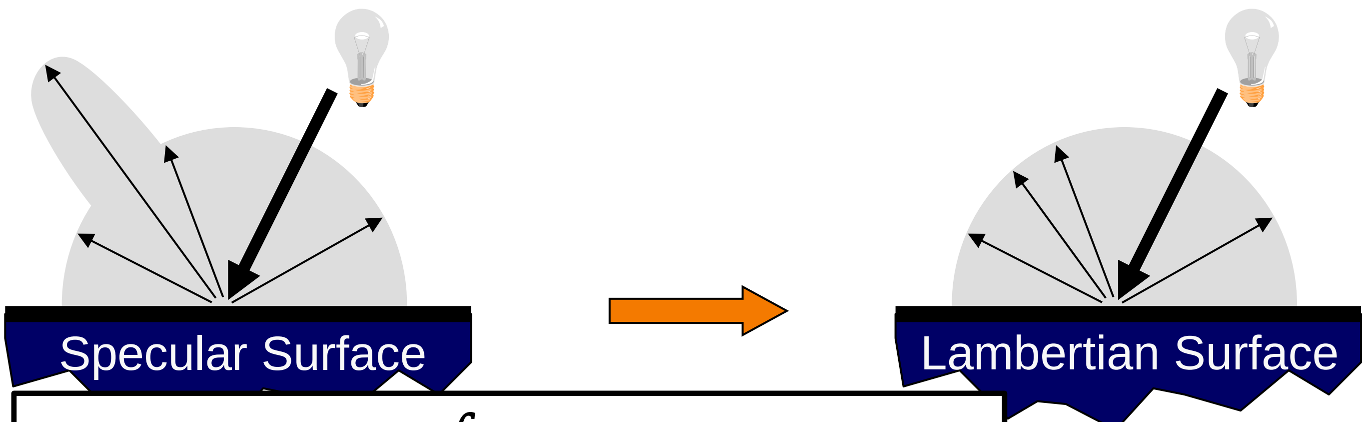
$\Rightarrow$ The amount of light emitted from a patch of area about $q$ in direction $p$ falls off as $\cos\theta_{q,p}$.

$$B(p) = E(p) + \int_\Omega F_r(q \to p) \cdot B(q) dq$$

# Radiosity

Lambertian assumption:

- The apparent brightness a patch of surface is constant (i.e. independent of the view direction).
  - » Emitters appear equally bright from all directions
  - » **Reflectors appear equally bright from all directions**




$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q)\, dq$$

# Lambertian Reflectors

How does the amount of light going from $q$ reflected through $p$ depend on:

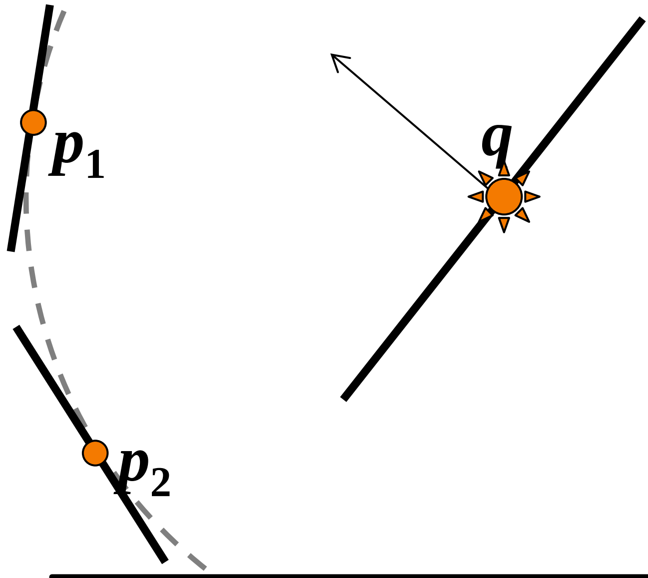
1. The direction to $p$ relative to the orientation at $q$,
2. The direction to $q$ relative to the orientation at $p$,
3. The distance between $p$ and $q$?

$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q) dq$$

# Lambertian Reflectors (1)

Treating $q$ as an emitter, the light emitted from $q$ in direction $p$ falls off as $\cos\theta_{q,p}$ – with $\theta_{q,p}$ the angle between the normal at $q$ and direction to $p$.

⇒ Reflected brightness at $p$ falls off as $\cos\theta_{q,p}$

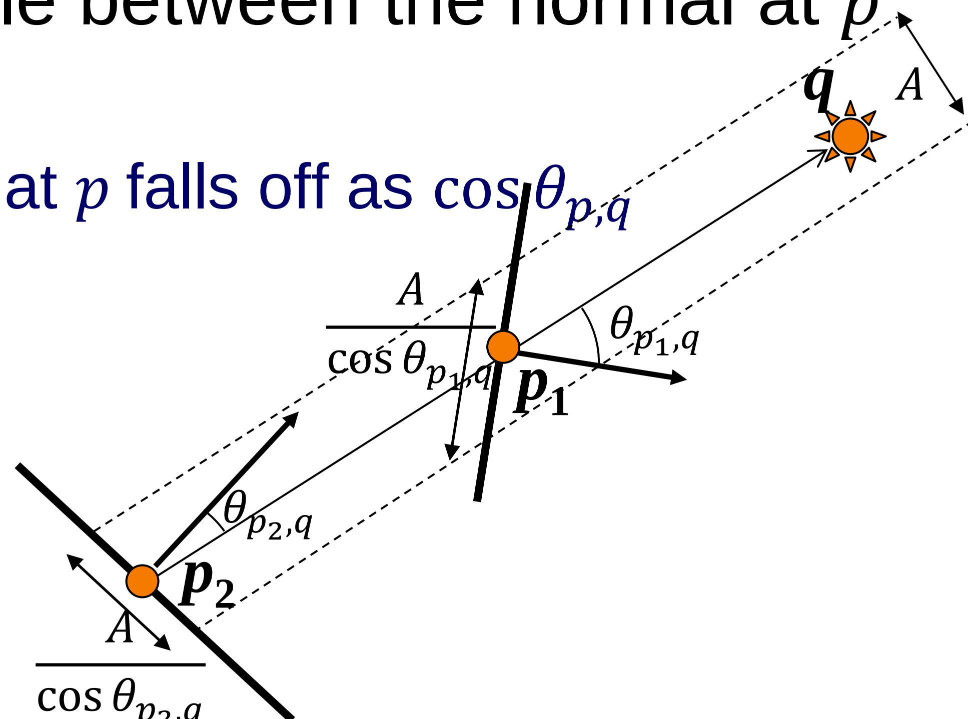



$$B(p) = E(p) + \int_\Omega F_r(q \to p) \cdot B(q) dq$$

# Lambertian Reflectors (2)

A beam of cross-sectional area $A$ leaving $q$ towards $p$, will spread out across a patch of area $A/\cos\theta_{p,q}$ at $p$ – with $\theta_{p,q}$ the angle between the normal at $p$ and direction to $q$.

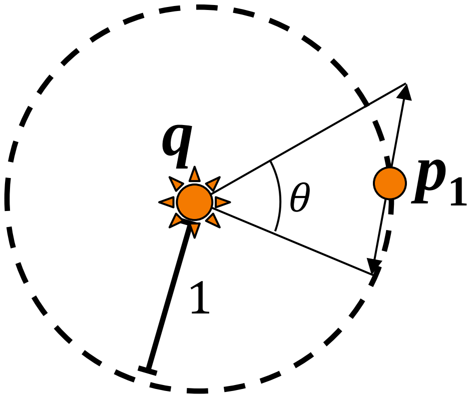
⇒ Reflected brightness at $p$ falls off as $\cos\theta_{p,q}$




$$B(p) = E(p) + \int_\Omega F_r(q \to p) \cdot B(q) dq$$

# Lambertian Reflectors (3)

The apparent brightness at $p$ is proportional to the subtended spherical angle by a unit area patch at $p$.

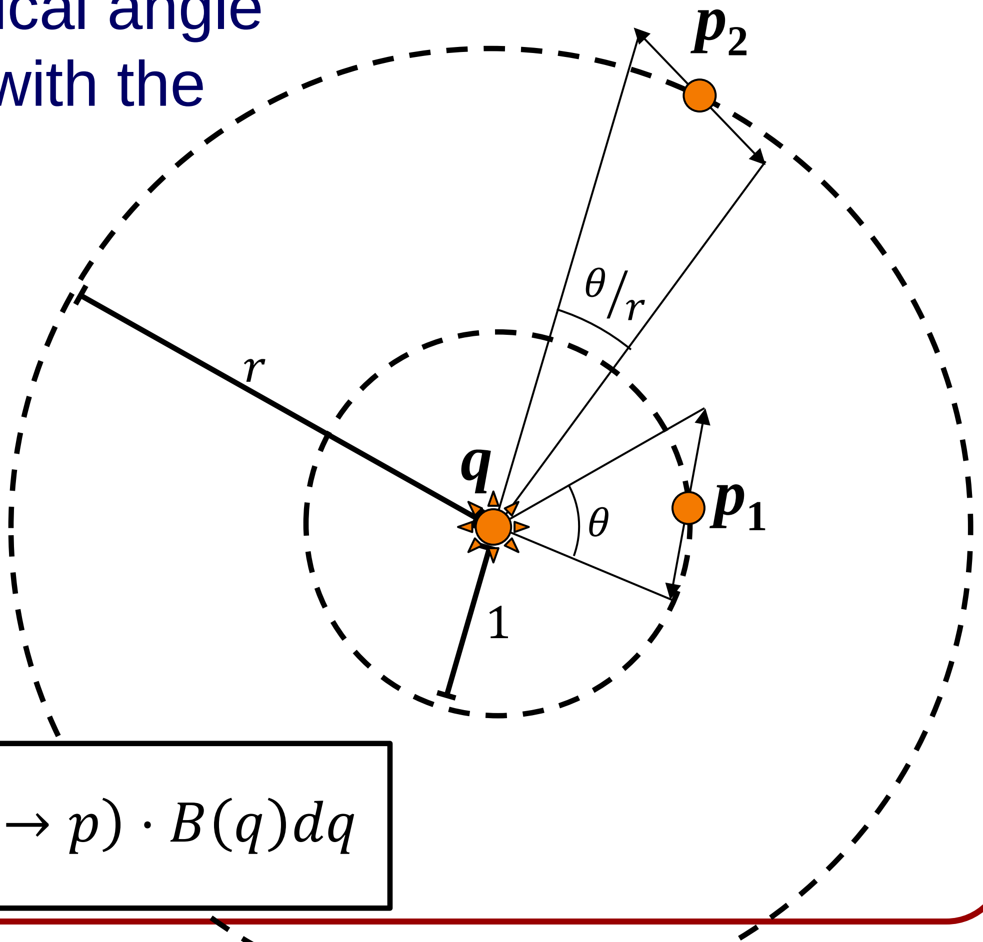



$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q)dq$$

# Lambertian Reflectors (3)

The apparent brightness at $p$ is proportional to the subtended spherical angle by a unit area patch at $p$.

- ⇒ The subtended spherical angle falls off quadratically with the distance of $p$ from $q$.
- ⇒ Perceived brightness decays as the square of the distance.

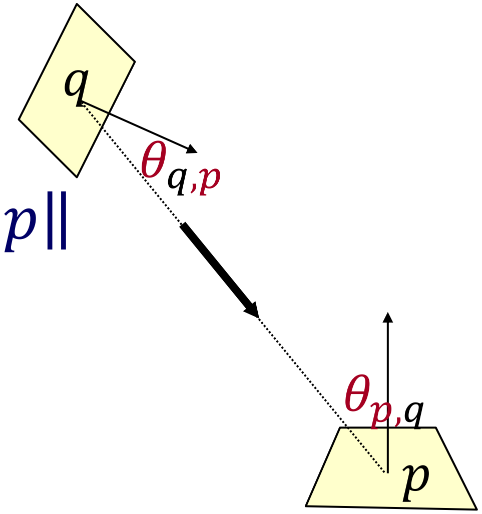



$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q)\,dq$$

# Lambertian Reflectors

⇒ The fraction of light from $q$ that is reflected off $p$ is determined by:

- The angle: $\theta_{q,p}$
- The angle: $\theta_{p,q}$
- The square distance from $q$ to $p$: $\|q-p\|$



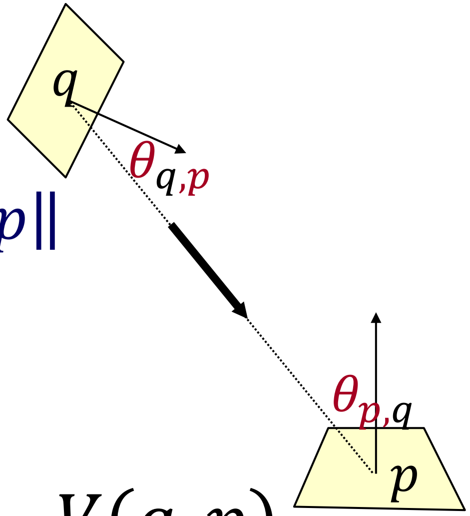

$$F_r(q \to p) = \frac{\cos\theta_{q,p} \cdot \cos\theta_{p,q}}{\|q-p\|^2}$$

$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q)\,dq$$

# Lambertian Reflectors

⇒ The fraction of light from $q$ that is reflected off $p$ is determined by:

- The angle: $\theta_{q,p}$
- The angle: $\theta_{p,q}$
- The square distance from $q$ to $p$: $\|q-p\|$
- The visibility of $p$ from $q$: $V(q,p)$
- The albedo at $p$: $\rho(p)$




$$F_r(q \to p) = \frac{\cos\theta_{q,p} \cdot \cos\theta_{p,q}}{\|q-p\|^2} \cdot \rho(p) \cdot V(q,p)$$

$$B(p) = E(p) + \int_\Omega F_r(q \to p) \cdot B(q)\,dq$$

# Radiosity

Lambertian assumption:

- The apparent brightness a patch of surface is constant (i.e. independent of the view direction).
  - » Emitters appear equally bright from all directions
  - » Reflectors appear equally bright from all directions

$$B(p) = E(p) + \rho(p)\int_{\Omega} V(q,p) \cdot \frac{\cos\theta_{q,p} \cdot \cos\theta_{p,q}}{\|q-p\|^2} \cdot B(q)dq$$

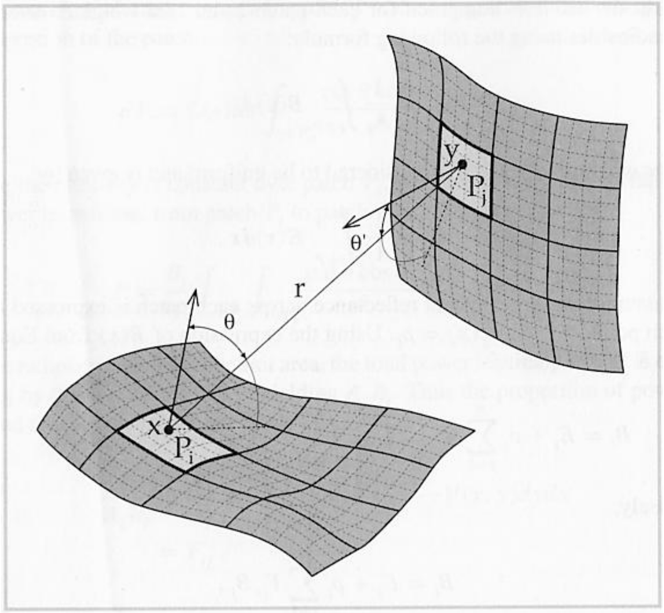
The radiosity equation

# Radiosity

Approximate the integral by decomposing surfaces into patches and doing a discrete summation:

$$B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} \cdot B_j$$

Form Factor

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For patch $i$:

- $B_i$: Total brightness
- $E_i$: Total emissivity
- $\rho_i$: Albedo

$$B(p) = E(p) + \rho(p) \int_{\Omega} V(q,p) \cdot \frac{\cos\theta_{q,p} \cdot \cos\theta_{p,q}}{\|q-p\|^2} \cdot B(q)dq$$

The radiosity equation

# Form Factor

The **form factor** $0 \leq F_{ij} \leq 1$ is the proportion of the power leaving patch $P_j$, received by patch $P_i$:

- Symmetry/Reciprocity: $A_j F_{ij} = A_i F_{ji}$
- Definiteness: $F_{ii} = 0$ unless the patch is concave
- Partition of unity: $\sum_i F_{ij} = 1$

# Radiosity

Approximate the integral by decomposing surfaces into patches and doing a discrete summation:

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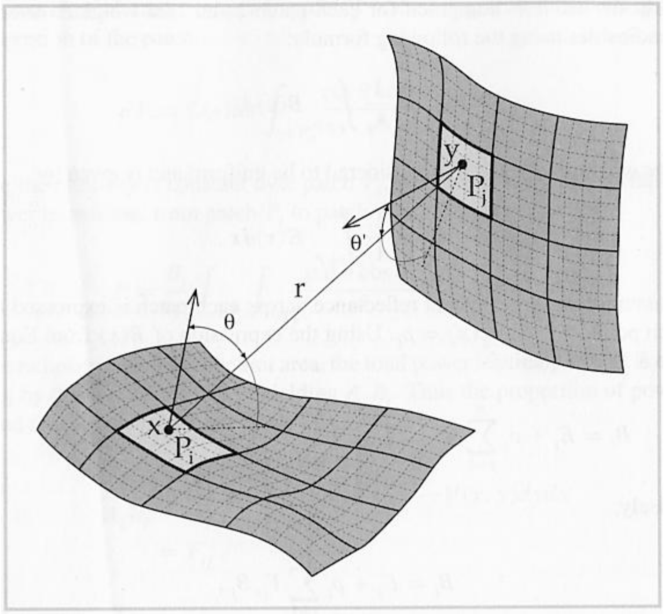
Form Factor



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This amounts to solving a linear system of equations

- $E_i$, $\rho_i$, and $F_{ij}$ are given
- $B_i$ are the unknowns.

# Radiosity

Re-ordering terms in the equation gives:

$$B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} \cdot B_j$$

$$\Downarrow$$

$$E_i = B_i - \rho_i \sum_{j=1}^{n} F_{ij} \cdot B_j$$

$$\Downarrow$$

$$\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix} = \begin{pmatrix} 1-\rho_1 \cdot F_{1,1} & -\rho_1 \cdot F_{2,1} & \cdots & -\rho_1 \cdot F_{n,1} \\ -\rho_2 \cdot F_{1,2} & 1-\rho_2 \cdot F_{2,2} & \cdots & -\rho_2 \cdot F_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n \cdot F_{1,n} & -\rho_n \cdot F_{2,n} & \cdots & 1-\rho_n \cdot F_{n,n} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix}$$

# Solving the System of Equations

- Challenges:
  - Size of matrix
  - Cost of computing form factors

$$
\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix} = \begin{pmatrix} 1-\rho_1\cdot F_{1,1} & -\rho_1\cdot F_{2,1} & \cdots & -\rho_1\cdot F_{n,1} \\ -\rho_2\cdot F_{1,2} & 1-\rho_2\cdot F_{2,2} & \cdots & -\rho_2\cdot F_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n\cdot F_{1,n} & -\rho_n\cdot F_{2,n} & \cdots & 1-\rho_n\cdot F_{n,n} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix}
$$

$$
\mathbf{e} = \mathbf{A} \cdot \mathbf{b}
$$

# Solving the System of Equations

- Solution methods:
  - ~~Invert the matrix – $O(n^3)$~~
  - Gathering methods – $O(n^2)$
  - Shooting methods – $< O(n^2)$

$$\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix} = \begin{pmatrix} 1-\rho_1 \cdot F_{1,1} & -\rho_1 \cdot F_{2,1} & \cdots & -\rho_1 \cdot F_{n,1} \\ -\rho_2 \cdot F_{1,2} & 1-\rho_2 \cdot F_{2,2} & \cdots & -\rho_2 \cdot F_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n \cdot F_{1,n} & -\rho_n \cdot F_{2,n} & \cdots & 1-\rho_n \cdot F_{n,n} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix}$$

$$\mathbf{e} = \mathbf{A} \cdot \mathbf{b}$$

# Gathering Iteration

Initialization:

- For each patch $P_i$, initialize its total radiosity to be equal to its total emissivity:

$$B_i = E_i$$

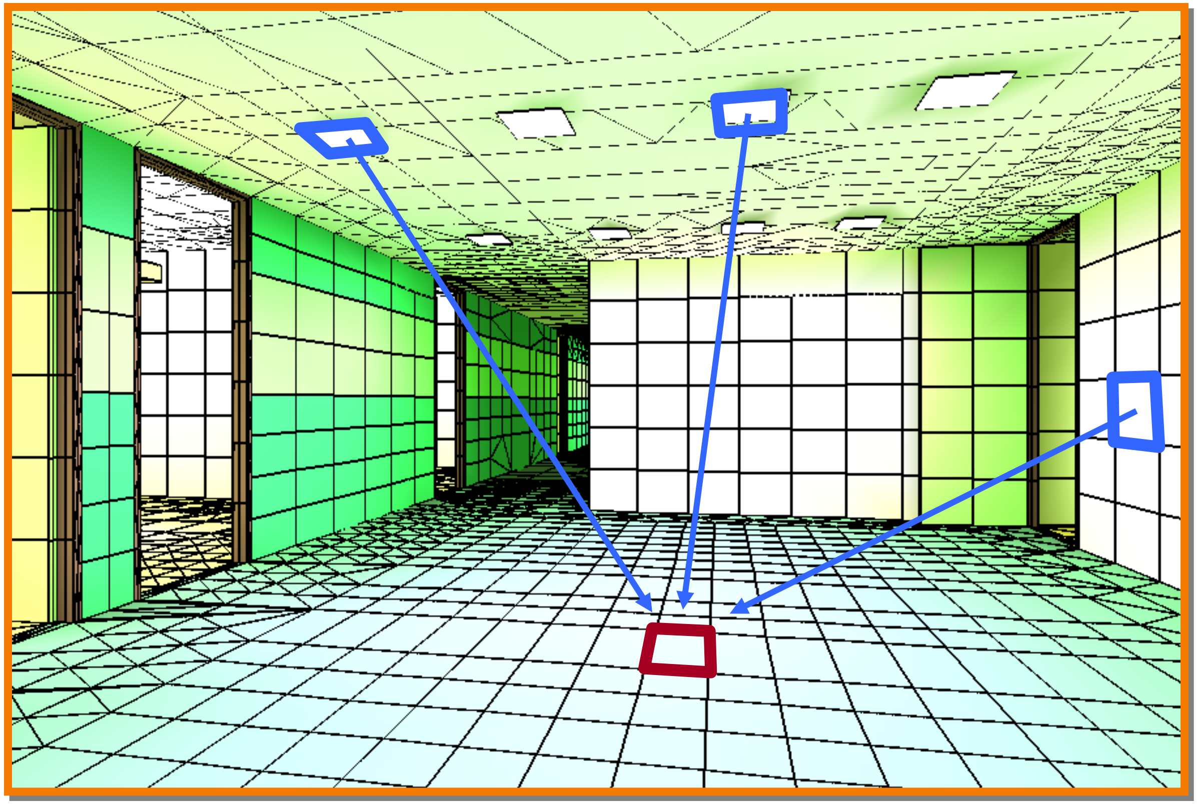
Iteration:

- At each iteration, update the values of each of the $B_i$ based on the values of all the other $B_j$:

$$B_i = E_i + \rho_i \sum_{j \neq i} F_{ij} \cdot B_j$$

# Gathering Iteration

- Geometric interpretation
  - Iteratively gather radiosity from elements



# Gathering Iteration

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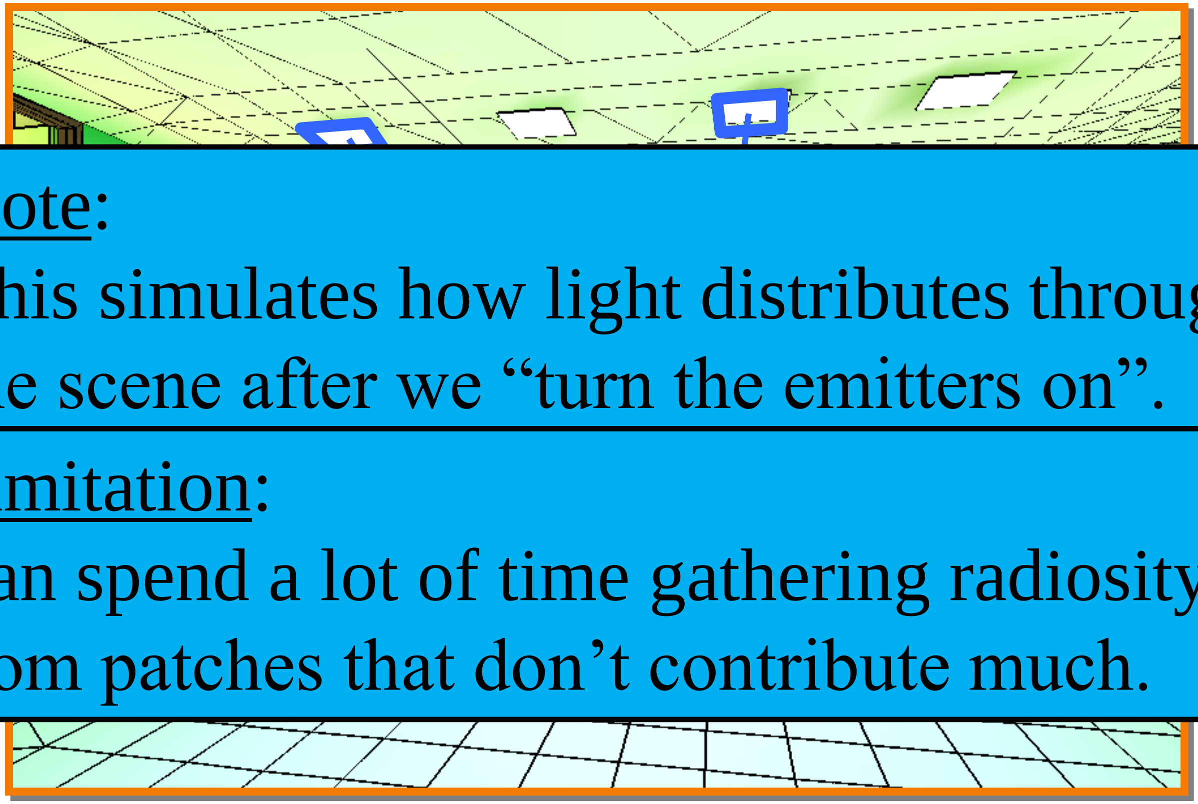
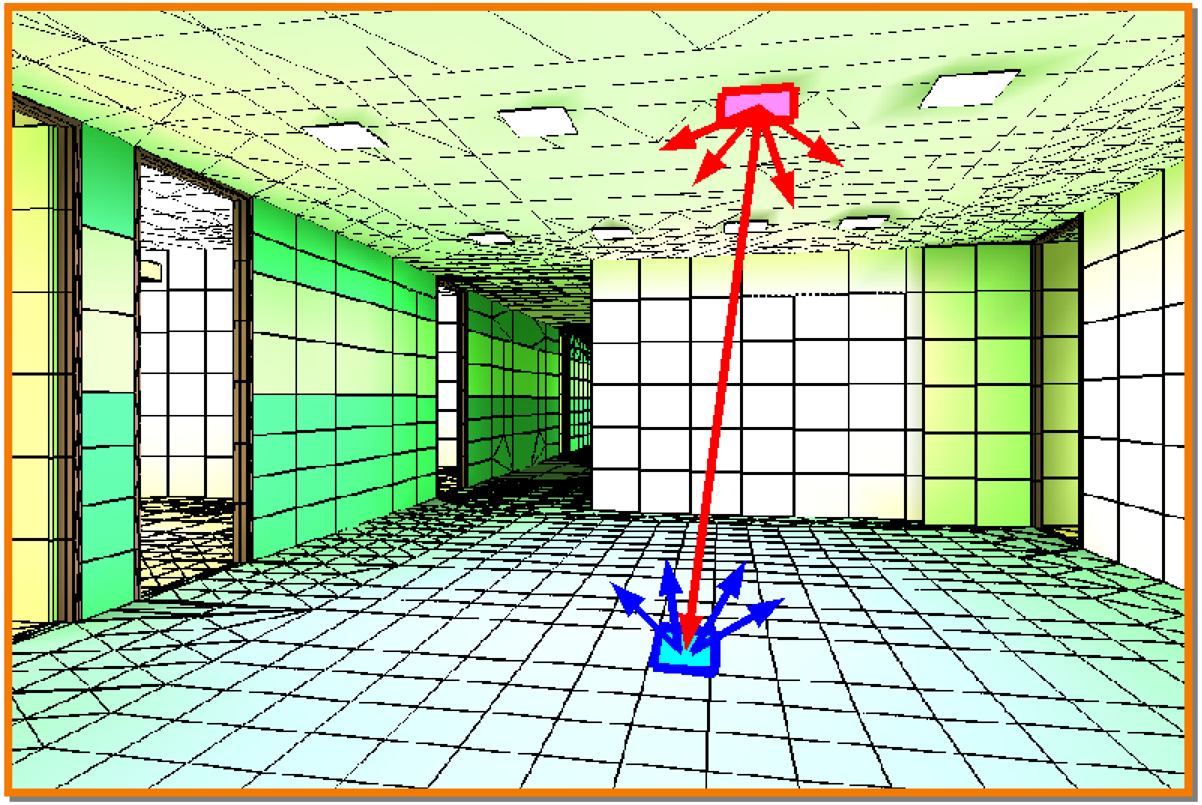


Note:
This simulates how light distributes through the scene after we "turn the emitters on".

Limitation:
Can spend a lot of time gathering radiosity from patches that don't contribute much.

# Shooting Iteration

- Geometric interpretation:
    - Iteratively shoot “unshot” radiosity from elements
    - Select shooters in order of unshot radiosity



# Summary

If we could, we would compute the lighting by recursively reflecting secondary rays in all directions to compute the brightness of a single point.

- Ray-Tracing:
  - Assume that surfaces are specular so that you only need to bounce in a single (specular) direction.
- Radiosity:
  - Assume that surfaces are Lambertian so that they reflect light in the same way in all directions.
- Reality:
  - Surfaces reflect in all directions, but not uniformly.