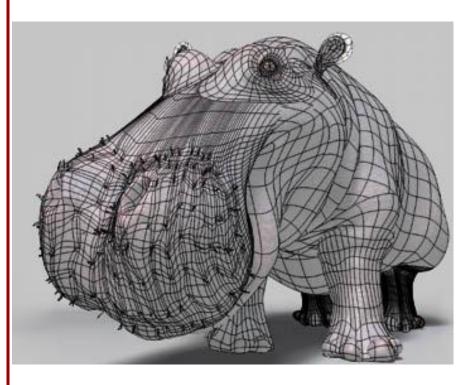


Michael Kazhdan

(601.457/657)



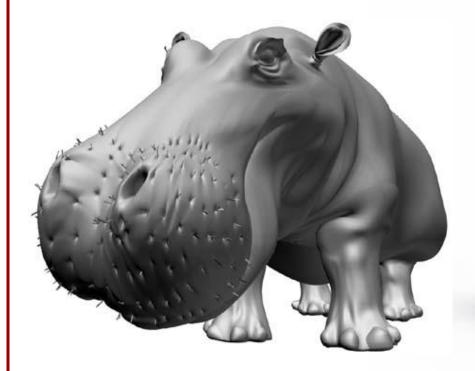




We know how to go from this...

to this
[J. Birn]







But what about this...

to this?

[J. Birn]



How do we draw surfaces with complex detail?



Target Model

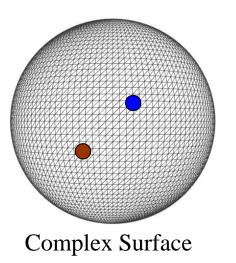


How do we draw surfaces with complex detail?

Direct:

Tessellate uniformly (finely)
 and then associate the
 appropriate material
 properties to each vertex



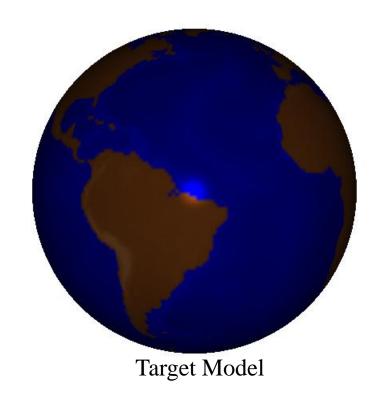


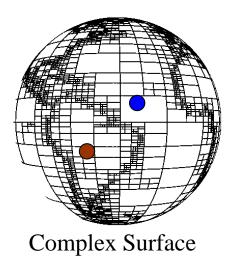


How do we draw surfaces with complex detail?

Direct:

 Tessellate adaptively and then associate the appropriate material properties to each vertex







How do we draw surfaces with complex detail?

Indirect:

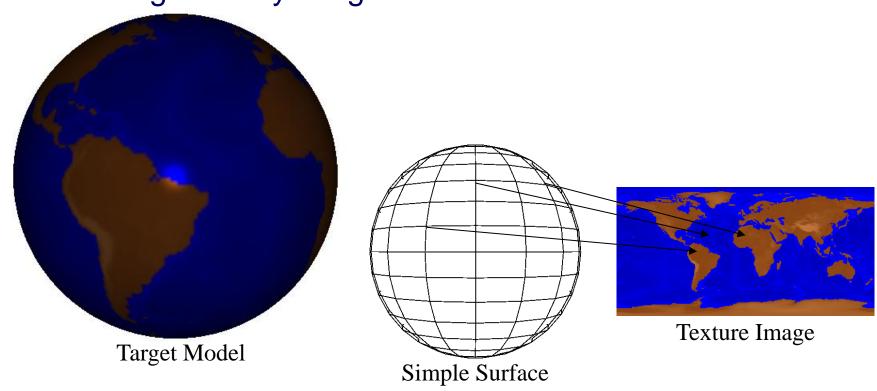
- Use a simple tessellation with an auxiliary texture image.
- Use texture coordinates stored at surface points to look up color values from the texture.



Texture Image
Simple Surface

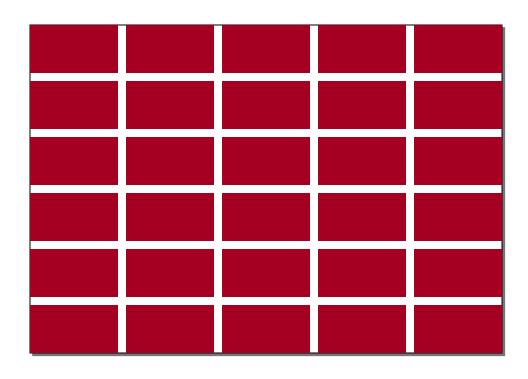


- Advantages:
 - The 3D model remains simple
 - It is easier to design/modify a texture image than it is to design/modify a signal on a surface.



Example: Brick Wall





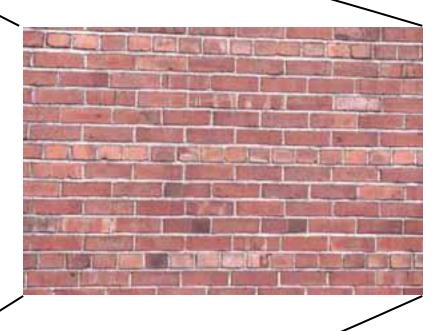
Example: Brick Wall



$$(s_v, t_v) = (0,1)$$
 $(s_v, t_v) = (1,1)$

+

$$(s_v, t_v) = (0.0)$$
 $(s_v, t_v) = (1.0)$



Textures (2 dimensions)



<u>Implementation</u>:

Associate a texture coordinate to each vertex v:

$$(s_v, t_v)$$
 with $(0 \le s_v, t_v \le 1)$

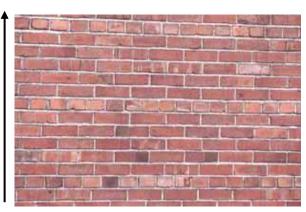
 When rasterizing, interpolate to get the texture coordinate to at a pixel:

$$(s_p, t_p)$$

• Sample the texture at (s_p, t_p) to get the color at p.

t.

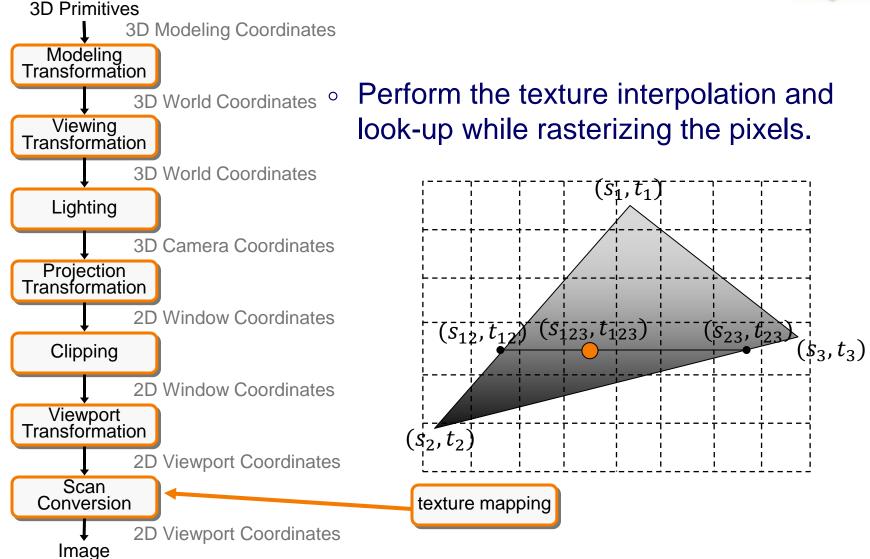
- Texture elements are called texels
- Often 4 bytes (rgba) per texel



S

3D Rendering Pipeline

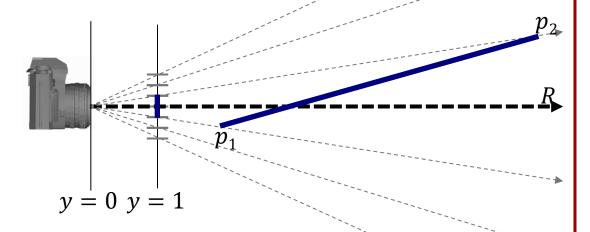




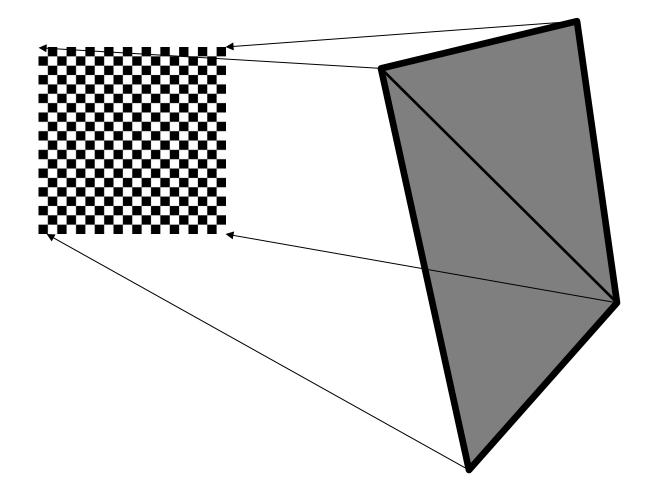


Recall (Perspective Divide):

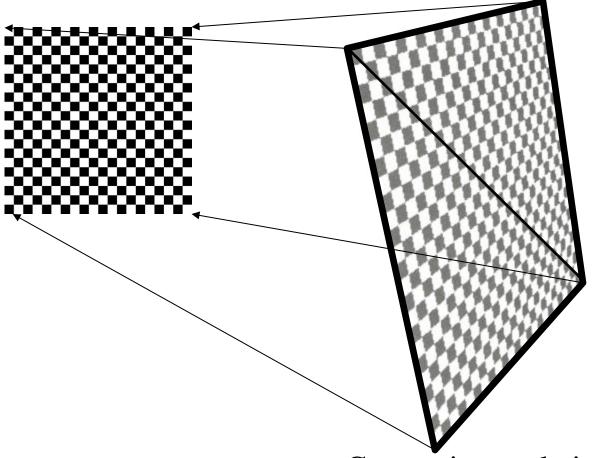
When performing scan-line rasterization and interpolating data from vertices, we need to compute the weights in 3D space.







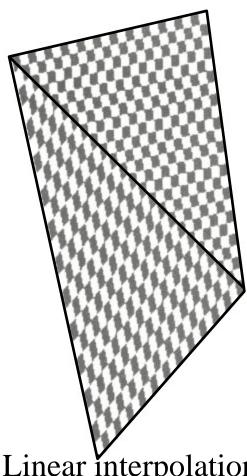




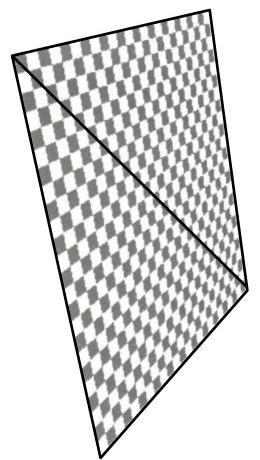
Correct interpolation of texture coordinates with perspective divide

Hill Figure 8.42





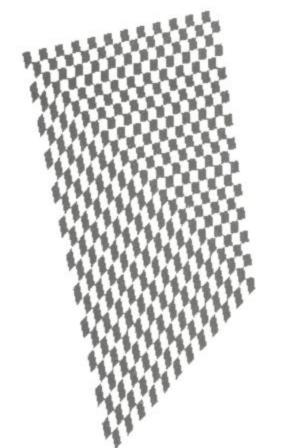
Linear interpolation of texture coordinates w/o perspective divide

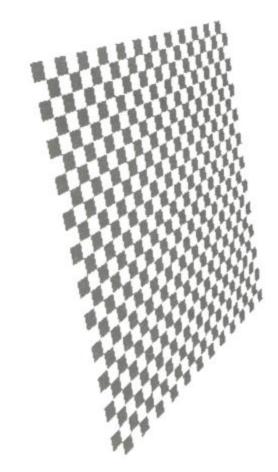


Correct interpolation of texture coordinates w/ perspective divide

Hill Figure 8.42







Linear interpolation of texture coordinates w/o perspective divide

Correct interpolation of texture coordinates w/ perspective divide

Hill Figure 8.42

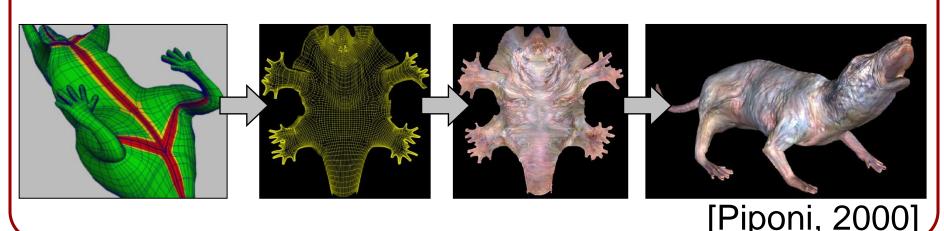
Overview



- Texture mapping methods
 - Parameterization
 - Sampling
- Texture mapping applications
 - Modulation textures
 - Illumination mapping
 - Bump mapping
 - Environment mapping
 - Shadow maps

Map to a 2D Domain (w/ Added Cuts)

- Introduce cuts to give the surface a disk topology
- Map the cut surface to the 2D plane
- Assign texture coordinates in the plane
- ✓ Good cut placement can reduce distortion
- Need to ensure cross-seam continuity
- * Have to contend with distortion

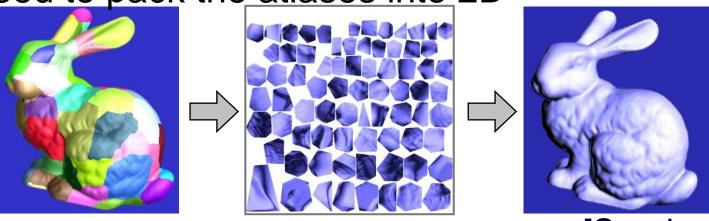


Texture Atlases



- Decompose the surface into multiple charts
- Map each chart to the 2D plane
- Assign texture coordinates in the plane
- ✓ Less distortion in the mapping
- Harder to ensure cross-seam continuity

Need to pack the atlases into 2D



[Sander, 2001]

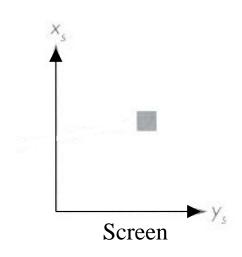
Overview



- Texture mapping methods
 - Parameterization
 - Sampling
- Texture mapping applications
 - Modulation textures
 - Illumination mapping
 - Bump mapping
 - Environment mapping
 - Shadow maps



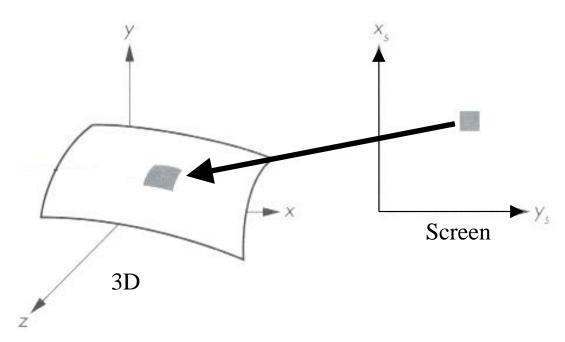
Given pixel on a screen:





Given pixel on a screen:

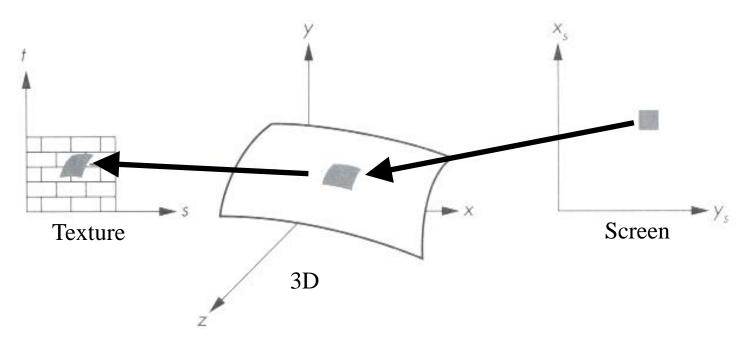
1. Determine the corresponding surface patch





Given pixel on a screen:

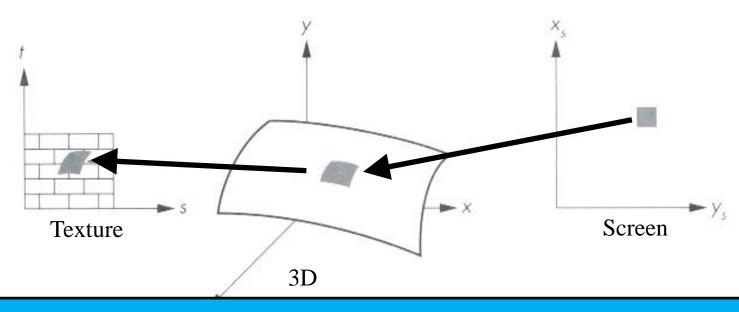
- 1. Determine the corresponding surface patch
- 2. Determine the corresponding texture patch





Given pixel on a screen:

- 1. Determine the corresponding surface patch
- 2. Determine the corresponding texture patch

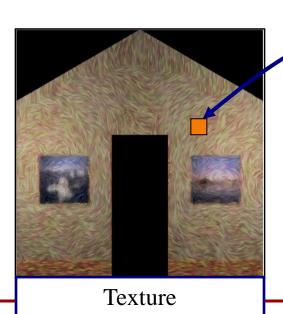


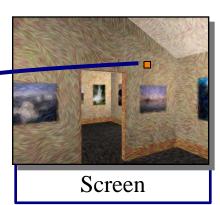
While the true shape of the texture patch mapping to a screen pixel may be hard to compute, we can approximate using the Jacobian.



Given pixel on a screen:

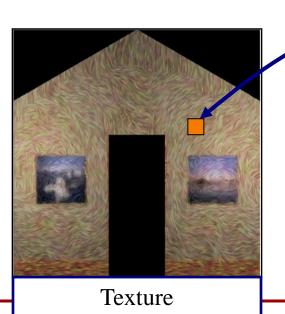
- 1. Determine the corresponding surface patch
- 2. Determine the corresponding texture patch
- 3. Average texel values over the texture patch



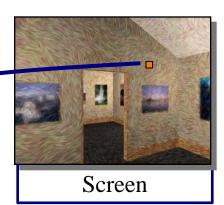




- Size of texture patch depends on the deformation
 - Computation is linear in the size of the pixel footprint
- Can pre-filter images for better performance
 - MIP (Multum In Parvo) maps
 - Summed area tables

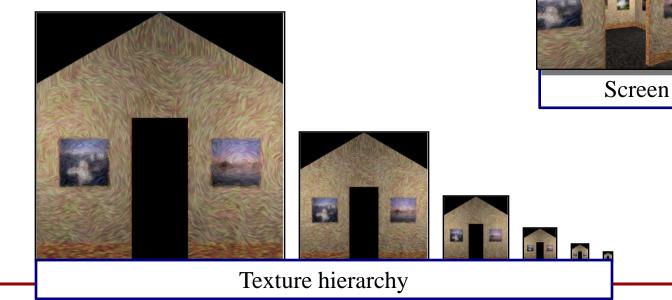


Average over many pixels





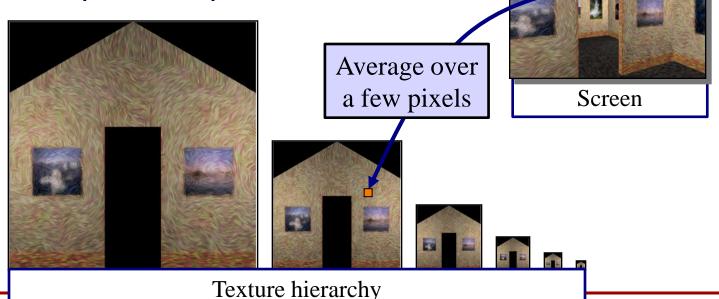
 Pre Processing: Compute a hierarchy of successively down-sampled texture images





- Pre Processing: Compute a hierarchy of successively down-sampled texture images
- Run-time: Sample the closest MIP map level(s)
 - Easy for hardware

 Computation is constant in the size of the pixel footprint





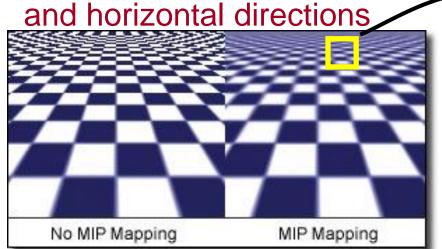
- Pre Processing: Compute a hierarchy of successively down-sampled texture images
 - ✓ Storage is only 4/3 the size of the input image

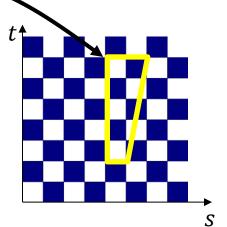




- Pre Processing: Compute a hierarchy of successively down-sampled texture images
 - ✓ Storage is only 4/3 the size of the input image
- Run-time: Sample the closest MIP map level(s)
 - * This type of filtering is isotropic:

» Assumes identical compression along the vertical





Again: we're trading aliasing for blurring!

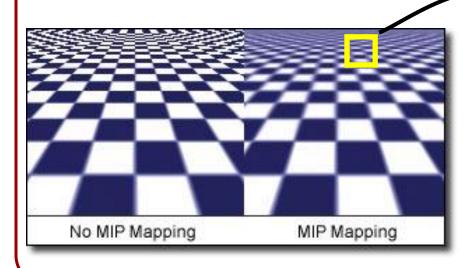
Summed-Area Tables

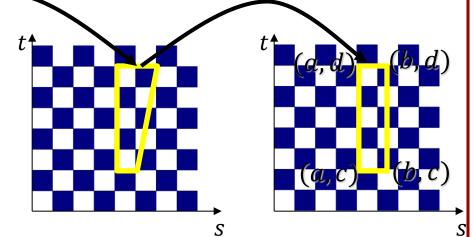


Key Idea:

 Approximate the summation/integration over an arbitrary region by a summation/integration over an axis-aligned rectangle:

$$Sum([a,b] \times [c,d]) = \int_a^b \int_c^d f(x,y) \, dy \, dx$$



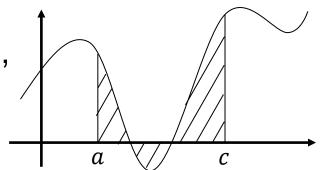




Integration:

Given a function f(x) and interval [a, c], the integral of f over the interval is:

$$\int_{a}^{c} f(x) dx$$



Naïve Approach:

Pre-compute $S(a,b) \equiv \int_a^b f(x) dx$ and evaluate that

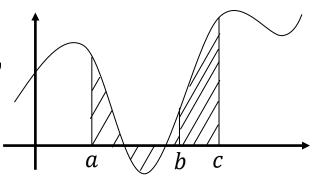
- ✓ Fast (constant time) look up
- **×** Replaces 1D function *f* with 2D function *S*.



Integration:

Given a function f(x) and interval [a, c], the integral of f over the interval is:

$$\int_{a}^{c} f(x) dx$$



Recall:

For any point $b \in [a, c]$ in the interval, we have:

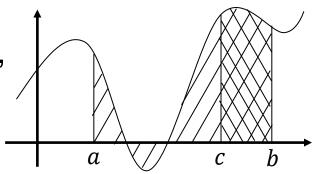
$$\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$$



Integration:

Given a function f(x) and interval [a, c], the integral of f over the interval is:

$$\int_{a}^{c} f(x) dx$$



Recall:

For any point $b \in [a, c]$ in the interval, we have:

$$\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$$

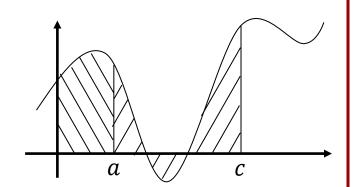
This is true even if c is outside the interval since:

$$\int_{a}^{b} f(x) dx = -\int_{b}^{a} f(x) dx$$



Approach:

Replace the integral over an interval, with two variable end-points with the difference between integrals with one variable end-point:



$$\int_{a}^{c} f(x)dx = \int_{0}^{c} f(x)dx - \int_{0}^{a} f(x)dx$$

 \Rightarrow Replace a look-up in the 2D function $S(a,c) = \int_a^c f(x) dx$ with two look-ups in the 1D function $S_0(b) = \int_0^b f(x) dx$



Integration:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds$$



Integration:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{a}^{b} \left(\int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$



Integration:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{a}^{b} \left(\int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$

$$= \int_{0}^{b} \left(\int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds - \int_{0}^{a} \left(\int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$



Integration:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{a}^{b} \left(\int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$

$$= \int_{0}^{b} \left(\int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds - \int_{0}^{a} \left(\int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$

$$= \int_{0}^{b} \int_{0}^{d} f(s,t)dt \, ds - \int_{0}^{b} \int_{0}^{c} f(s,t)dt \, ds - \int_{0}^{a} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{a} \int_{0}^{c} f(s,t)dt \, ds$$



Precomputing the 2D function:

$$S_{(0,0)}(x,y) \equiv \int_0^x \int_0^y f(s,t) dt ds$$

lets us evaluate integrals with a constant number of look-ups:

$$\int_{a}^{b} \int_{c}^{d} f(s,t) dt ds = S_{(0,0)}(b,d) - S_{(0,0)}(b,c) - S_{(0,0)}(a,d) + S_{(0,0)}(a,c)$$

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{a}^{b} \left(\int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$

$$= \int_{0}^{b} \left(\int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds - \int_{0}^{a} \left(\int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$

$$= \int_{0}^{b} \int_{0}^{d} f(s,t)dt \, ds - \int_{0}^{b} \int_{0}^{c} f(s,t)dt \, ds - \int_{0}^{a} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{a} \int_{0}^{c} f(s,t)dt \, ds$$



Integration:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{0}^{b} \int_{0}^{d} f(s,t)dt \, ds - \int_{0}^{b} \int_{0}^{c} f(s,t)dt \, ds - \int_{0}^{a} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{a} \int_{0}^{c} f(s,t)dt \, ds$$



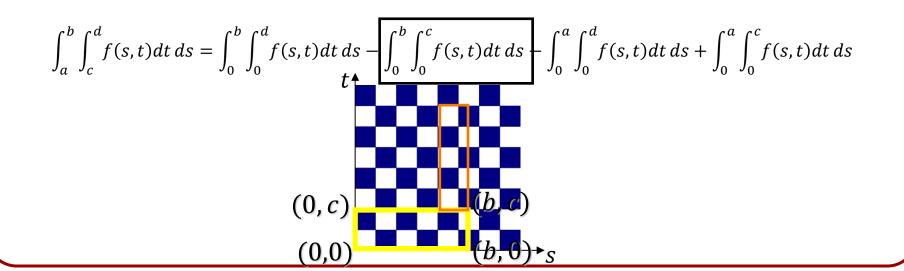
Integration:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{0}^{b} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{b} \int_{0}^{c} f(s,t)dt \, ds - \int_{0}^{a} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{a} \int_{0}^{c} f(s,t)dt \, ds$$

$$(0,d)$$



Integration:





Integration:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{0}^{b} \int_{0}^{d} f(s,t)dt \, ds - \int_{0}^{b} \int_{0}^{c} f(s,t)dt \, ds - \int_{0}^{a} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{a} \int_{0}^{c} f(s,t)dt \, ds$$

$$(0,d)$$



Integration:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{0}^{b} \int_{0}^{d} f(s,t)dt \, ds - \int_{0}^{b} \int_{0}^{c} f(s,t)dt \, ds - \int_{0}^{a} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{a} \int_{0}^{c} f(s,t)dt \, ds$$

$$(0,c)$$



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

 Each summed-area table texel is the sum of all input texels below and to the left

Input image

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

 Each summed-area table texel is the sum of all input texels below and to the left

Input image

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

1		



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

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Input image

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

1	3	



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

 Each summed-area table texel is the sum of all input texels below and to the left

Input image

	0-0-		
1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

1	3	4	



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

 Each summed-area table texel is the sum of all input texels below and to the left

Input image

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

1	3	4	7



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

 Each summed-area table texel is the sum of all input texels below and to the left

Input image

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

5			
1	3	4	7



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

 Each summed-area table texel is the sum of all input texels below and to the left

Input image

			<u> </u>
1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

5	9		
1	3	4	7



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

 Each summed-area table texel is the sum of all input texels below and to the left

Input image

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7



Example:

Compute the average in the rectangle $[1,3] \times [2,3]$.

⇒ Compute the sum and divide by the area

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

Input image



Example:

Compute the average in the rectangle $[1,3] \times [2,3]$.

 \Rightarrow Compute the sum and divide by the area $Sum([1,3] \times [2,3]) = S_{(0,0)}(3,3)$

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

Input image



Example:

Compute the average in the rectangle $[1,3] \times [2,3]$.

⇒ Compute the sum and divide by the area

$$Sum([1,3] \times [2,3]) = S_{(0,0)}(3,3) - S_{(0,0)}(0,3)$$

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

Input image



Example:

Compute the average in the rectangle $[1,3] \times [2,3]$.

⇒ Compute the sum and divide by the area

$$Sum([1,3] \times [2,3]) = S_{(0,0)}(3,3) - S_{(0,0)}(0,3) - S_{(0,0)}(3,1)$$

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

Input image



Example:

Compute the average in the rectangle $[1,3] \times [2,3]$.

⇒ Compute the sum and divide by the area

$$Sum([1,3] \times [2,3]) = S_{(0,0)}(3,3) - S_{(0,0)}(0,3) - S_{(0,0)}(3,1) + S_{(0,0)}(0,1)$$

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

Input image



Example:

Compute the average in the rectangle $[1,3] \times [2,3]$.

⇒ Compute the sum and divide by the area

$$Sum([1,3] \times [2,3]) = S_{(0,0)}(3,3) - S_{(0,0)}(0,3) - S_{(0,0)}(3,1) + S_{(0,0)}(0,1)$$

$$= 26 - 6 - 14 + 5 = 11$$

$$Average([1,3] \times [2,3]) = \frac{Sum([1,3] \times [2,3])}{Area([1,3] \times [2,3])} = \frac{11}{6}$$

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

Input image



- Precompute the values of the integral
- ✓ Constant time averaging, regardless of rectangle size
- ★ If the input image has values in the range [0,255] (i.e. one byte per channel), the summed area table can have values in the range [0,255 · width · height]

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

Input image

Overview

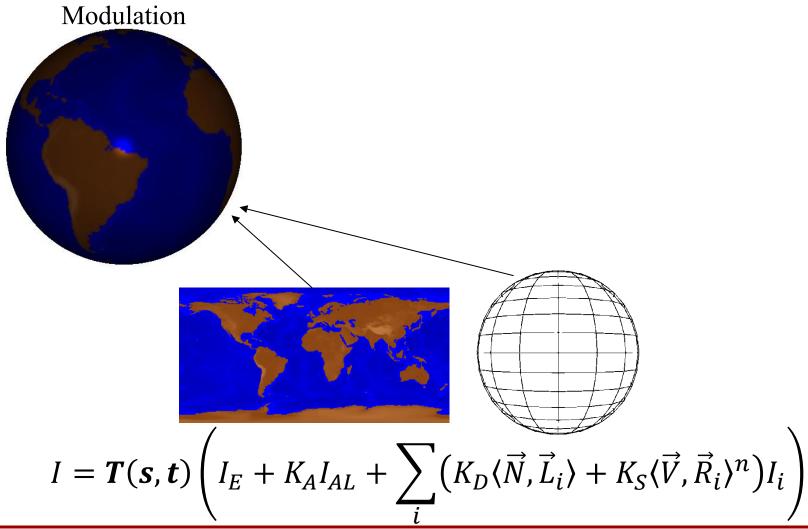


- Texture mapping methods
 - Parameterization
 - Sampling
- Texture mapping applications
 - Modulation textures
 - Illumination mapping
 - Bump mapping
 - Environment mapping
 - Shadow mapping

Modulation textures



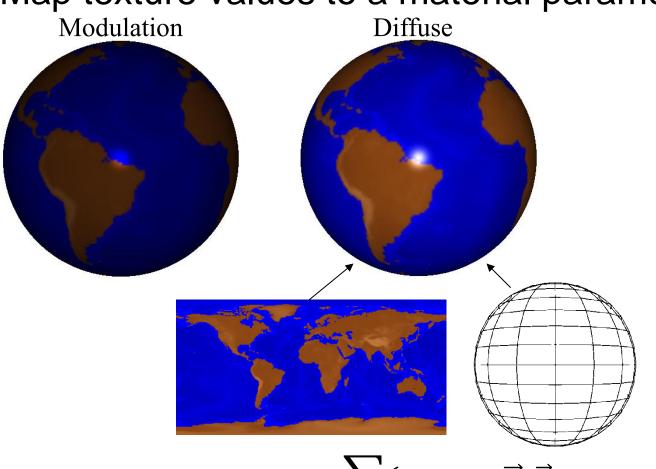
Map texture values to scale factor



Illumination Mapping



Map texture values to a material parameter



$$I = I_E + K_A I_{AL} + \sum_{i} (T(s, t) \langle \vec{N}, \vec{L}_i \rangle + K_S \langle \vec{V}, \vec{R}_i \rangle^n) I_i$$

Illumination Mapping



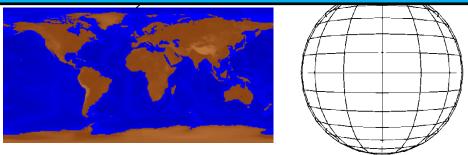
Map texture values to a material parameter

Modulation

Diffuse

Note that we need to evaluate the texture at each pixel but can still use the interpolated lighting values $\langle \vec{N}, \vec{L}_i \rangle$

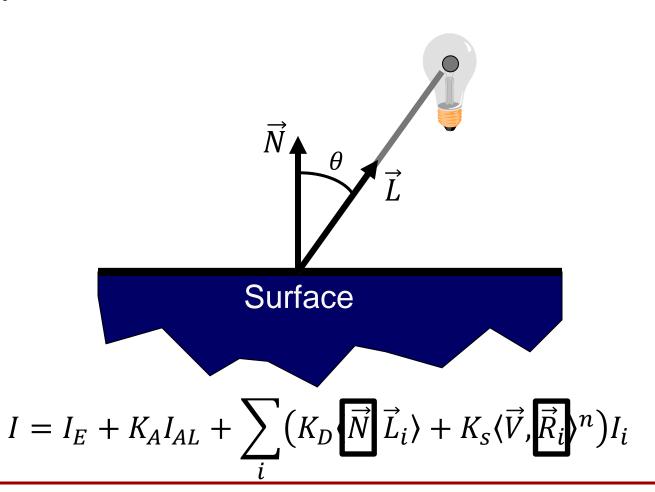
This requires the graphics card to separately store the diffuse component of the lighting at each vertex



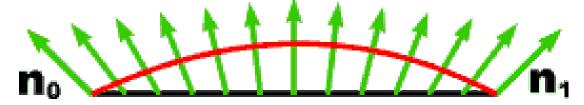
$$I = I_E + K_A I_{AL} + \sum_{i} (T(s, t) \langle \vec{N}, \vec{L}_i \rangle + K_S \langle \vec{V}, \vec{R}_i \rangle^n) I_i$$



 Recall that many parts of our lighting calculation depend on surface normals







Phong shading performs per-pixel lighting calculations with the interpolated normal

approximates a smoothly curved surface



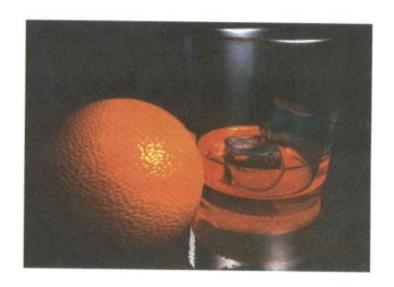
Bump maps encode the normals in the texture

approximates a more complex undulating surface

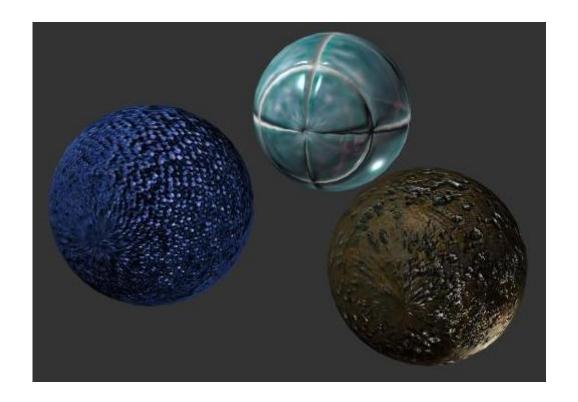
P. Rheingans











Note that bump mapping does not change object silhouette

Siggraph.org



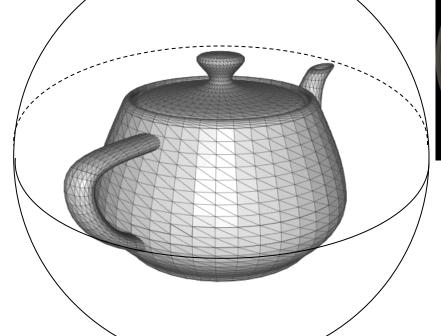
Simulate reflective materials through pre-computed texture maps representing the environment around a shape.



 Generate a spherical/cubic map of the environment around the model.

Texture coordinates are computed <u>dynamically</u>

through reflection





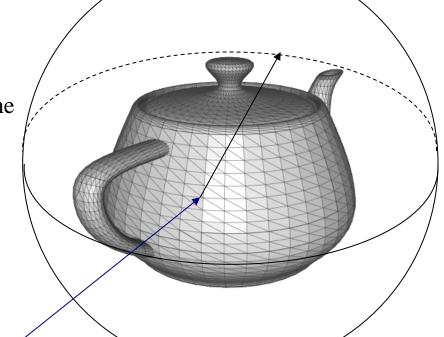


 Generate a spherical/cubic map of the environment around the model.

Texture coordinates are computed <u>dynamically</u>

through reflection

Set the texture coordinates based on the direction of the reflected view direction



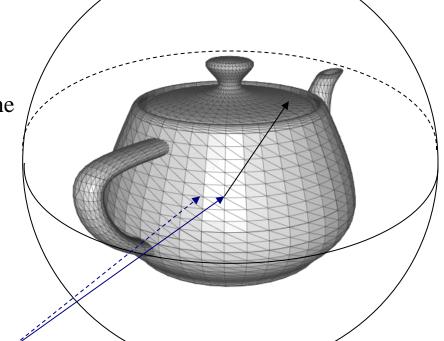


 Generate a spherical/cubic map of the environment around the model.

Texture coordinates are computed <u>dynamically</u>

through reflection

Set the texture coordinates based on the direction of the reflected view direction



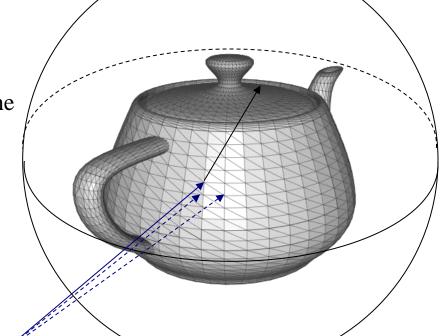


 Generate a spherical/cubic map of the environment around the model.

Texture coordinates are computed <u>dynamically</u>

through reflection

Set the texture coordinates based on the direction of the reflected view direction





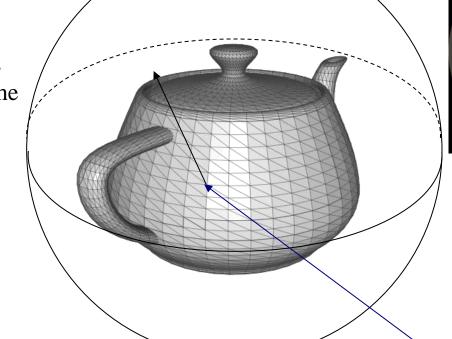
 Generate a spherical/cubic map of the environment around the model.

Texture coordinates are computed <u>dynamically</u>

through reflection

Set the texture coordinates based on the direction of the reflected view direction

At the same triangle, changing the position of the camera changes the texture coordinates.





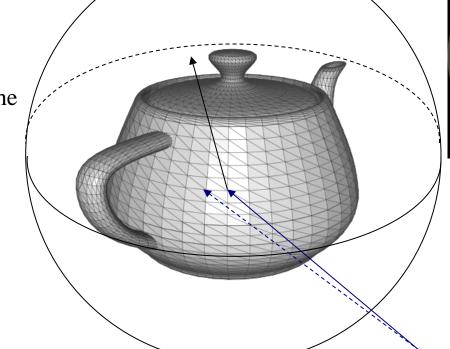
 Generate a spherical/cubic map of the environment around the model.

Texture coordinates are computed <u>dynamically</u>

through reflection

Set the texture coordinates based on the direction of the reflected view direction

At the same triangle, changing the position of the camera changes the texture coordinates.





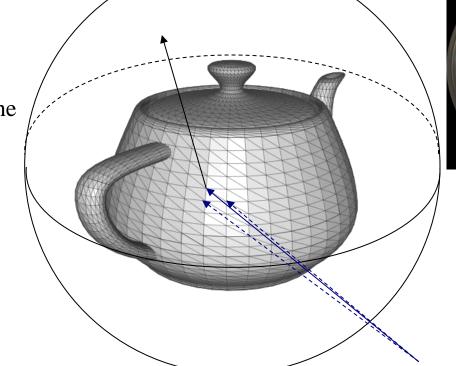
 Generate a spherical/cubic map of the environment around the model.

Texture coordinates are computed <u>dynamically</u>

through reflection

Set the texture coordinates based on the direction of the reflected view direction

At the same triangle, changing the position of the camera changes the texture coordinates.





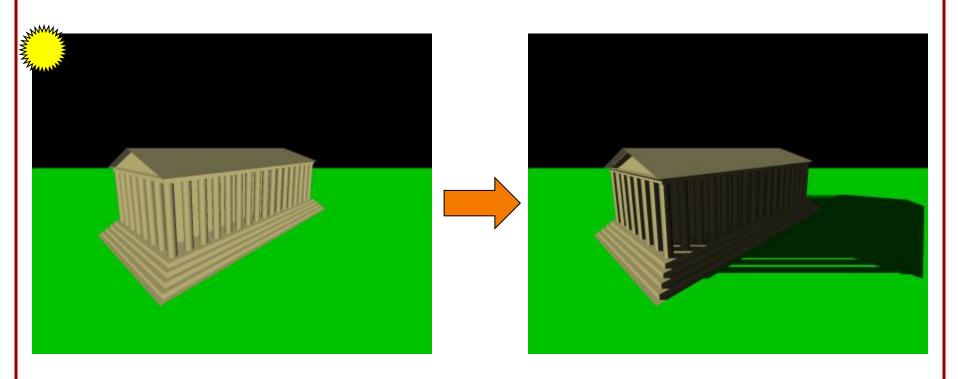
Texture coordinates are computed <u>dynamically</u> through reflection of the view direction through the surface normal



P. Debevec



Test if surface is visible to the light when computing the contribution to the lighting equation.



Images courtesy of https://en.wikipedia.org/wiki/Shadow_mapping

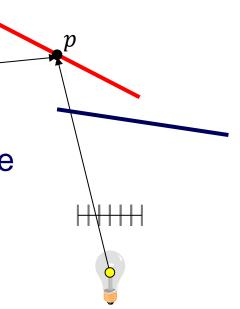


Q: Is the point that is seen by the camera visible (i.e. not in shadow) to the light?

A: The point p is visible if:

 The light can "see" the point p.

⇒ Rendering the scene from the light's perspective, p's z-coordinate is the value stored in the z-buffer.



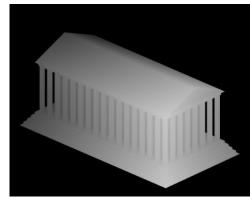


Test if surface is visible to the light when computing contribution to the lighting equation.

- Render the scene from the light's perspective and read back the z-buffer/shadow map.
- For each pixel in the camera view, compute its z-coordinate relative to the light



Camera view

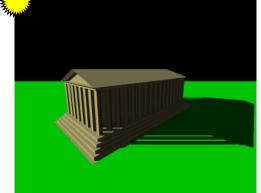


Shadow map

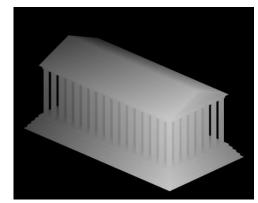


Test if surface is visible to the light when computing contribution to the lighting equation.

- Render the scene from the light's perspective and read back the z-buffer/shadow map.
- For each pixel in the camera view, compute its z-coordinate relative to the light
 - If it's further back than the value in the shadow map, it's in shadow
 - Otherwise, it's illuminated



Camera view

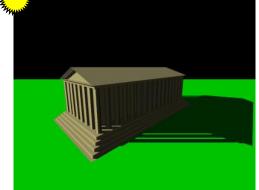


Shadow map

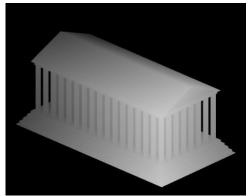


Test if surface is visible to the light when computing contribution to the lighting equation.

- The projection used for rendering from the light-source depends on the type of light:
 - Directional → Parallel
 - Point → Perspective
- Need to use multiple shadow maps if there are multiple lights in the scene



Camera view



Shadow map