

3D Polygon Rendering Pipeline

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(601.457/657)

3D Polygon Rendering



 Many applications require interactive rendering of 3D polygons with direct illumination

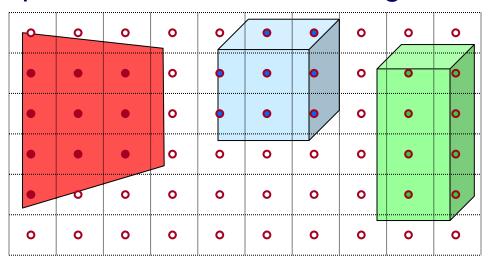


God of War (Santa Monica Studio, 2018)

Ray Casting



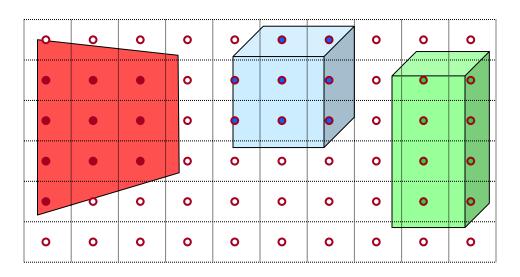
- For each sample:
 - Construct ray <u>from the camera into the scene</u>
 - Find first surface intersected by ray through pixel
 - Compute color of sample based on surface radiance
 - Send 2D pixels into the scene and get color



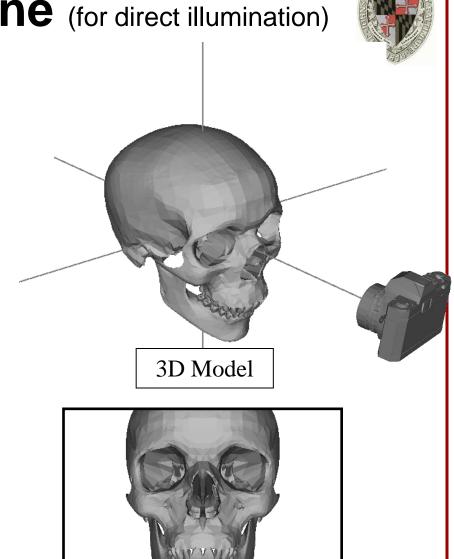
3D Polygon Rendering



- For each primitive:
 - Send 3D points to the camera and set the pixel color

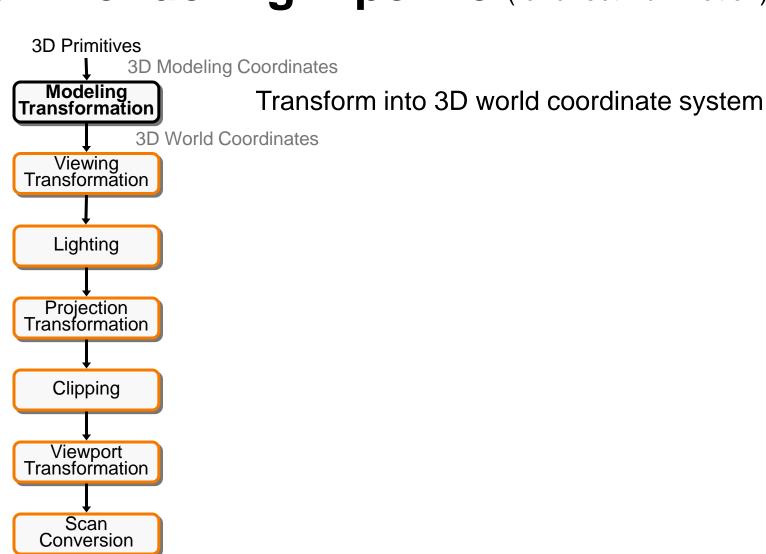




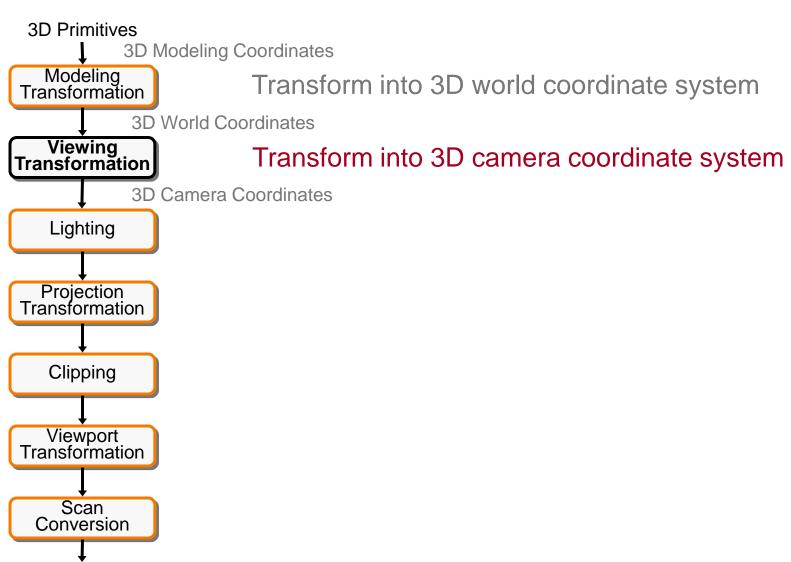


2D Viewport

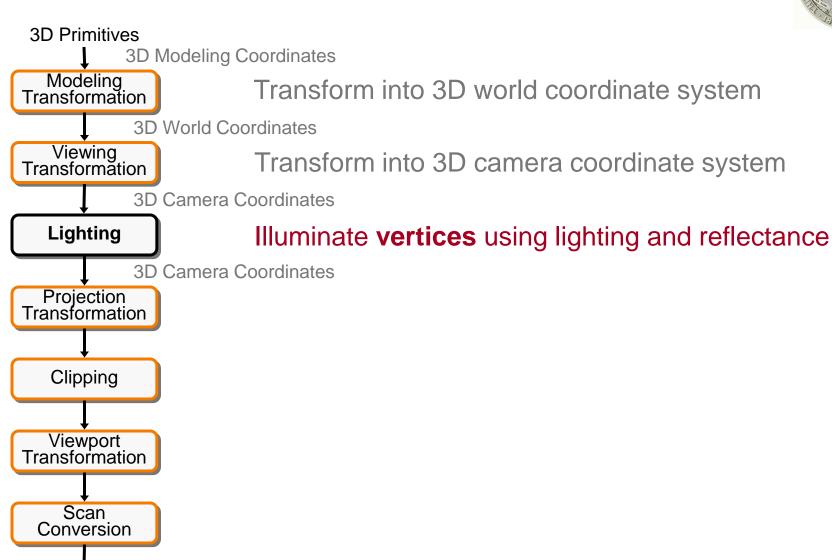




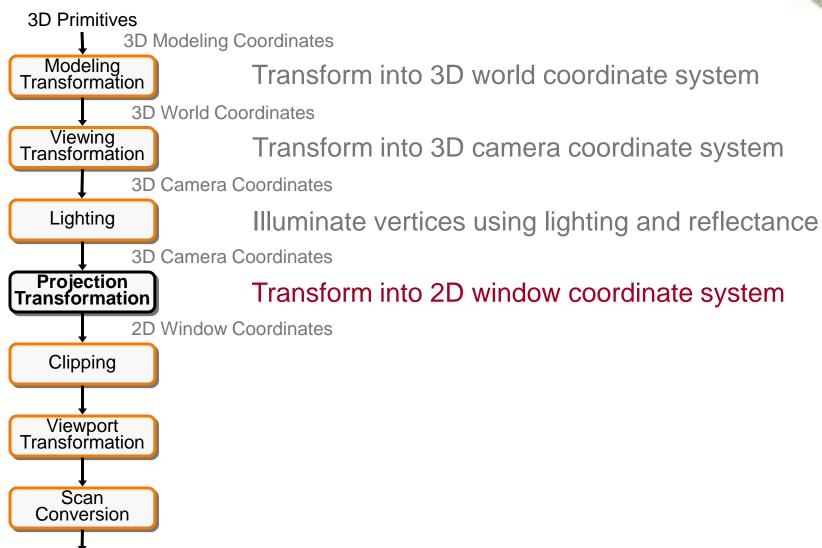




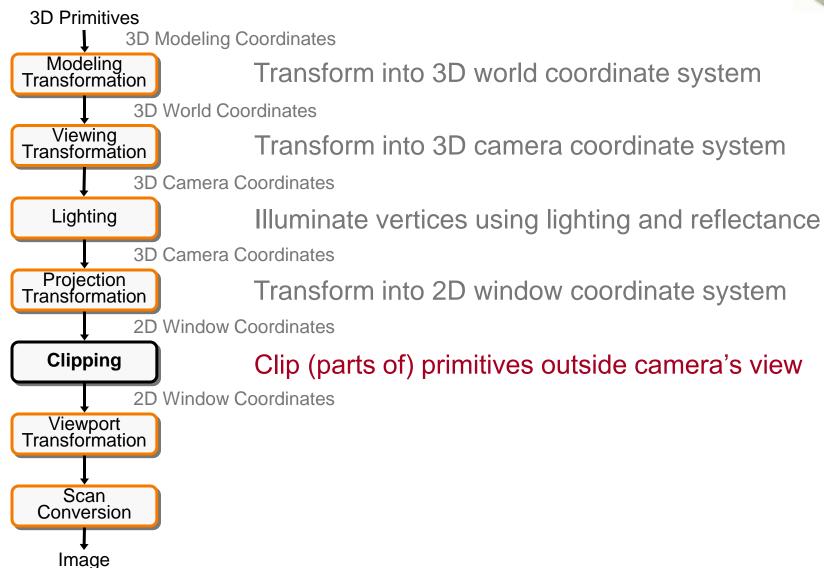




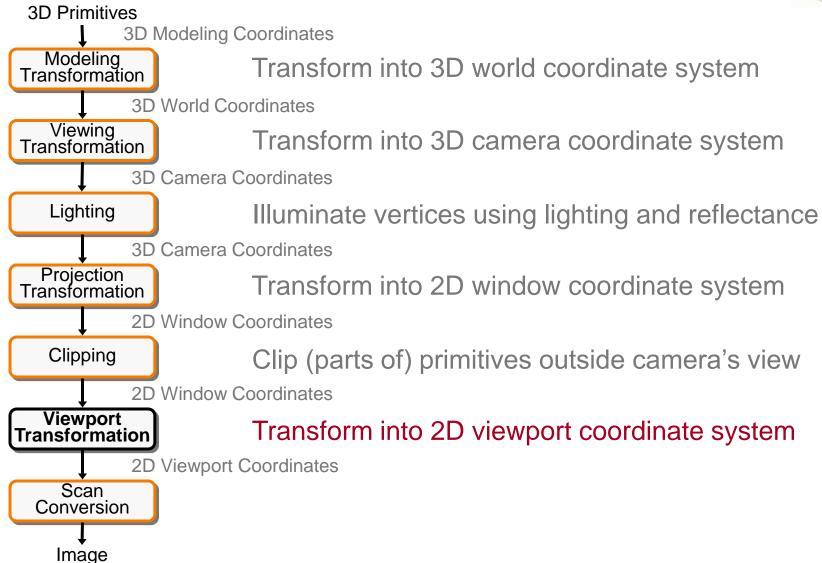




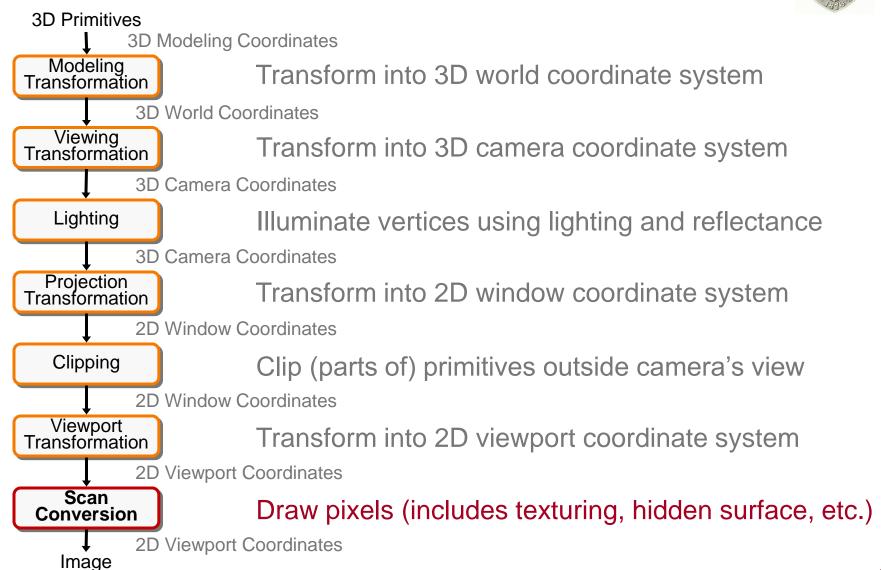




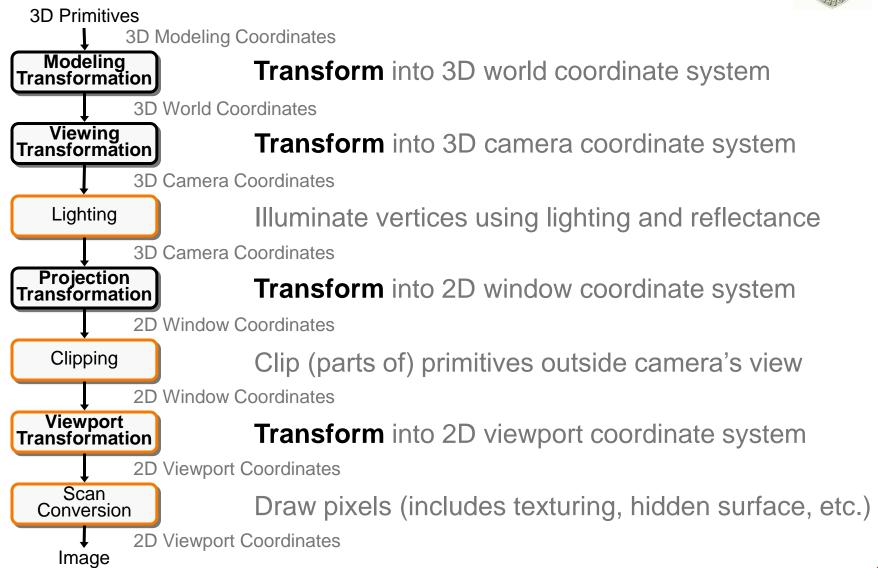












Recall: Homogeneous Coordinates



- Add a 4th coordinate to every 3D point
 - (x, y, z, w) represents a point at location $(\frac{x}{w}, \frac{y}{w}, \frac{z}{w})$
 - (x, y, z, 0) represents the (unsigned) direction $\frac{\pm (x, y, z)}{\sqrt{x^2 + y^2 + z^2}}$
 - (0,0,0,0) is not allowed

Recall: 3D Transformations



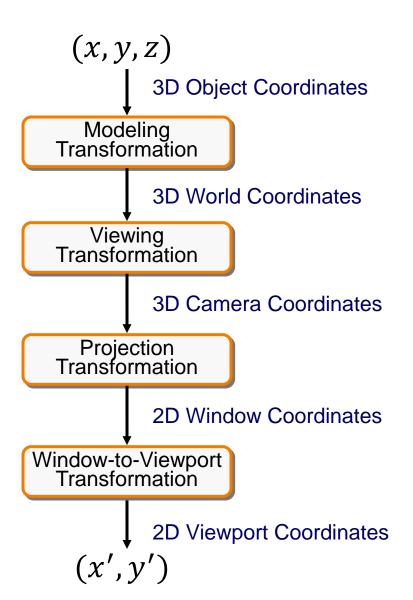
- Using homogenous coordinates, we have two types of transformations:
 - Affine

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

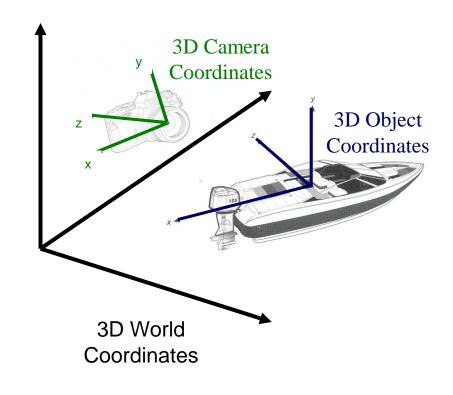
Projective

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

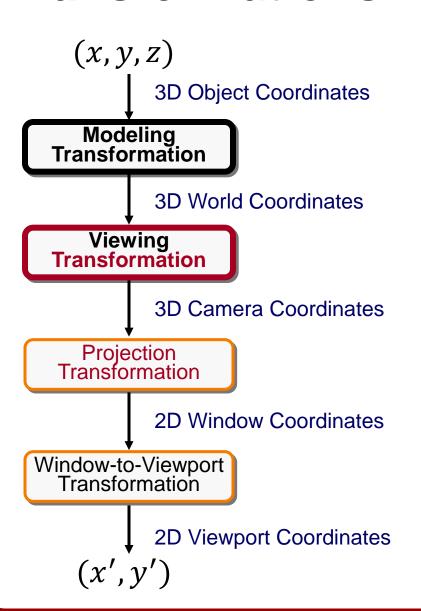




Transformations map points from one coordinate system to another

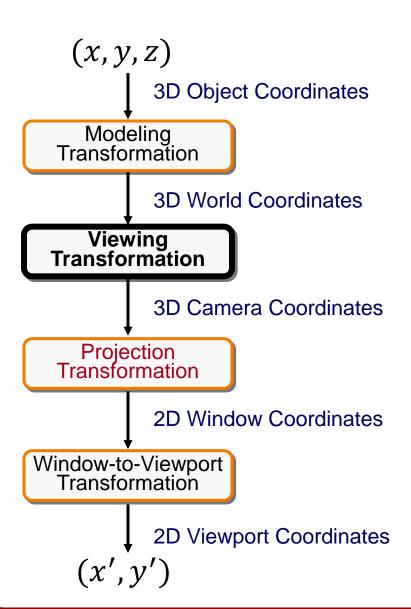






Modelview Transformations

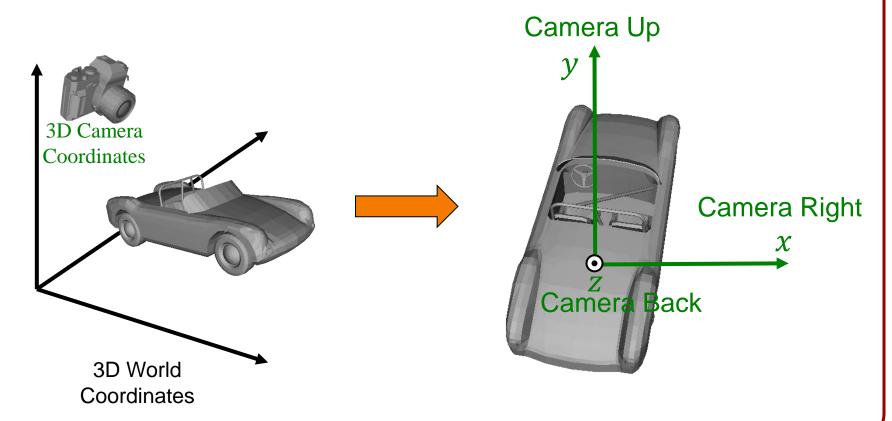




Viewing Transformation



- Canonical coordinate system
 - \circ Convention is right-handed (looking down -z axis)
 - Convenient for projection, clipping, etc.



Viewing Transformation



- The transformation, $T_{W\to C}$, taking us from world coordinates to camera coordinates should map:
 - The right vector to the *x*-axis:

$$(R_x, R_y, R_z, 0) \rightarrow (1,0,0,0)$$

• The up vector to the *y*-axis:

$$(U_x, U_y, U_z, 0) \rightarrow (0,1,0,0)$$

The back vector to the z-axis:

$$(B_x, B_y, B_z, 0) \rightarrow (0,0,1,0)$$

The eye position to the origin:

$$(E_x, E_y, E_z, 1) \rightarrow (0,0,0,1)$$

How should we define this transformation/matrix?

Viewing Transformation



• Consider the inverse transformation, $T_{C\to W}$, taking us from camera coordinates to world coordinates:

$$(R_{\chi}, R_{y}, R_{z}, 0) \leftarrow (1,0,0,0)$$
$$(U_{\chi}, U_{y}, U_{z}, 0) \leftarrow (0,1,0,0)$$
$$(B_{\chi}, B_{y}, B_{z}, 0) \leftarrow (0,0,1,0)$$
$$(E_{\chi}, E_{y}, E_{z}, 1) \leftarrow (0,0,0,1)$$

This is described by the matrix:

$$\begin{pmatrix} x^{w} \\ y^{w} \\ z^{w} \end{pmatrix} = \begin{pmatrix} R_{x} & U_{x} & B_{x} & E_{x} \\ R_{y} & U_{y} & B_{y} & E_{y} \\ R_{z} & U_{z} & B_{z} & E_{z} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^{c} \\ y^{c} \\ z^{c} \\ 1 \end{pmatrix}$$

Finding the Viewing Transformation



The camera-to-world matrix:

$$\begin{pmatrix} x^{w} \\ y^{w} \\ z^{w} \end{pmatrix} = \begin{pmatrix} R_{x} & U_{x} & B_{x} & E_{x} \\ R_{y} & U_{y} & B_{y} & E_{y} \\ R_{z} & U_{z} & B_{z} & E_{z} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^{c} \\ y^{c} \\ z^{c} \\ 1 \end{pmatrix}$$

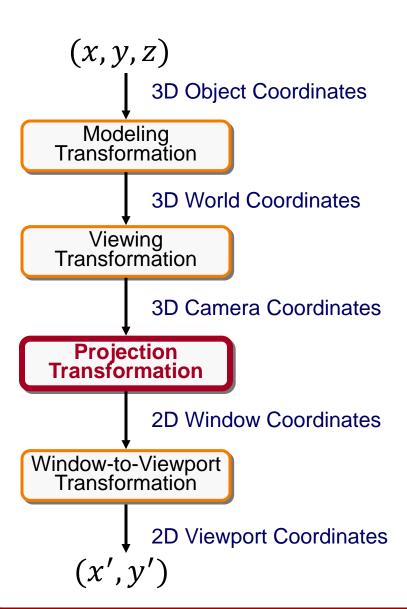
The world-to-camera matrix is its inverse:

$$\begin{pmatrix} x^{c} \\ y^{c} \\ z^{c} \\ 1 \end{pmatrix} = \begin{pmatrix} R_{x} & U_{x} & B_{x} & E_{x} \\ R_{y} & U_{y} & B_{y} & E_{y} \\ R_{z} & U_{z} & B_{z} & E_{z} \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} x^{w} \\ y^{w} \\ z^{w} \\ 1 \end{pmatrix}$$

$$\mathbf{T}_{W \to c} = \mathbf{T}_{c \to W}^{-1}$$

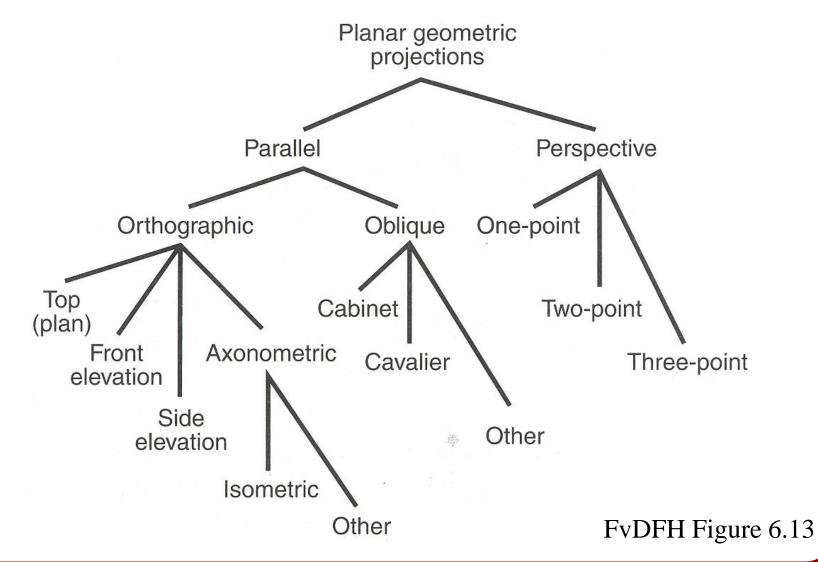
$$\mathbf{T}_{W\to C} = \mathbf{T}_{C\to W}^{-1}$$





Taxonomy of Projections

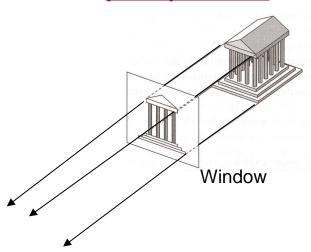


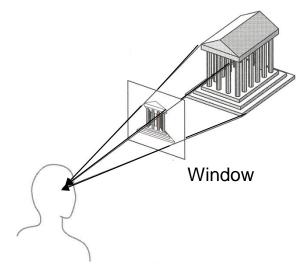


Projection



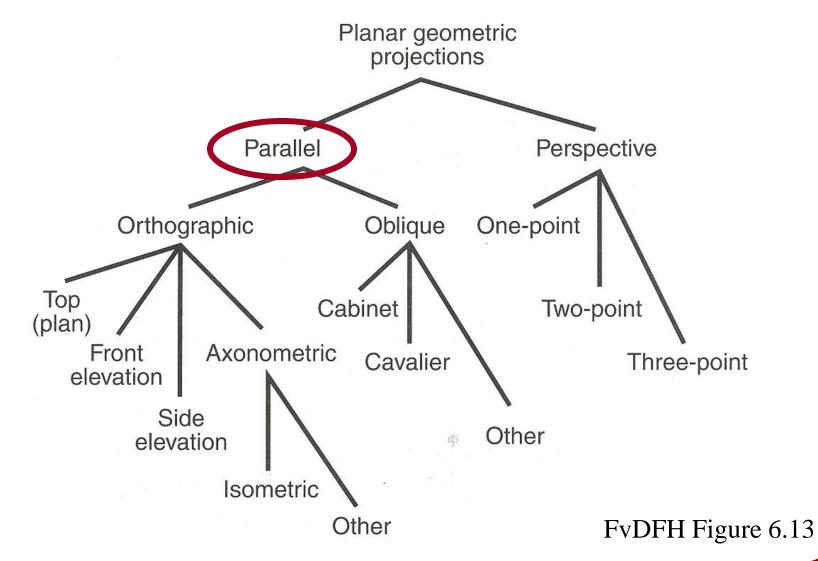
- Two general classes of projections, which shoot rays from the 3D scene, through the 2D window:
 - Parallel Projection:
 - » Rays converge at a point at infinity and are parallel
 - Perspective "Projection":
 - » Rays converge at a finite point, giving rise to perspective distortion





Taxonomy of Projections

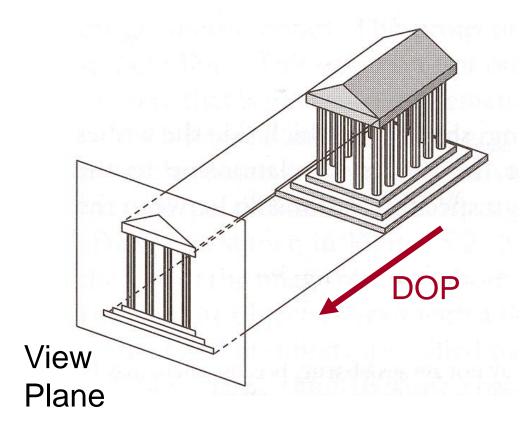




Parallel Projection



- Center of projection is at infinity
 - Direction of projection (DoP) same for all points

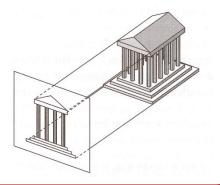


Angel Figure 5.4

Parallel Projection

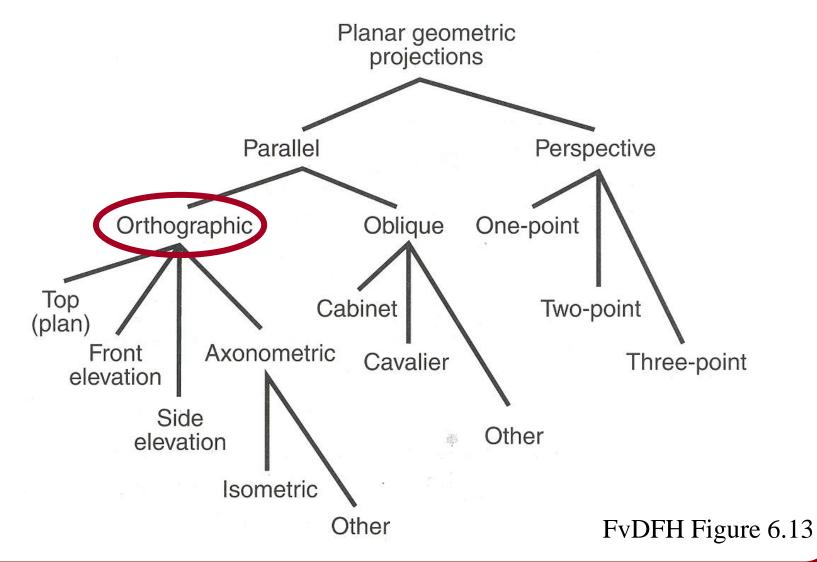


- ✓ Parallel lines remain parallel
- ✓ Proportions are preserved (no foreshortening)
- Some angles are not preserved
- Less realistic looking



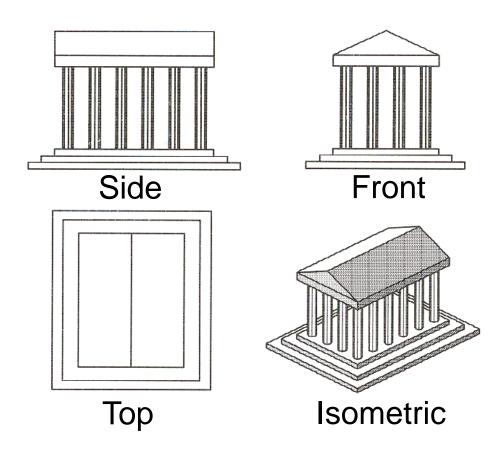
Taxonomy of Projections







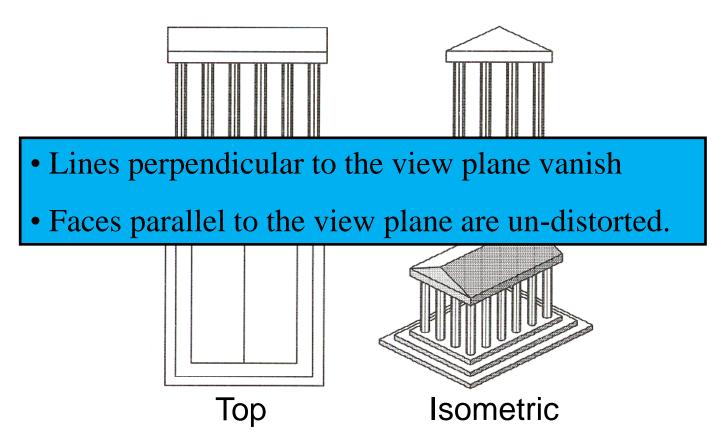
DoP perpendicular to view plane



Angel Figure 5.5



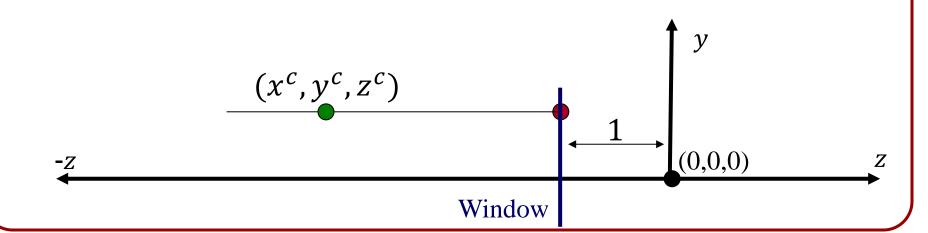
DoP perpendicular to view plane





- DoP perpendicular to view plane
 - Maps a point in 3D space to the (x, y, -1)-plane, by projecting out the z-component:

$$(x^c, y^c, z^c) \rightarrow (x^c, y^c, -1)$$



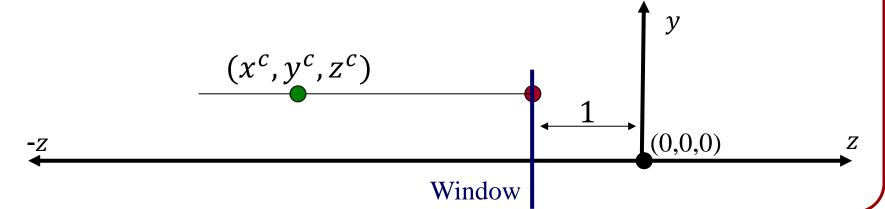


- DoP perpendicular to view plane
 - Maps a point in 3D space to the (x, y, -1)-plane, by projecting out the z-component:

$$(x^c, y^c, z^c, 1) \rightarrow (x^c, y^c, -1, 1)$$

In terms of homogenous coordinates:

$$\begin{bmatrix} x^c \\ y^c \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^c \\ y^c \\ z^c \\ 1 \end{bmatrix}$$



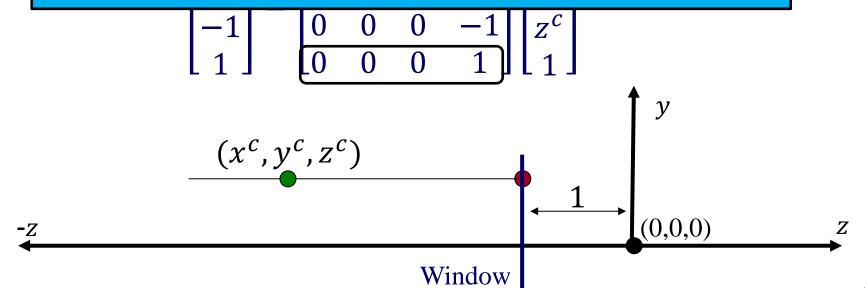


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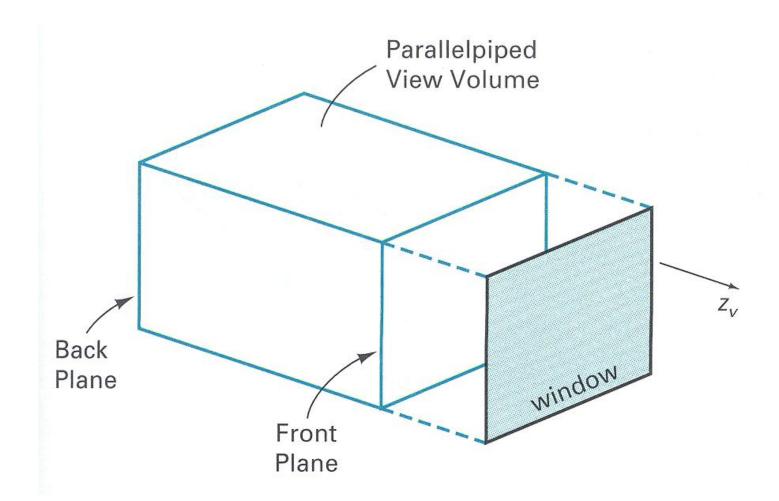
Note:

This matrix describes an affine transformation



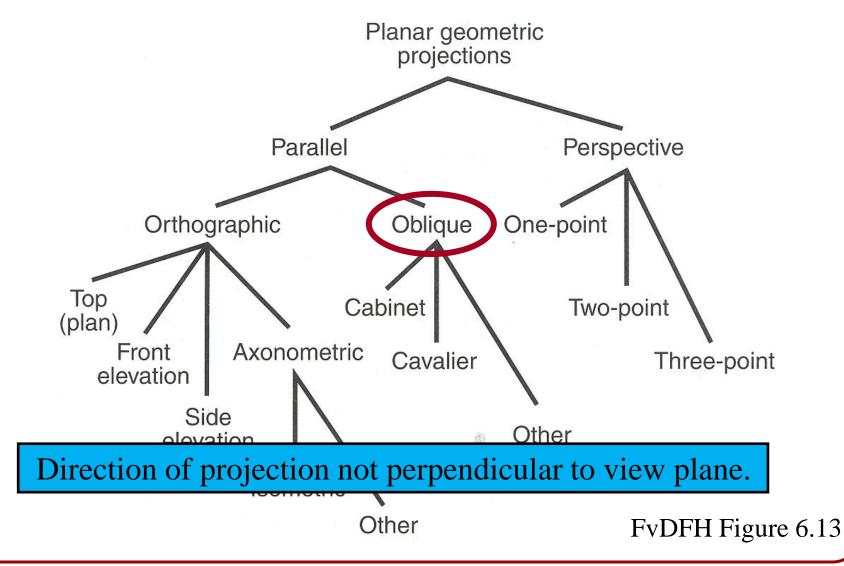
Parallel Projection View Volume





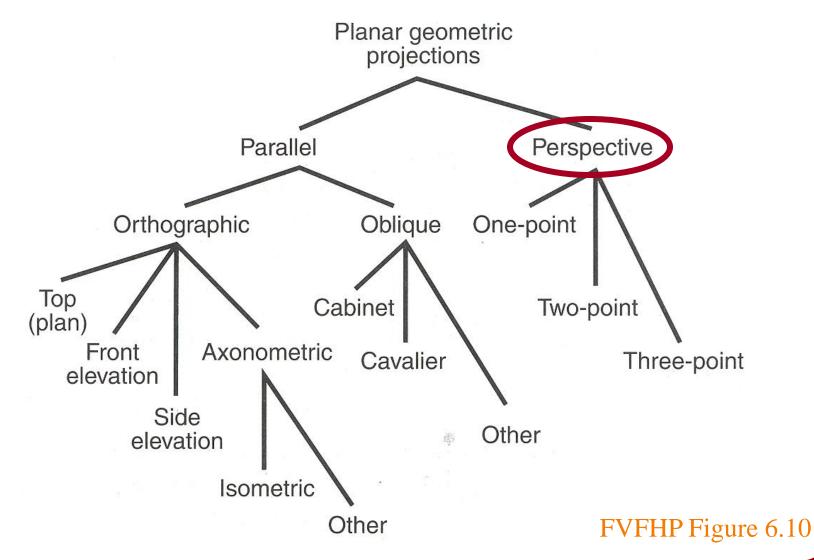
Taxonomy of Projections





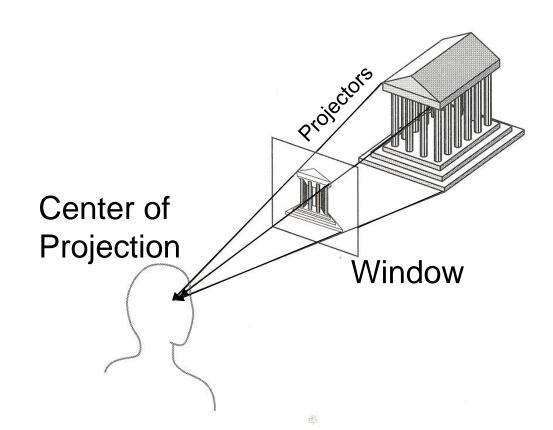
Taxonomy of Projections





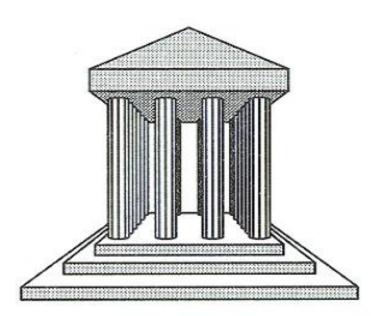


 Map points onto "view plane" along "projectors" emanating from "center of projection" (CoP)



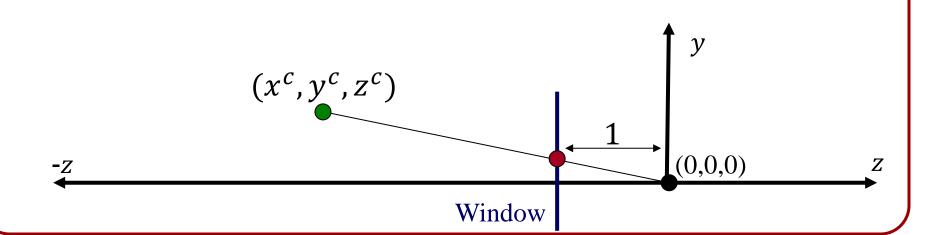


Not all parallel lines remain parallel!



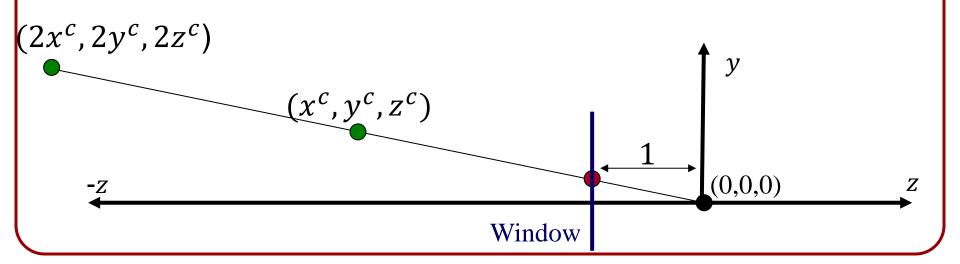


• What are the coordinates of the point resulting from projection of (x^c, y^c, z^c) onto the camera screen a <u>unit</u> distance back along the z-axis?





• For any point (x^c, y^c, z^c) and any scalar α , the points (x^c, y^c, z^c) and $(\alpha x^c, \alpha y^c, \alpha z^c)$ map to the same location.





- For any point (x^c, y^c, z^c) and any scalar α , the points (x^c, y^c, z^c) and $(\alpha x^c, \alpha y^c, \alpha z^c)$ map to the same location.
- Since we want the position on the window that intersects the line from (x^c, y^c, z^c) to the origin:

$$(x^{c}, y^{c}, z^{c}) \rightarrow \left(\frac{x^{c}}{-z^{c}}, \frac{y^{c}}{-z^{c}}, -1\right)$$

$$(x^{c}, y^{c}, z^{c})$$

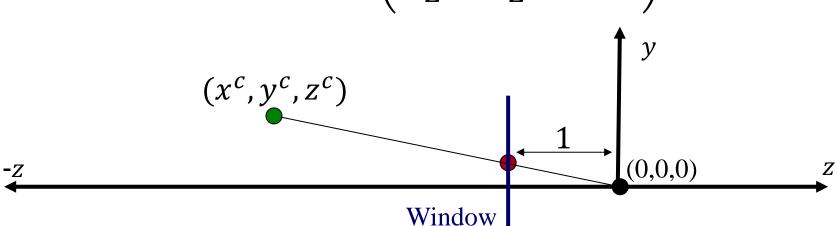
$$(0,0,0)$$

Window



- For any point (x^c, y^c, z^c) and any scalar α , the points (x^c, y^c, z^c) and $(\alpha x^c, \alpha y^c, \alpha z^c)$ map to the same location.
- Since we want the position on the window that intersects the line from (x^c, y^c, z^c) to the origin:

$$(x^c, y^c, z^c, 1) \rightarrow \left(\frac{x^c}{-z^c}, \frac{y^c}{-z^c}, -1, 1\right)$$



Perspective Projection Matrix



$$(x^c, y^c, z^c, 1) \rightarrow \left(\frac{x^c}{-z^c}, \frac{y^c}{-z^c}, -1, 1\right)$$

Division by z^c can't represented with a 3×3 matrix!

In homogenous coordinates, we can write this as:

$$(x^{c}, y^{c}, z^{c}, 1) \rightarrow (x^{c}, y^{c}, z^{c}, -z^{c})$$

In matrix form, this gives:

$$\begin{bmatrix} -x^c/z^c \\ -y^c/z^c \\ -1 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} x^c \\ y^c \\ z^c \\ -z^c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x^c \\ y^c \\ z^c \\ 1 \end{bmatrix}$$

Perspective Projection Matrix



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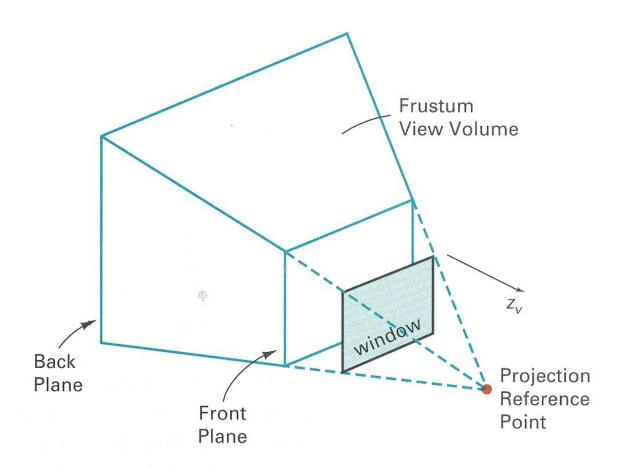
$$(x^c, y^c, z^c, 1) \rightarrow (x^c, y^c, z^c, -z^c)$$

In Note:

This matrix describes a projective transformation

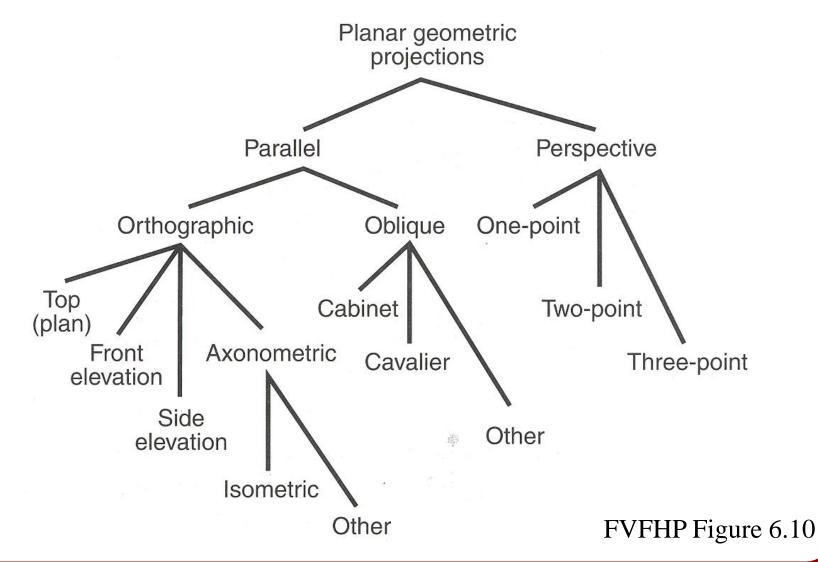
$$\begin{bmatrix} -y^{c}/z^{c} \\ -1 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} y^{c} \\ z^{c} \\ -z^{c} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} y^{c} \\ z^{c} \\ 1 \end{bmatrix}$$

Perspective Projection View Volume



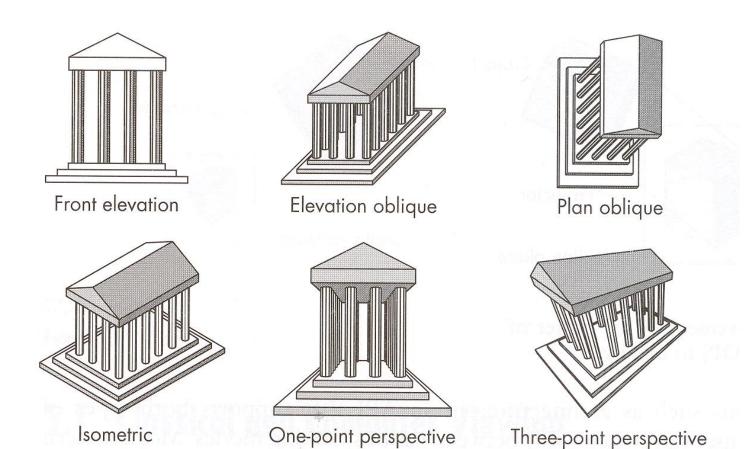
Taxonomy of Projections





Classical Projections

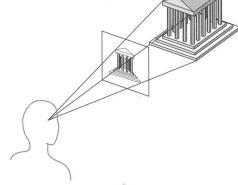




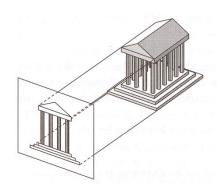
Perspective vs. Parallel



- Perspective projection
 - ✓ Size varies inversely with distance looks realistic
 - ✓ Angles are preserved on faces parallel to the view plane
 - Distance are not preserved

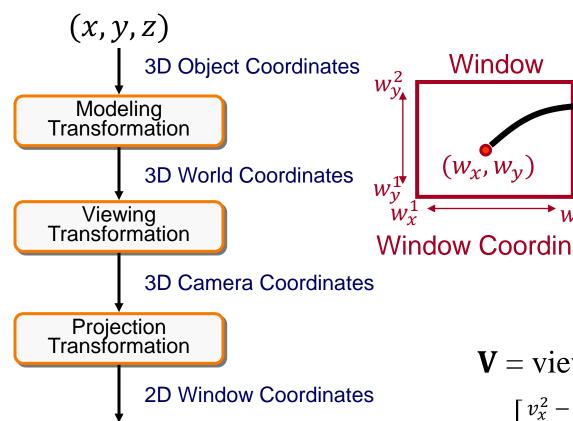


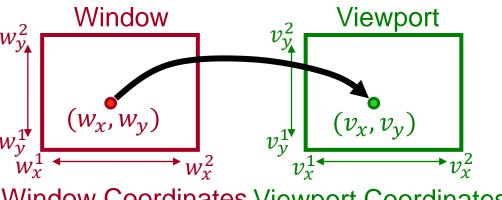
- Parallel (orthographic) projection
 - ✓ Parallel lines remain parallel
 - ✓ Angles and distances are preserved on faces parallel to the view plane
 - Less realistic looking
 - ✓ Good for exact measurements



Transformations







Window Coordinates Viewport Coordinates

Window-to-Viewport Transformation

2D Viewport Coordinates

(x',y')

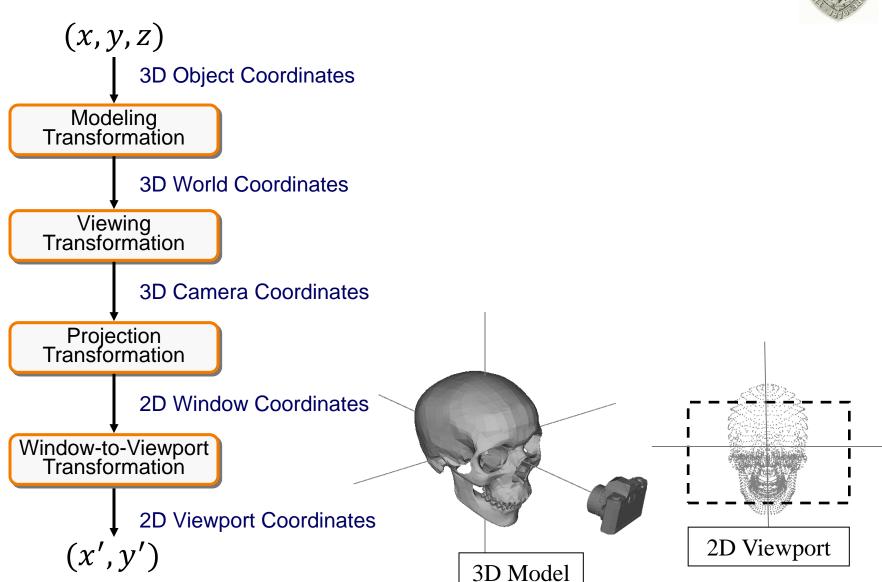
V = viewport transform

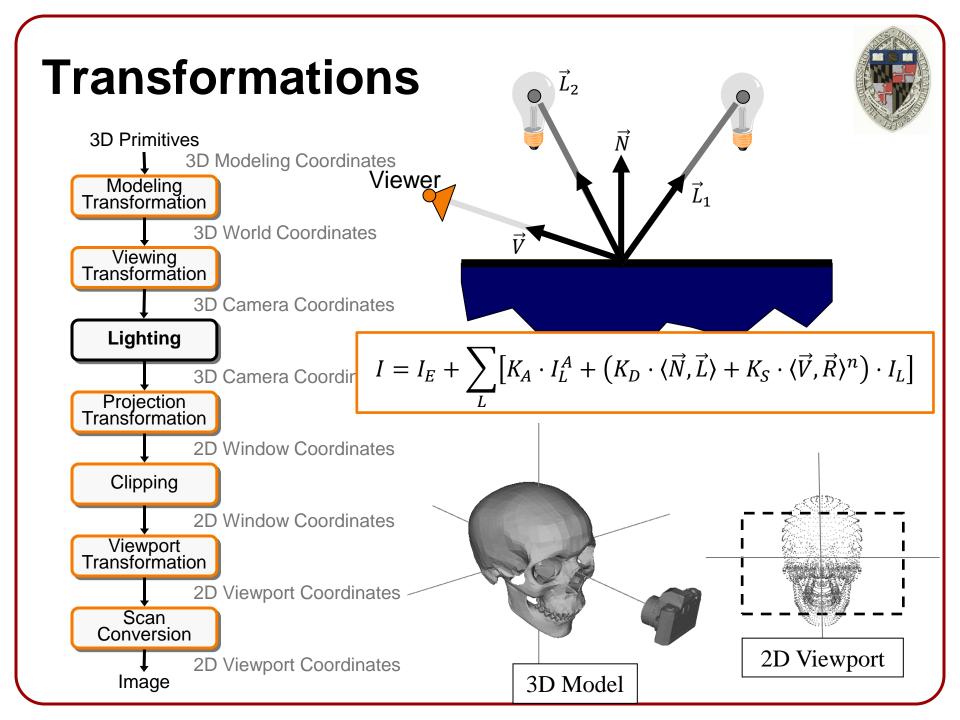
es
$$\mathbf{V} = \begin{bmatrix} 1 & 0 & v_x^1 \\ 0 & 1 & v_x^2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{v_x^2 - v_x^1}{w_x^2 - w_x^1} & 0 & 0 \\ w_x^2 - w_x^1 & 0 & 0 \\ 0 & \frac{v_y^2 - v_y^1}{w_y^2 - w_y^1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -w_x^1 \\ 0 & 1 & -w_y^1 \\ 0 & 0 & 1 \end{bmatrix}$$
tes

Note that this may scale non-uniformly.

3D Rendering Pipeline (for direct illumination)

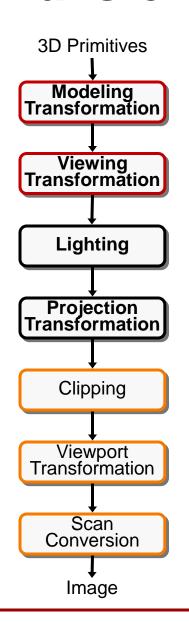






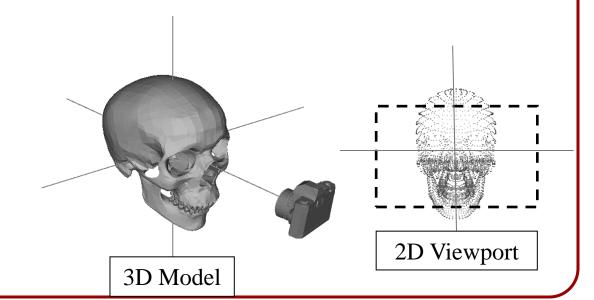
Transformations





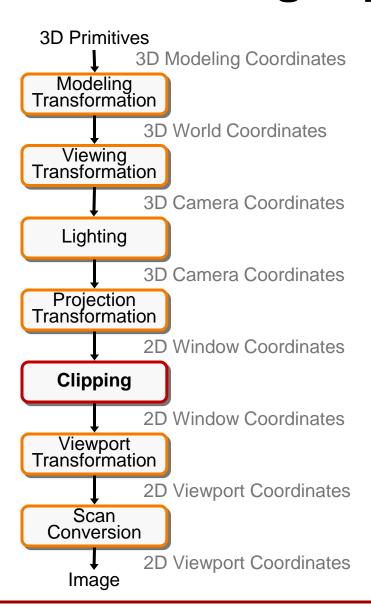
Vertex processing

- Originally, vertex processing was <u>fixed</u>
- Now this is programmable in the <u>vertex shader</u>



3D Rendering Pipeline (for direct illumination)





Clipping



- Avoid drawing parts of primitives outside window
 - Window defines the subset of the scene being viewed
 - Must draw geometric primitives only inside window



Clipping



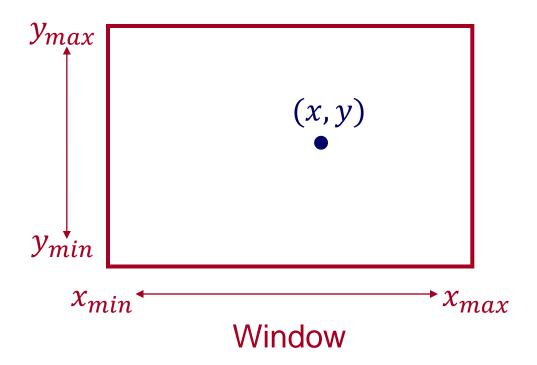
- Avoid drawing parts of primitives outside window
 - Points
 - Line Segments
 - Polygons



Point Clipping



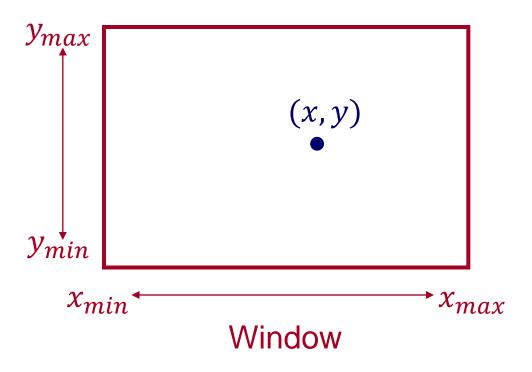
• Is point (x, y) inside the clip window?



Point Clipping



• Is point (x, y) inside the clip window?



```
inside =
  (x >= x_min) &&
  (x < x_max) &&
  (y >= y_min) &&
  (y < y_max);
```

Clipping



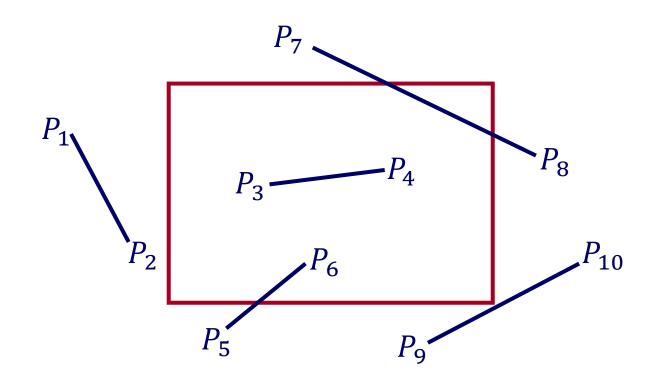
- Avoid drawing parts of primitives outside window
 - Points
 - Line Segments
 - Polygons



Line Segment Clipping



- Find the part of a line inside the clip window
 - Do this as <u>efficiently</u> as possible by identifying the easiest cases first

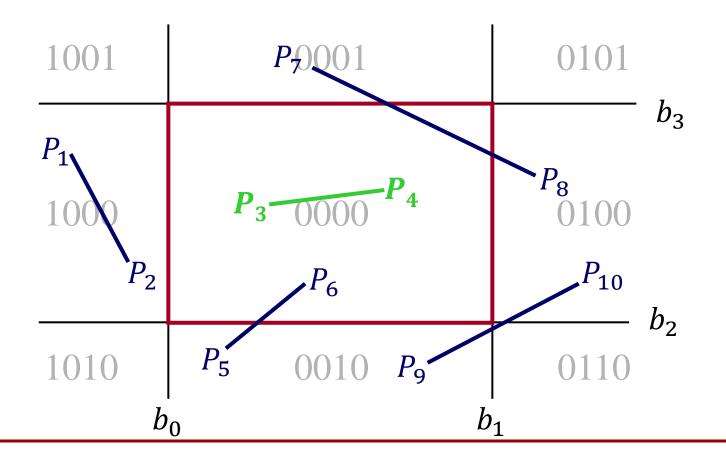




- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
 - $b_0 = 1$ if the vertex is <u>left of</u> the window
 - $b_1 = 1$ if the vertex is <u>right of</u> the window
 - $b_2 = 1$ if the vertex is <u>below</u> the window
 - $b_3 = 1$ if the vertex is above the window 1001 0101 0001 b_3 1000 0100 0000 1010 0010 0110

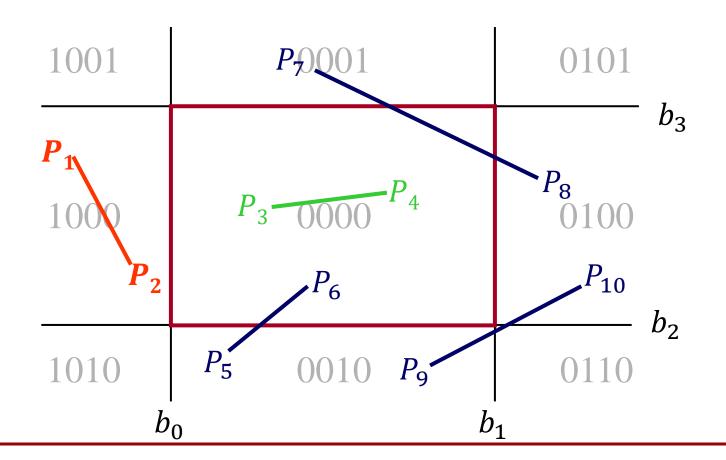


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside



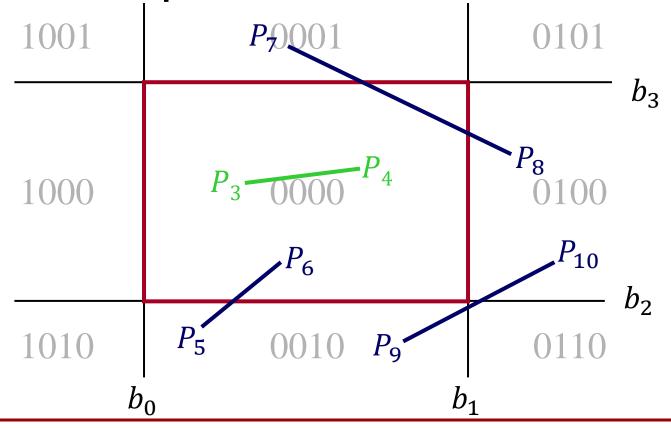


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside



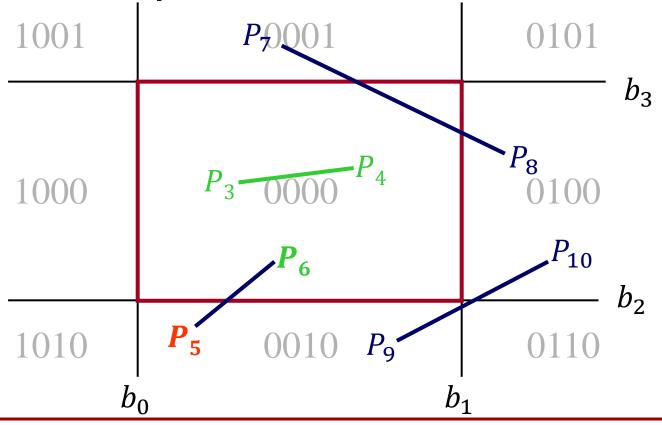


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



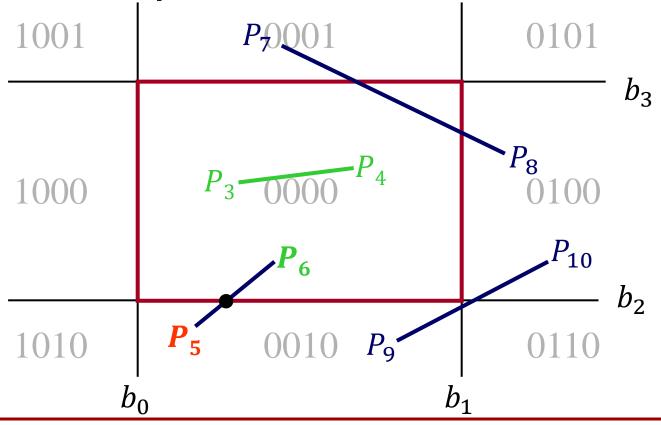


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



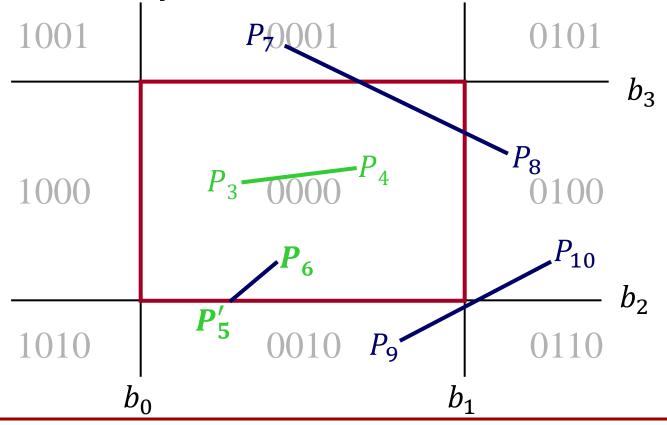


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



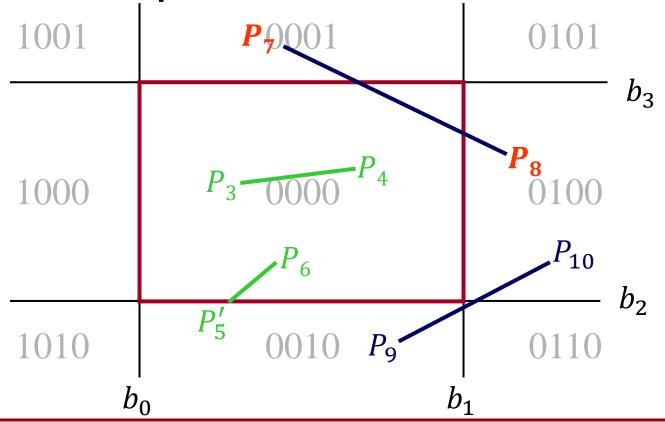


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



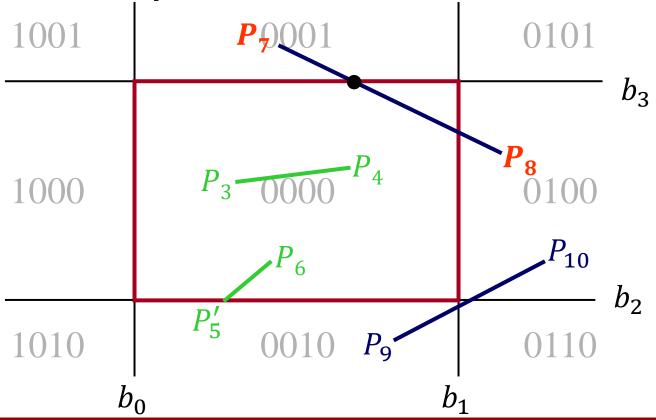


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



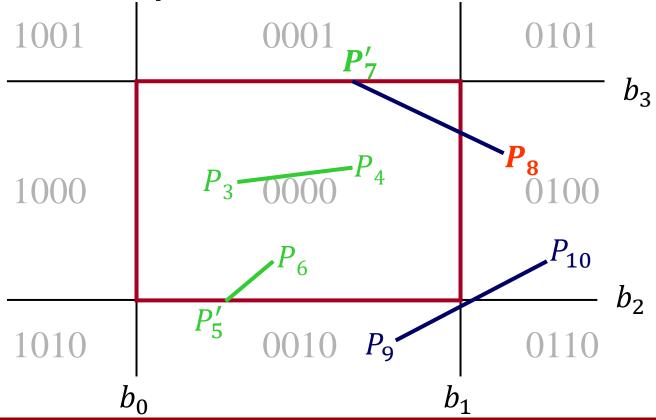


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



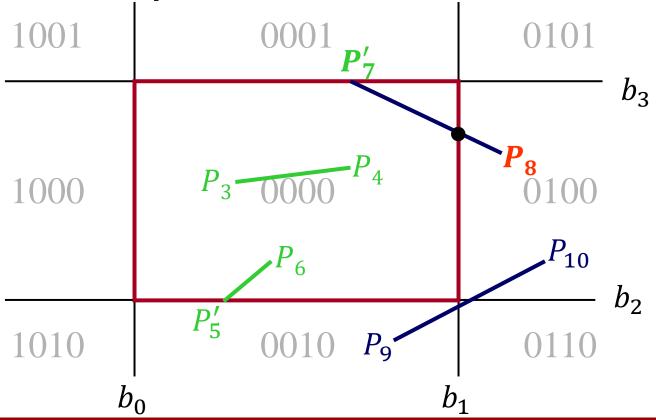


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



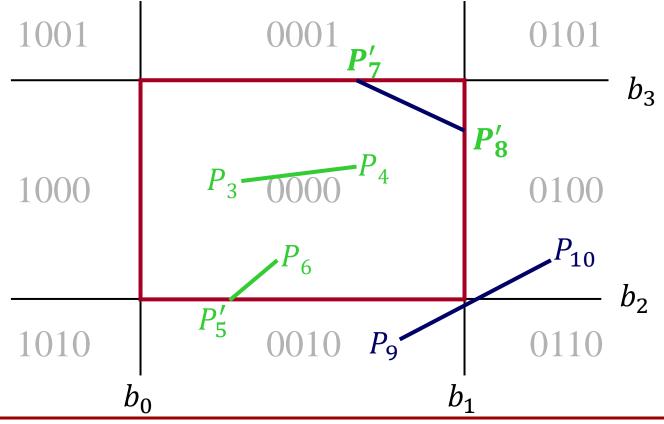


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



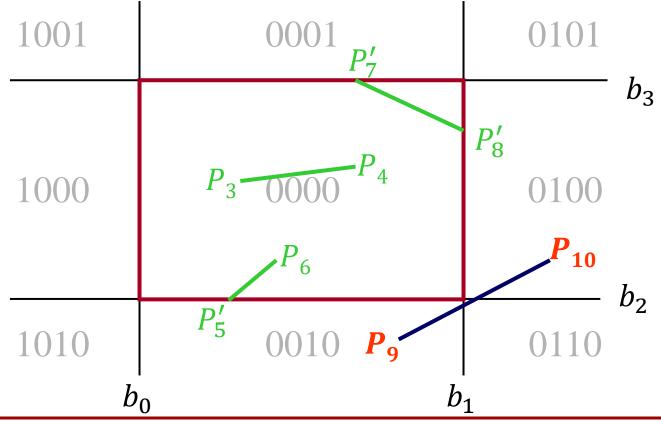


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



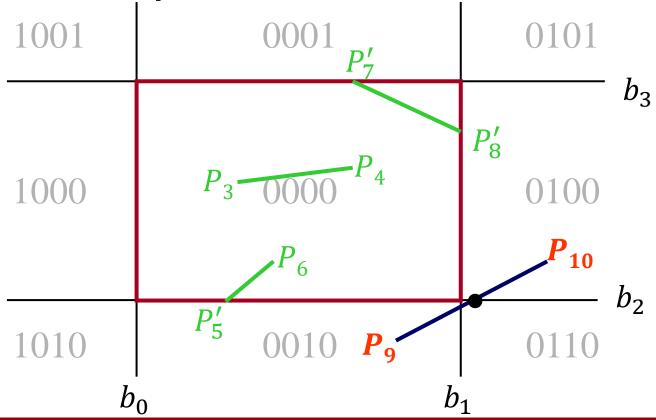


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



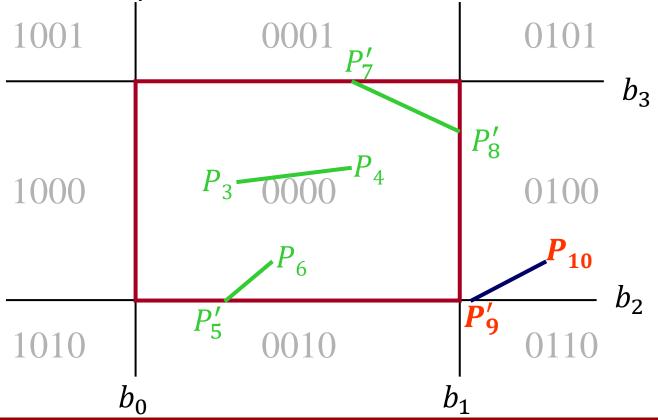


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



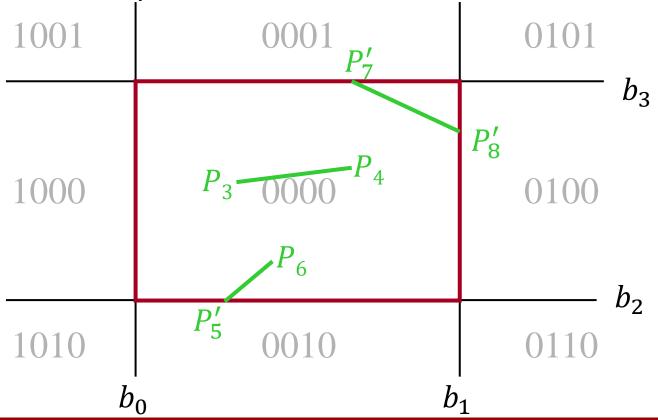


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
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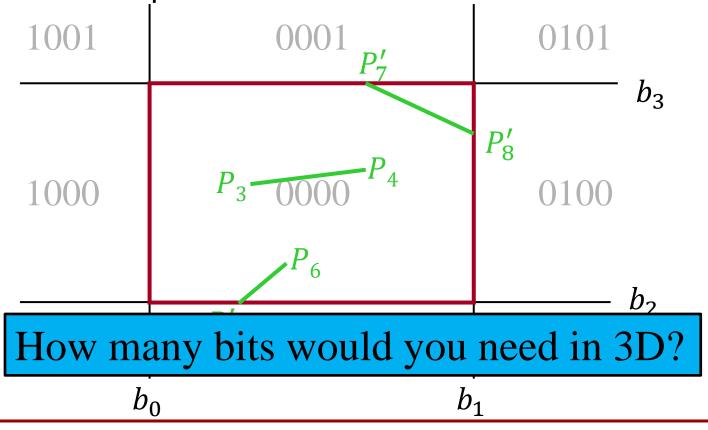


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
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- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



Clipping



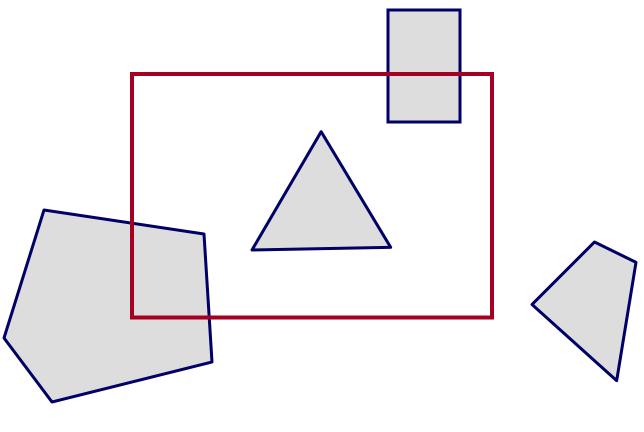
- Avoid drawing parts of primitives outside window
 - Points
 - Line Segments
 - Polygons



Polygon Clipping

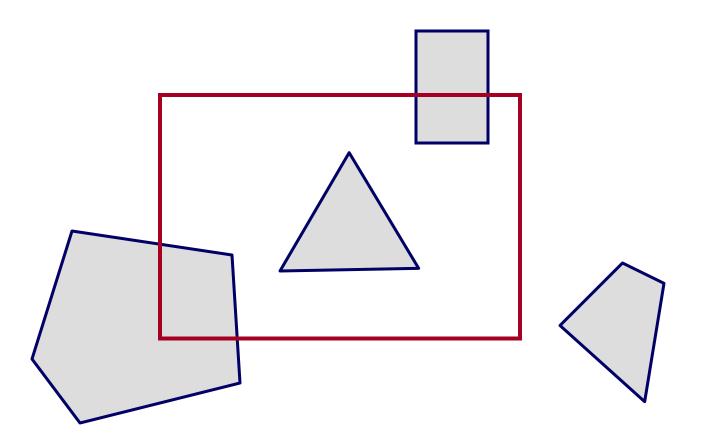


Find the part of a polygon inside the clip window

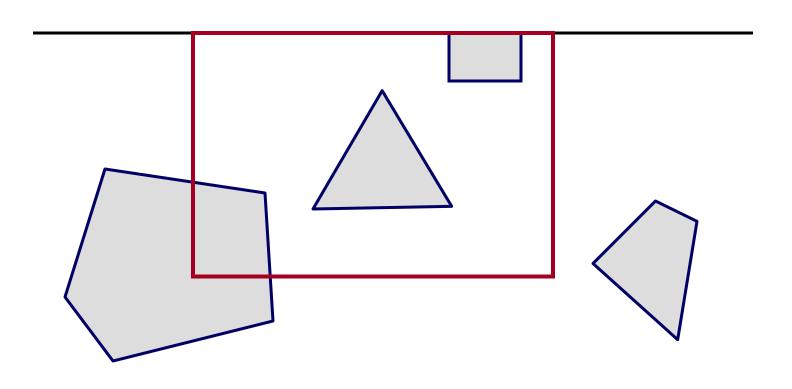


Before Clipping

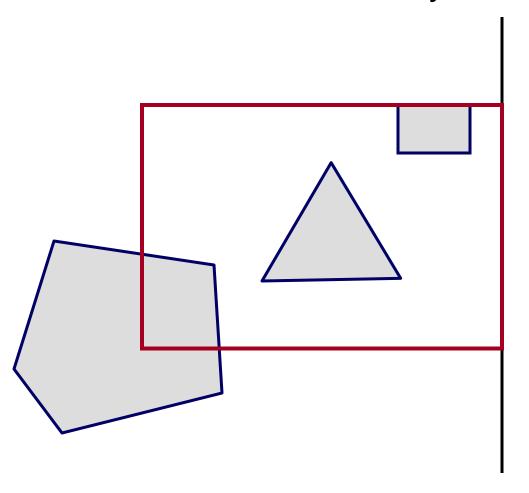




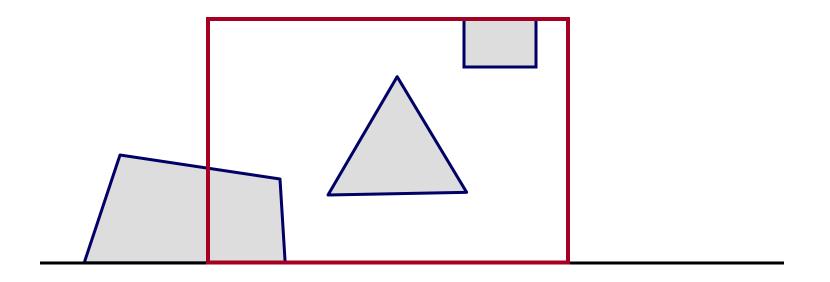




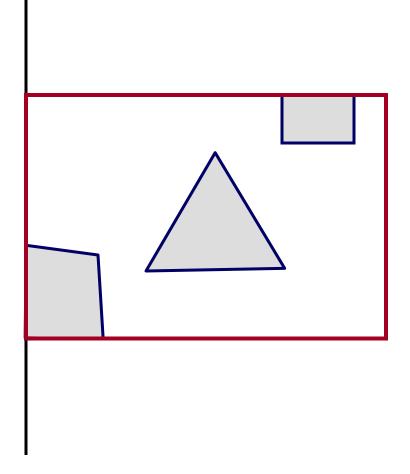






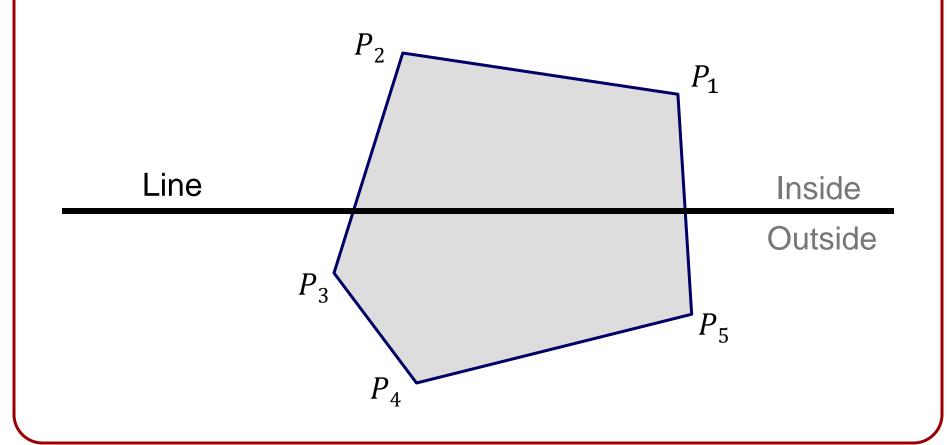






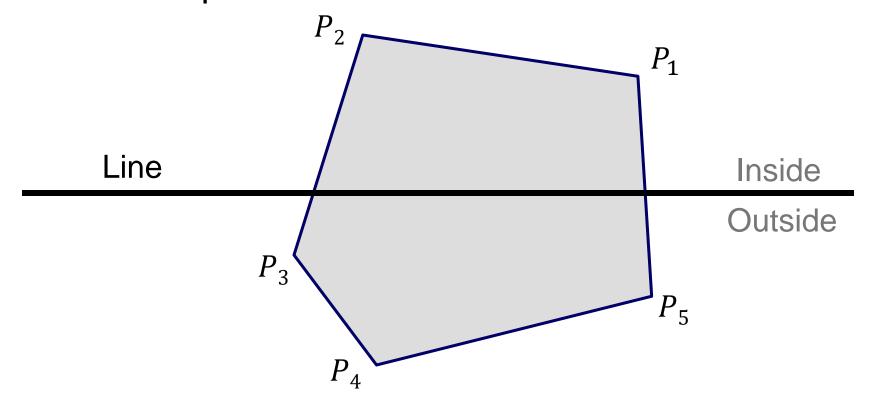


 How do we clip a <u>convex</u> polygon with respect to a (window boundary) line?



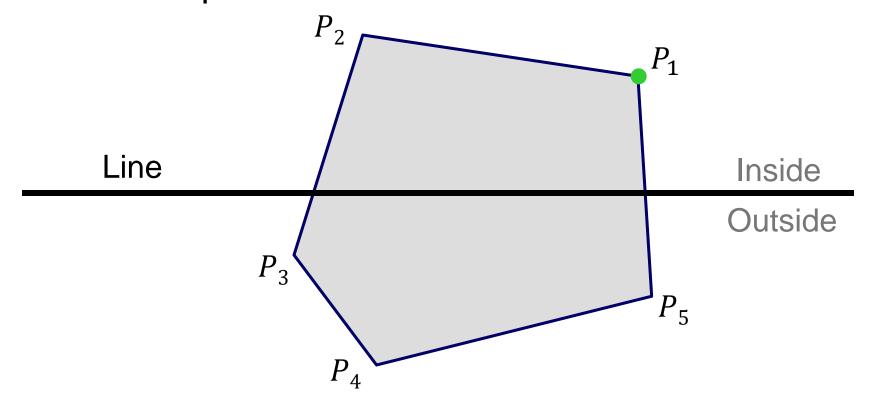


- Do interior test for each point in sequence.
- Insert a new point when crossing the line.
- Remove points outside the line.



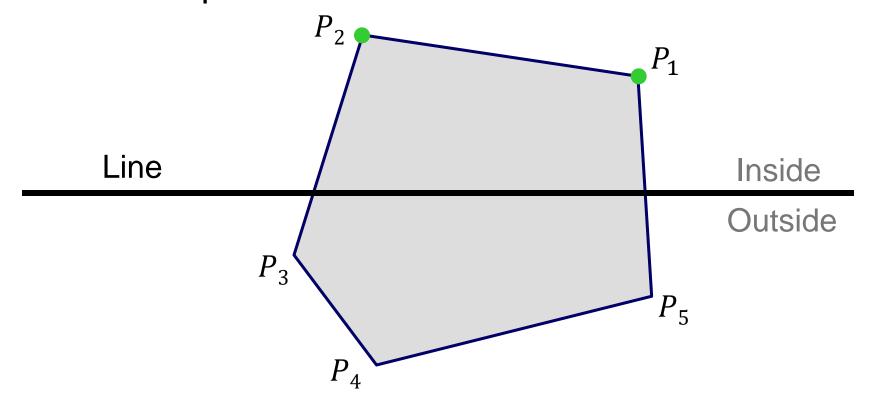


- Do interior test for each point in sequence.
- Insert a new point when crossing the line.
- Remove points outside the line.



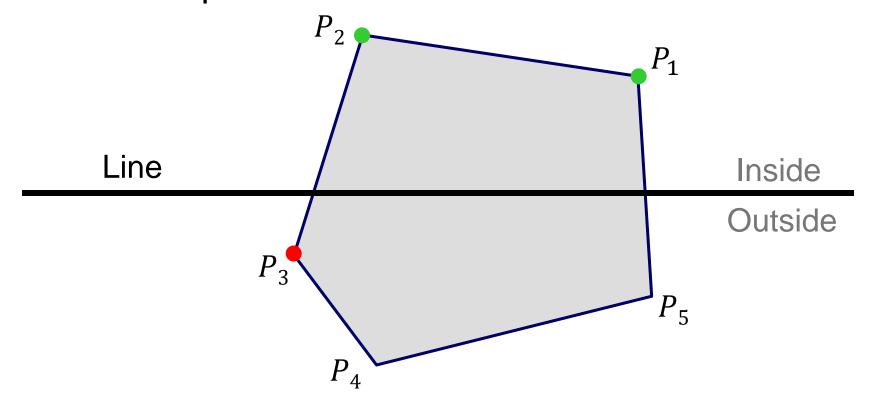


- Do interior test for each point in sequence.
- Insert a new point when crossing the line.
- Remove points outside the line.



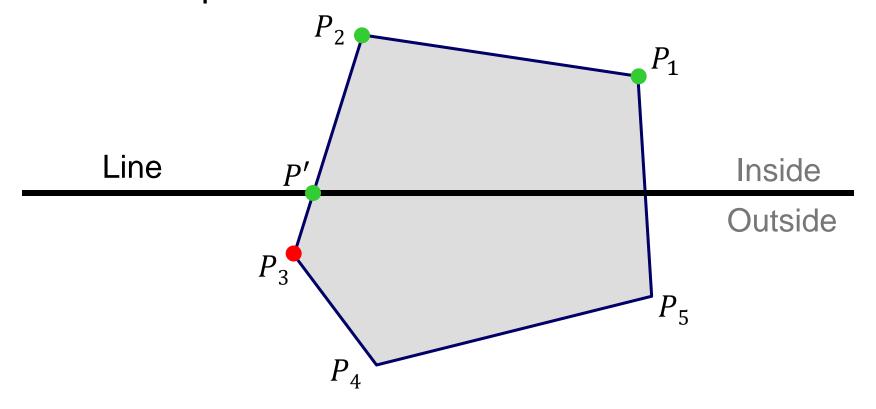


- Do interior test for each point in sequence.
- Insert a new point when crossing the line.
- Remove points outside the line.



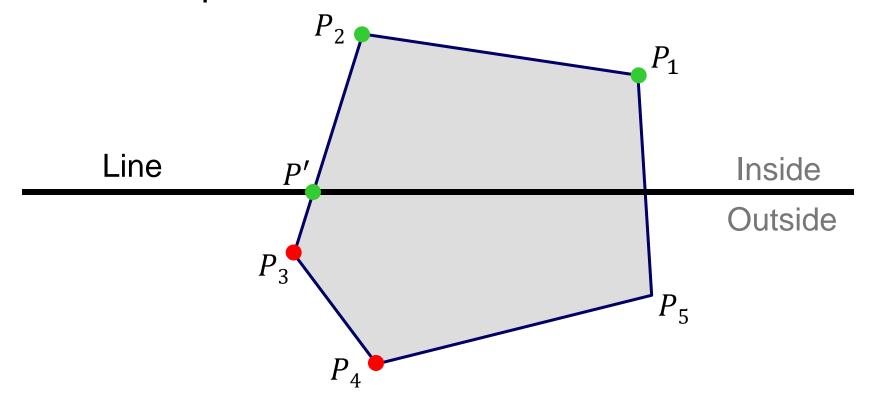


- Do interior test for each point in sequence.
- Insert a new point when crossing the line.
- Remove points outside the line.



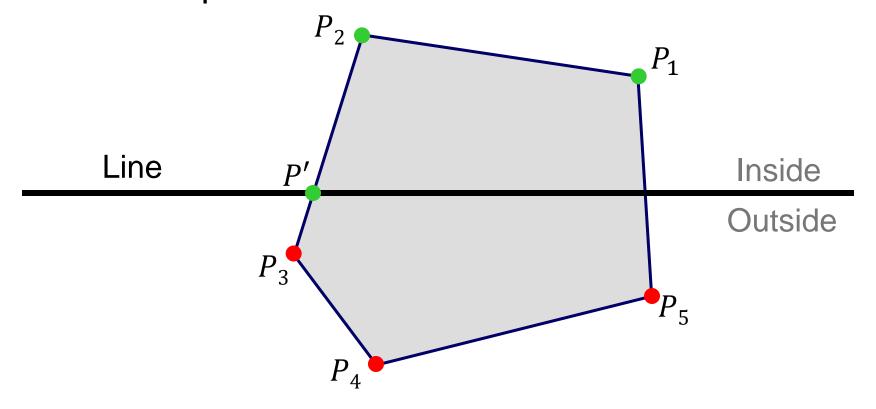


- Do interior test for each point in sequence.
- Insert a new point when crossing the line.
- Remove points outside the line.



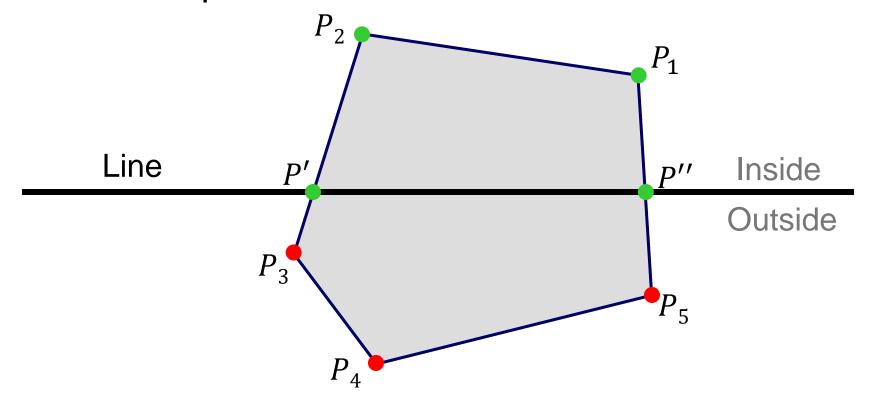


- Do interior test for each point in sequence.
- Insert a new point when crossing the line.
- Remove points outside the line.



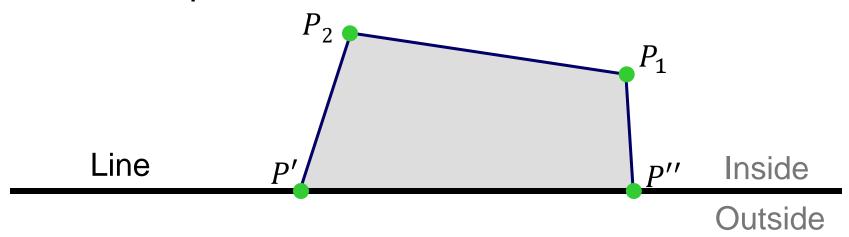


- Do interior test for each point in sequence.
- Insert a new point when crossing the line.
- Remove points outside the line.



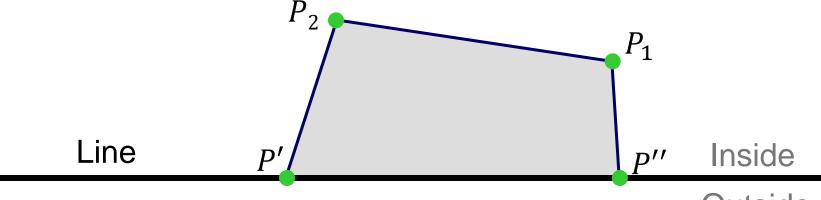


- Do interior test for each point in sequence.
- Insert a new point when crossing the line.
- Remove points outside the line.





- Do interior test for each point in sequence.
- Insert a new point when crossing the line.
- Remove points outside the line.

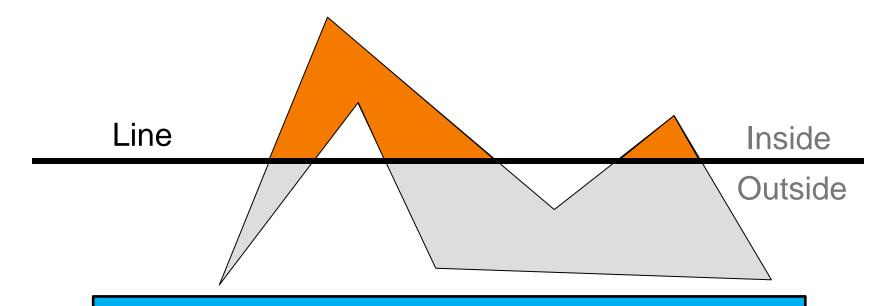


Outside

When polygons are clipped, per-vertex properties (e.g. lighting) is interpolated to the new vertices.



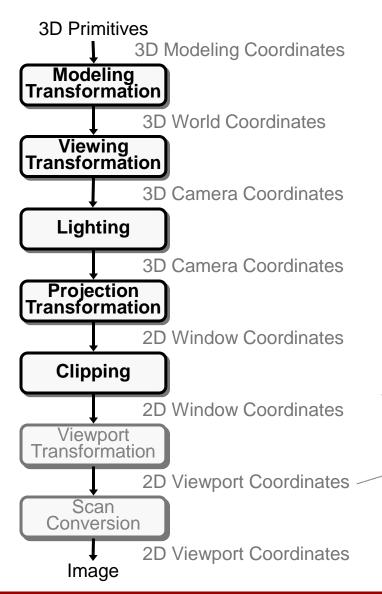
- Do interior test for each point in sequence.
- Insert a new point when crossing the line.
- Remove points outside the line.



[WARNING] If the polygon is not convex, we may end up with more than one polygon!

3D Rendering Pipeline (for direct illumination)





At this point we have the:

- Positions of the mesh vertices (including new vertices obtained through clipping)
- Color information at each vertex.
- A list of (possibly clipped) polygons describing the intersection of the projected 3D polygons with the window.

