Image Filtering, Warping, and Morphing

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(601.457/657)

Outline

- Image Filtering
- Image Warping
- Image Morphing

Image Filtering

- What about the case when the modification that we would like to make to a pixel depends on the pixels around it?
  - Blurring
  - Edge Detection
  - Etc.

Multi-Pixel Operations

Stationary/Local Filtering

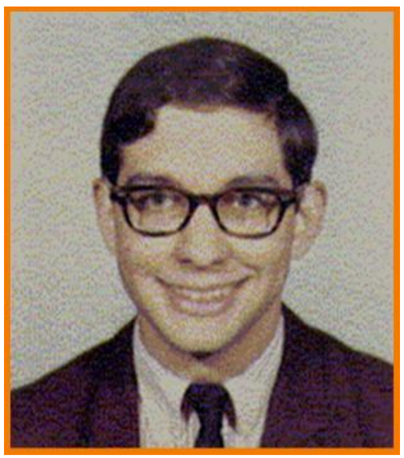
A general approach is to:

1. Define a *mask* of weights telling us how values at adjacent pixels should be combined to generate the new value.
2. Apply the (same) mask at every pixel.*

*Care is needed around at boundary pixels.

Blurring

- To blur across pixels, define a mask:
  - Whose entries sum to one
  - Whose value is larger near the center of the mask
  - Whose values are non-negative



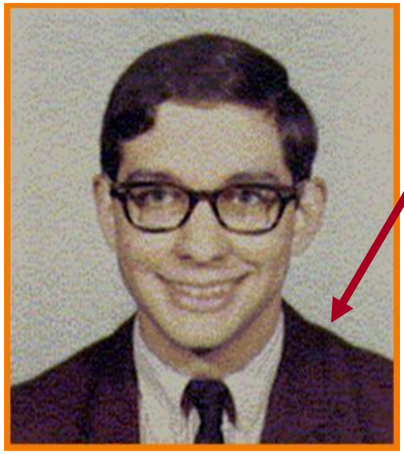
Original

Blur

$$\text{Filter} = \begin{bmatrix} 1/16 & 2/16 & 1/16 \\ 2/16 & 4/16 & 2/16 \\ 1/16 & 2/16 & 1/16 \end{bmatrix}$$

Blurring

Pixel(x,y): red = 36
green = 36
blue = 0

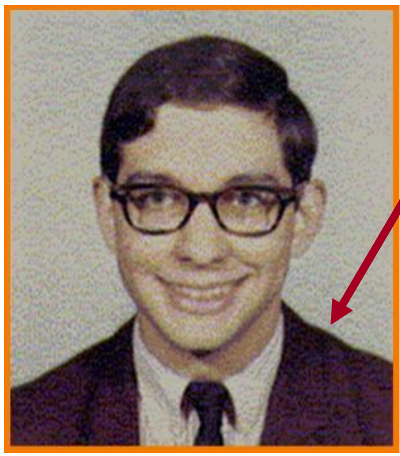


Original

$$\text{Filter} = \begin{bmatrix} \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \\ \frac{2}{16} & \frac{4}{16} & \frac{2}{16} \\ \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \end{bmatrix}$$

Blurring

Pixel(x,y): red = 36
green = 36
blue = 0



Original

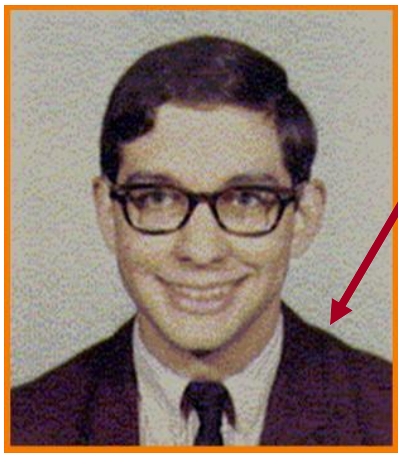
	X - 1	X	X + 1
Y - 1	36	109	146
Y	32	36	109
Y + 1	32	36	73

Pixel(x,y).red and its red neighbors

$$\text{Filter} = \begin{bmatrix} \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \\ \frac{2}{16} & \frac{4}{16} & \frac{2}{16} \\ \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \end{bmatrix}$$

Blurring

New value for Pixel(x,y).red =
(36 * 1/16) + (109 * 2/16) + (146 * 1/16)
(32 * 2/16) + (36 * 4/16) + (109 * 2/16)
(32 * 1/16) + (36 * 2/16) + (73 * 1/16)



Original

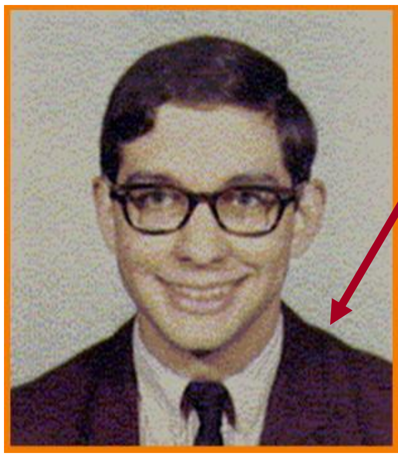
	X - 1	X	X + 1
Y - 1	36	109	146
Y	32	36	109
Y + 1	32	36	73

Pixel(x,y).red and its red neighbors

$$\text{Filter} = \begin{bmatrix} \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \\ \frac{2}{16} & \frac{4}{16} & \frac{2}{16} \\ \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \end{bmatrix}$$

Blurring

New value for Pixel(x,y).red = 62.69



Original

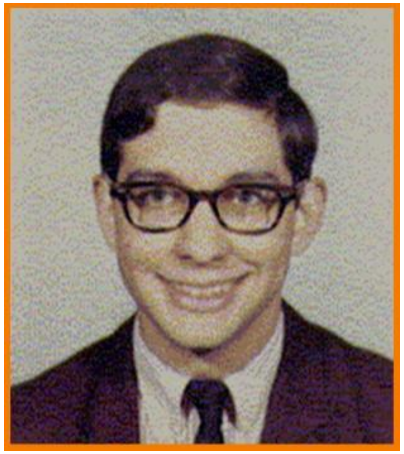
	X - 1	X	X + 1
Y - 1	36	109	146
Y	32	36	109
Y + 1	32	36	73

Pixel(x,y).red and its red neighbors

$$\text{Filter} = \begin{bmatrix} \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \\ \frac{2}{16} & \frac{4}{16} & \frac{2}{16} \\ \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \end{bmatrix}$$

Blurring

New value for Pixel(x,y).red = 63



Original

Blur

$$\begin{bmatrix} \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \\ \frac{2}{16} & \frac{4}{16} & \frac{2}{16} \\ \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \end{bmatrix}$$

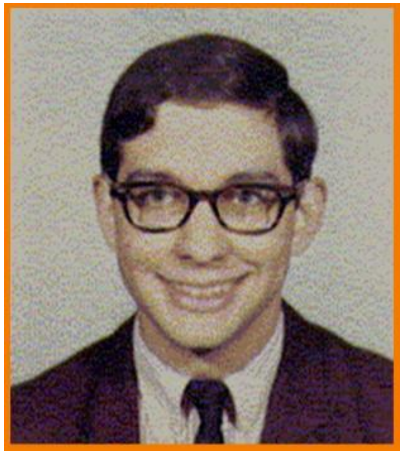
Blurring

- Repeat for each color channel of each pixel.
- Keep source and destination separate to avoid drift.
- For boundary pixels, not all neighbors are used:
  - » Normalize the mask so the values sum to one, or
  - » Assume that the exterior values are black, or
  - » Assume the exterior values can be obtained by reflecting the image across the boundary, or
  - » Assume…

Blurring

- Masks can have arbitrary size:
  - Can expand smaller masks by zero-padding

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 4 & 2 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} /16 \Leftrightarrow \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} /16$$



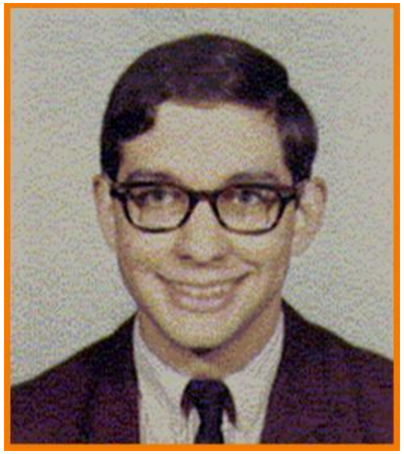
Original

Narrow Blur

Blurring

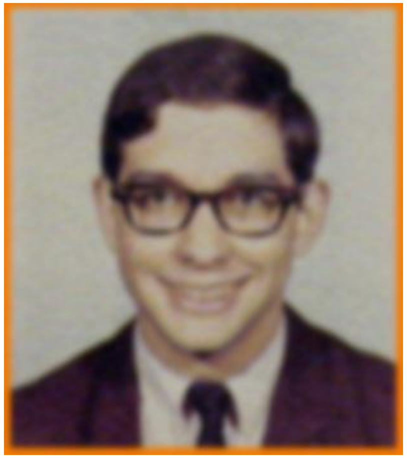
- Masks can have arbitrary size:
  - Can expand smaller masks by zero-padding
  - Can use more non-zero entries to get a wider blur

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 4 & 2 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} /16 \qquad \begin{bmatrix} 0 & 1 & 2 & 1 & 0 \\ 1 & 2 & 4 & 2 & 1 \\ 2 & 4 & 8 & 4 & 2 \\ 1 & 2 & 4 & 2 & 1 \\ 0 & 1 & 2 & 1 & 0 \end{bmatrix} /48$$



Original

Narrow Blur



Wide Blur

Blurring

A general way for defining the entries of an $n \times n$ mask is to use the values of a Gaussian:

$$\text{GaussianMask}[i][j] \sim e^{-\frac{(i-r)^2+(j-r)^2}{4r^2}}$$

$$i, j \in [0, 2r]$$

- r is the (integer) mask radius
- $n = 2r + 1$ is the width
- $0 \leq i \leq 2r$ is the horizontal position in the mask
- $0 \leq j \leq 2r$ is the vertical position in the mask
- **Don't forget to normalize!**

Note:

- The center of the mask is at index (r, r).
- The Gaussian itself has standard deviation $\sigma = \sqrt{2} \cdot r$

Edge Detection

An edge is where the image is far from constant:

⇒ The difference between the value at the pixel and the average value of neighboring pixels is large (in absolute value)

Edge Detection

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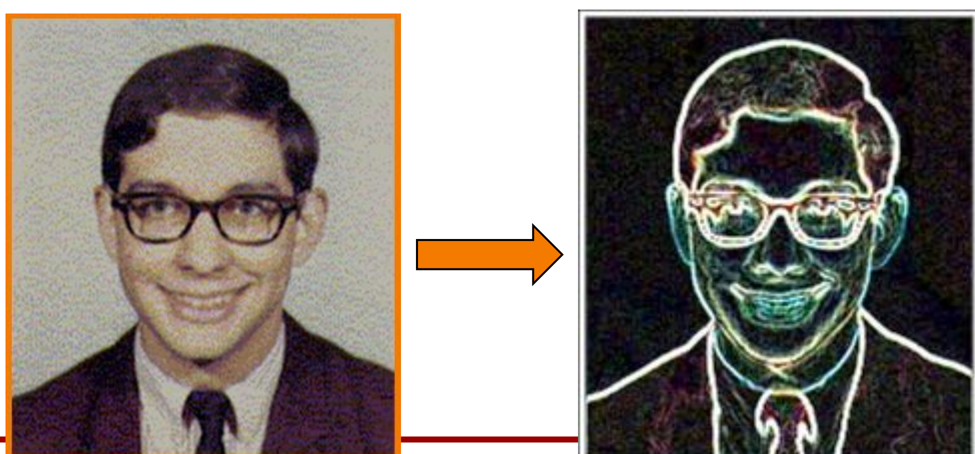
⇒ The difference between the value at the pixel and the average value of neighboring pixels is large (in absolute value)

Define a mask whose:

- Entries sum to zero
- Value is one at the center pixel



Filter $= \frac{1}{8}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$

Edge Detection

An edge is where the image is far from constant:

⇒ The difference between the value at the pixel and the average value of neighboring pixels is large (in absolute value)

Define a mask whose:

- Entries sum to zero
- Value is one at the center pixel

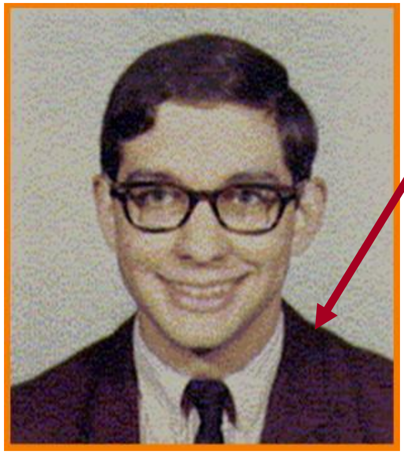
$$\text{Filter} = \frac{1}{\;}\begin{bmatrix} -1 & -1 & -1 \\ & & \\ & & \end{bmatrix}$$

Pixels with large absolute values correspond to edges:

- Positive values: “upper” edges
- Negative values: “lower” edges

Edge Detection

Pixel(x,y): red = 36
green = 36
blue = 0



Original

Filter = $\frac{1}{8}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$

Edge Detection

Pixel(x,y): red = 36
green = 36
blue = 0

Original

	X - 1	X	X + 1
Y - 1	36	109	146
Y	32	$\boxed{36}$	109
Y + 1	32	36	73

Pixel(x,y).red and its red neighbors

$$\text{Filter} = \frac{1}{8}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Edge Detection

New value for Pixel(x,y).red =
(36 * -1/8) + (109 * -1/8) + (146 * -1/8)
(32 * -1/8) + (36 * 1) + (109 * -1/8)
(32 * -1/8) + (36 * -1/8) + (73 * -1/8)

Original

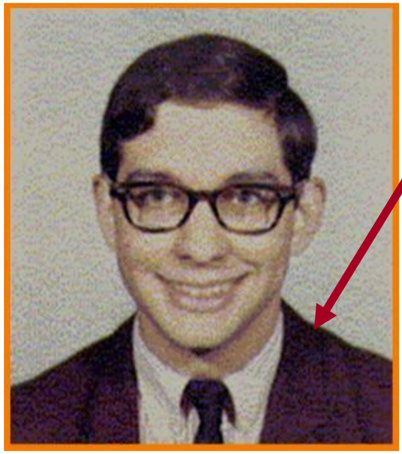
	X - 1	X	X + 1
Y - 1	36	109	146
Y	32	36	109
Y + 1	32	36	73

Pixel(x,y).red and its red neighbors

$$\text{Filter} = \frac{1}{8}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Edge Detection

New value for Pixel(x,y).red = -285/8



Original

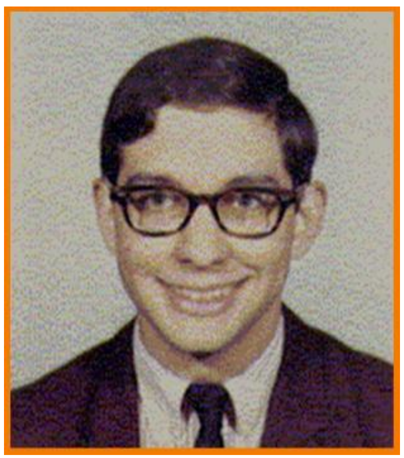
	X - 1	X	X + 1
Y - 1	36	109	146
Y	32	$\boxed{36}$	109
Y + 1	32	36	73

Pixel(x,y).red and its red neighbors

$$\text{Filter} = \frac{1}{8}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Edge Detection

New value for Pixel(x,y).red = -35.625



Original

Detected Edges

Filter = $\frac{1}{8}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$

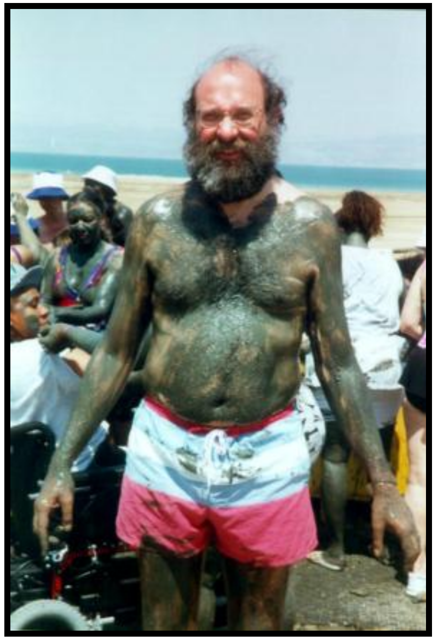
<u>Note</u>:
Output values are not colors, so need to find a way to remap for visualization.

Outline

- Image Filtering
- Image Warping
- Image Morphing

Image Warping

- Move pixels of image
  - Mapping
  - Resampling



Warp

Source image

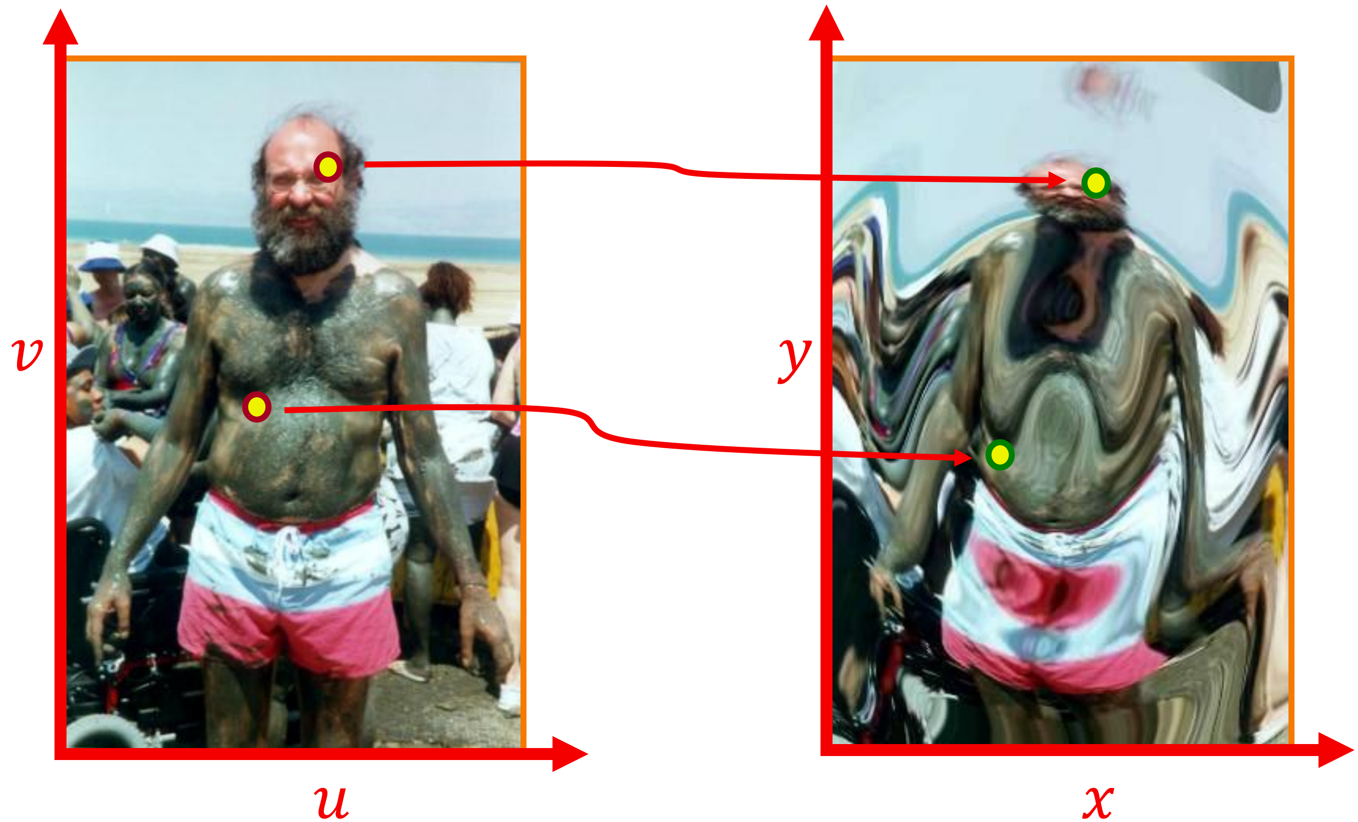
Destination image

Overview

- Mapping
  - Forward
  - Inverse
- Resampling
  - Point sampling
  - Triangle filter
  - Gaussian filter

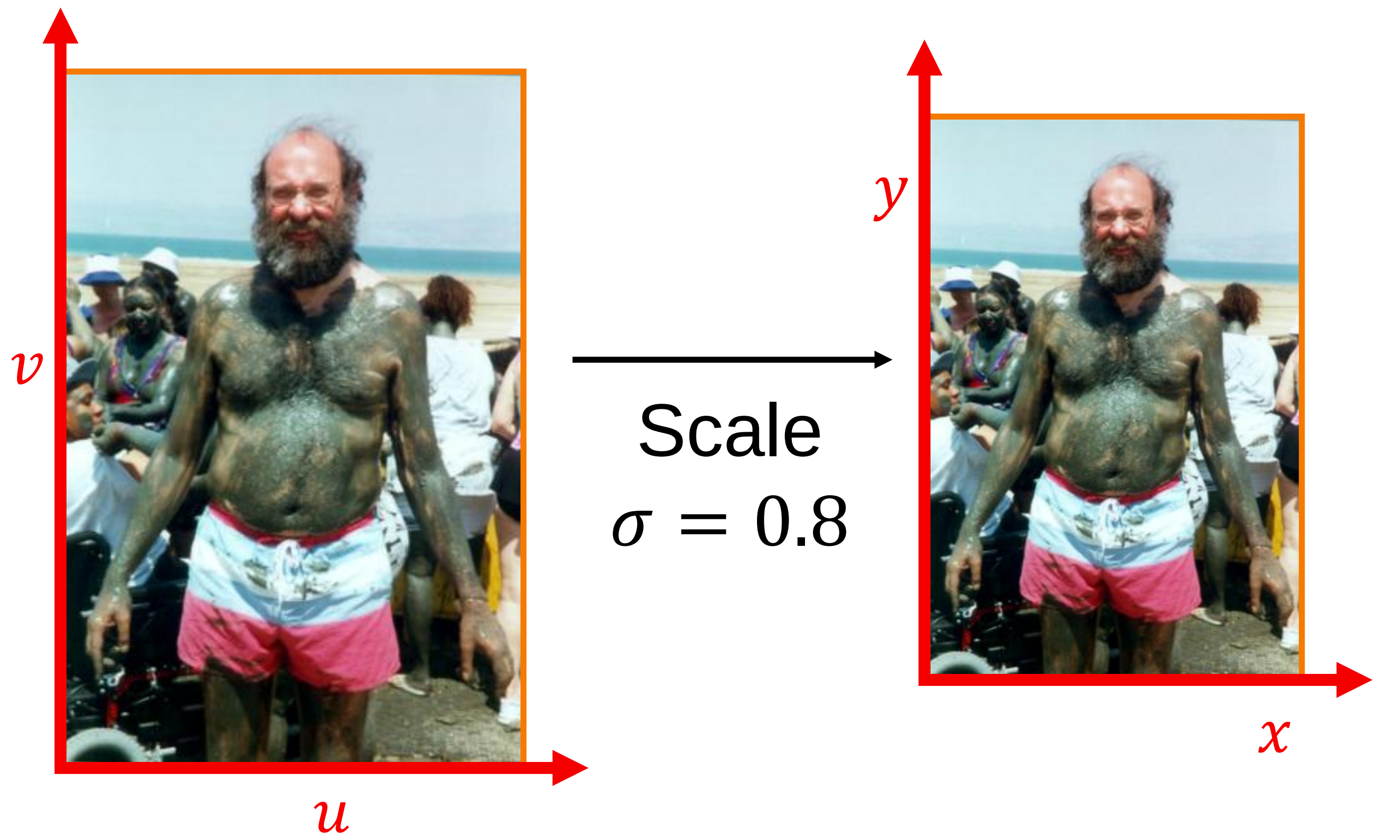
Mapping

- Define transformation
  - Describe the destination $(x, y) = \Phi(u, v)$ for every location (u, v) in the source



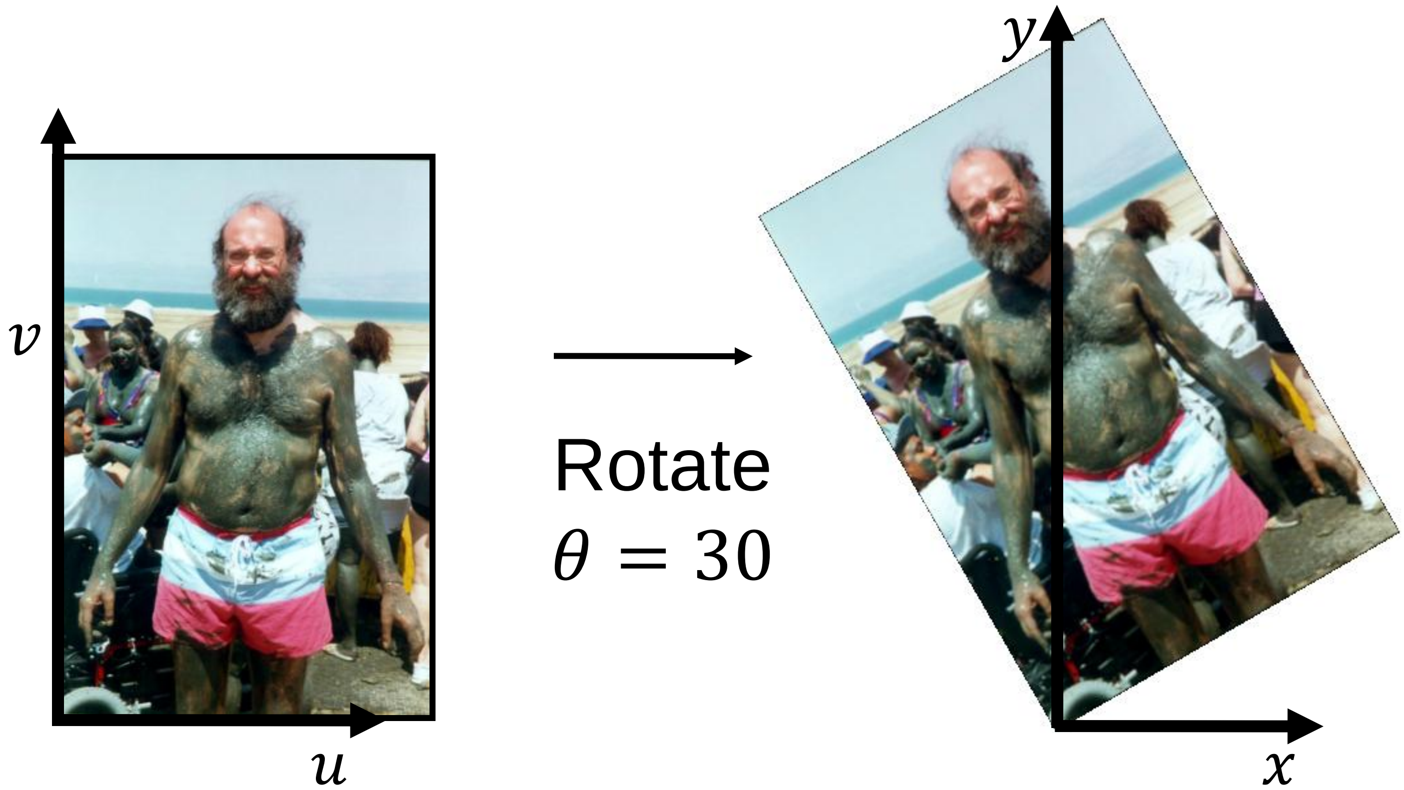
Example Mappings

- Scale by σ:
  - $\Phi(u, v) = (\sigma u, \sigma v)$



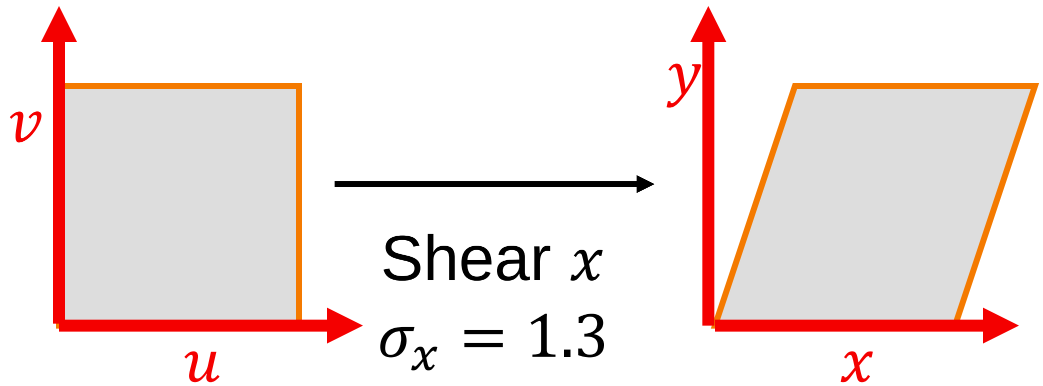
Example Mappings

- Rotate by θ degrees:
  - $\Phi(u, v) = (u\cos\theta - v\sin\theta\,, u\sin\theta + v\cos\theta)$

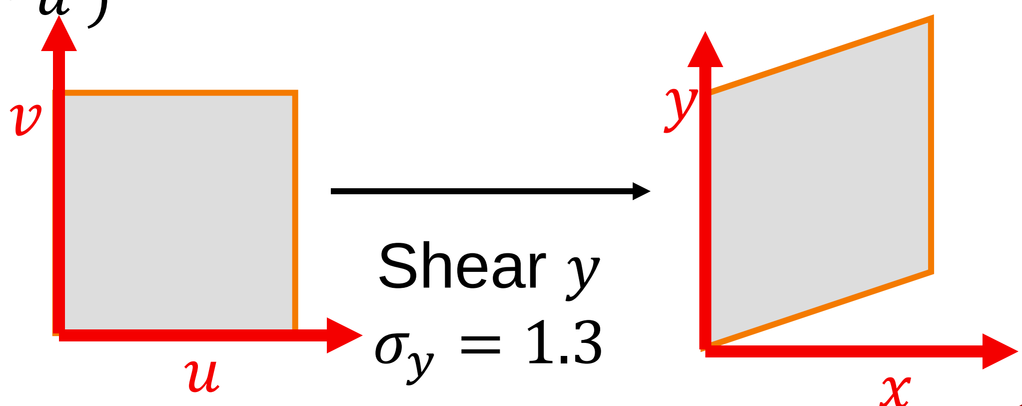


Example Mappings

- Shear in x by σ_x:
  - $\Phi(u, v) = (u + \sigma_x \cdot v, v)$

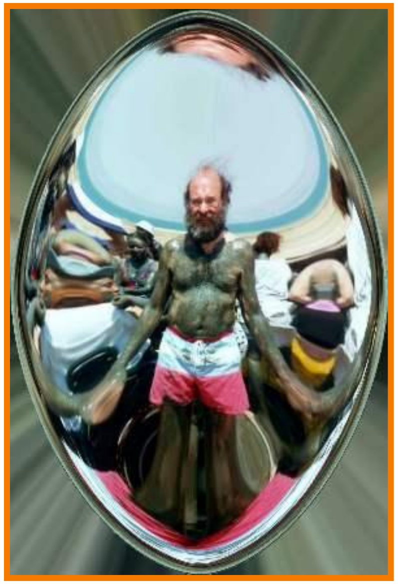


- Shear in y by σ_y:
  - $\Phi(u, v) = (u, v + \sigma_y \cdot u\ )$



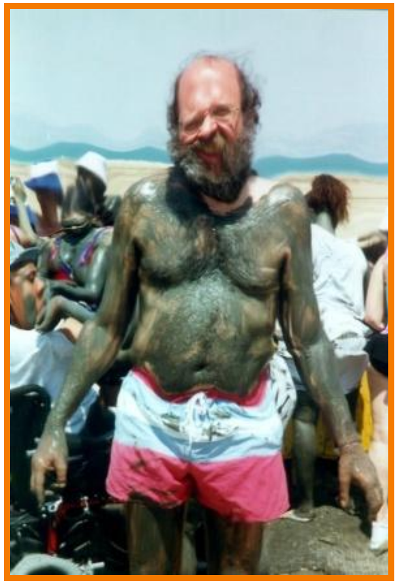
Other Mappings

- Any function of u and v:
  - $\Phi(u, v) = \cdots$



Fish-eye

"Swirl"

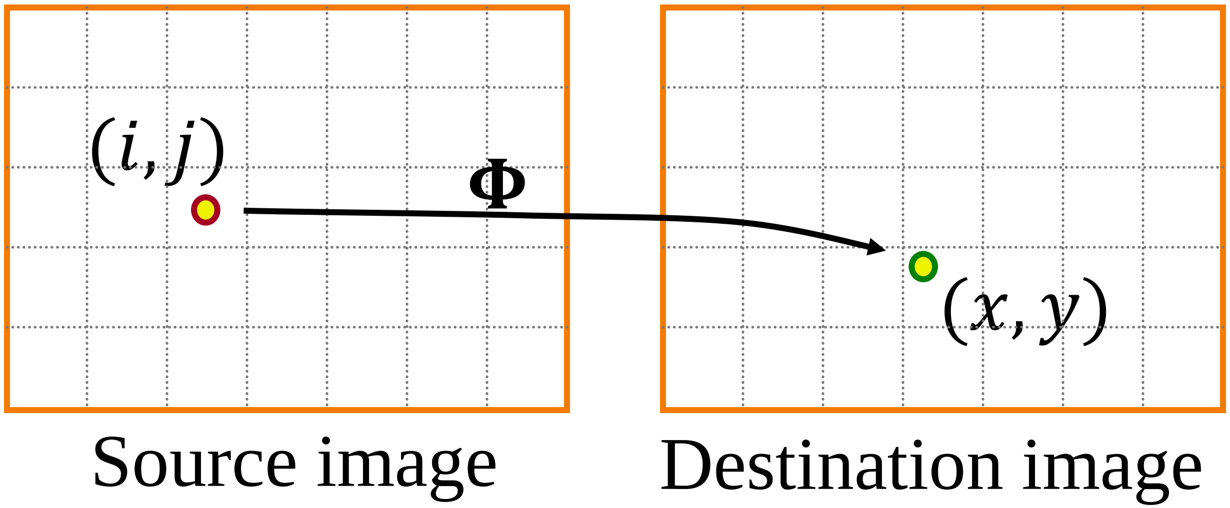


"Rain"

Image Warping Implementation I

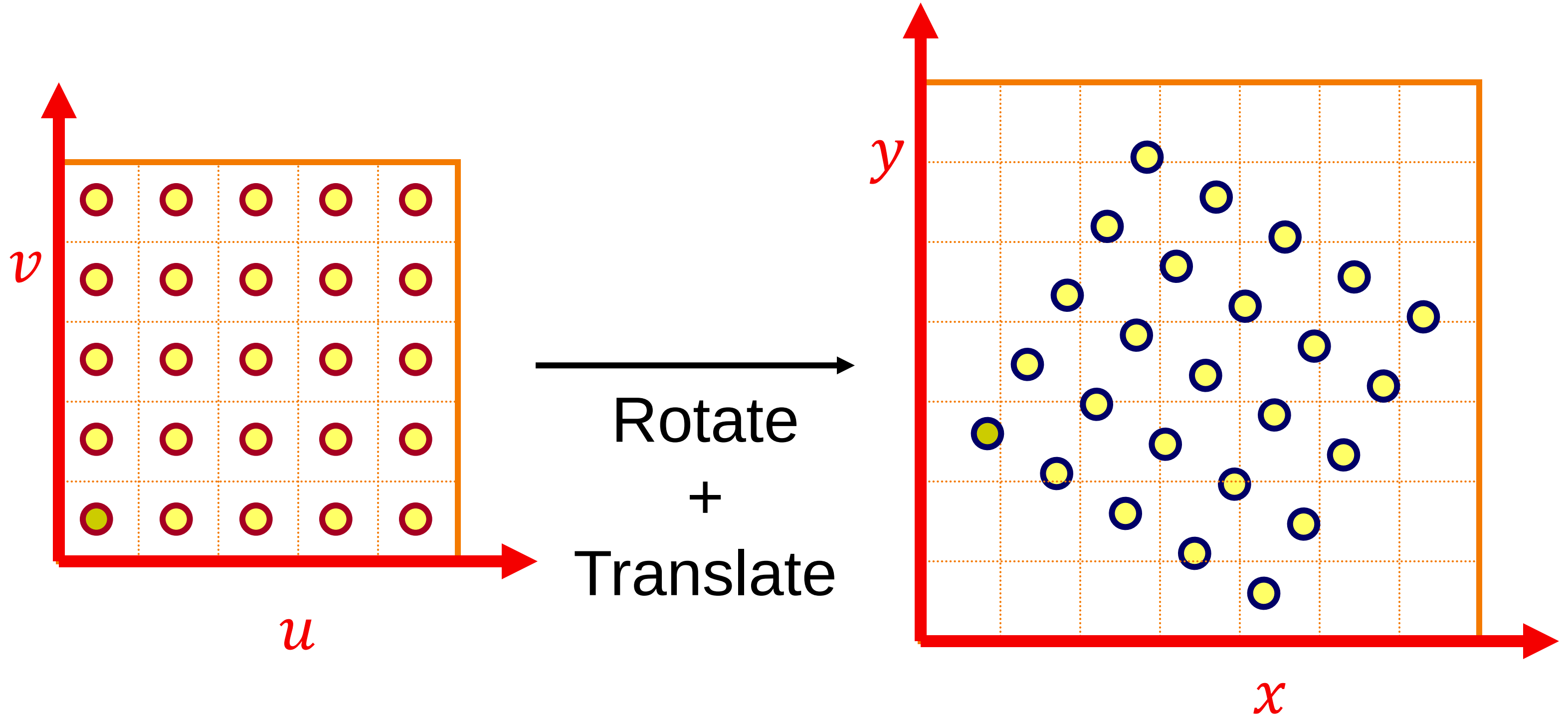
- Forward mapping:

```
for( j=0 ; j<srcHeight ; j++ )
  for( i=0 ; i<srcWidth ; i++ )
    (x,y) = Φ(i,j);
    dst(x,y) = src(i,j);
```



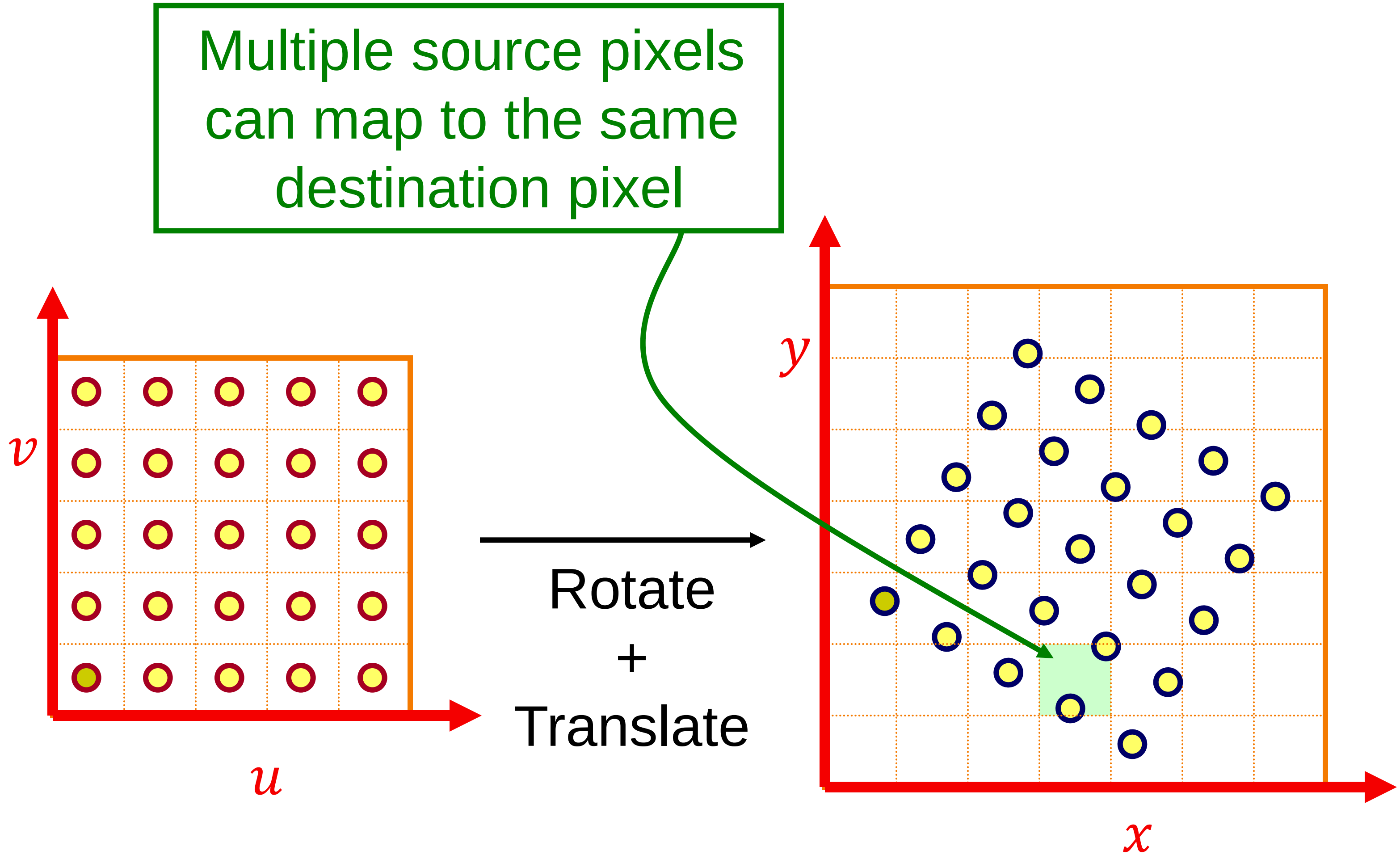
Forward Mapping

- Iterate over source image



Forward Mapping – BAD!

- Iterate over source image



Forward Mapping – BAD!

- Iterate over source image

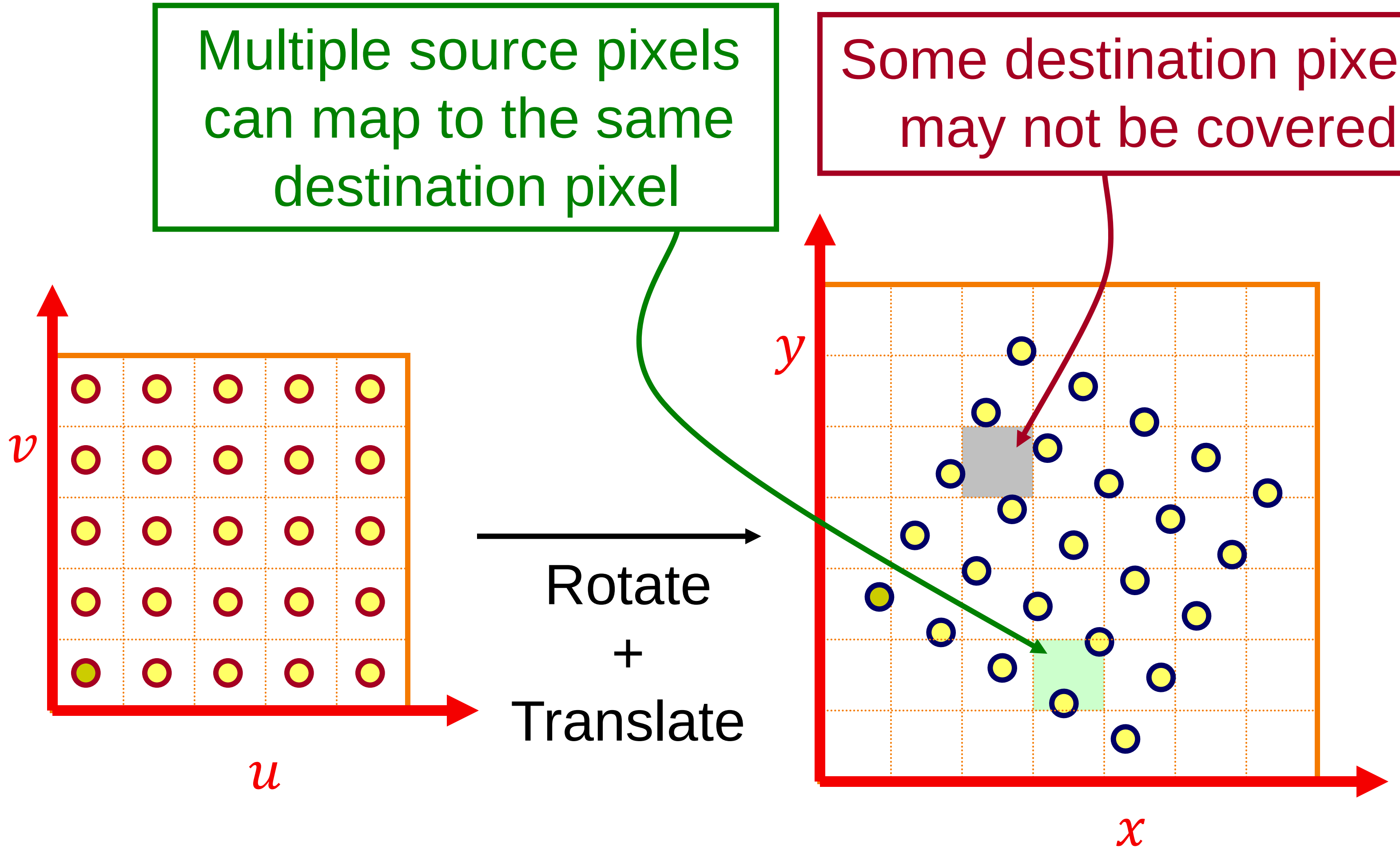
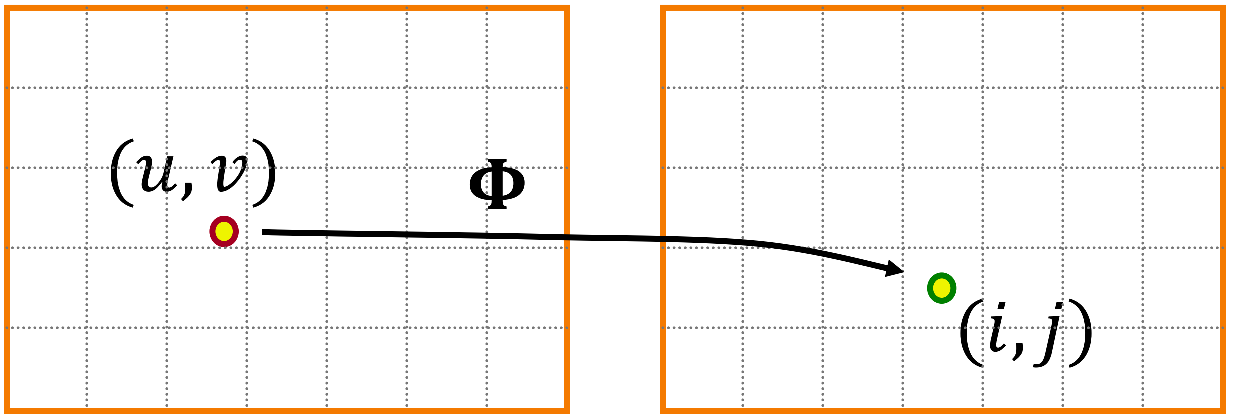


Image Warping Implementation II

- Inverse mapping:

```
for( j=0 ; j<dstHeight ; j++ )
   for( i=0 ; i<dstWidth ; i++ )
      (u,v) = Φ^-1(i,j);
      dst(i,j) = src(u,v);
```

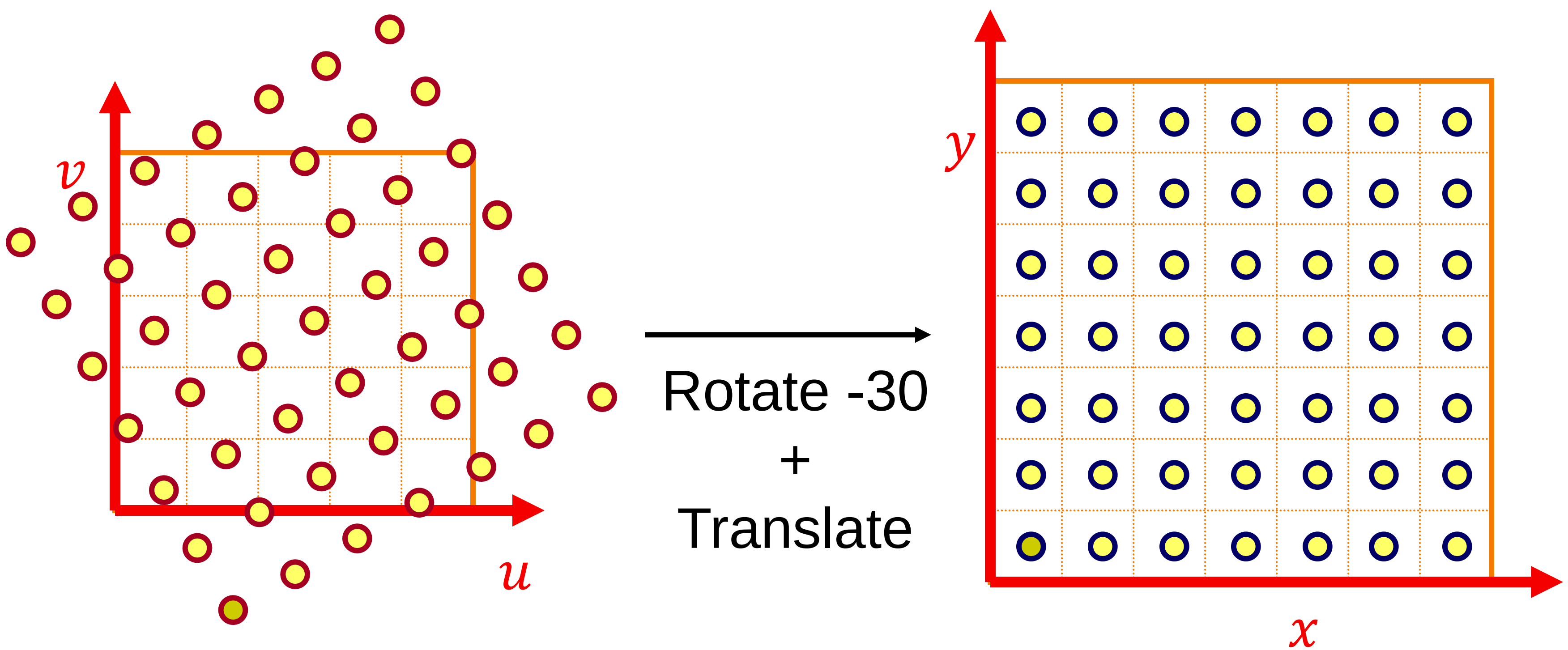


Source image

Destination image

Reverse Mapping – GOOD!

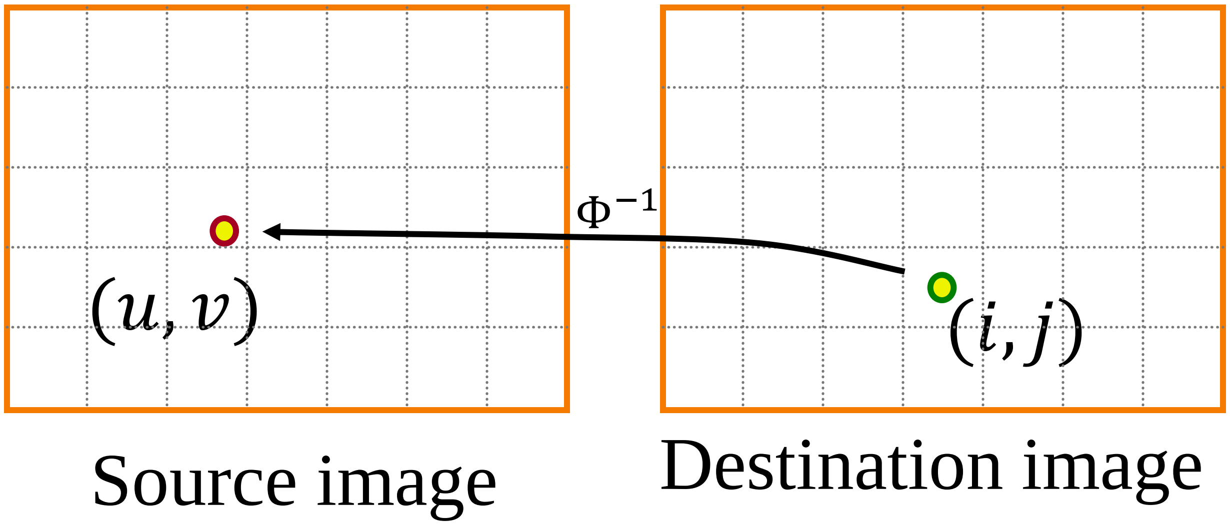
- Iterate over destination image
  - Must resample source



Resampling

- Evaluate source image at $(u, v) = \Phi^{-1}(i, j)$

(u, v) does not usually have integer coordinates



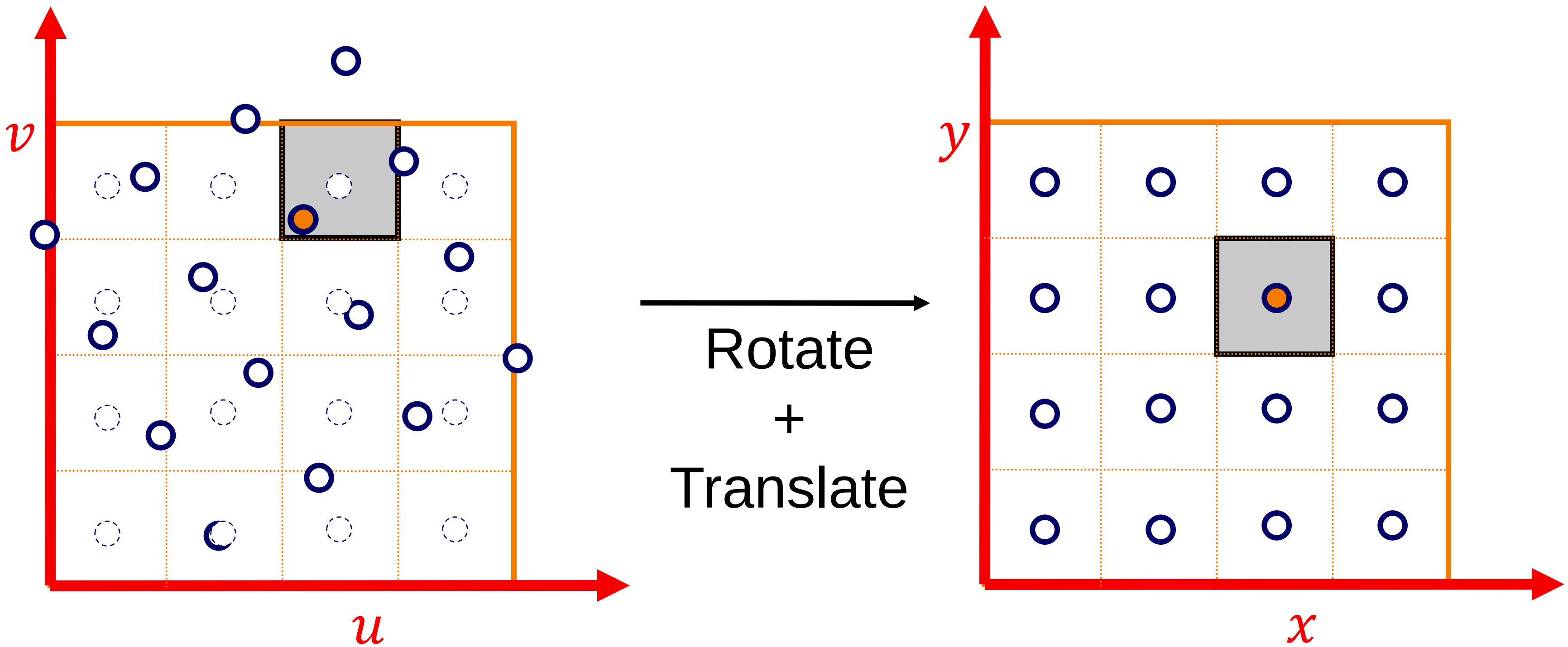
Overview

- Mapping
  - Forward
  - Inverse
- Resampling
  - Nearest Point Sampling
  - Bilinear Sampling
  - Gaussian Sampling

Nearest Point Sampling

- Take value at closest pixel:

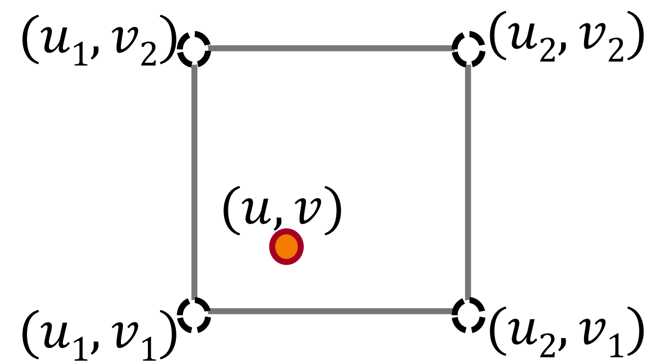
```
int intU = floor(u+0.5);
int intV = floor(v+0.5);
dst(i,j) = src(intU,intV);
```

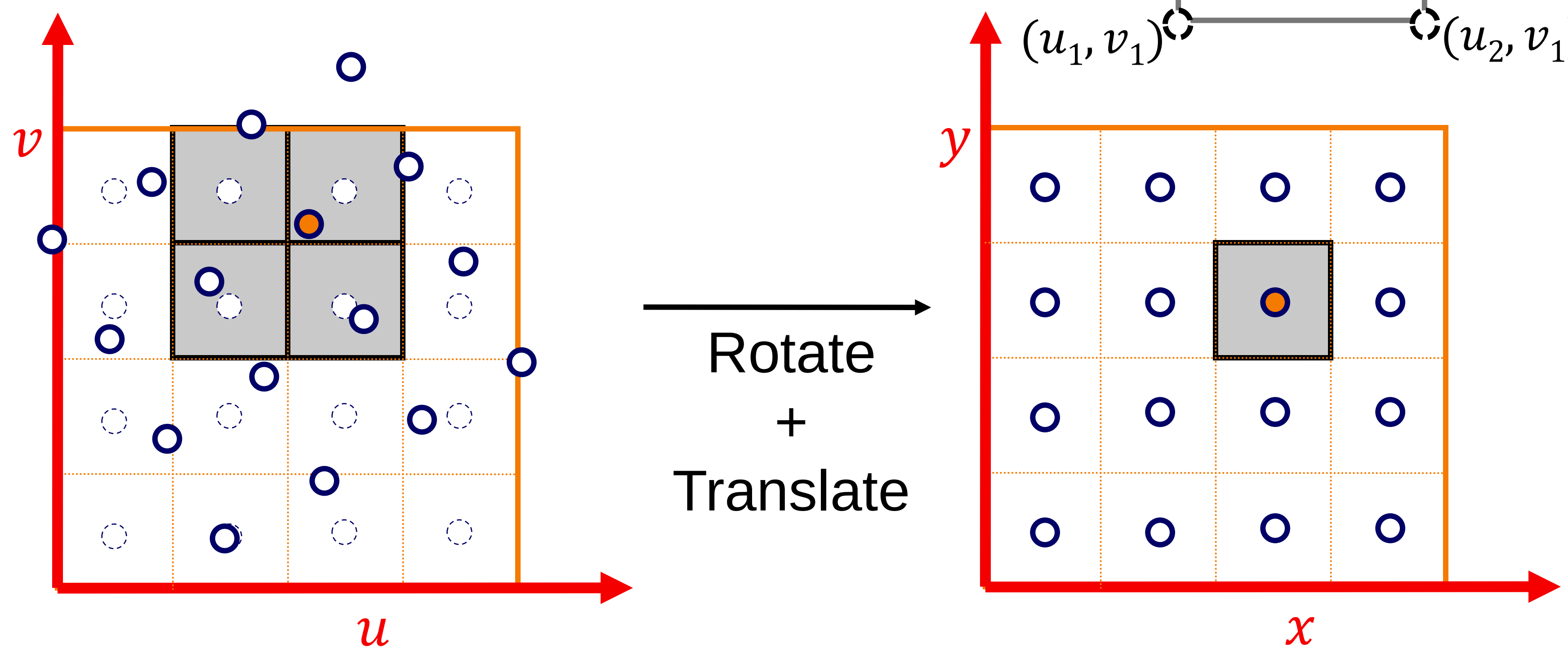


Bilinear Sampling

- Bilinearly interpolate four closest source pixels

$\mathrm{dst}(i, j)$ = Weighted average of source at
(u_1, v_1), (u_2, v_1), (u_1, v_2), and (u_2, v_2)

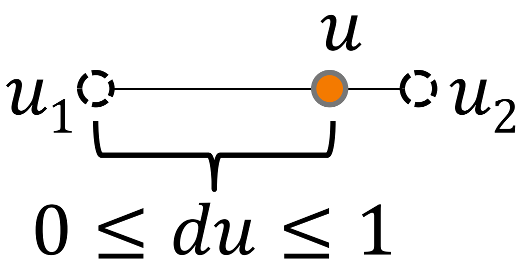




Linear Sampling

- Linearly interpolate two closest source pixels

  dst(i) = linear interpolation of u_1 and u_2




```
u = Φ^{-1}(i)
u1 = floor(u);
u2 = u1 + 1;
du = u - u1;
dst(i) = src(u1)*(1-du) + src(u2)*du;
```

Bilinear Sampling

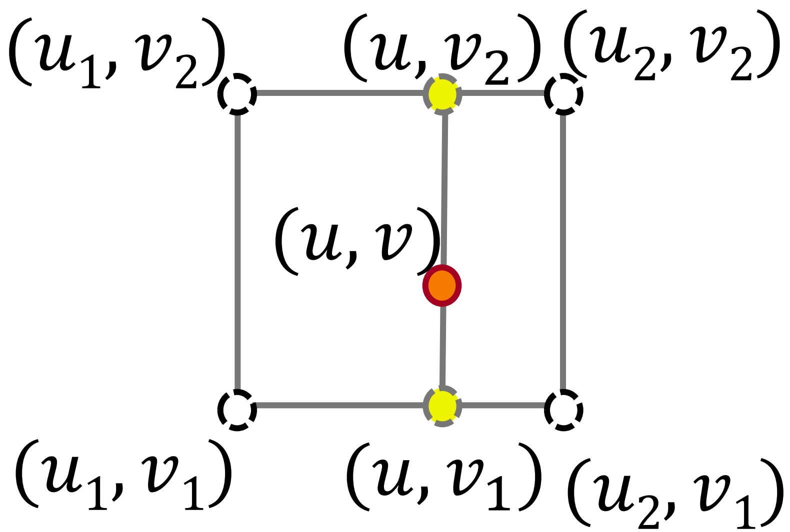
- Bilinearly interpolate four closest source pixels

  a = linear interpolation of $\mathrm{src}(u_1, v_1)$ and $\mathrm{src}(u_2, v_1)$

  b = linear interpolation of $\mathrm{src}(u_1, v_2)$ and $\mathrm{src}(u_2, v_2)$

  $\mathrm{dst}(i, j)$ = linear interpolation of a and b

```
(u,v) = Φ^{-1}(i,j)

u1 = floor( u ) , u2 = u1 + 1;
v1 = floor( v ) , v2 = v1 + 1;

du = u - u1;

a = src(u1,v1)*(1-du)
  + src(u2,v1)*(  du);

b = src(u1,v2)*(1-du)
  + src(u2,v2)*du;

dv = v - v1;
dst(i,j) = a*(1-dv) + b*dv;
```



Bilinear Sampling

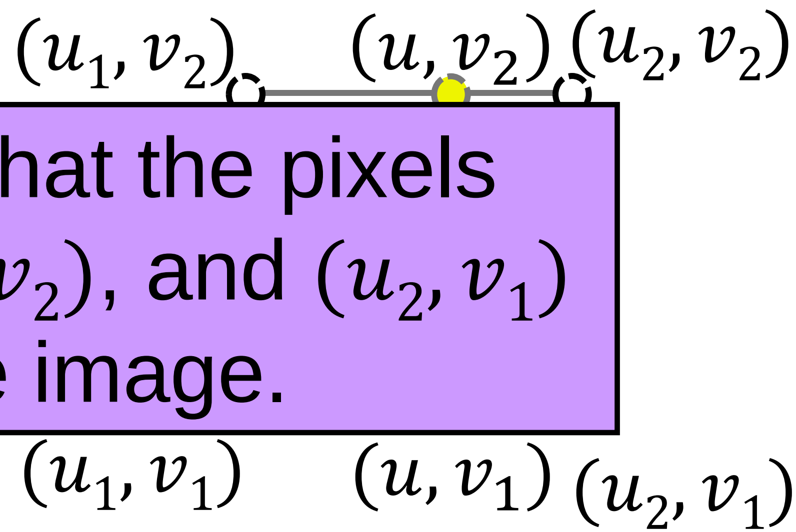
- Bilinearly interpolate four closest source pixels

  a = linear interpolation of $\mathrm{src}(u_1, v_1)$ and $\mathrm{src}(u_2, v_1)$

  b = linear interpolation of $\mathrm{src}(u_1, v_2)$ and $\mathrm{src}(u_2, v_2)$

  $\mathrm{dst}(i, j)$ = linear interpolation of a and b

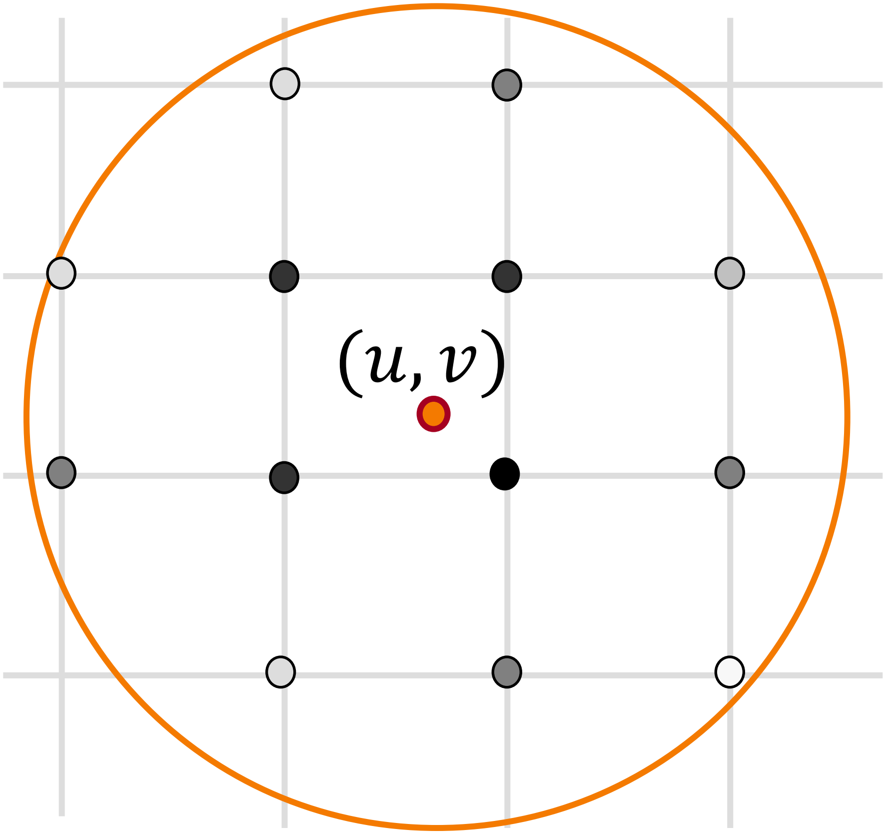
```
(u,v) = Φ^{-1}(i,j)

u1 =
v1 =

du =

a =
   + src(u2,v1)*(  du);

b = src(u1,v2)*(1-du)
   + src(u2,v2)*du;

dv = v - v1;
dst(i,j) = a*(1-dv) + b*dv;
```



Make sure to test that the pixels (u_1, v_1), (u_2, v_2), (u_1, v_2), and (u_2, v_1) are within the image.

Gaussian Sampling

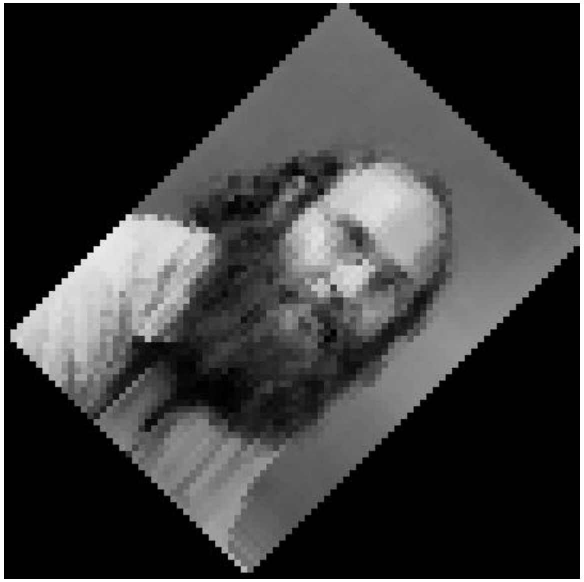
- Compute weighted sum of pixel neighborhood:
  - The blending weights are the normalized values of a Gaussian function.



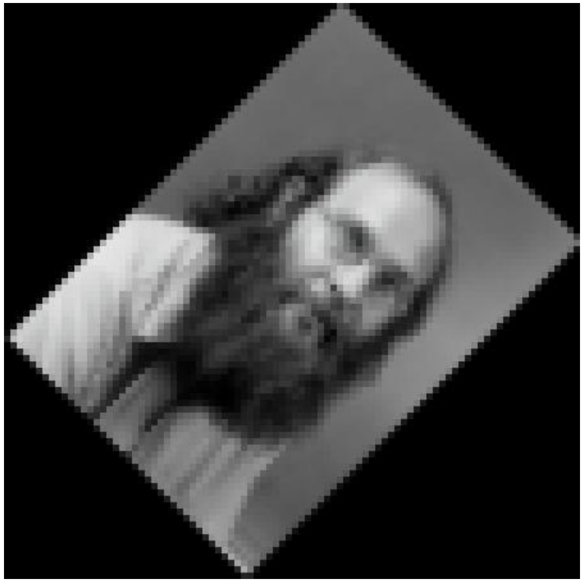
Filtering Methods Comparison

- Trade-offs
  - Jagged edges versus blurring
  - Computation speed

We'll talk more about trade-offs next time.



Nearest



Bilinear

Gaussian

Image Warping Implementation

- Inverse mapping:

```
for( j=0 ; j<dstHeight ; j++ )
  for( i=0 ; i<dstWidth ; i++ )
    (u,v) = Φ^-1(i,j);
    dst(i,j) = resample_src(u,v,r);
```

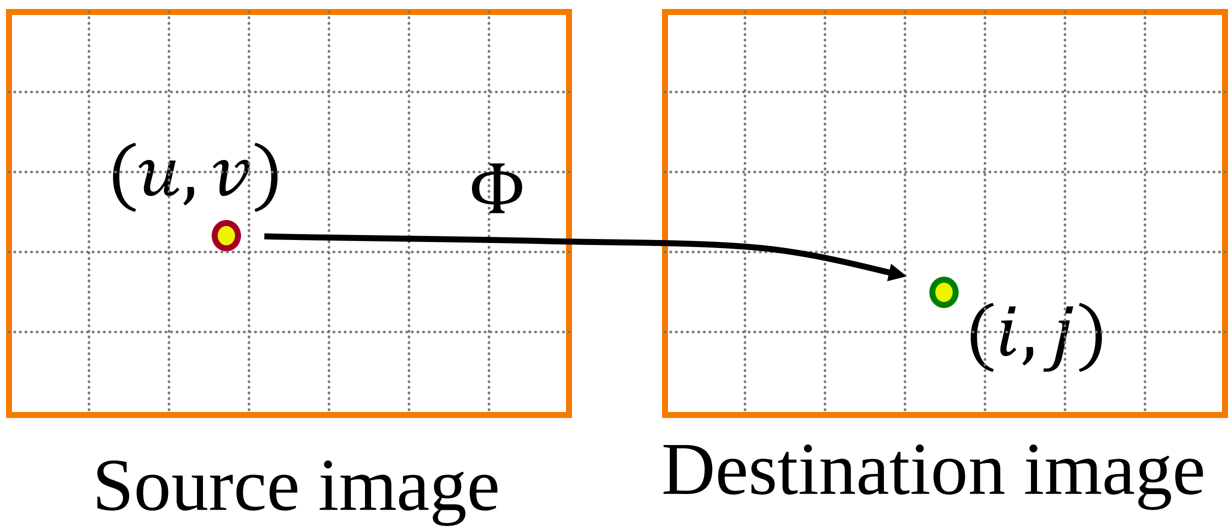
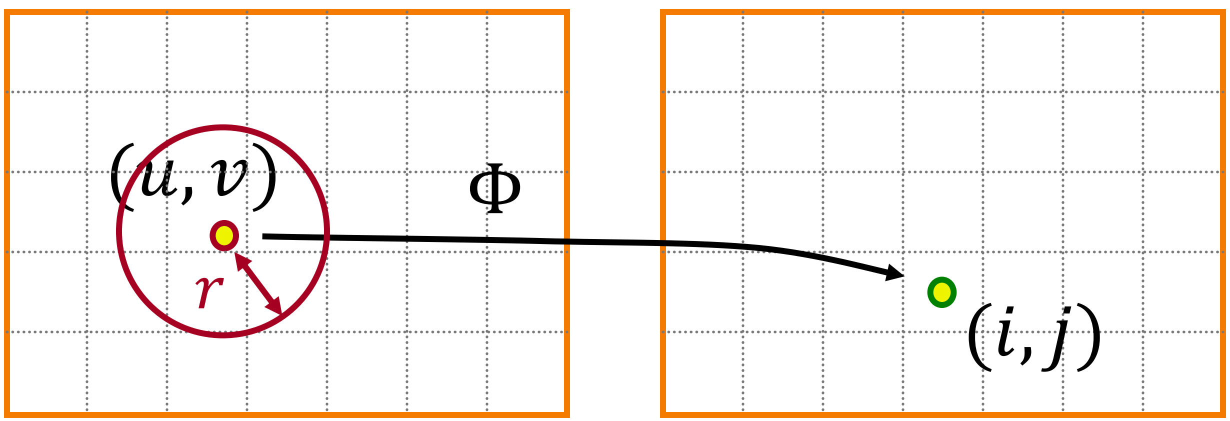


Image Warping Implementation

- Inverse mapping:

```
for( j=0 ; j<dstHeight ; j++ )
  for( i=0 ; i<dstWidth ; i++ )
    (u,v) = Φ^-1(i,j);
    dst(i,j) = resample_src(u,v,r);
```



Source image

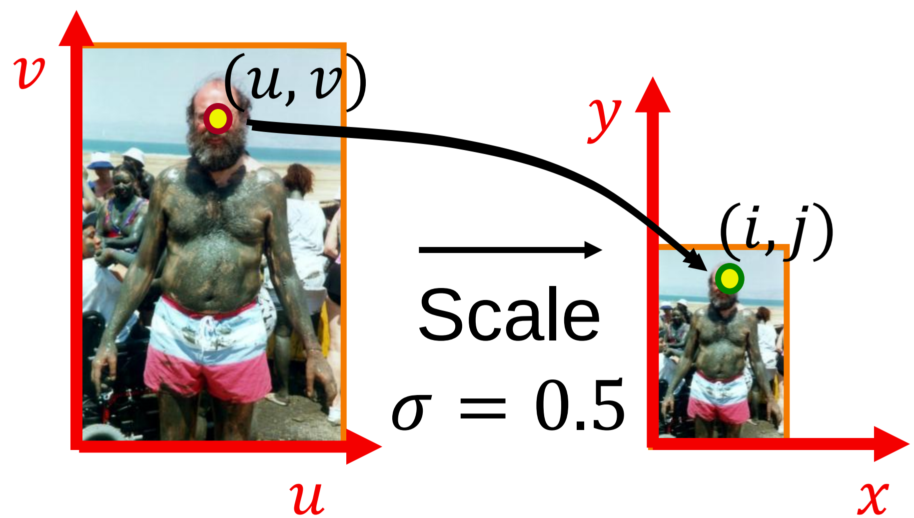
Destination image

Example: Scale

Scale(src , dst , σ):

```
r ≅ ?;
for( j=0 ; j<dstHeight ; j++ )
  for( i=0 ; i<dstWidth ; i++ )
    (u,v) = (i,j) / σ;
    dst(i,j) = resample_src(u,v,r);
```

$$\boxed{r = \frac{1}{\sigma}}$$

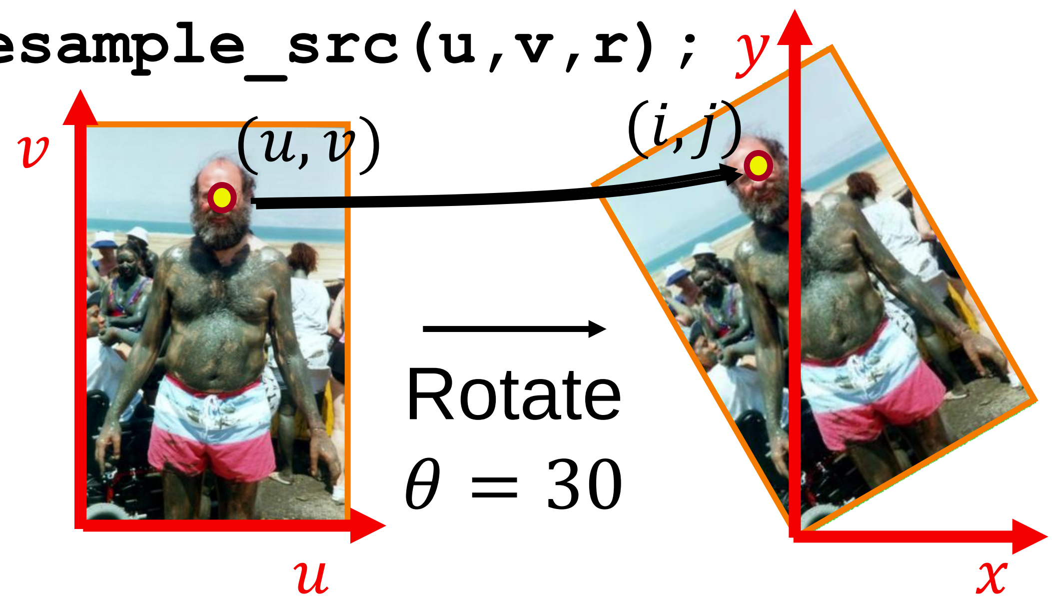


Example: Rotate

Rotate(src , dst , θ):

```
r ≅ ?;
for( j=0 ; j<dstHeight ; j++ )
   for( i=0 ; i<dstWidth ; i++ )
      (u,v) = ( i*cos(-θ) - j*sin(-θ) ,
                i*sin(-θ) + j*cos(-θ) );
      dst(x,y) = resample_src(u,v,r);
```

$\boxed{r = 1}$

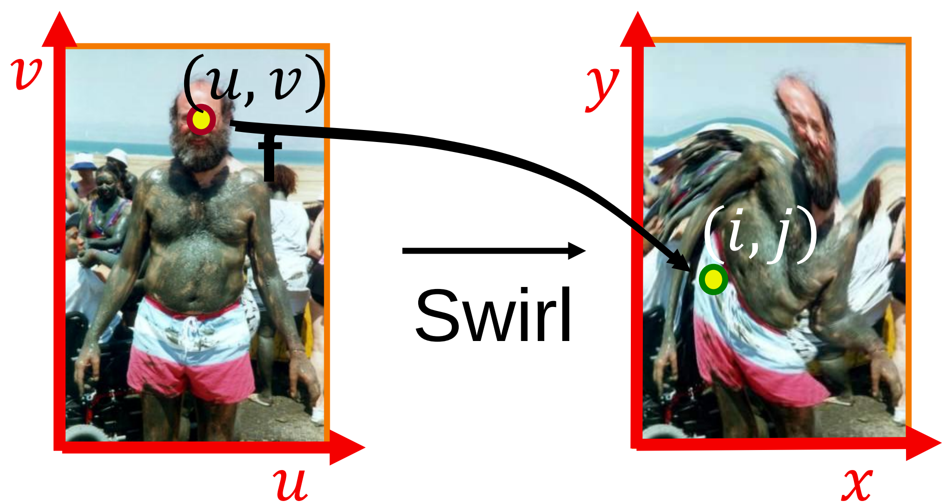


Example:

General(src , dst , Φ):

```
r ≅ ?;
for( j=0 ; j<dstHeight ; j++ )
  for( i=0 ; i<dstWidth ; i++ )
    (u,v) = Φ-1(i,j);
     dst(i,j) = resample_src(u,v,r);
```

$$\boxed{r = ?}$$



Example:

General(src , dst , Φ):

```
r ≅ ?;
for( j=0 ; j<dstHeight ; j++ )
   for( i=0 ; i<dstWidth ; i++ )
      (u,v) = Φ⁻¹(i,j);
       dst(i,j) = resample_src(u,v,r);
```

Instead of using a fixed radius circle to sample the source, we can:

1. Have the radius changes
2. Use an ellipse.

For example, the parameters can be determined by looking at the derivative/Jacobean of Φ.

Outline

- Image Filtering
- Image Warping
- Image Morphing

Image Warping

Recall:

1. For a vector $\vec{v} = (x, y)$, its $90°$ (ccw) rotation is given by:
$$\vec{v}^{\perp} = (-y, x)$$

Image Warping

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1. For a vector $\vec{v} = (x, y)$, its $90°$ (ccw) rotation is given by:
$$\vec{v}^{\perp} = (-y, x)$$

2. For two vectors $\vec{v}_1 = (x_1, y_1)$ and $\vec{v}_2 = (x_2, y_2)$
$$\langle \vec{v}_1, \vec{v}_2 \rangle \equiv x_1 \cdot x_2 + y_1 \cdot y_2 = \|\vec{v}_1\| \cdot \|\vec{v}_2\| \cdot \cos\theta$$

$\Rightarrow$ The signed length of the projection of v_1 onto the line through v_2 is:
$$\|v_1\| \cdot \cos\theta = \frac{\langle v_1, v_2 \rangle}{\|v_2\|}$$

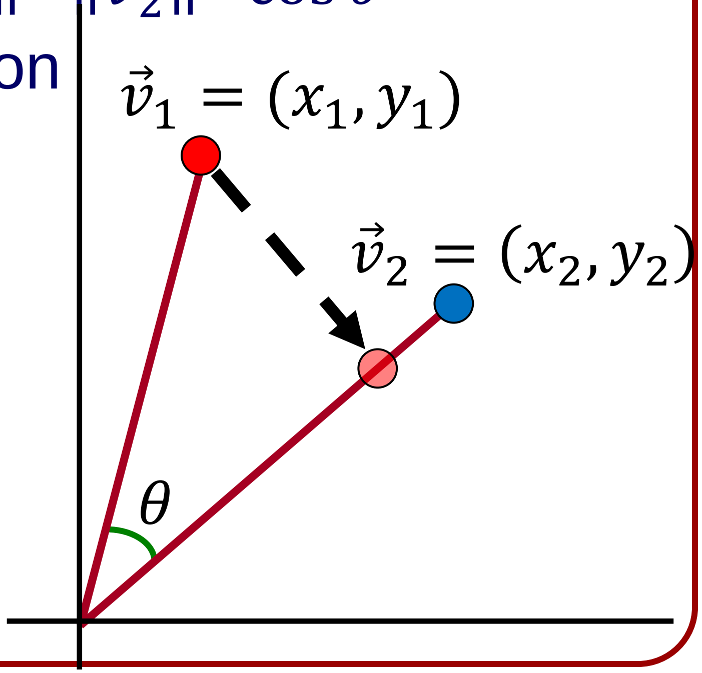


Image Warping

Recall:

1. For a vector $\vec{v} = (x, y)$, its $90°$ (ccw) rotation is given by:
$$\vec{v}^{\perp} = (-y, x)$$

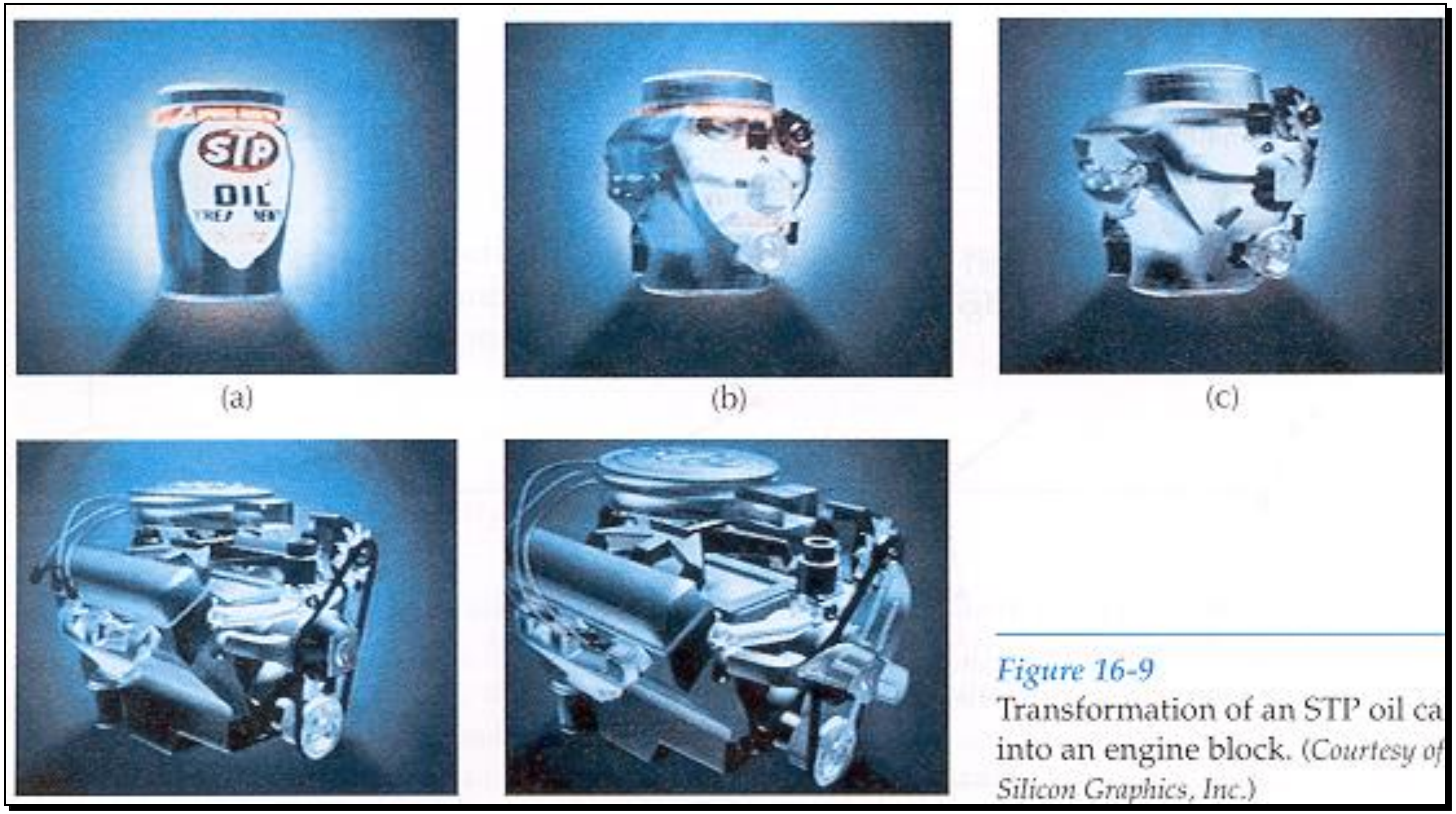
2. For two vectors $\vec{v}_1 = (x_1, y_1)$ and $\vec{v}_2 = (x_2, y_2)$
$$\langle \vec{v}_1, \vec{v}_2 \rangle \equiv x_1 \cdot x_2 + y_1 \cdot y_2 = \|\vec{v}_1\| \cdot \|\vec{v}_2\| \cdot \cos\theta$$

3. The inner-product of two vectors is *isometry-invariant*:
If $\mathbf{R} \in \mathbb{R}^{2\times 2}$ is a rotation, then:
$$\langle \mathbf{R}(\vec{v}_1), \mathbf{R}(\vec{v}_2) \rangle = \langle \vec{v}_1, \vec{v}_2 \rangle$$

$\Rightarrow$ In particular:
$$\left\langle \vec{v}_1^{\perp}, \vec{v}_2^{\perp} \right\rangle = \langle \vec{v}_1, \vec{v}_2 \rangle$$

Image Morphing

- Animate transition between two images



H&B Figure 16.9

Image Morphing

- Animate transition between two images

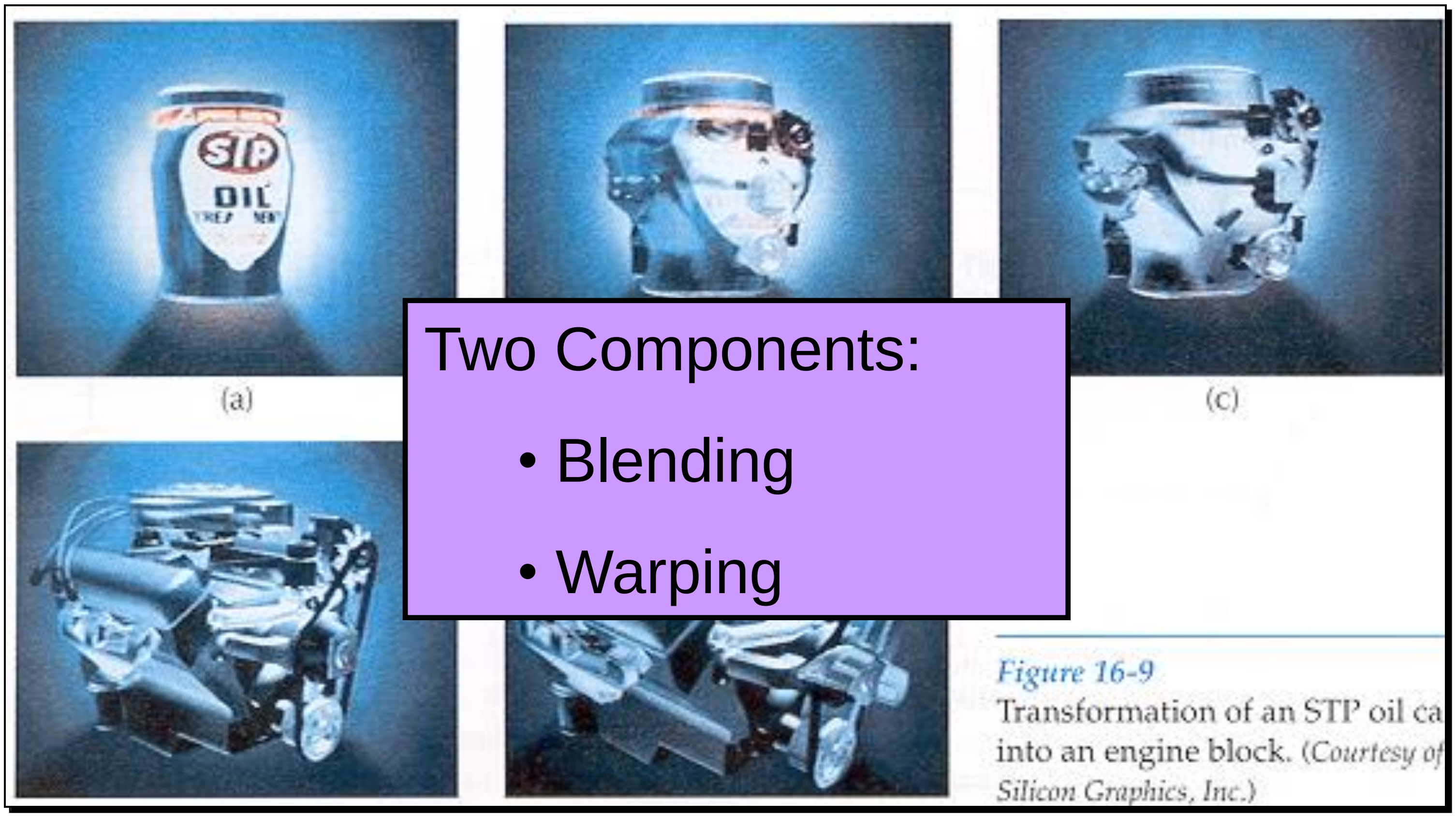


Figure 16-9
Transformation of an STP oil ca
into an engine block. (Courtesy of
Silicon Graphics, Inc.)

Two Components:

- Blending
- Warping

H&B Figure 16.9

Blending

Blend **colors** using an α-blend ($\alpha \in [0,1]$):

$$\text{blend}(i,j,\alpha) = (1-\alpha)\cdot \text{Img}_0(i,j) + \alpha \cdot \text{Img}_1(i,j)$$

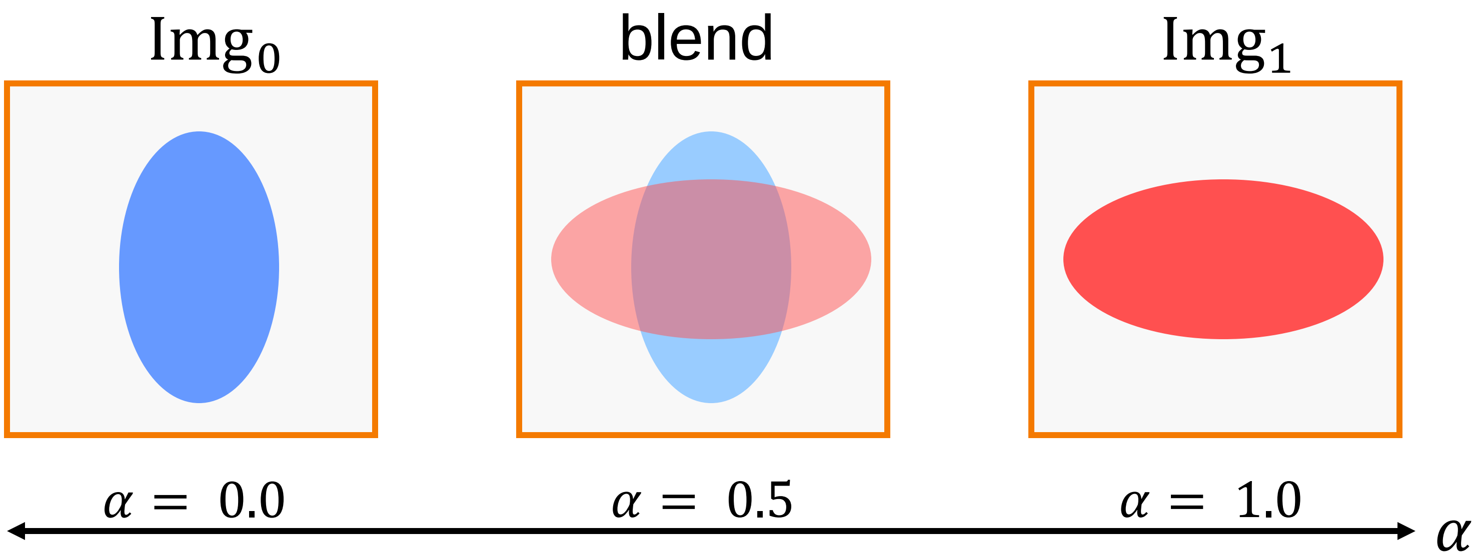


Image Warping

Deform Img_0 so its **shape** matches that of Img_1…

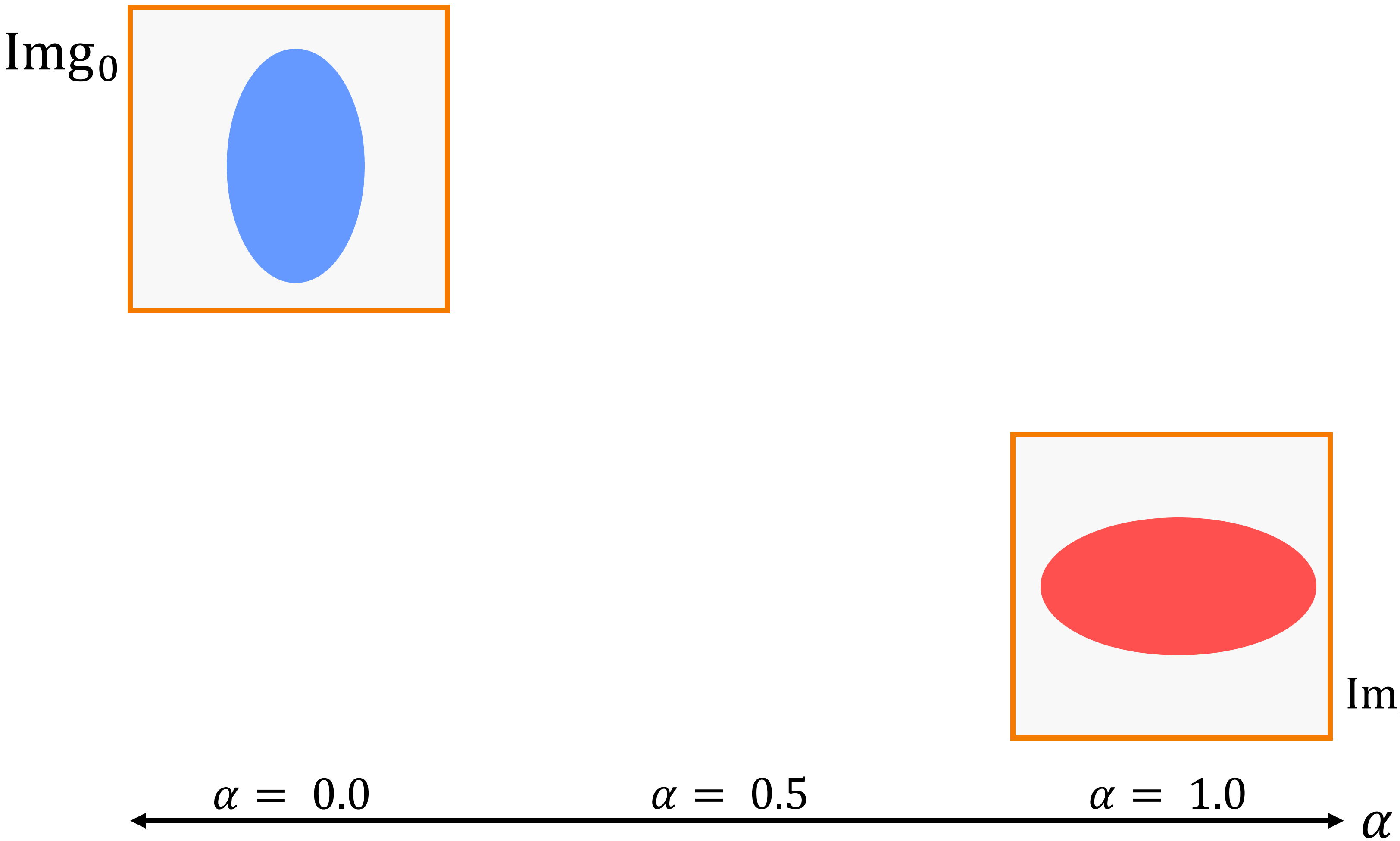


Image Warping

Deform Img_0 so its **shape** matches that of Img_1…

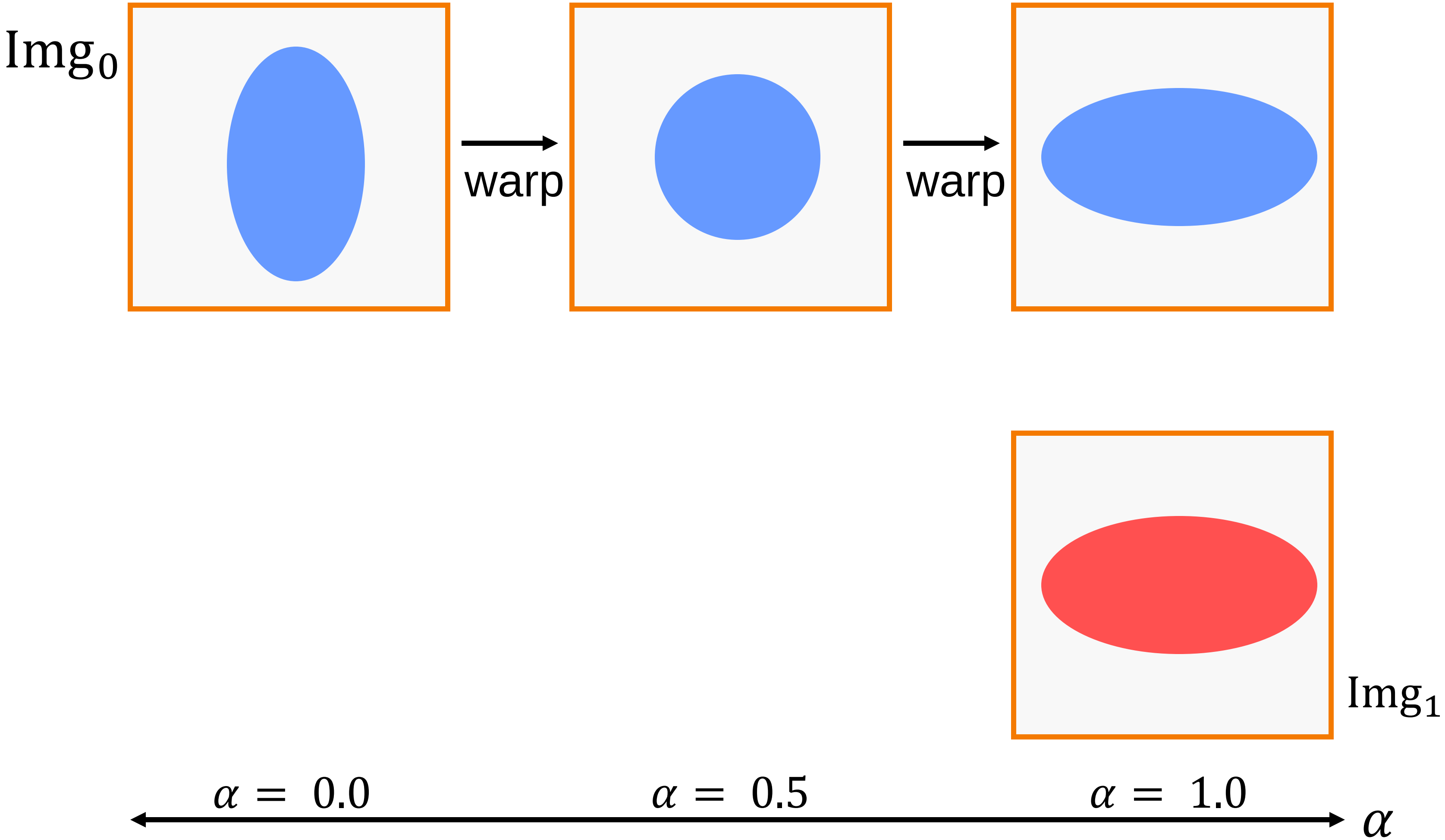


Image Warping

Deform Img_1 so its **shape** matches that of Img_0…

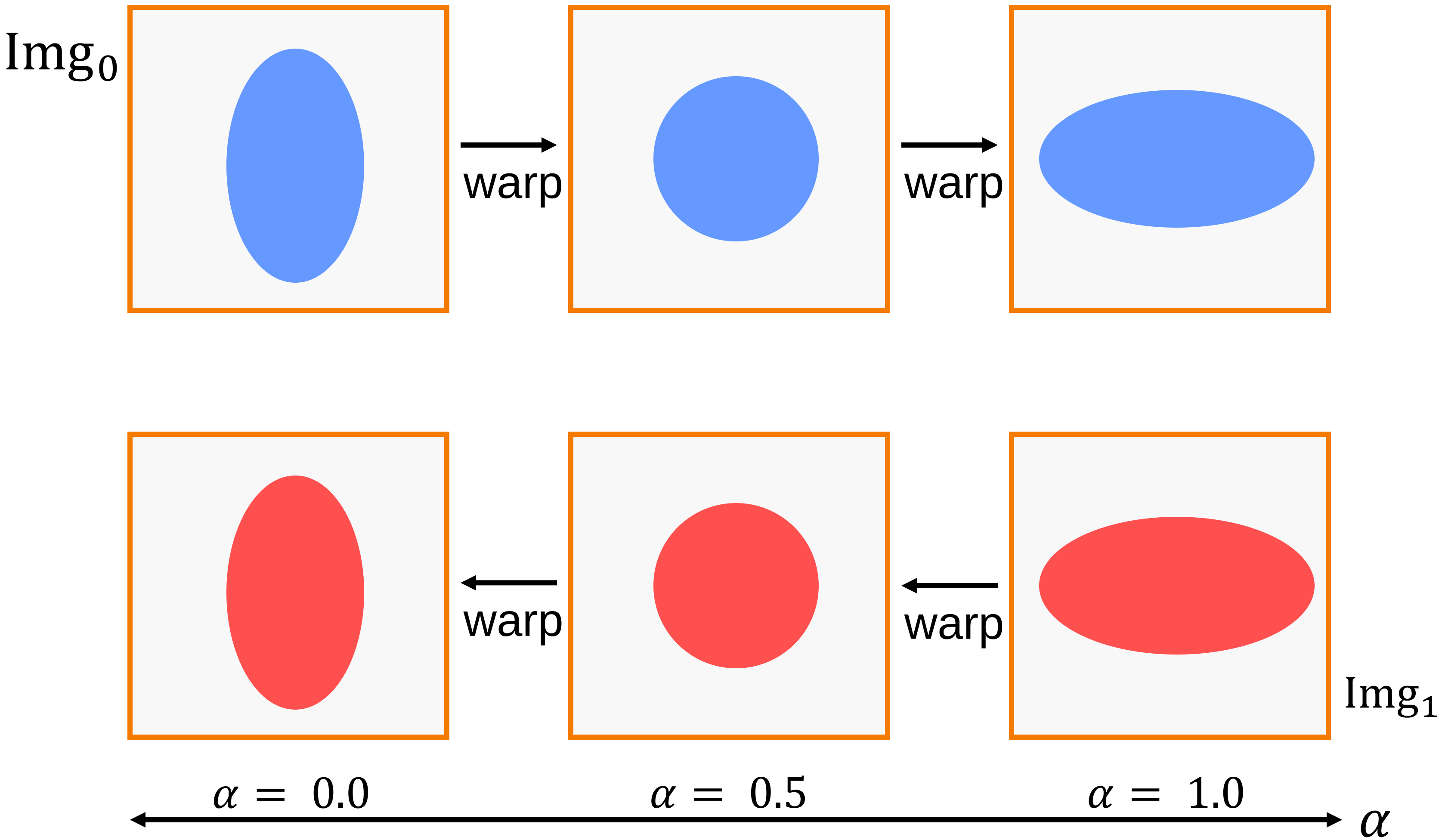


Image Morphing

… then blend **colors**

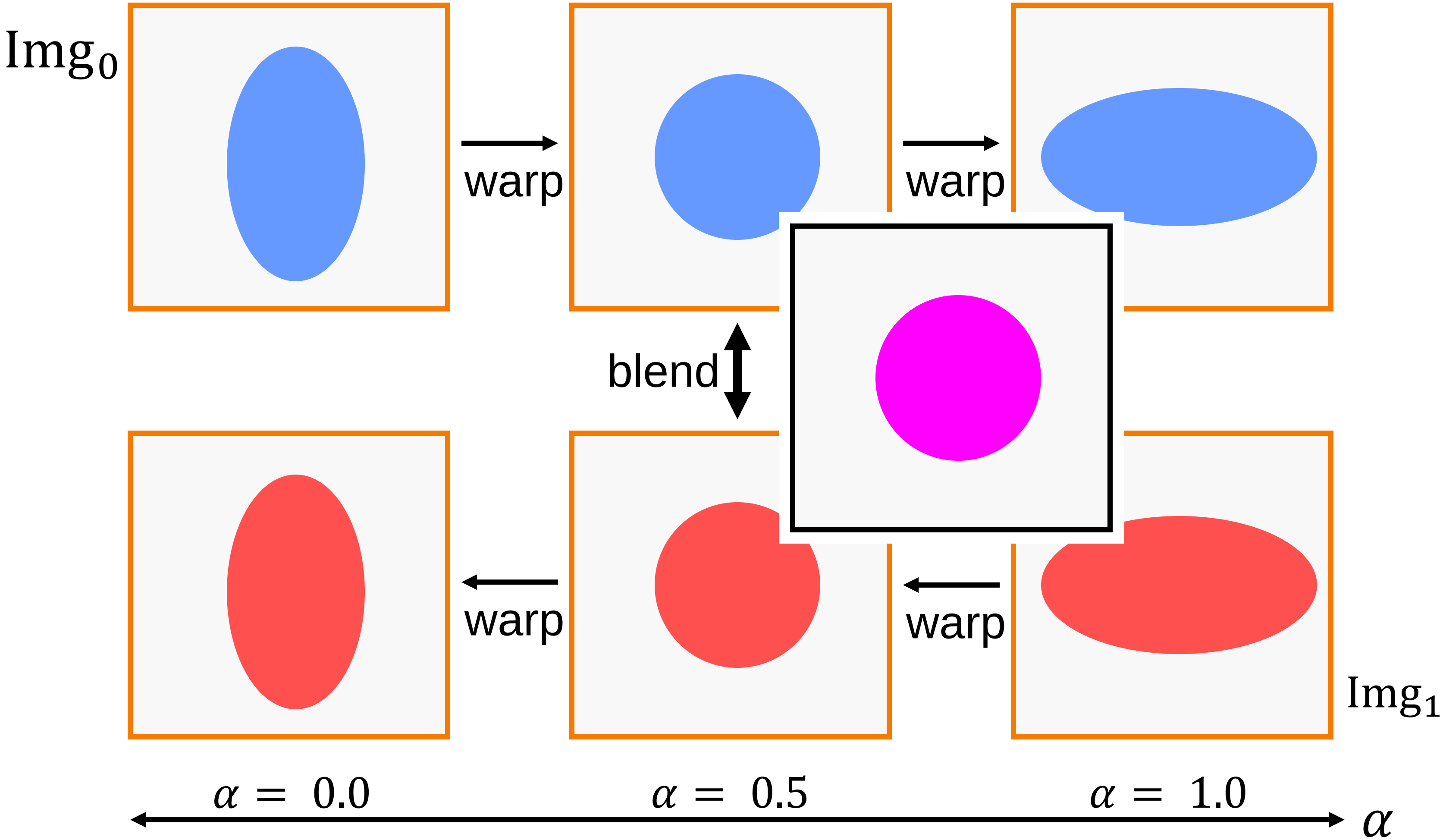
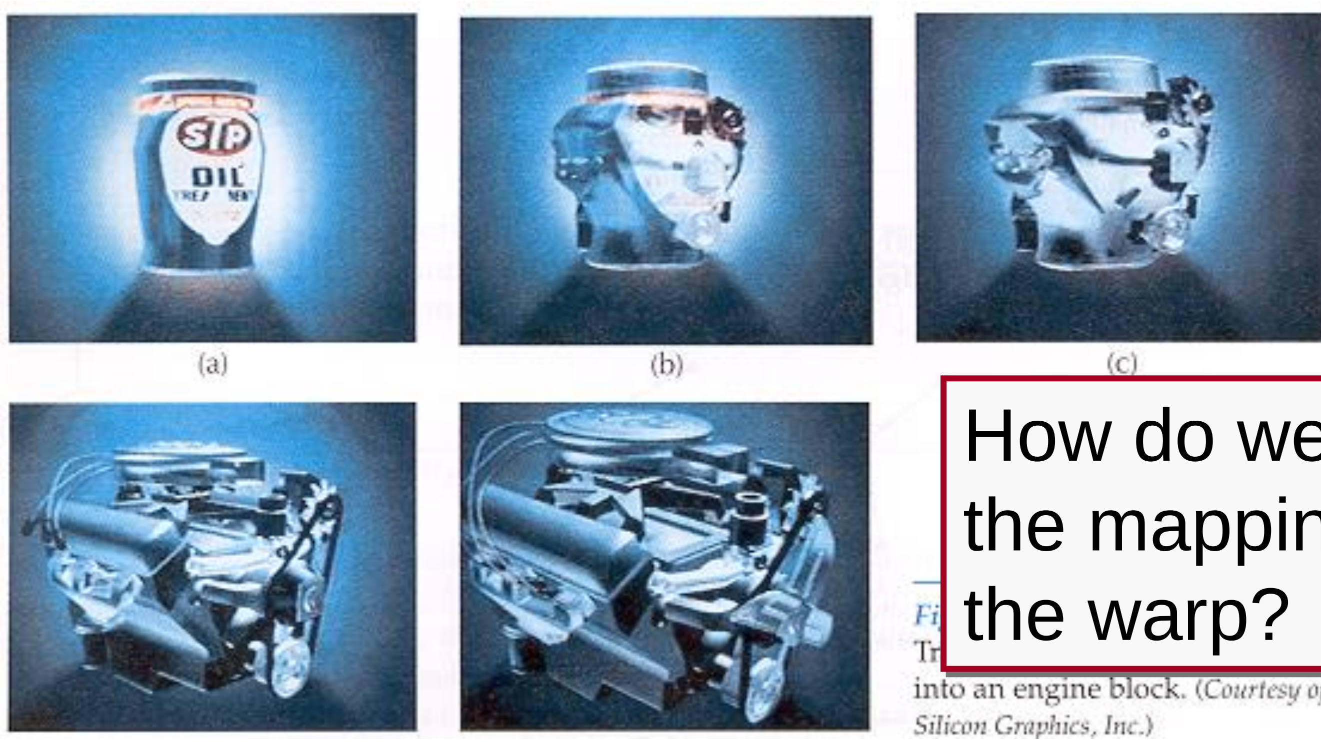


Image Morphing

- The warping step is the hard one
  - Aim to align features in images



Fi
Tr
into an engine block. (*Courtesy of Silicon Graphics, Inc.*)

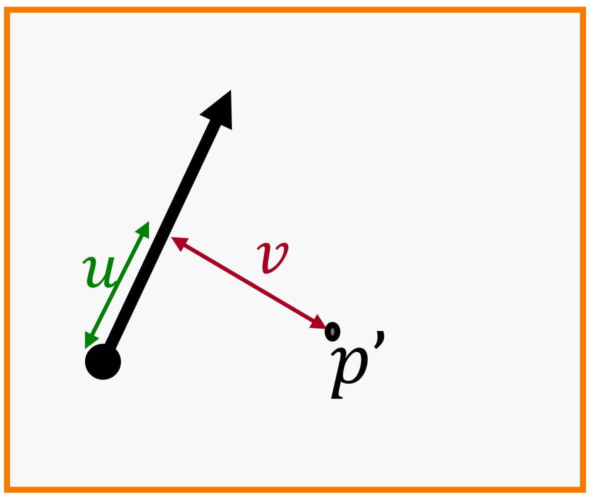
How do we specify the mapping for the warp?

H&B Figure 16.9

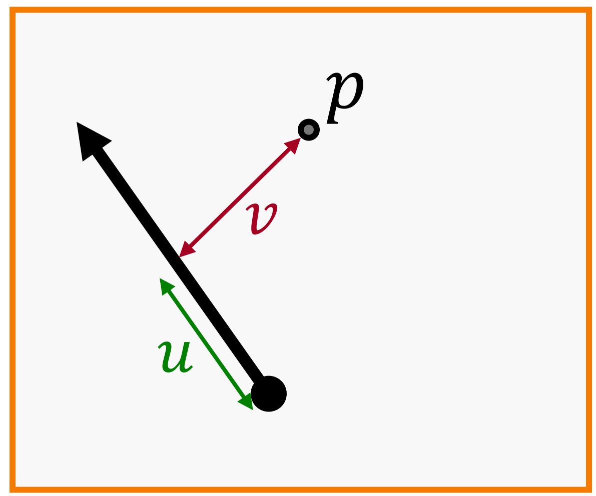
Feature-Based Warping

[Beier & Neeley, 1992] use a pair of lines:

- Given p in the destination image, where is p' in the source?
- Describe p relative to the destination line
- Map the description to the source



Source image



Destination image

u is a signed <u>fraction</u>

v is a signed <u>length</u> (in pixels)

Beier & Neeley
SIGGRAPH 92

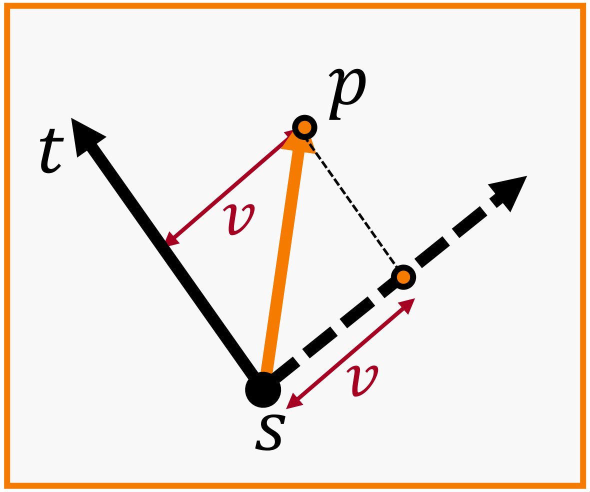
Feature-Based Warping

How do we calculate v (perp. pixel distance)?

Recall:

$\frac{\langle \vec{v}_1, \vec{v}_2 \rangle}{\|\vec{v}_2\|}$ is the signed length of the projection of $\vec{v}_1$ on the line through $\vec{v}_2$.

$$v = \frac{\langle \overbrace{p - s}^{\vec{v}_1}, \overbrace{(t - s)^{\perp}}^{\vec{v}_2} \rangle}{\|(t - s)^{\perp}\|}$$



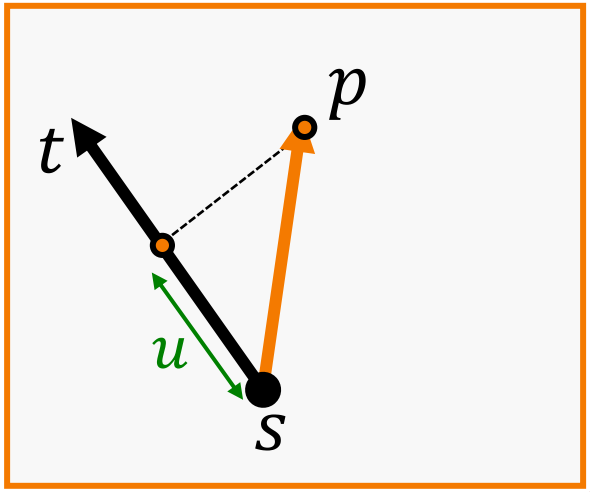
Feature-Based Warping

How do we calculate u (parallel fractional distance)?

Recall:

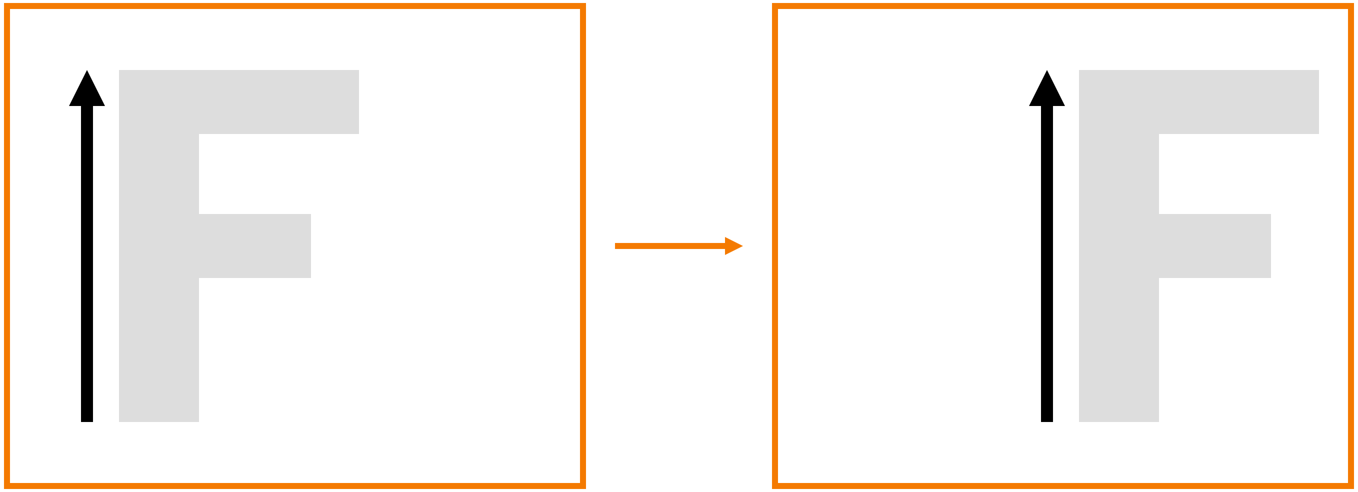
$\frac{\langle \vec{v}_1, \vec{v}_2 \rangle}{\|\vec{v}_2\|}$ is the signed length of the projection of $\vec{v}_1$ on the line through $\vec{v}_2$.

$$u = \frac{\langle \overbrace{p - s}^{\vec{v}_1}, \overbrace{t - s}^{\vec{v}_2} \rangle}{\|t - s\|} \cdot \frac{1}{\|t - s\|}$$



Warping with One Line Pair

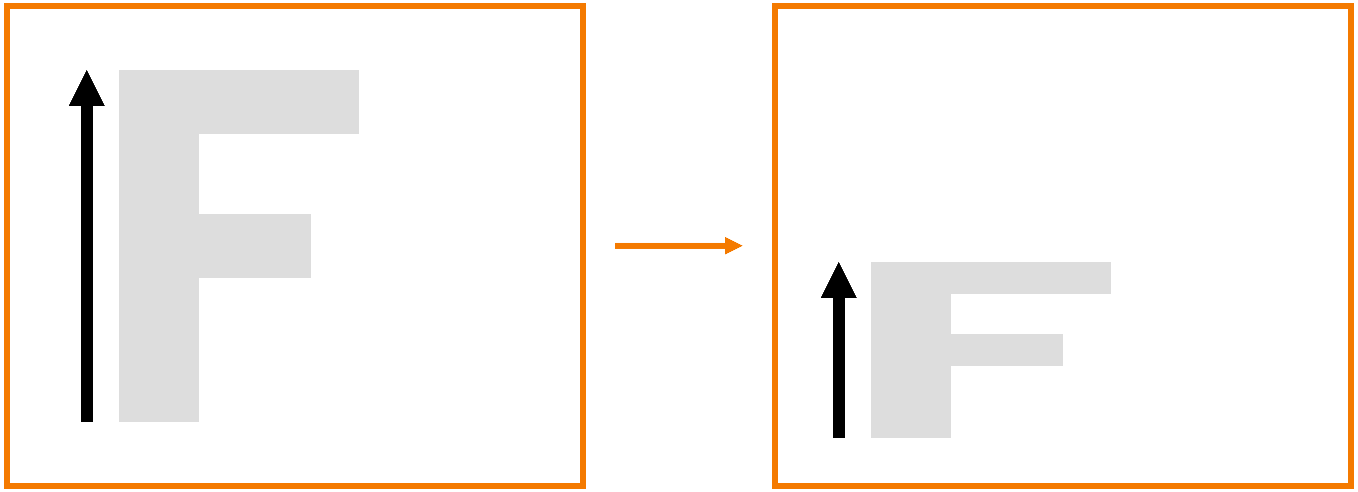
- What happens to the "F"?



Translation!

Warping with One Line Pair

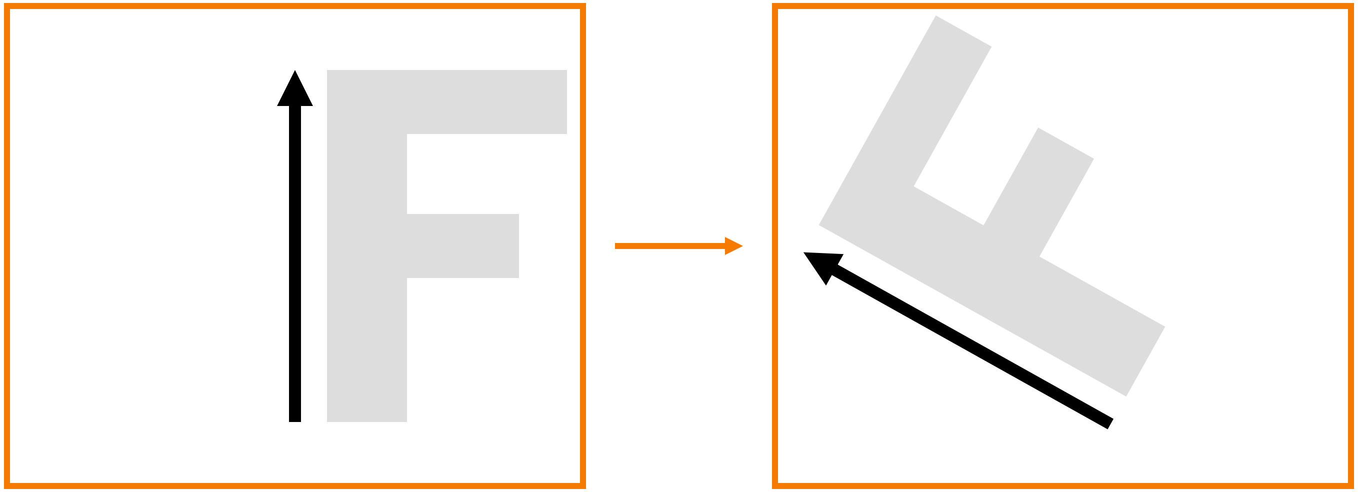
- What happens to the "F"?



<u>Non-uniform</u> scale!

Warping with One Line Pair

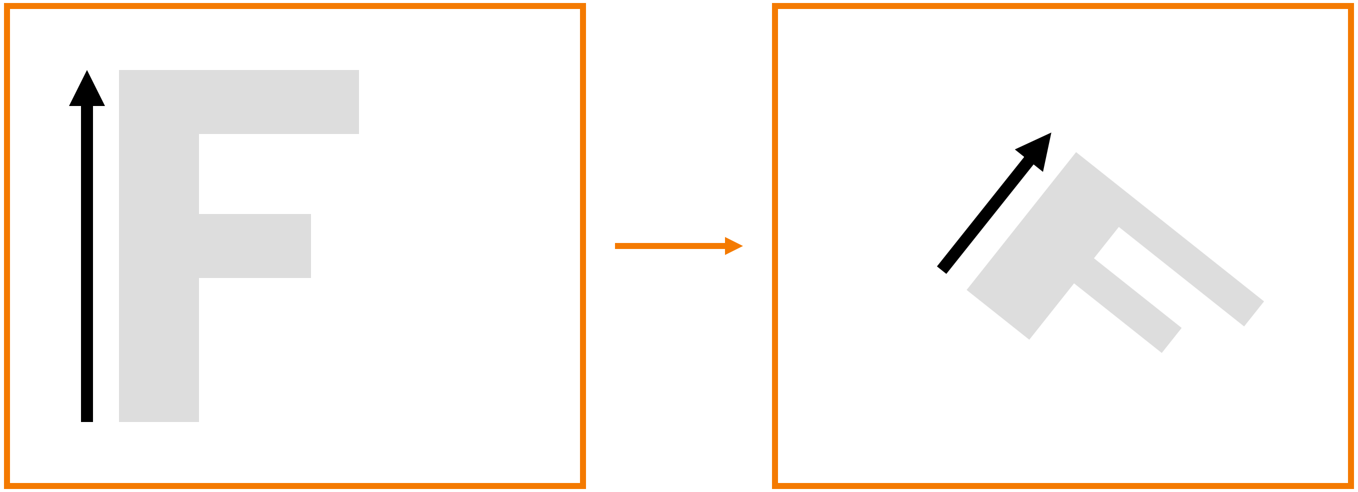
- What happens to the “F”?



Rotation!

Warping with One Line Pair

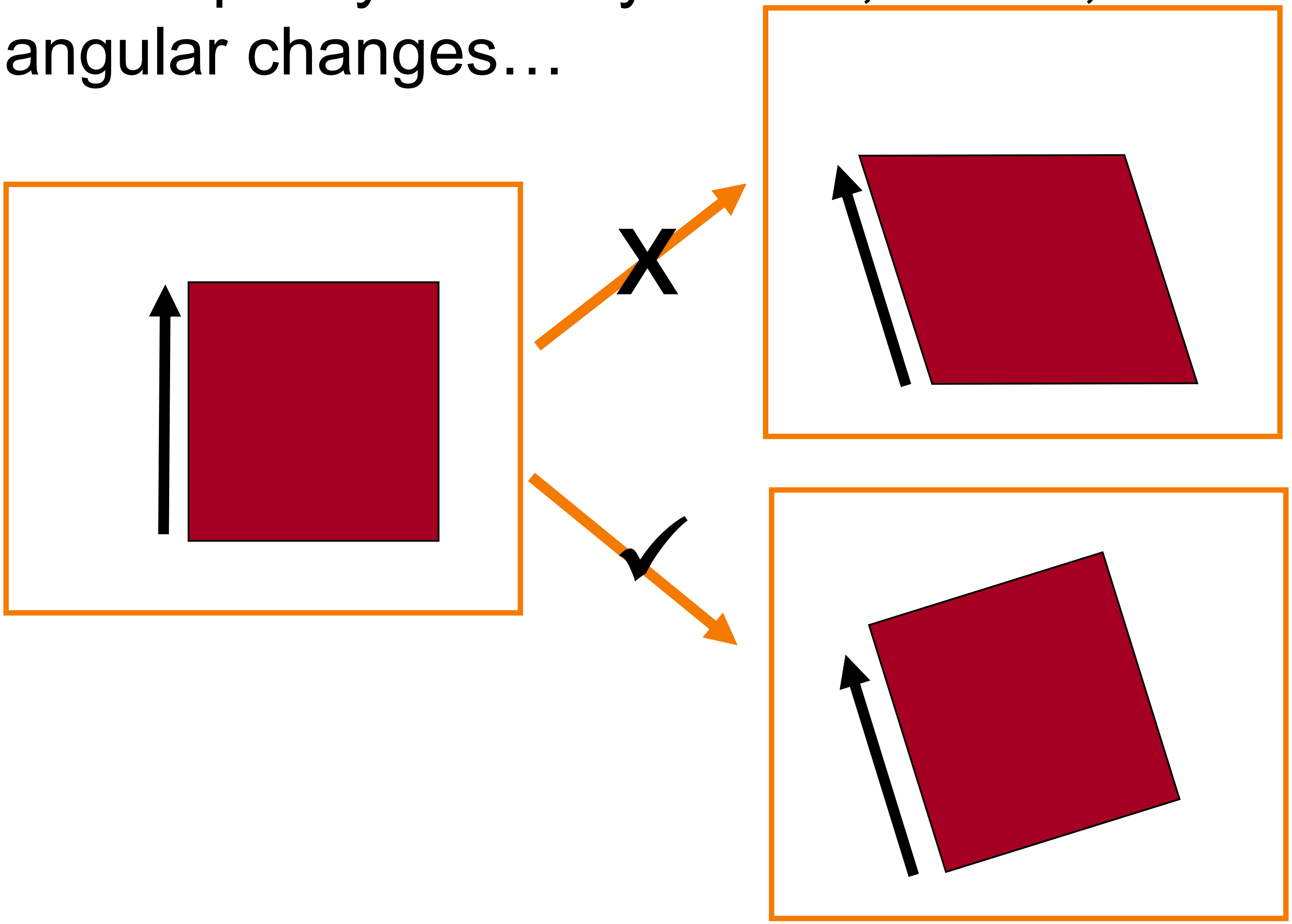
- What happens to the "F"?



What types of <u>affine</u> transformations can't be specified?

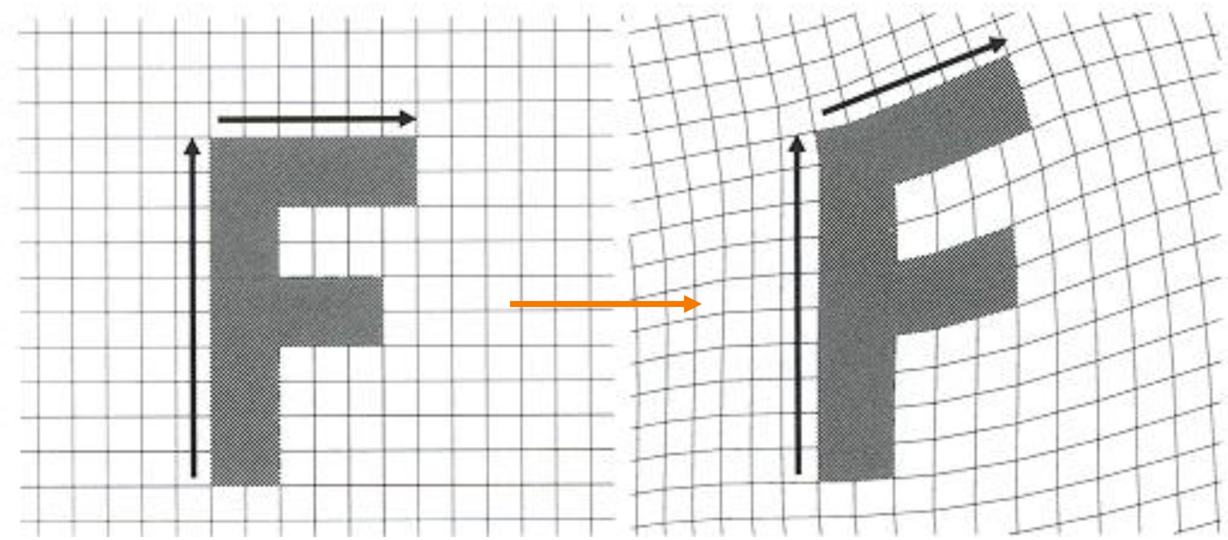
Warping with One Line Pair

- Can’t specify arbitrary scales, skews, mirrors, angular changes…



Warping with Multiple Line Pairs

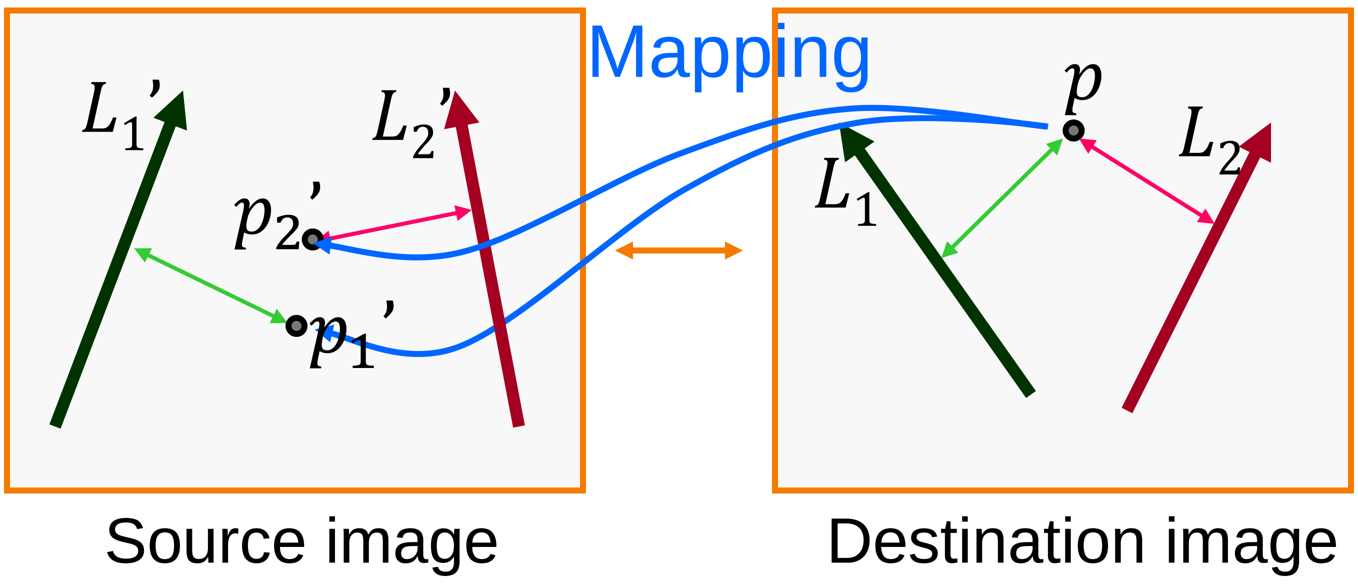
- Use weighted combination of points defined by each pair of corresponding lines



Beier & Neeley, Figure 4

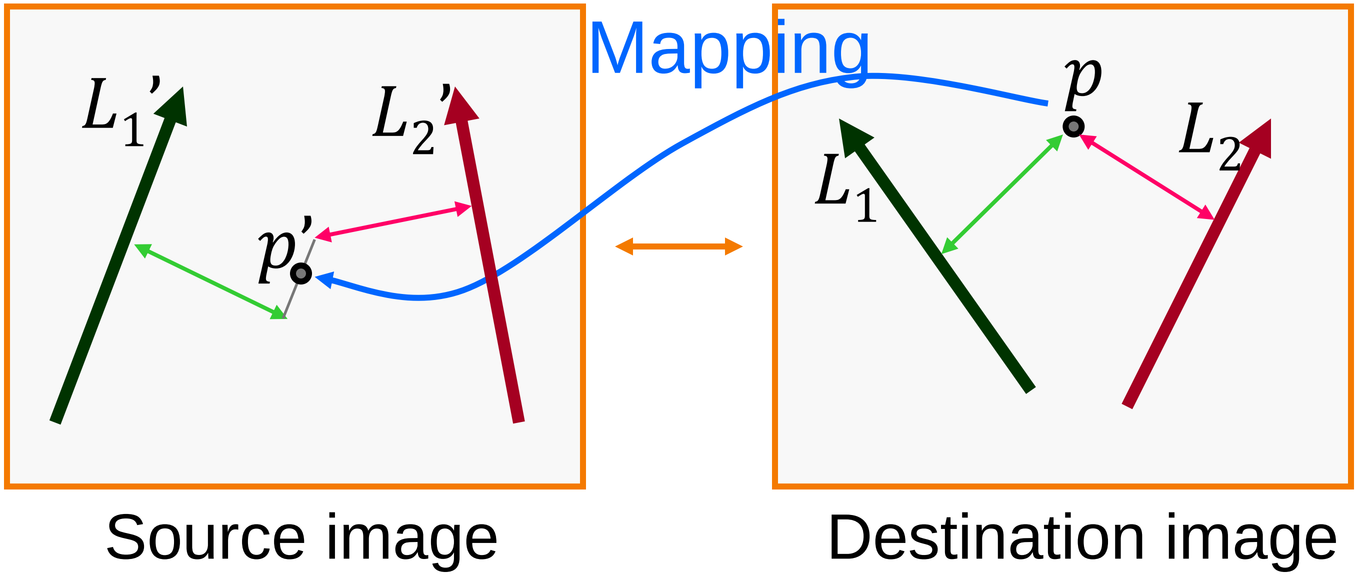
Warping with Multiple Line Pairs

- Use weighted combination of points defined by each pair of corresponding lines



Warping with Multiple Line Pairs

- Use weighted combination of points defined by each pair of corresponding lines



p' is a weighted average

Weighting Effect of Each Line Pair

- Given a set of line pairs $\{L_{in}[0], \ldots, L_{in}[N]\}$ and $\{L_{out}[0], \ldots, L_{out}[N]\}$, to weight the contribution of each line pair, [Beier & Neely, 1992] use:

$$\text{weight}[i](p) \sim \left(\frac{\text{length}[i]^c}{a + \text{dist}[i](p)} \right)^b$$

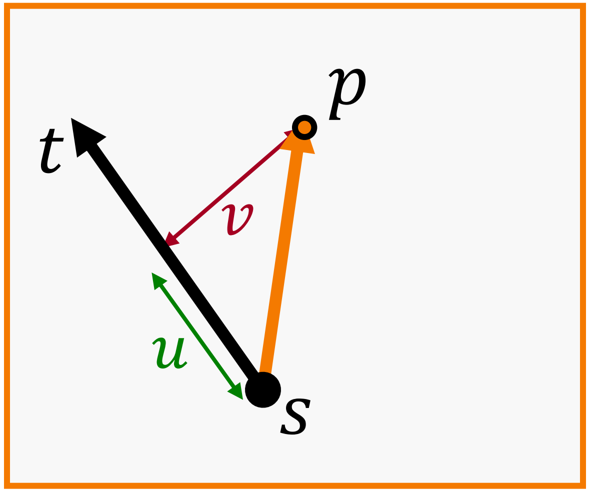
where:

- $\text{length}[i]$ is the length of line $L_{out}[i]$
- $\text{dist}[i](p)$ is the distance from p to $L_{out}[i]$
- a (small), $b \in [0.5,2.0]$, $c \in [0.0,1.0]$ are constants that control the warp

Feature-Based Warping

How do we calculate the unsigned distance from a point p to the line segment from s to t?

$$\text{dist}(p) = \begin{cases} |v| & \text{if } u \in [0,1] \\ \|p - s\| & \text{if } u < 0 \\ \|p - t\| & \text{if } u > 1 \end{cases}$$



Warping

Given a source image and set of corresponding line segment pairs, warp the source to the target by:

- Iterating over each pixel in the target
  - For each pair of line segments
    - » Compute the corresponding position in the source
    - » Compute the weights
  - Average to get the final source position
  - Sample the source (at the source position) to get the color at the target pixel

Warping Pseudocode

Warp(Img_{src} , L_{src}[N] , L_{tgt}[N])
{
    foreach destination pixel p_{trgt}:
        $p_{src} = (0,0)$
        $sum = 0$
        for i = 0 to N:
            $q_{src} = p_{trgt}$ transformed by ($L_{src}[i]$, $L_{trgt}[i]$)
            p_{src} += q_{src} * $weight[i](p_{trgt})$
            sum += $weight[i](p_{trgt})$
        p_{src} /= sum
        $Img_{trgt}(p_{trgt}) = Img_{src}(p_{src})$
    return Img_{trgt}
}

Morphing at $\boldsymbol{\alpha} \in [\mathbf{0}, \mathbf{1}]$

Given two images, given a set of corresponding line segment pairs, and an interpolation time $\alpha \in [0,1]$:

- Compute the α-blend of the of line segments (by blending the end-points)
- Warp the first image using the first set of line segments and the blended line segments
- Warp the second image using the second set of line segments and the blended line segments
- Compute the α-blend of the warped images

Morphing at $\alpha \in [0,1]$ Pseudocode

Morph(Img_0 , $L_0[N]$, Img_1 , $L_1[N]$, α)
{
  foreach $i \in \{1,\dots,N\}$:
    $L_\alpha[i]$ = line α-th of the way from $L_0[i]$ to $L_1[i]$

$$\left.\begin{array}{l} Warp_0 = \text{Warp}(\ Img_0\ ,\ L_0[\,]\ ,\ L_\alpha[\,]\ ) \\ Warp_1 = \text{Warp}(\ Img_1\ ,\ L_1[\,]\ ,\ L_\alpha[\,]\ ) \end{array}\right\}\text{warp}$$

$$\left.\text{return } (1-\alpha) * Warp_0 + \alpha * Warp_1 \right\}\alpha\text{-blend}$$

}

[Beier & Neely, 1992] Example $(\alpha = 0.5)$

Img$_0$

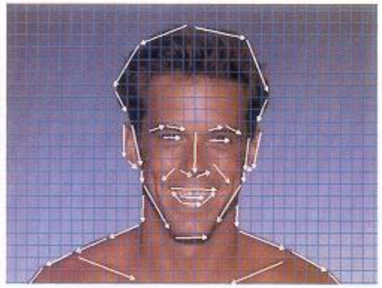
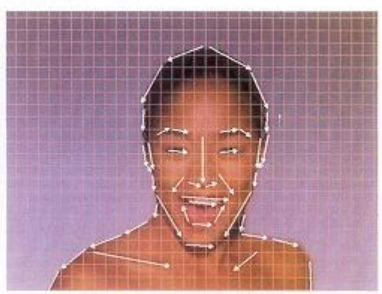


Figure 7

Warp$_0$

Img$_1$

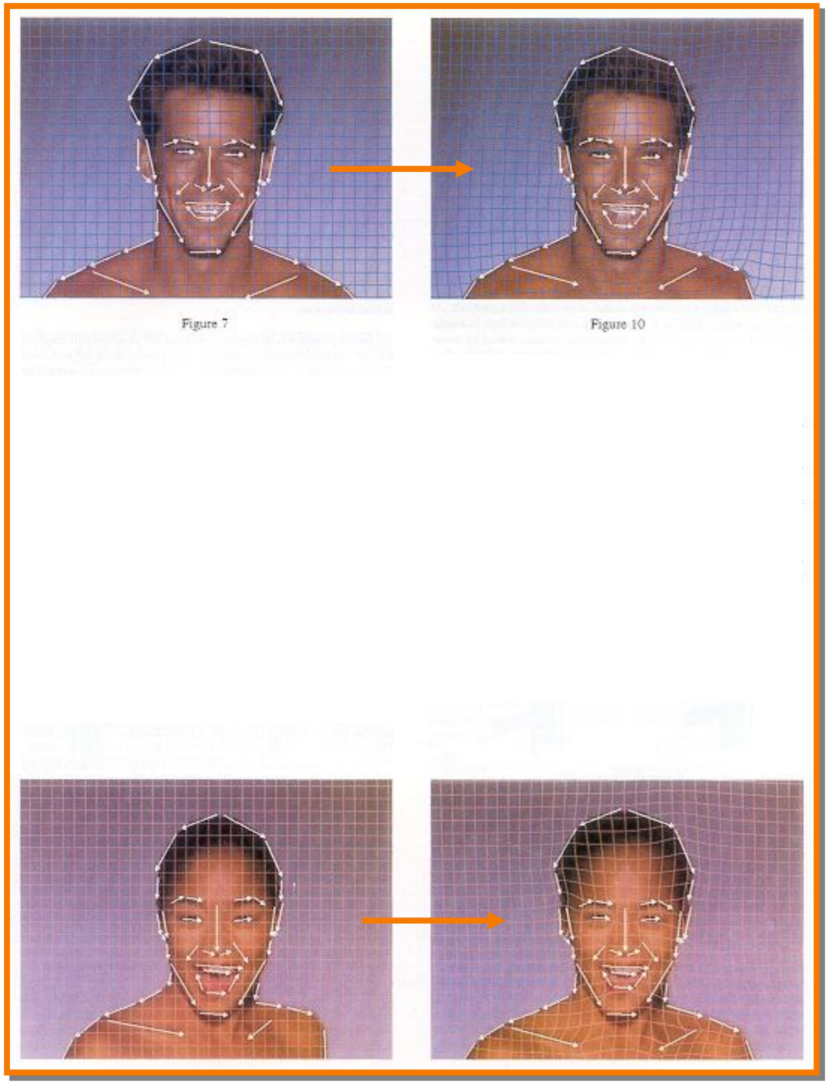


Warp$_1$

[Beier & Neely, 1992] Example ($\alpha = 0.5$)

Img_0

Warp_0



Img_1

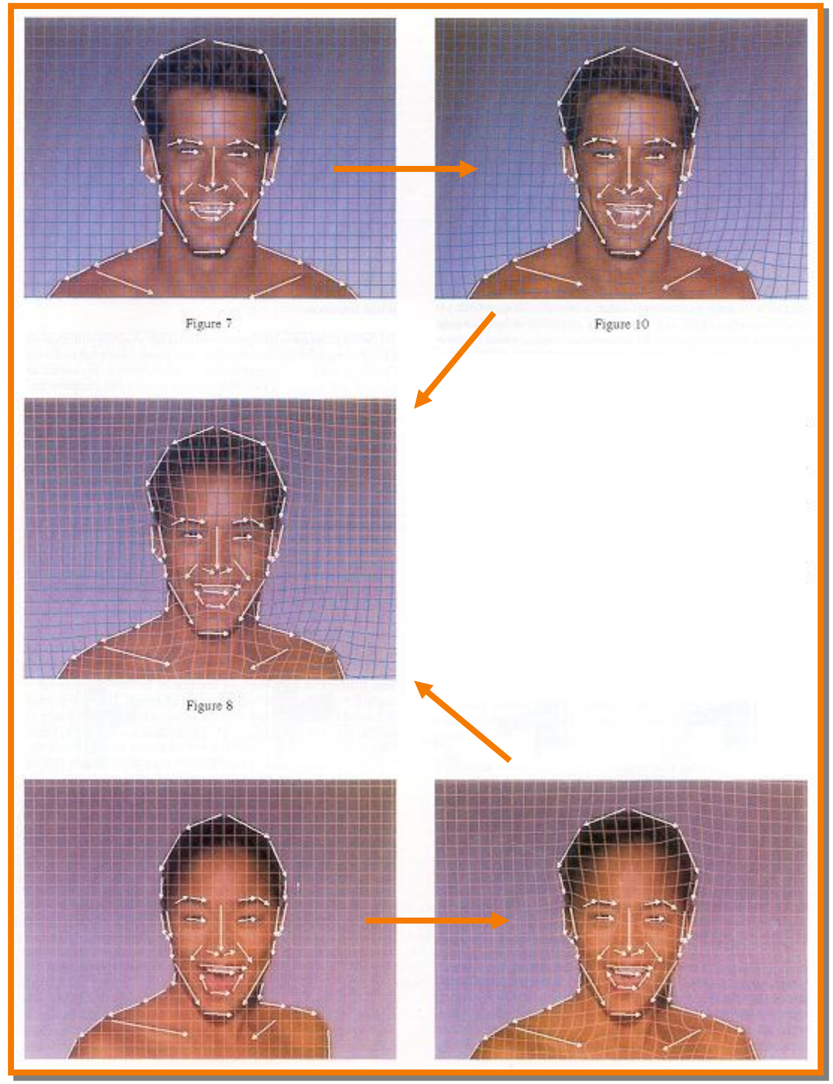
Warp_1

[Beier & Neely, 1992] Example ($\alpha = 0.5$)

Img$_0$

Warp$_0$

Result



Img$_1$

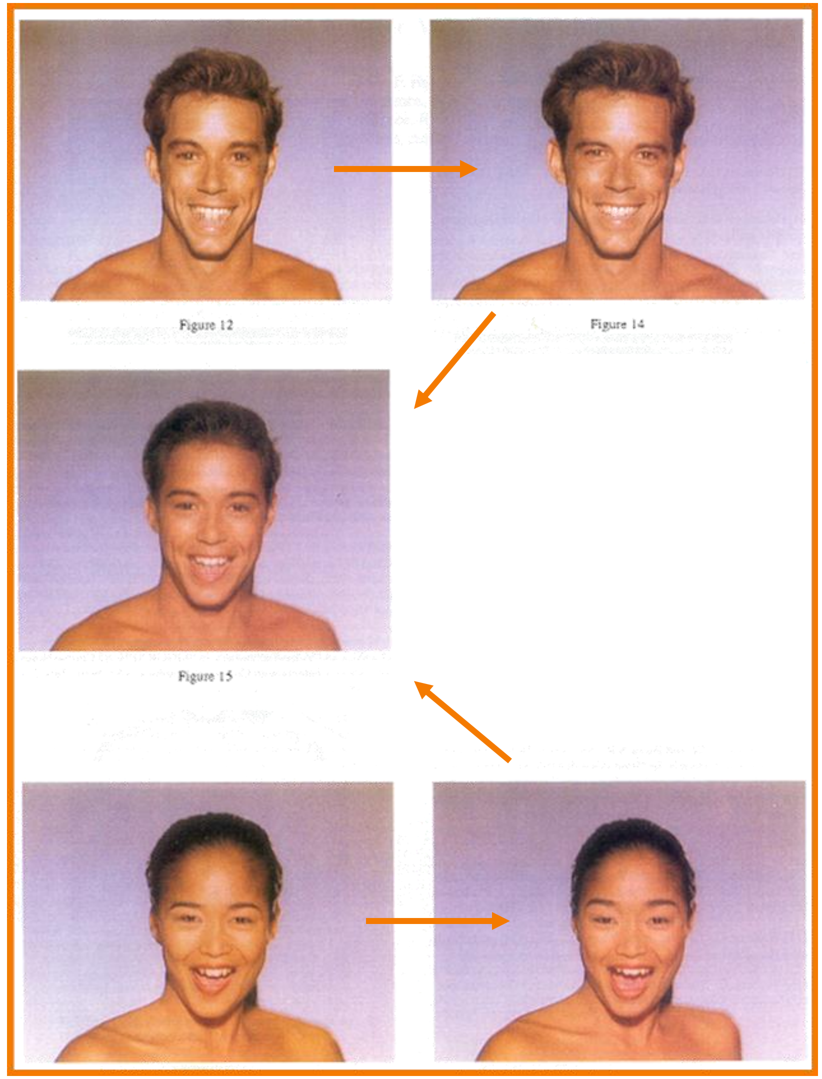
Warp$_1$

[Beier & Neely, 1992] Example ($\alpha = 0.5$)

Img$_0$

Warp$_0$

Result



Img$_1$

Warp$_1$

Animation Pseudocode

Animate(Img_0 , $L_0[N]$, Img_1 , $L_1[N]$, $Imgs_{out}[T+1]$)
{
    foreach $t \in \{0,\ldots,T\}$:
        $Imgs_{out}[t]$ = Morph(Img_0 , $L_0[N]$, Img_1 , $L_1[N]$, t/T)
}

Morphing

Check out Michael Jackson’s “Black or White” video at:

https://www.youtube.com/watch?v=pTFE8cirkdQ

Or the earlier Plymouth Voyager commercial at:

https://www.youtube.com/watch?v=0b939O7dGqQ