FFTs in Graphics and Vision

Conclusion
Announcements

Assignment 3 due May 6th.
Key Ideas

Functions Are Vectors
We can think of functions as (complex) vectors in a high-dimensional space.

Representations
Translations and rotations of functions can be thought of as linear, norm-preserving, transformations.

Irreducible Representations
To understand how translations and rotations act on functions, we strive to decompose the space of functions into the smallest sub-spaces in which these transformations are contained.
Essential Facts

Schur’s Lemma (Corollary)
If the group is commutative, the irreducible representations are all one-dimensional.

Homogenous Polynomials
Homogenous polynomials of fixed degree are sub-representations.

Self-Adjoint Operators
If a linear operator is self-adjoint (symmetric) there is an orthogonal basis of eigenvectors.

Commuting Self-Adjoint Operators
The spaces of eigenvectors of the operator with the same eigenvalue are sub-representation.
Implications / Applications

Fast Correlation:

For correlation, we only need to perform cross-multiplication of coefficients in the same frequency:

\[ D_{f,g}(\alpha) = \sum_{k} \hat{f}_k \cdot \hat{g}_k \cdot D_k(\alpha) \]
Implications / Applications

Fast Correlation:

For correlation, we only need to perform cross-multiplication of coefficients in the same frequency:

\[ D_{f,g}(\theta, \phi, \psi) = \sum_{l} \sum_{m=-l}^{m} \hat{f}_{lm} \cdot \overline{\hat{g}_{lm}} \cdot D_{l}^{mm'}(\theta, \phi, \psi) \]
Implications / Applications

Fast Correlation:

For correlation, we only need to perform cross-multiplication of coefficients in the same frequency.

This reduces the implementation of correlation to:

1. Computing a forward frequency transform
2. Doing the intra-frequency cross multiplication
3. Computing an inverse frequency transform
Implications / Applications

Fast Correlation:

For correlation, we only need to perform cross-multiplication of coefficients in the same frequency.

For Translational Correlation:

1. Computing the FFT: \( O(N \log N) \)
2. Doing the multiplication: \( O(N) \)
3. Computing the inverse FFT: \( O(N \log N) \)

Brute force would have been \( O(N^2) \).

Optimal would be \( O(N) \).
Implications / Applications

Fast Correlation:
For correlation, we only need to perform cross-multiplication of coefficients in the same frequency.

For Rotational Correlation:
1. Computing the SHT: $O(N^2 \log^2 N)$
2. Doing the multiplication: $O(N^3)$
3. Computing the inverse WDT: $O(N^3 \log^2 N)$

Brute force would have been $O(N^5)$.
Optimal would have been $O(N^3)$. 
Implications / Applications

Fast Correlation (Applications):

For image processing, we have seen applications of correlation in:

- Filtering

![Image of zebras]

Edge Detection
Implications / Applications

Fast Correlation (Applications):

For image processing, we have seen applications of correlation in:

- Filtering
- Pattern Recognition

Template Matching
Implications / Applications

Fast Correlation (Applications):

For image processing, we have seen applications of correlation in:

- Filtering
- Pattern Recognition
- Symmetry Detection

Rotational Symmetry

Reflective Symmetry
Implications / Applications

Fast Correlation (Applications):

We have even seen some unexpected applications resulting from the fact that the Laplacian is a symmetric operator commuting with translation in:

- Solving PDEs
Implications / Applications

Fast Correlation (Applications):

For the group of rotations in 3D, we have seen applications in:

- Shape Alignment
Implications / Applications

Fast Correlation (Applications):

For the group of rotations in 3D, we have seen applications in:

- Shape Alignment
- Symmetry Detection
Thank You!