Convex Hulls (3D)

O'Rourke, Chapter 4

Outline

- Correction
- Polyhedra
  - Polytopes
  - Euler Characteristic
- (Oriented) Mesh Representation

Correction

For implementing the trapezoidalization, we described using a sorting function which changes dynamically:

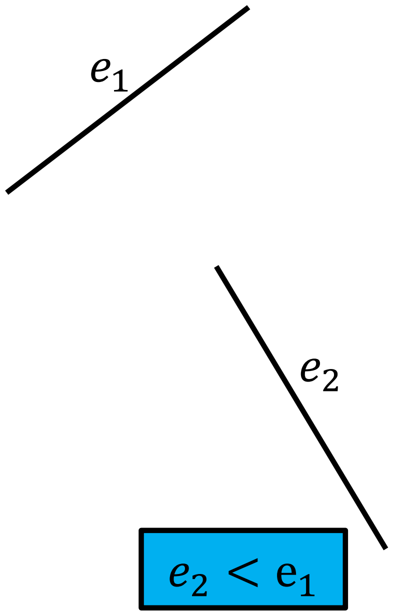
```
float sweepHeight;
typedef function< bool ( const EKey & , const EKey & ) > EComparator;
EComparator eComparator = [&]( const EKey &k1 , const EKey &k2 )
{
        // Compare the keys using the current value of sweepHeight
};
```

Correction

This is not necessary.

We could check if the y-spans of the two edges overlap.

- If they do not, call the lower edge "first".

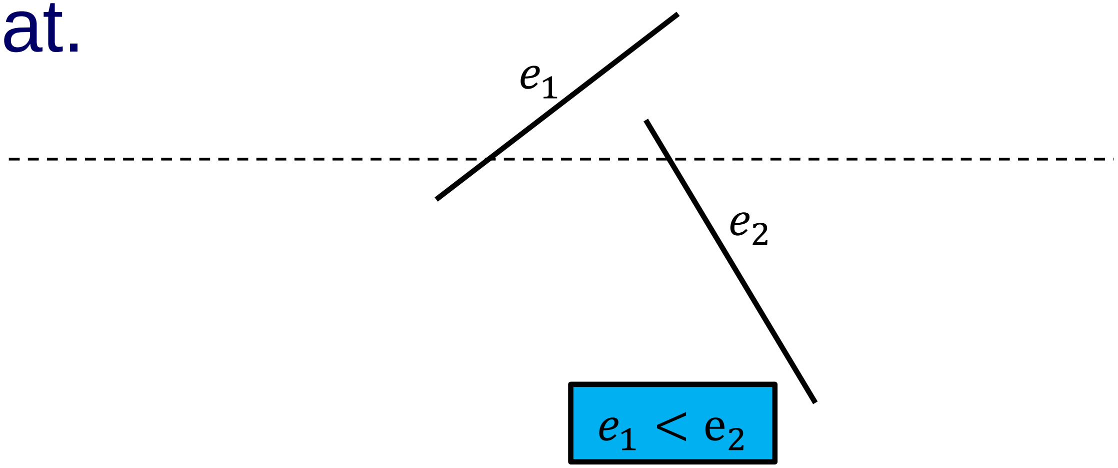


Correction

This is not necessary.

We could check if the y-spans of the two edges overlap.

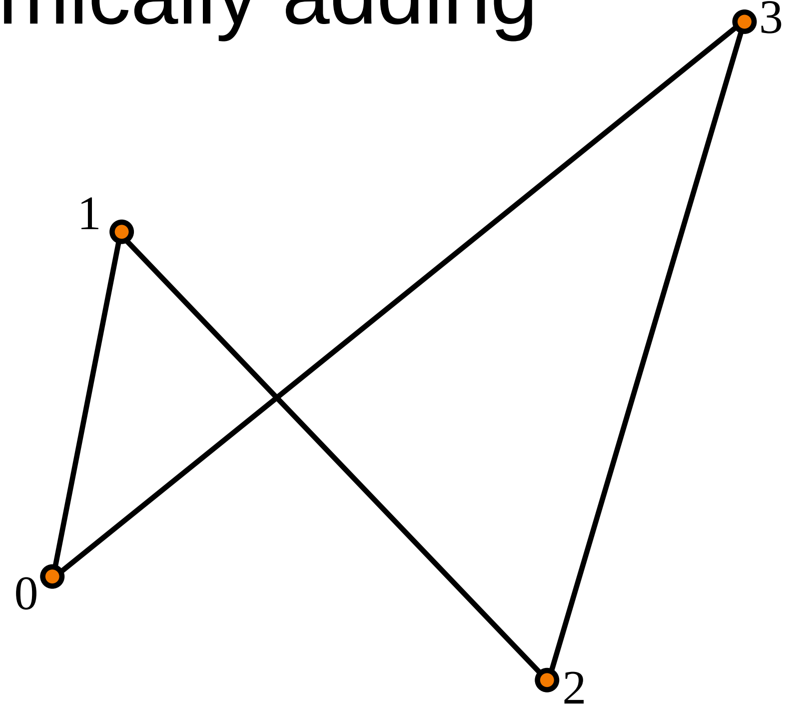
- If they do not, call the lower edge "first".
- Otherwise, draw a horizontal line through some point on the overlap of the y-spans and sort based on that.



Correction

However…

We can also use sweep-line algorithm to check if a closed (piecewise-linear) curve self-intersects by dynamically adding intersection events.



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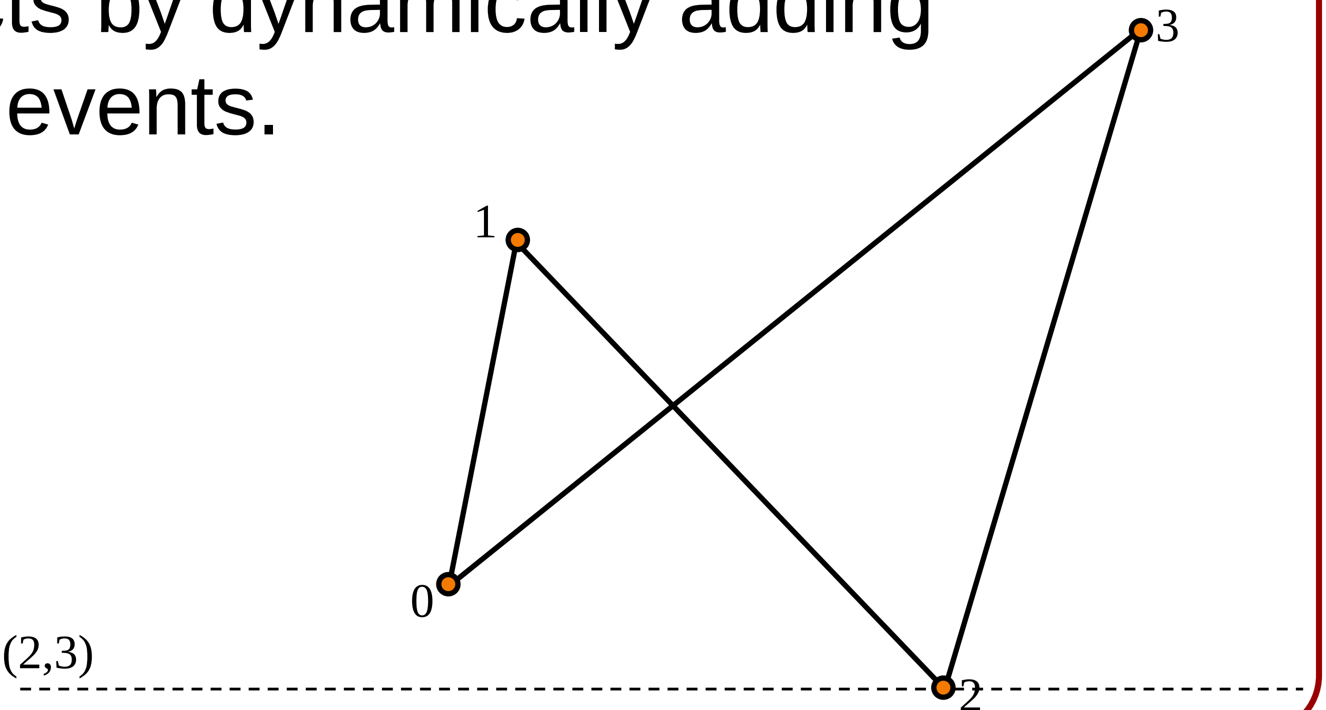

(2,1) - (2,3)

Correction

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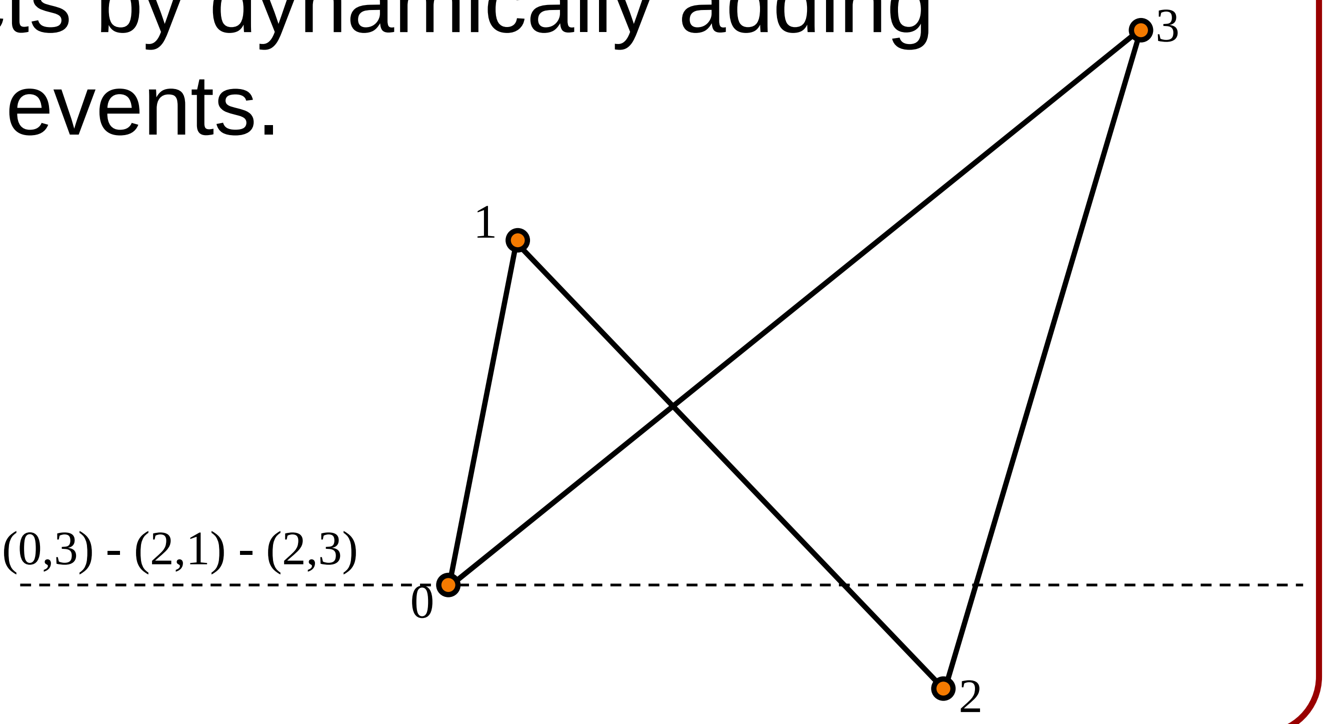

(0,1) - (0,3) - (2,1) - (2,3)

Correction

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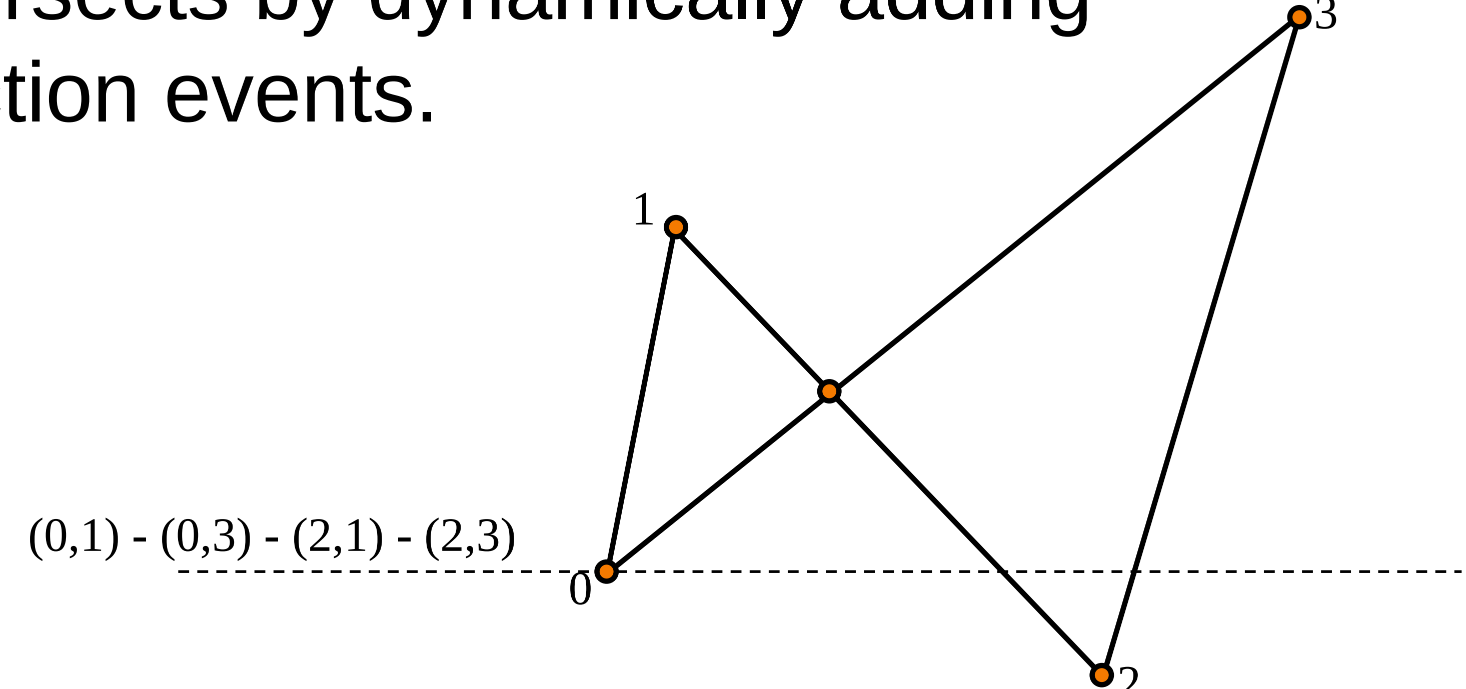

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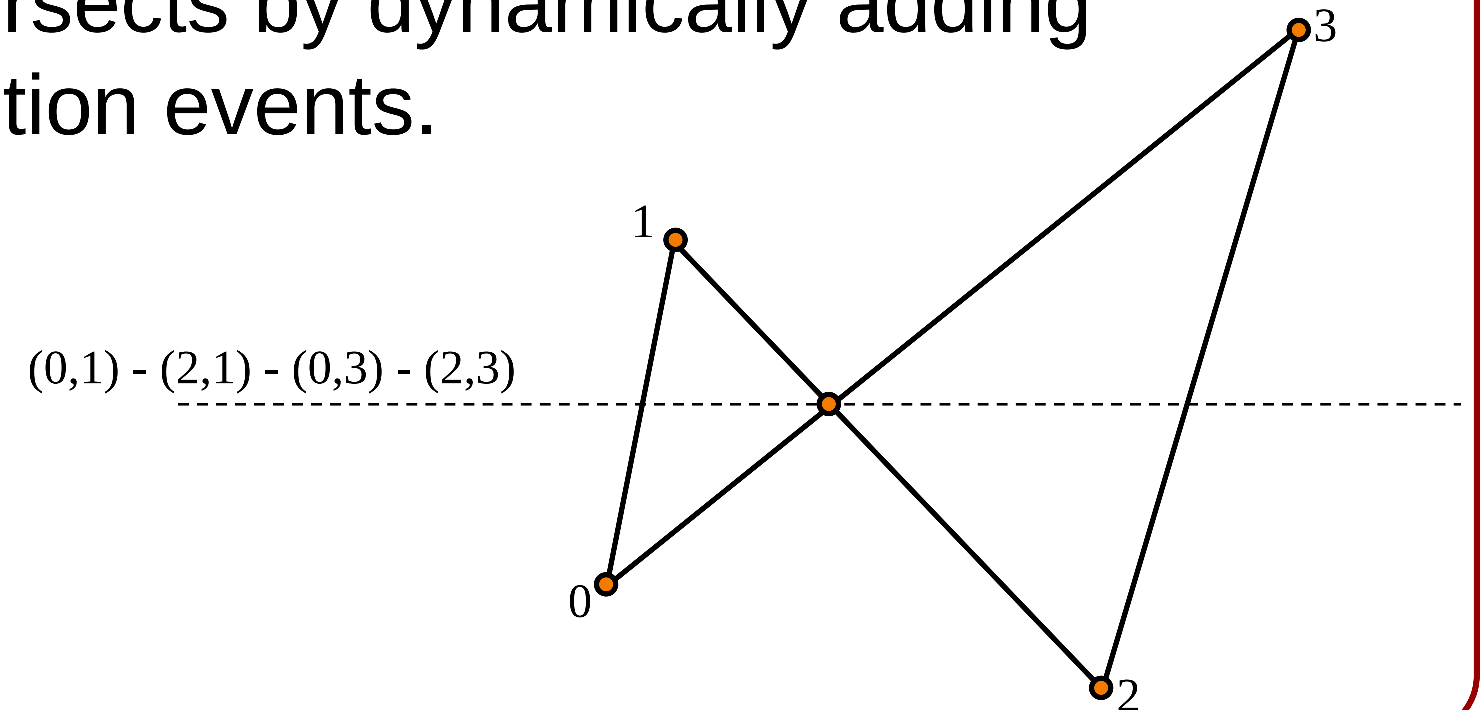

Correction

However…

We
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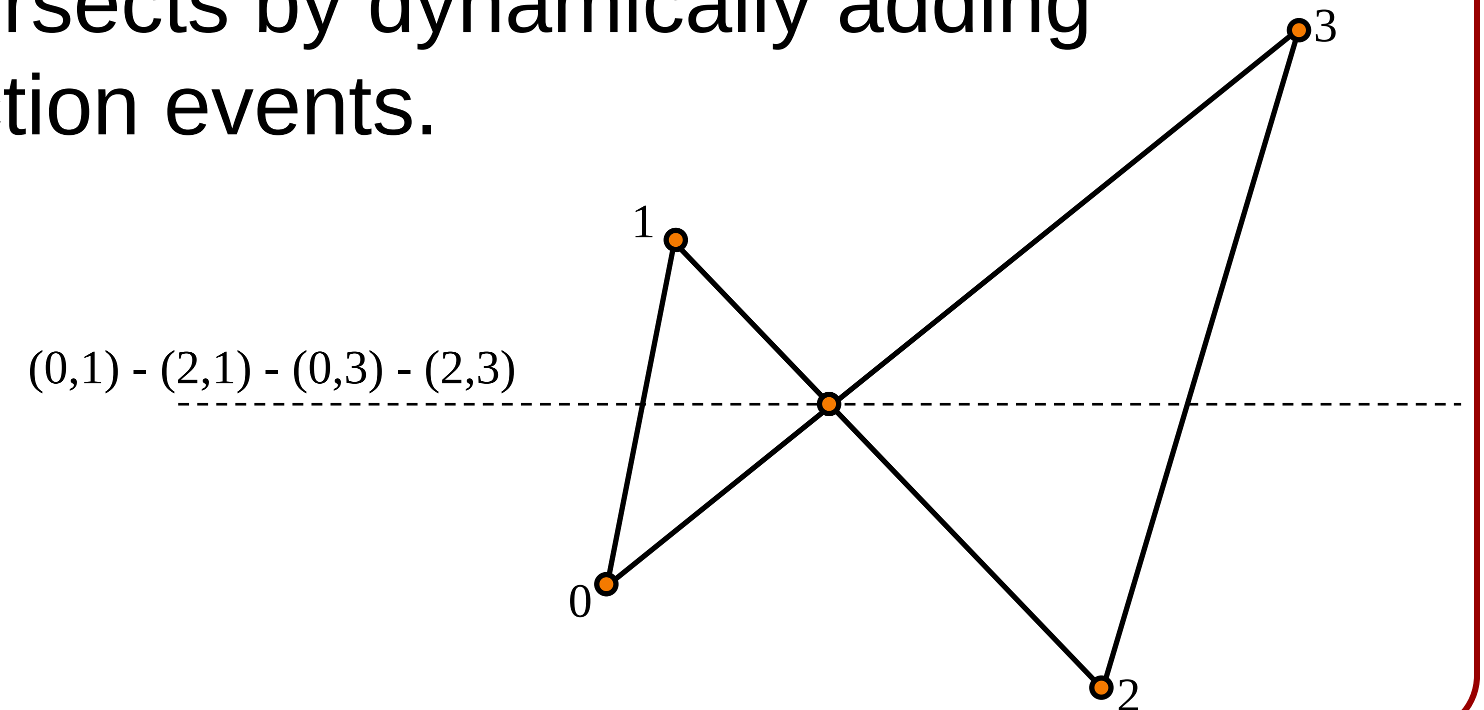
self-intersects by dynamically adding intersection events.

> Note: Only need to check for intersections
> 1. Between adjacent edges in the active-edge list
> 2. Around newly added/removed edges



Polyhedra

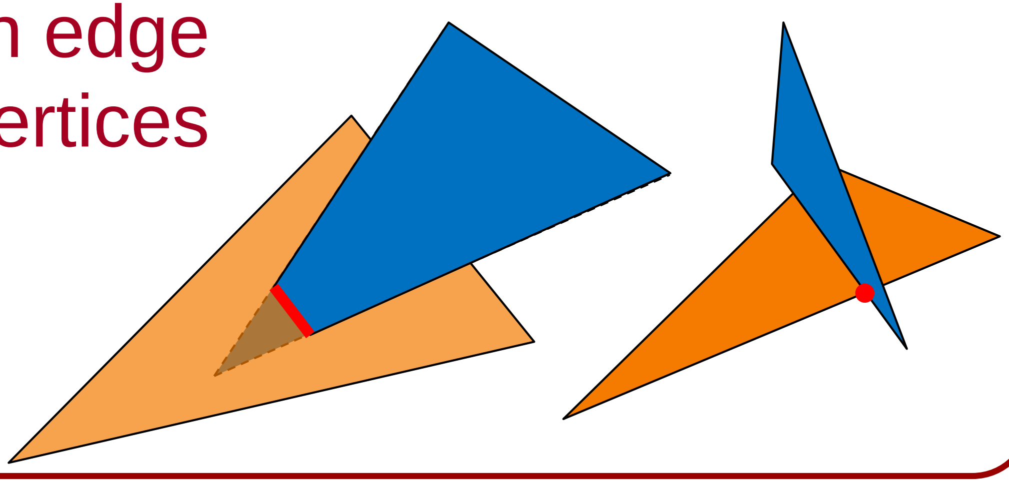
Definition:

A *polyhedron* is a solid region in 3D space whose boundary is made up of planar polygonal faces comprising a connected 2D manifold.

Polyhedra

The boundary of a polyhedron can be expressed as a combination of vertices, edges, and faces:

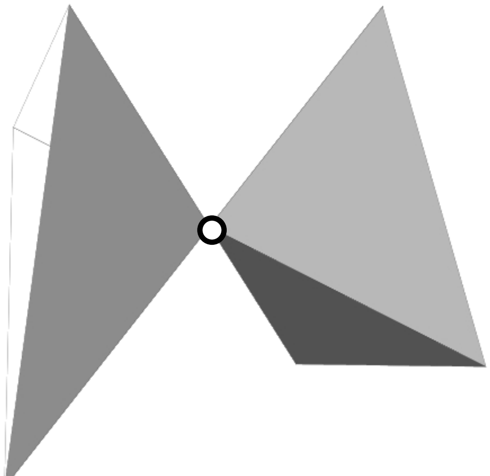
- Intersections are proper:
  - Elements don't overlap, or
  - They share a single vertex, or
  - They share an edge and the two vertices



Polyhedra

The boundary of a polyhedron can be expressed as a combination of vertices, edges, and faces:

- Intersections are proper
- Locally manifold:
  - Edges around a vertex can be sorted to match their incidence on adjacent faces.



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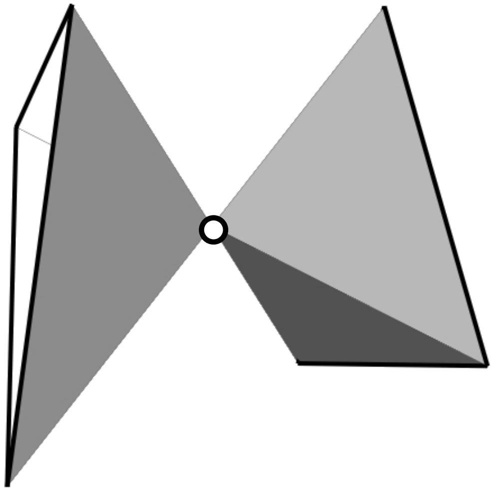
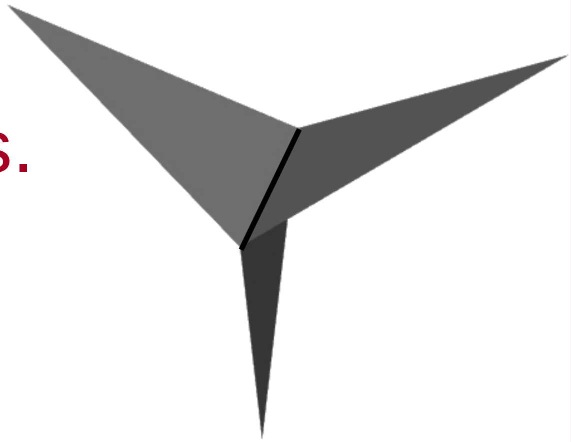


Alternatively, the subgraph of the dual obtained by restricting to the adjacent faces (the link) is connected.

Polyhedra

The boundary of a polyhedron can be expressed as a combination of vertices, edges, and faces:

- Intersections are proper
- Locally manifold:
  - Edges around a vertex can be sorted to match their incidence on adjacent faces.
  - Exactly two faces meet at each edge.



Polyhedra

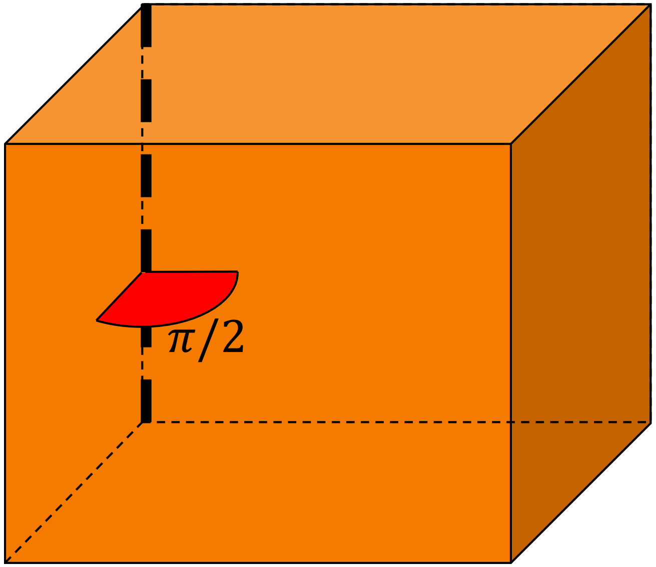
The boundary of a polyhedron can be expressed as a combination of vertices, edges, and faces:

- Intersections are proper
- Locally manifold
- Globally connected

Definition

Definition:

Given an edge on a polyhedron, the *dihedral angle* of the edge is the internal angle between the two adjacent faces.

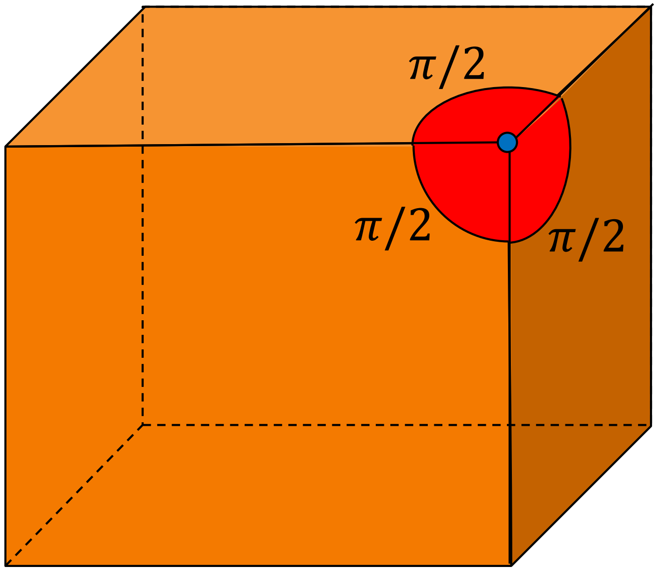


Aside:
The dihedral angle is a discrete measure of mean curvature.

Definition

<u>Definition</u>:

Given a vertex on a polyhedron, the *deficit angle* at the vertex is 2π minus the sum of angles around the vertex.



$\Rightarrow \pi/2$

<u>Aside</u>:
The deficit angle is a discrete measure of Gauss curvature.

Polytopes

A convex polyhedron is a *polytope*:

- Non-negative mean curvature:
  All dihedral angles are less than or equal to π.
  (Necessary and sufficient.)
- Non-negative Gaussian curvature:
  Sum of angles around a vertex is at most 2π.
  (Necessary but not sufficient).

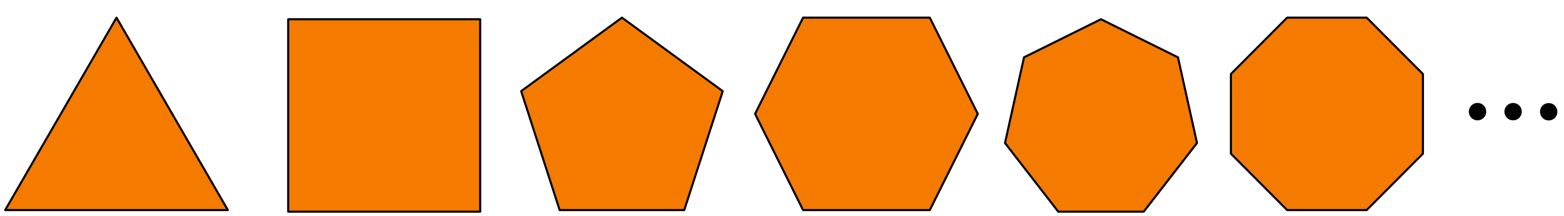
Schwarz Lantern

Image courtesy of https://www.cut-the-knot.org/

Platonic Solids

Definition:

A *regular polygon* is a polygon with equal sides and equal angles.



Platonic Solids

Definition:

A *regular polygon* is a polygon with equal sides and equal angles.

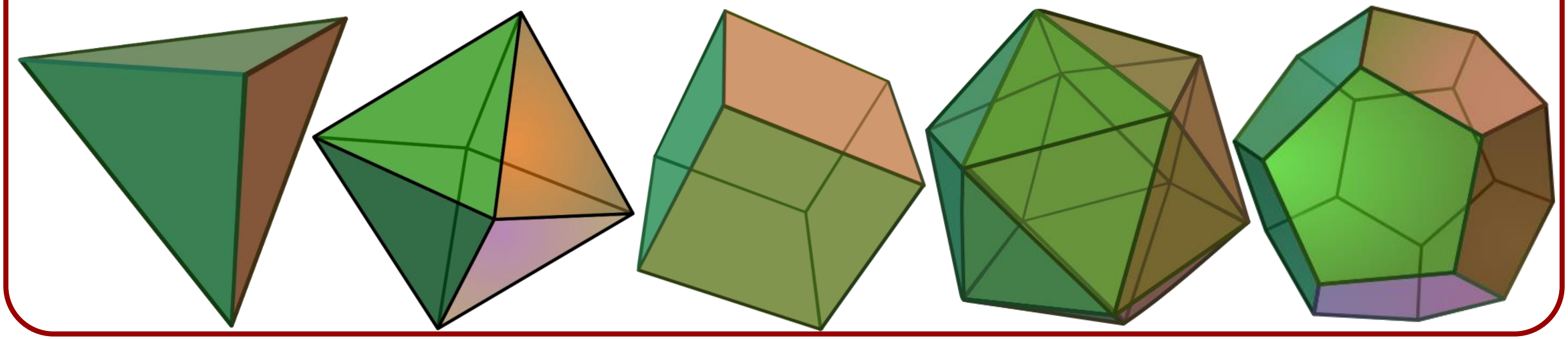
A *regular polyhedron* is a convex polyhedron, with all faces congruent regular polygons and vertices having the same valence.

Platonic Solids

Claim:

The five platonic solids are the only regular polyhedra.

[Images courtesy of Wikipedia]



Platonic Solids

Proof:

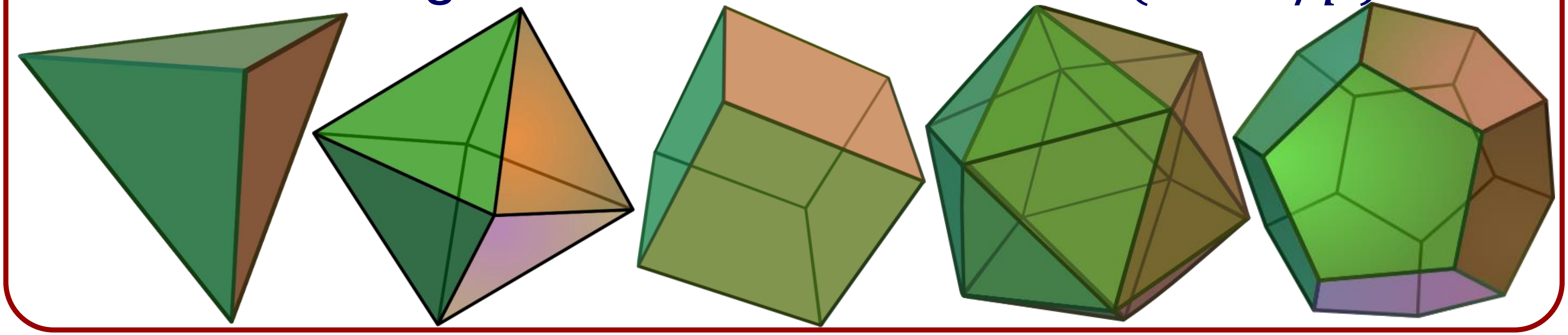
Assume each face is p-sided:

$\Rightarrow$ The sum of angles in a face is $\pi(p-2)$

$\Rightarrow$ The angle at each vertex is $\pi(1-2/p)$

Assume each vertex has valence v:

$\Rightarrow$ The angle-sum at a vertex is $v\pi\,(1-2/p)$

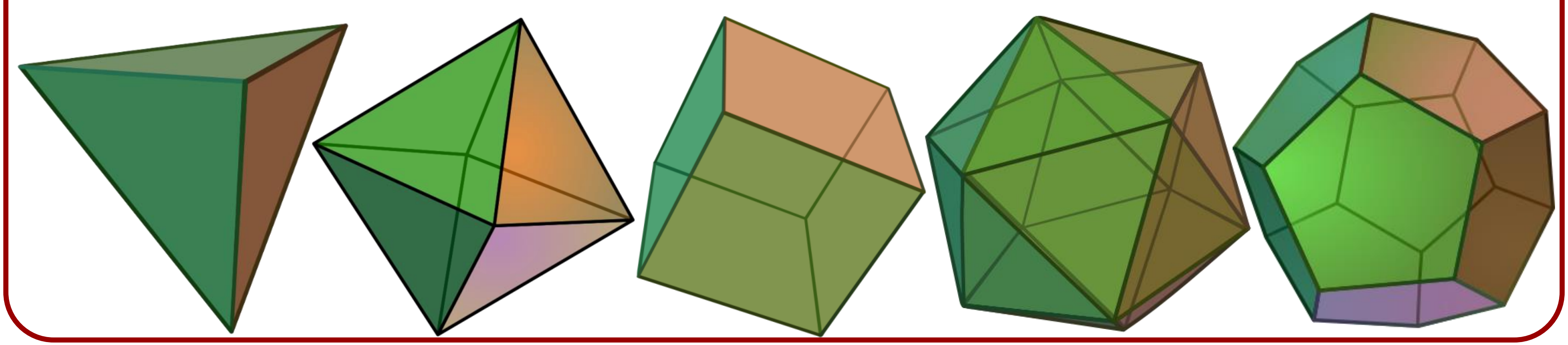


Platonic Solids

Proof:

Since the polyhedron is convex:

$$v\pi(1 - 2/p) < 2\pi \iff v(1 - 2/p) < 2$$
$$\iff v(p - 2) < 2p$$
$$\iff vp - 2v - 2p < 0$$
$$\iff (p - 2)(v - 2) - 4 < 0$$



Platonic Solids

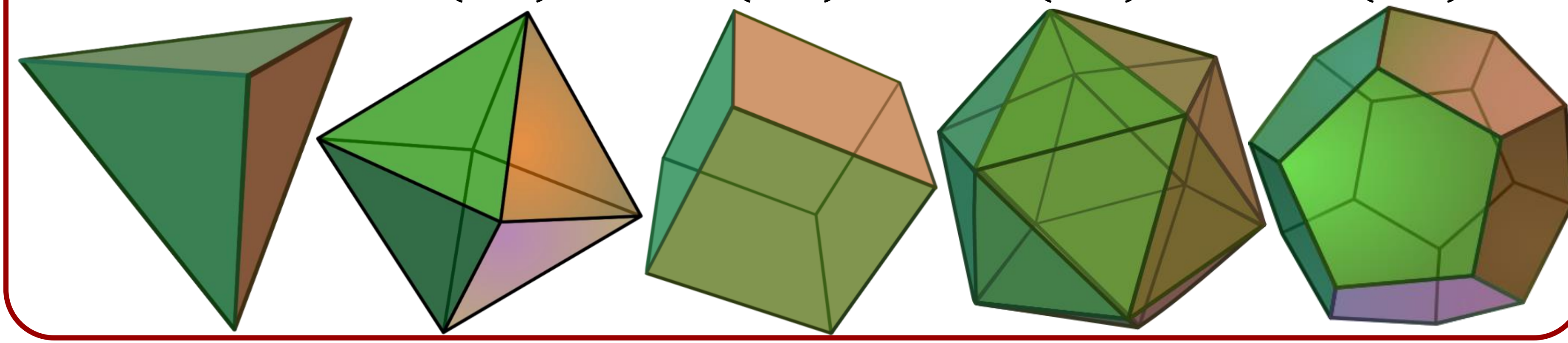
Proof:

Since the polyhedron is convex:

$$(p-2)(v-2)-4<0$$

Since $p, v \geq 3$, valid options are (p, v):

$(3,3)$ $(3,4)$ $(4,3)$ $(3,5)$ $(5,3)$



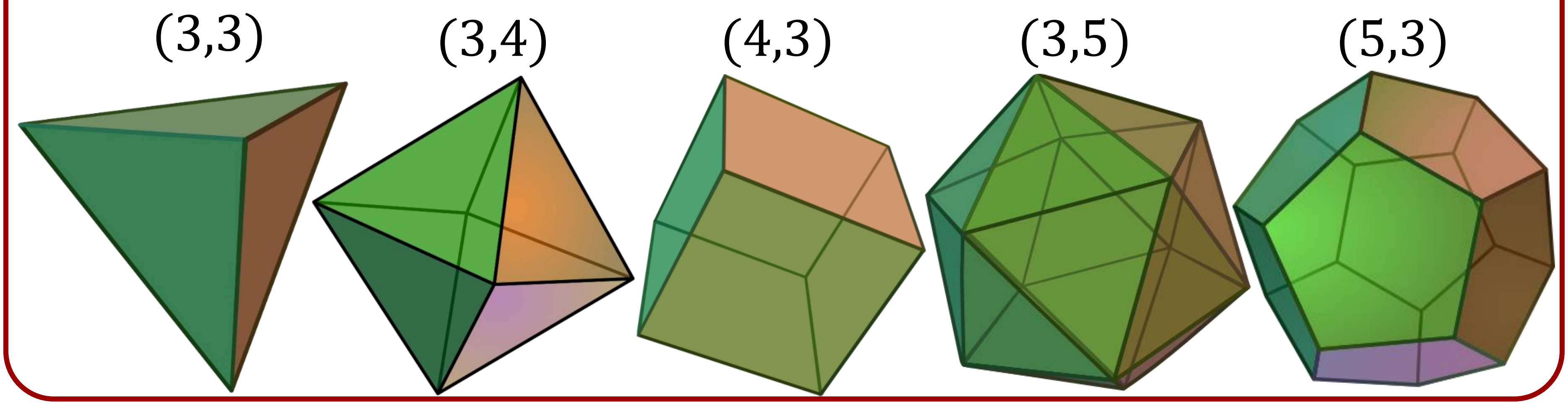
Platonic Solids

The platonic solids come in dual pairs, where one solid is obtained from the other by replacing faces with vertices:

Cube ↔ Octahedron

Icosahedron ↔ Dodecahedron

Tetrahedron ↔ Tetrahedron



Topological Polyhedra

The boundary of a polyhedron can be expressed as a combination of vertices, edges, and faces:

- Intersections are proper } Geometric
- Locally manifold
- Globally connected } Topological

Topological Polyhedra

If we ignore the vertex positions, we get a combinatorial structure composed of faces (cells), edges, and vertices.*

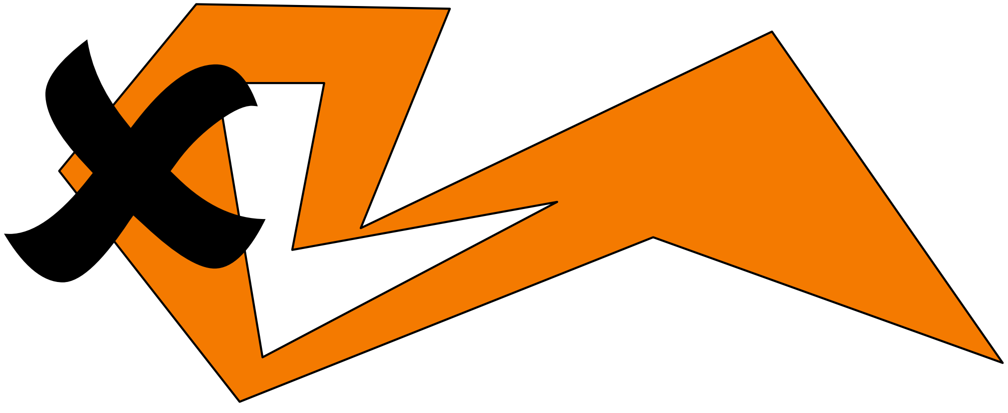
[Nivoliers and Levy, 2013]

*These are CW complexes. (And, if faces are triangles, these are simplicial complexes).

Topological Polyhedra

<u>Properties (CW Complex)</u>:

- Faces intersect at edges and vertices.
- Edges are topologically line segments and intersect at vertices.
- Interiors of faces have disk-topology and the boundary is a polygon made up of edges.



Topological Polyhedra

Properties (Manifold):

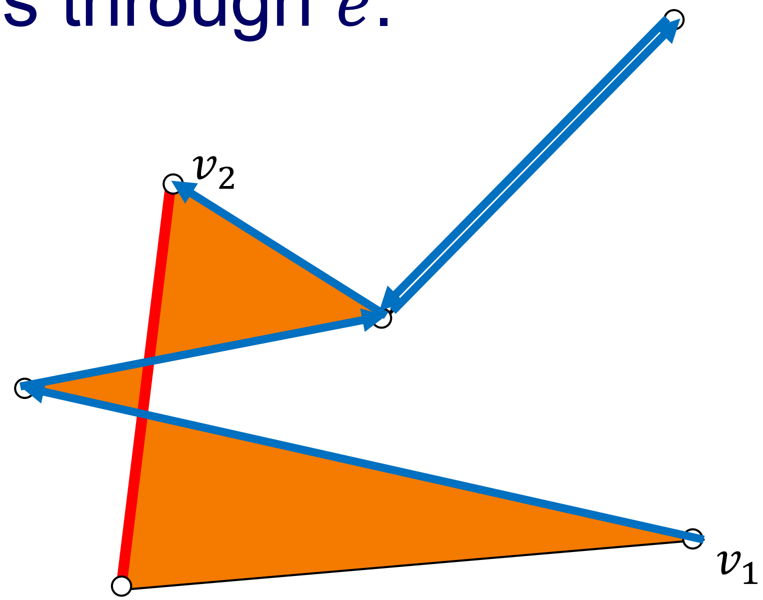
- Each vertex is on the boundary of some edge.
- Each edge is on the boundary of some face.
- Edges around a vertex can be sorted.
- An edge is on the boundary of two faces.

Topological Polyhedra

<u>Note</u>:

Given a topological polygon P, and given an edge $e \in P$ that only occurs once on P:

For any vertices $v_1, v_2 \in P$ there is a path from v_1 to v_2 that doesn't pass through e.



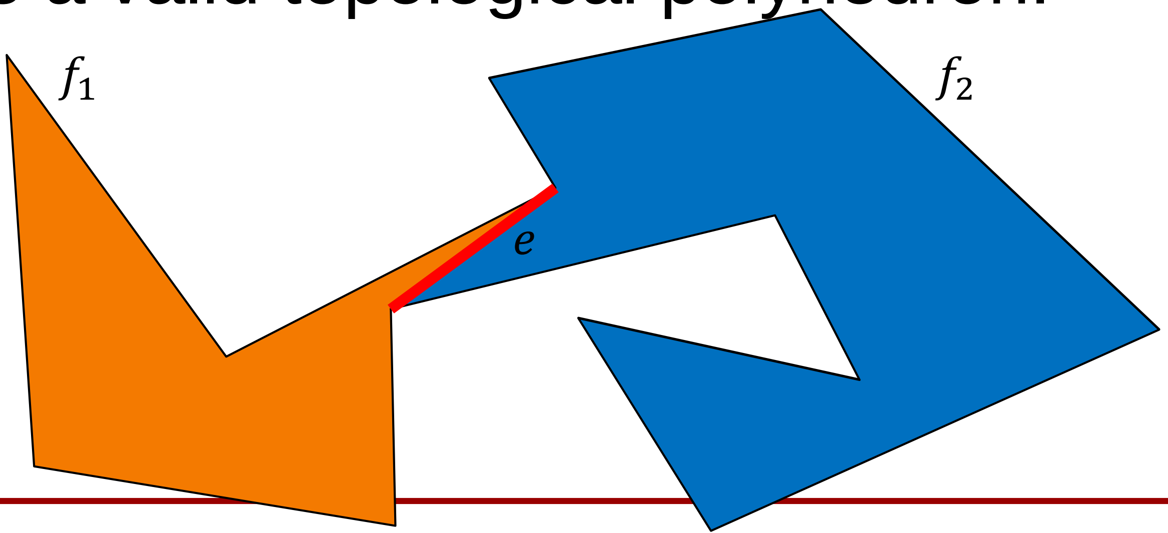
Topological Polyhedra

<u>Claim</u>:

If f_1 and f_2 are distinct faces of a topological polyhedron which share an edge e, then:

- replacing f_1 and f_2 with $f_1 \cup f_2$, and
- removing e from the edge list,

we still have a valid topological polyhedron.



Topological Polyhedra

Proof (CW Complex):

The edges/vertices of $f_1 \cup f_2$ are in the complex.

*Since the intersection $f_1 \cap f_2$ is connected and the interiors of f_1 and f_2 have disk-topology, the interior of $f_1 \cup f_2$ also has disk-topology.

*This is just a sketch of the proof.

Topological Polyhedra

Proof (CW Complex):

The boundary of $f_1 \cup f_2$ is connected.

- Let $v \in e$ be an end-point.
- For $v_1, v_2 \in f_1 \cup f_2$, there is a curve connecting v to each v_i that does not contain the edge e.
- Concatenating the two curves we connect v_1 to v_2 along the boundary of $f_1 \cup f_2$.

Topological Polyhedra

Proof (Manifold):

The smaller polyhedron still passes through all the vertices.

The edge e is removed and all other edges remain adjacent to a face.

Topological Polyhedra

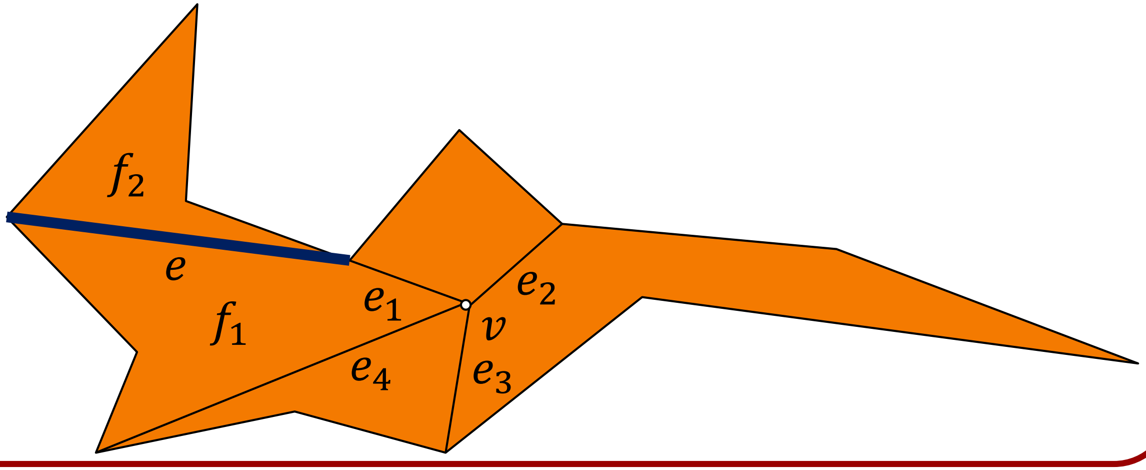
Proof (Manifold Edges):

The old edges still have only two faces on them (or one face twice).

Topological Polyhedra

Proof (Manifold Vertices):

If $v \notin e$, we can use the old edge ordering.



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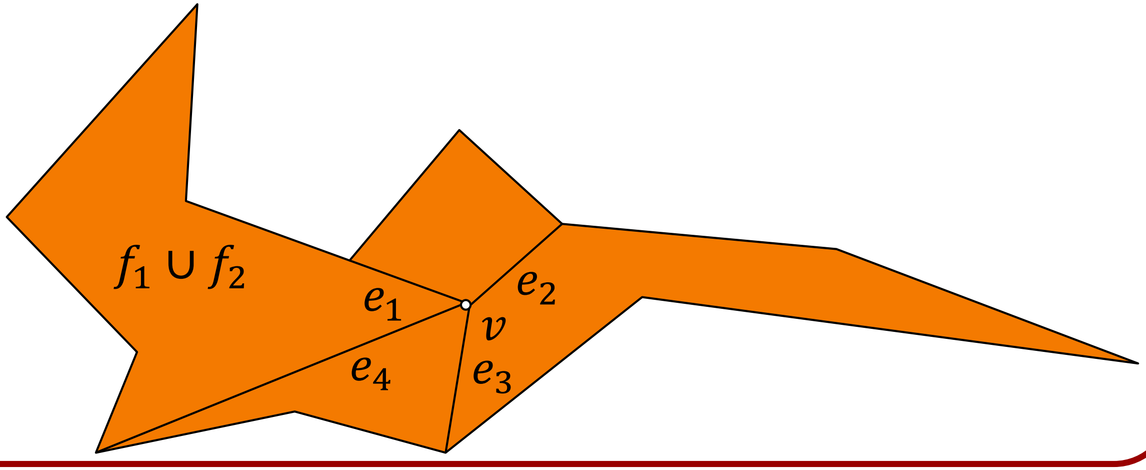


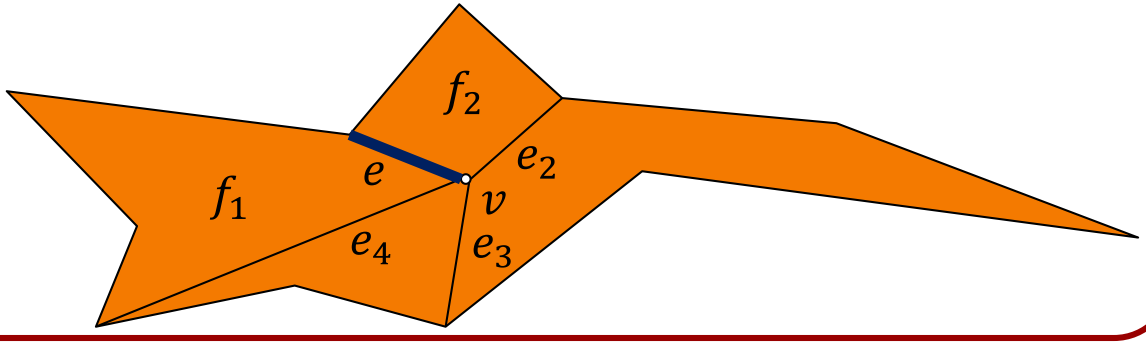
Topological Polyhedra

Proof (Manifold Vertices):

If $v \notin e$, we can use the old edge ordering.

If $v \in e$ let $\{e_1, e_2, \ldots, e_k\}$ be the old ordered edges around v, shifted so that $e_1 = e$.

Then e_k and e_2 are consecutive edges on $f_1 \cup f_2$ so $\{e_2, \ldots, e_k\}$ is a valid ordering.



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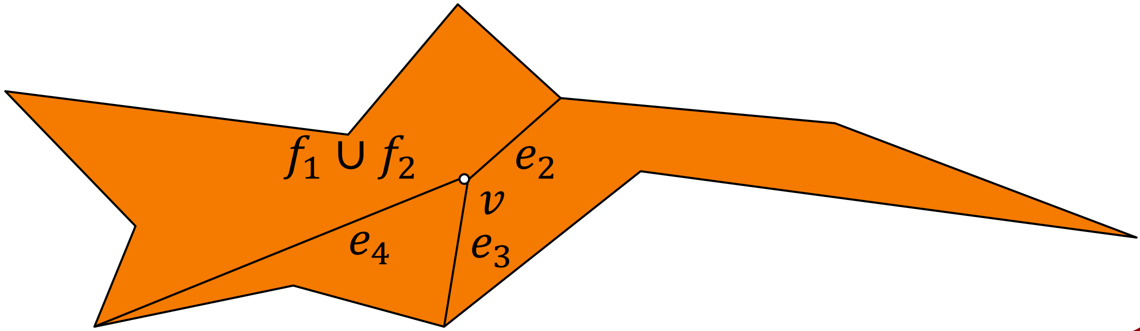
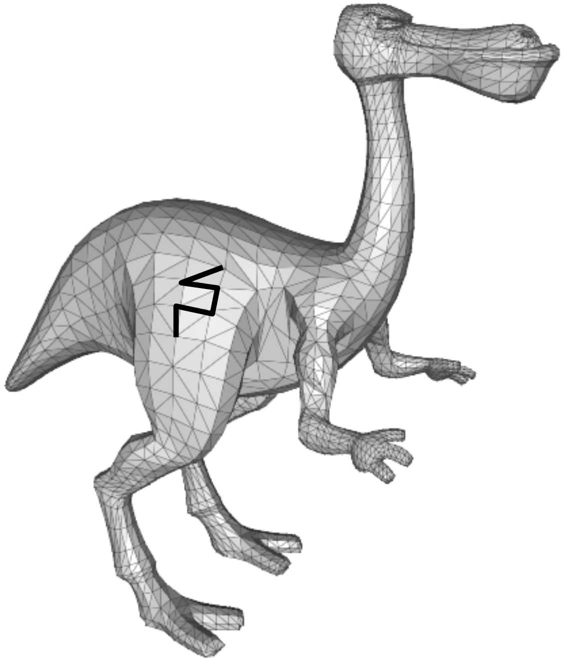


Curves

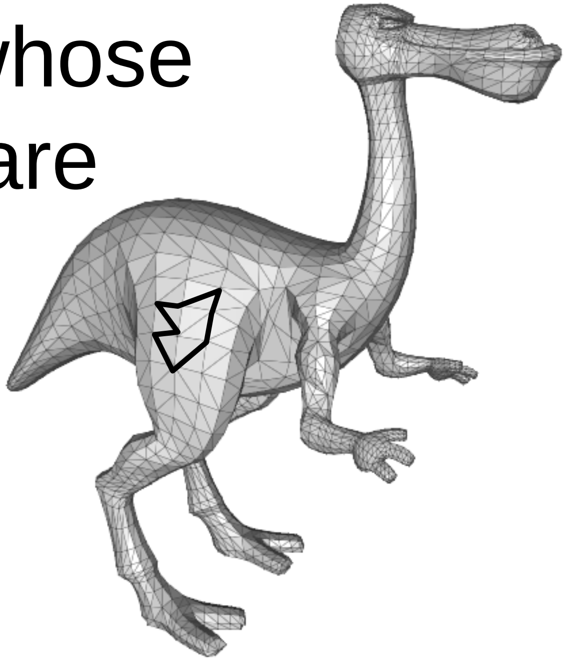
A (connected) *curve* on a topological polyhedron is a list of edges such that the ending vertex of one edge is the starting vertex of the next.



Curves

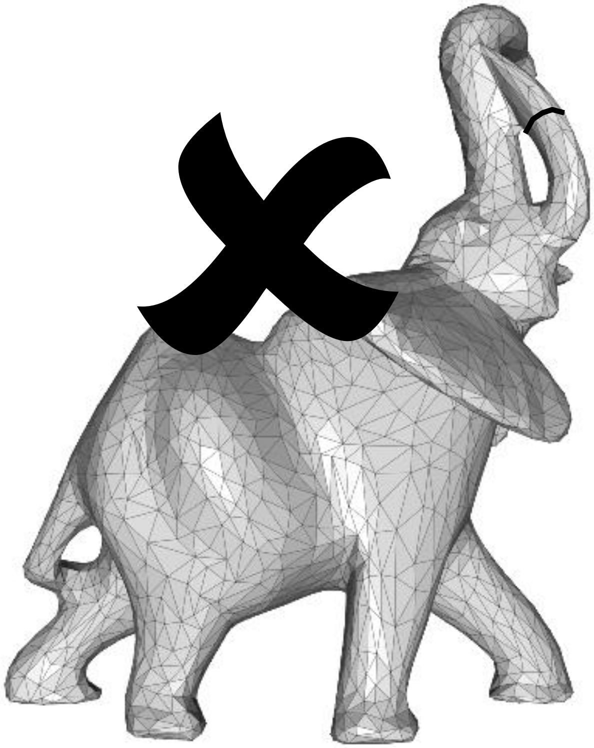
A (connected) *curve* on a topological polyhedron is a list of edges such that the ending vertex of one edge is the starting vertex of the next.

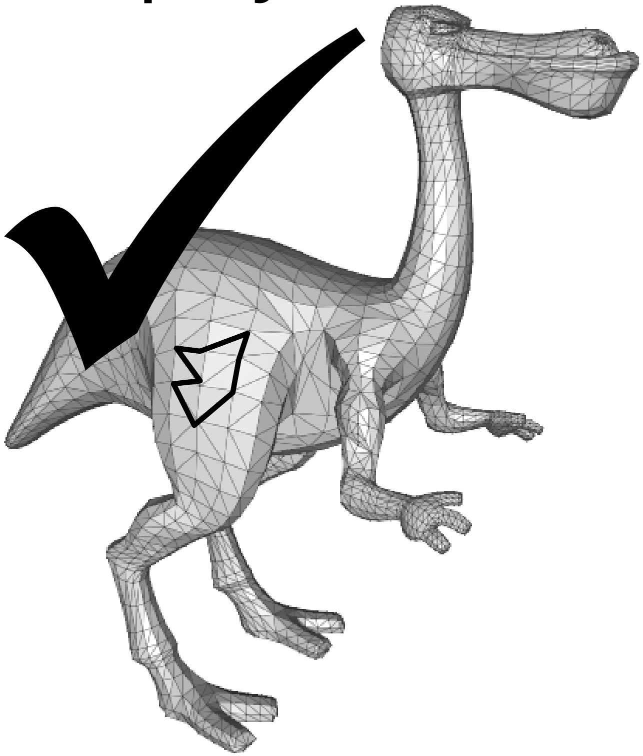
A *closed curve* is a curve whose starting and ending points are the same.



Genus-0 Polyhedra

A polyhedron is *genus-0* (or *simply connected*) if every non-trivial closed curve disconnects the faces of the polyhedron.

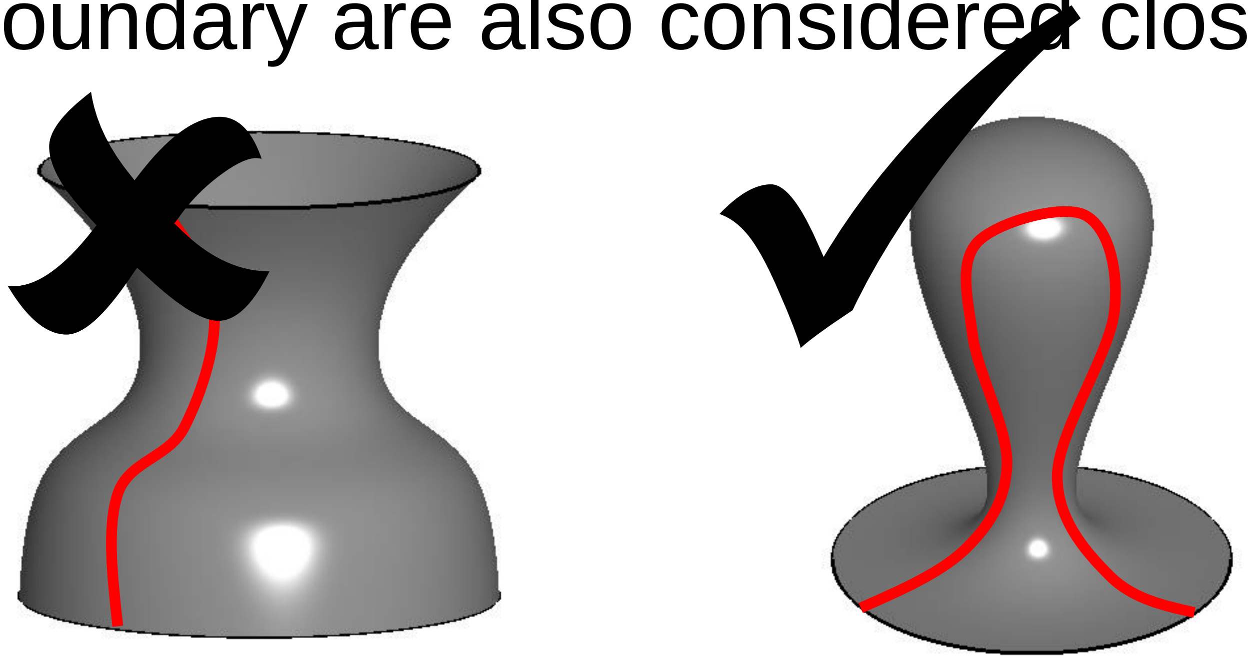




Genus-0 Polyhedra

Aside:

The definition can be extended to surfaces with boundary if curves that start and end at the boundary are also considered closed.



Genus-0 Polyhedra

Equivalently, given a topological polyhedron P we can define the dual graph $P^* = (V^*, E^*)$.

⇒ A curve $C \subset E$ corresponds to a set of dual edges $C^* \subset E^*$ of the dual.

⇒ P is genus-0 if removing C^* disconnects P^*.

Genus-0 Polyhedra

1. There is a continuous map from a polytope to a sphere.
   (e.g. Put the center of mass at the origin and normalize the positions.)

2. By the Jordan Curve Theorem the sphere is genus-zero.

<u>One Can Show</u>:

$\Rightarrow$ The polytope must also be genus-0.

Euler's Formula

For a genus-0 polyhedron P, the number of vertices, $|V|$, the number of edges, $|E|$, and the number of faces, $|F|$, satisfy:

$$|V| - |E| + |F| = 2$$

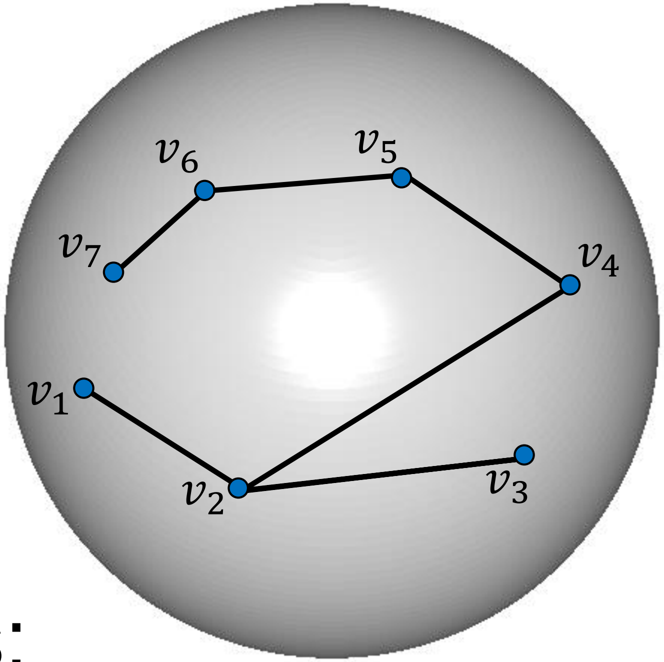
Euler's Formula (by Induction on $|F|$)

<u>Base case</u>: $|F| = 1$

We have:

- $V = \{v_1, \ldots, v_n\}$

*The edges on the boundary of the face form a connected tree (otherwise there is a closed loop and the interior of the face is disconnected).



Then there are $n - 1$ edges:

$$|V| - |E| + |F| = n - (n - 1) + 1 = 2$$

*This is just a sketch of the proof.

Euler's Formula (by Induction on $|F|$)

Induction: Assume true for $|F| = n - 1$

Find $e \in E$ shared by two distinct faces.

- If no such e exists, then all faces are adjacent to themselves, which contradicts the assumption that the polyhedron is connected.

Euler's Formula (by Induction on $|F|$)

Induction: Assume true for $|F| = n - 1$

Find $e \in E$ shared by two distinct faces.

- Remove e and merge the two adjoining faces, f_1 and f_2.

Claim:

The new polyhedron, P', is still genus-0.

Euler's Formula (by Induction on $|F|$)

Proof (P' is genus-zero):

Let C be a non-trivial curve on P'.

$\Rightarrow C$ is a non-trivial curve on P with $e \notin C$.

$\Rightarrow f_1$ and f_2 are in the same component.

$\Rightarrow C$ disconnects $f_1 \cup f_2$ from a face g on P.

$\Rightarrow C$ disconnects $f_1 \cup f_2$ from g in P'.

Euler's Formula (by Induction on $|F|$)

Induction: Assume true for $|F| = n - 1$

Find $e \in E$ shared by two distinct faces.

Remove e and merge the two adjoining faces.

P' is genus-0 with $|E| - 1$ edges, $|F| - 1$ faces, and $|V|$ vertices.

By the induction hypothesis we have:

$$|V| - (|E| - 1) + (|F| - 1) = 2$$

$$\Updownarrow$$

$$|V| - |E| + |F| = 2$$

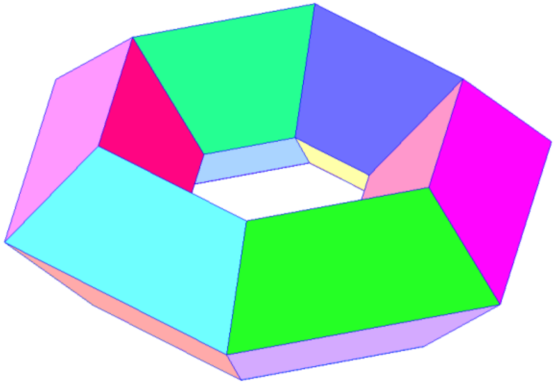
Euler's Formula

$$|V| - |E| + |F| = 2$$

More Generally:

If a polygon mesh is genus-g (g is the number of handles) then:

$$|V| - |E| + |F| = 2 - 2g.$$



$|V| = 24, |E| = 48, |F| = 24$

[Wikipedia: Toroidal Polyhedron]

Euler's Formula

Implication:

The number of faces and edges is linear in the number of vertices.

Euler's Formula

Proof:

Assume all faces are triangles.
(Triangulating only increase $|F|$ and $|E|$.)

Since each edge is shared by two triangles:

$$|E| = 3|F|/2$$

Using Euler's Formula:

$$|V| - |E| + |F| = 2$$
$$\Updownarrow$$
$$|F| = 2|V| - 4 \quad \text{and} \quad |E| = 3|V| - 6$$

Outline

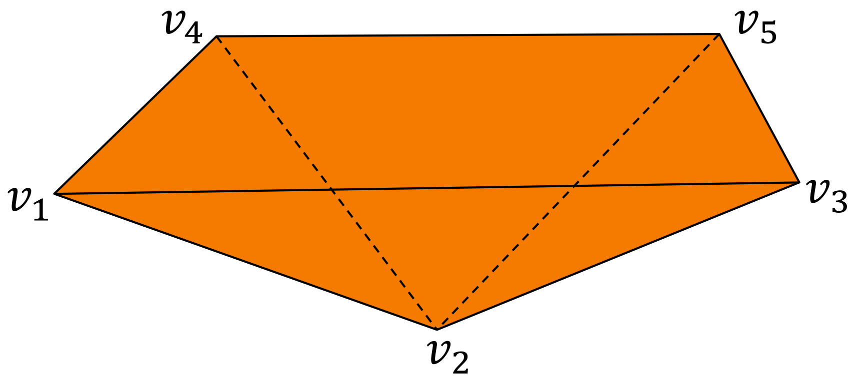
- Polyhedra
- (Oriented) Mesh Representation
  - Face-vertex data-structure
  - Winged-edge data-structure

(Oriented) Mesh Representation

Face-Vertex Lists:

Most often (e.g. ply, obj, etc. formats) polygon meshes are represented using vertex and face lists:

- **Vertex Entry**: (x, y, z) coordinates.
- **Face Entry**: Count and CCW indices of the vertices.



(Oriented) Mesh Representation

Face-Vertex Lists:

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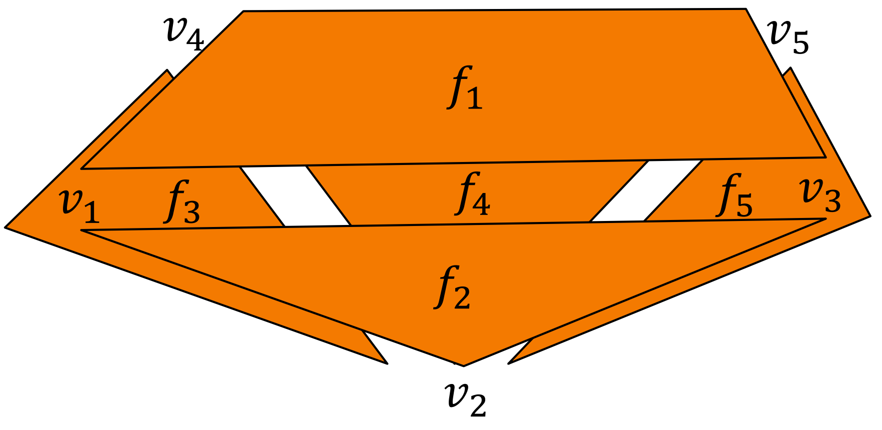
- **Face Entry**: Count and CCW indices of the vertices.

Vertex List

Id	x	y	z
1	-1	-1	0
2	0	0	-1
3	1	-1	0
4	-1	1	0
5	1	1	0

Face List

Id	#	Indices			
1	4	1	3	5	4
2	3	1	2	3	
3	3	4	2	1	
4	3	5	2	4	
5	3	3	2	5	



(Oriented) Mesh Representation

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Face List

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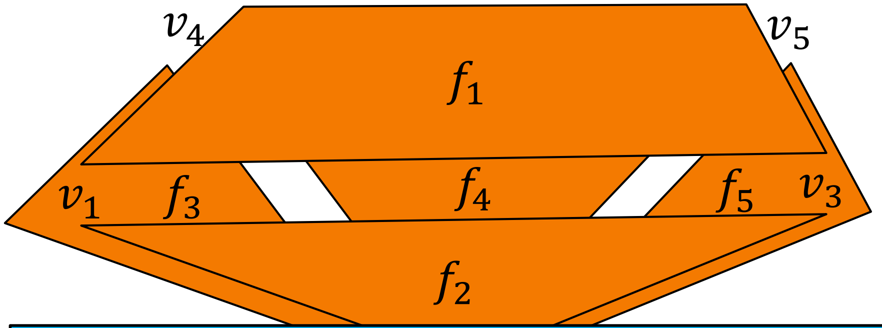
3	3	4	2	1	
4	3	5	2	4	
5	3	3	2	5	



<u>Limitation</u>:

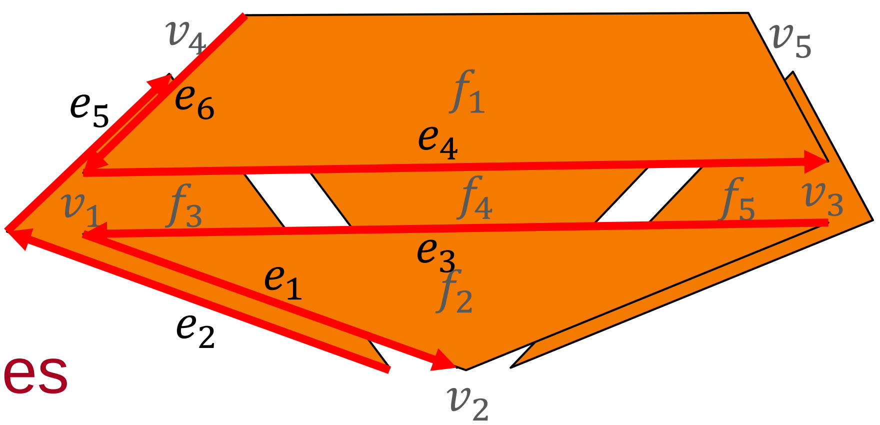
- Variable sized rows
- No explicit connectivity

(Oriented) Mesh Representation

Winged-Edge List:

Common representation for connectivity querying, represented using vertex, half-edge, and face lists:

- **Vertex Entry**:
  - » (x, y, z) coordinates
  - » Outgoing h.e. index
- **Face Entry**:
  - » h.e. index
- **Half-Edge Entry**:
  - » in/out wing h.e. indices
  - » opposite h.e. index
  - » end vertex index
  - » face index



Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	0	…

) Mesh Representation

e List:

presentation for connectivity querying, using vertex, half-edge, and face lists:

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  - (x, y, z) coordinates
  - Outgoing h.e. index
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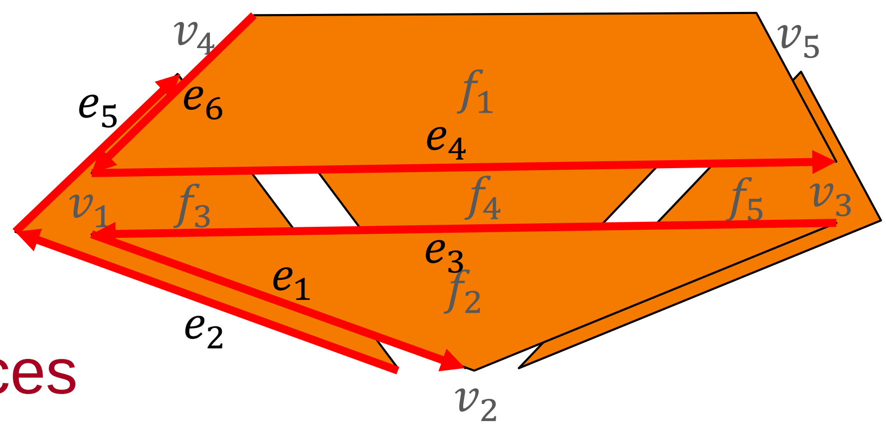

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	0	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

…n Representation

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- **Vertex Entry**:
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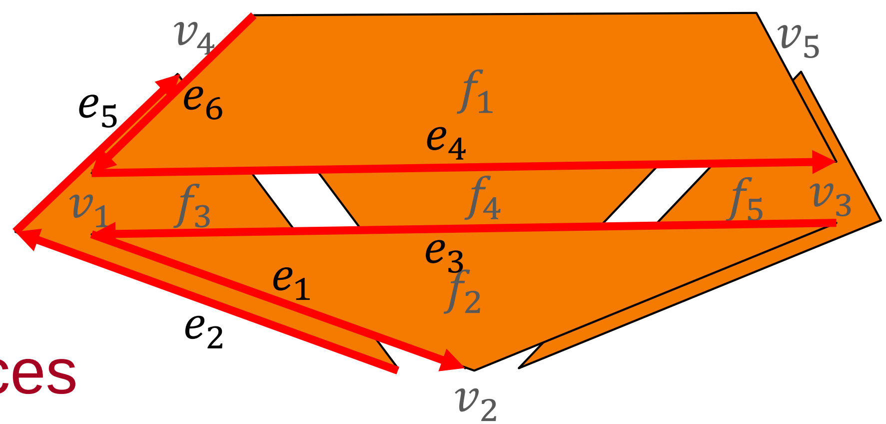

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3	1	-1	0	3
4	-1	1	0	6
5	1	1	0	…

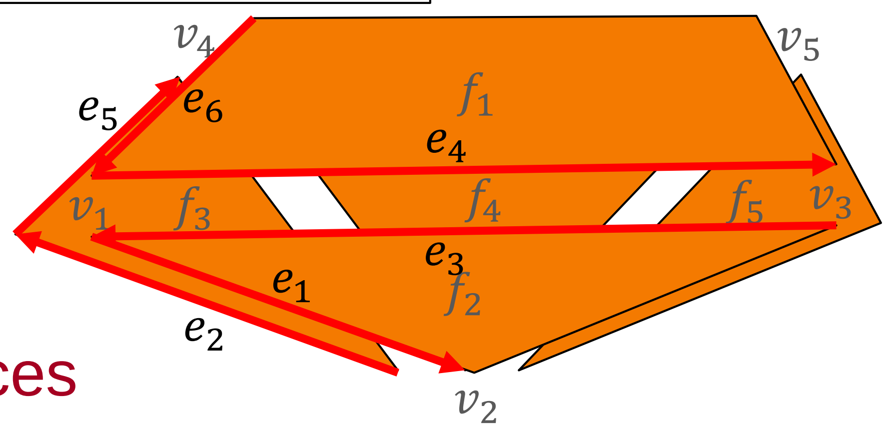
Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	...	4	1	1
…					

  - » Outgoing h.e. index
- ◦ **Face Entry**:
  - » h.e. index
- ◦ **Half-Edge Entry**:
  - » in/out wing h.e. indices
  - » opposite h.e. index
  - » end vertex index
  - » face index



Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	0	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

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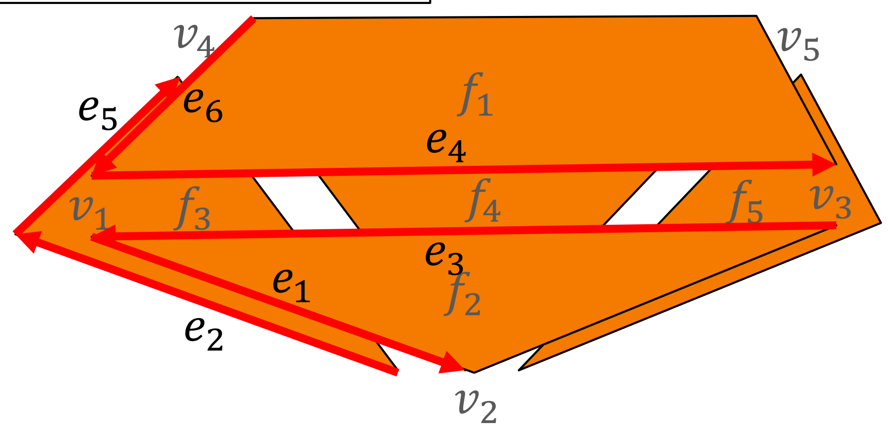
- ◦ **Face Entry**:
  - » h.e. index
- ◦ **Half-Edge Entry**:



<u>Example</u>:

Find CCW vertices around v_1:

tion

ity querying,
nd face lists:

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	0	…

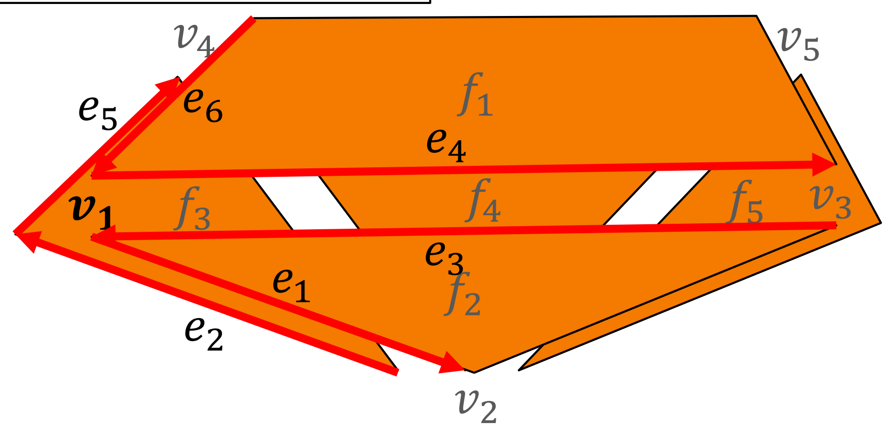
Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

- » Outgoing h.e. index
- ○ **Face Entry**:
  - » h.e. index
- ○ **Half-Edge Entry**:



<u>Example</u>:

Find CCW vertices around v_1:

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	0	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	**3**	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

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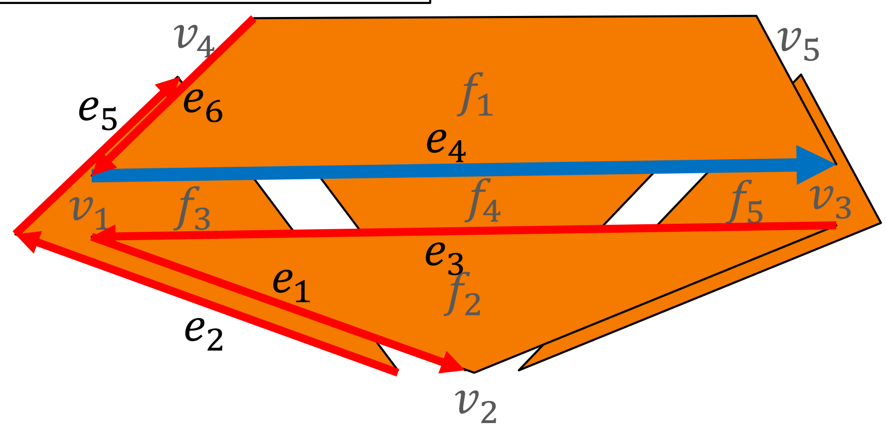
nd face lists:

» Outgoing h.e. index

◦ **Face Entry**:

» h.e. index

◦ **Half-Edge Entry**:



Example:

Find CCW vertices around v_1: v_3

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	0	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	**6**	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

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…ity querying,

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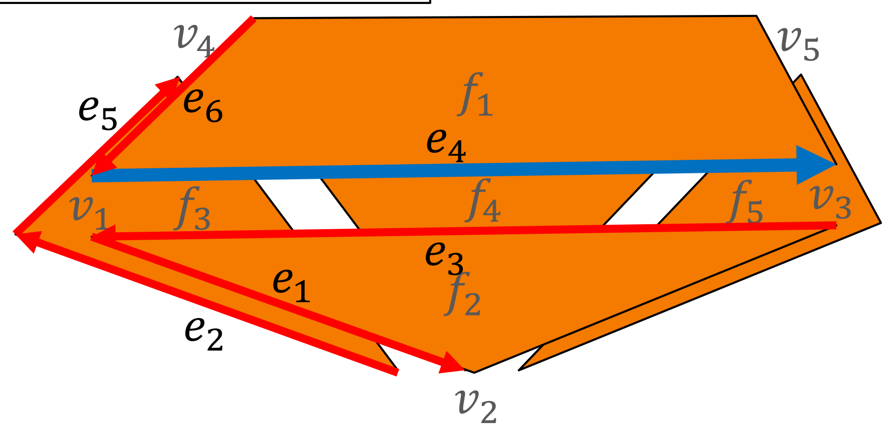
   » Outgoing h.e. index

- **Face Entry**:
   » h.e. index
- **Half-Edge Entry**:



<u>Example</u>:

Find CCW vertices around v_1: v_3

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	0	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

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- » Outgoing h.e. index
- ○ **Face Entry**:
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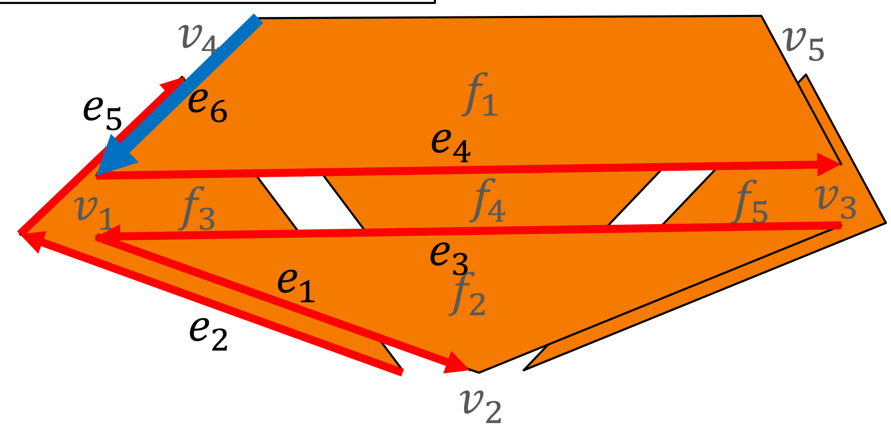
- ○ **Half-Edge Entry**:



<u>Example</u>:

Find CCW vertices around v_1: v_3

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	0	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

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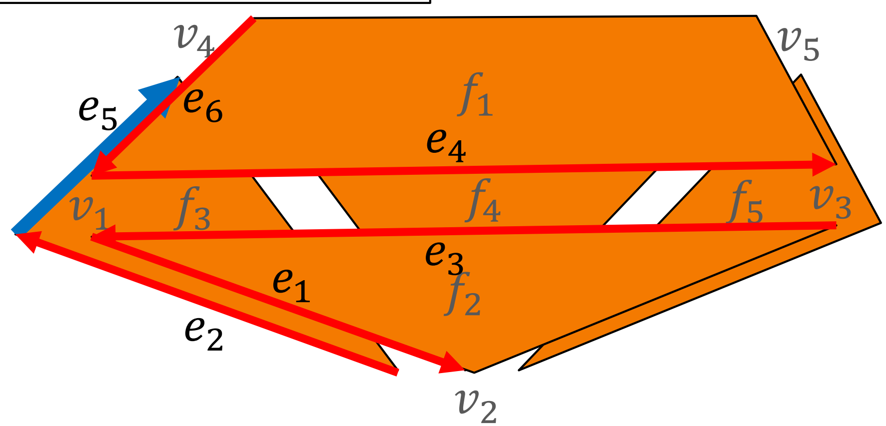

Example:

Find CCW vertices around v_1: v_3, v_4

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	0	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

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- **Face Entry**:
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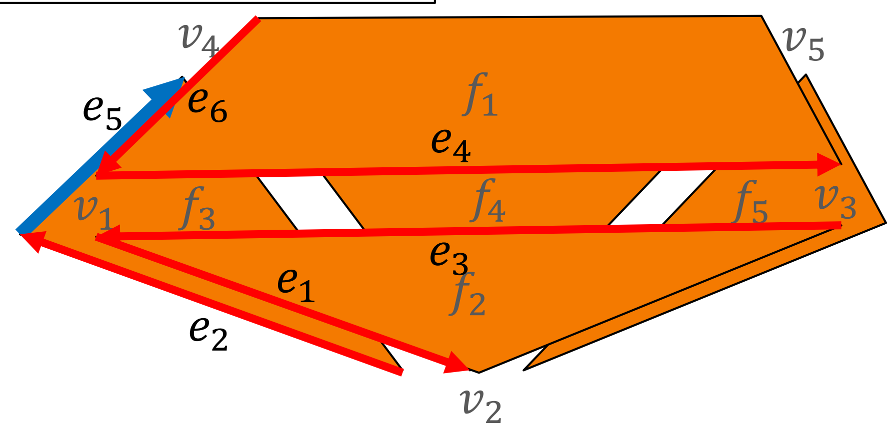
- **Half-Edge Entry**:



<u>Example</u>:

Find CCW vertices around v_1: v_3, v_4

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	0	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	...	4	1	1
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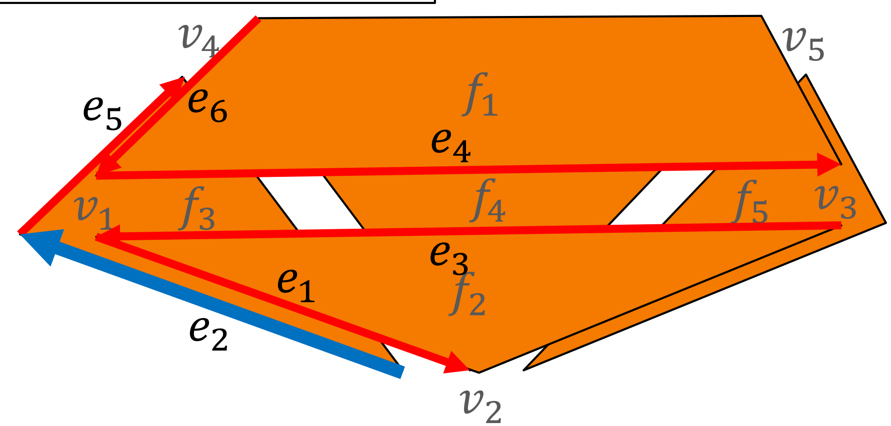

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Find CCW vertices around v_1: v_3, v_4

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Half-Edge List

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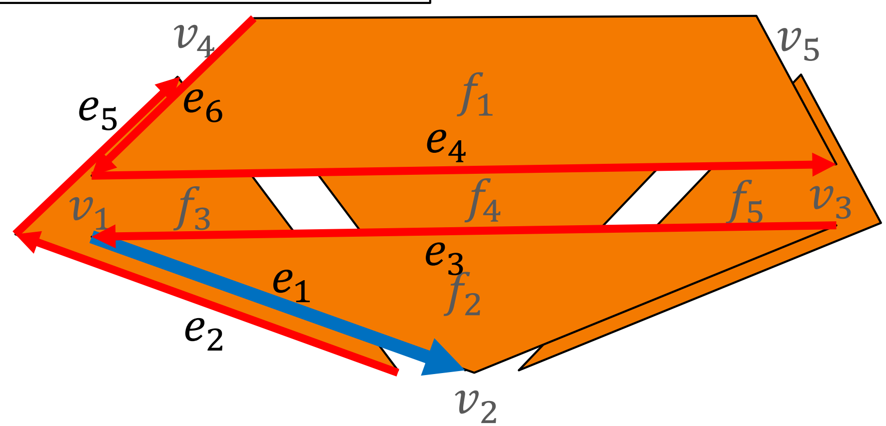

<u>Example</u>:

Find CCW vertices around v_1: v_3, v_4, v_2

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	0	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	**3**	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
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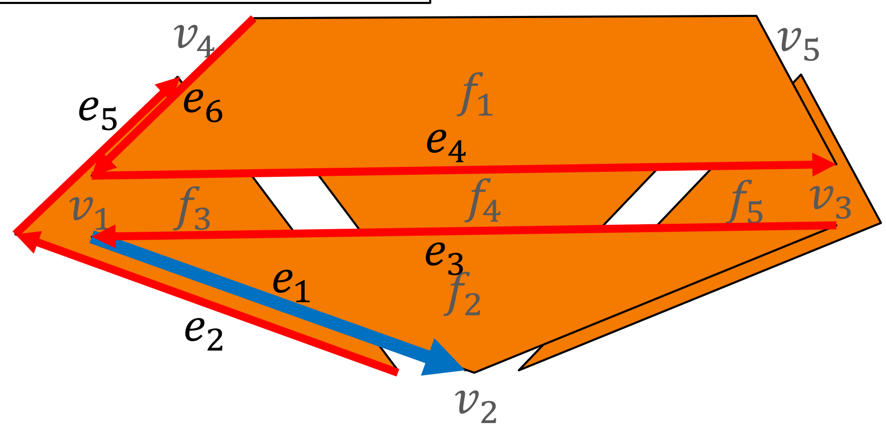

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Face List

Id	h
1	4
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Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
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3	**4**	…	1	1	2
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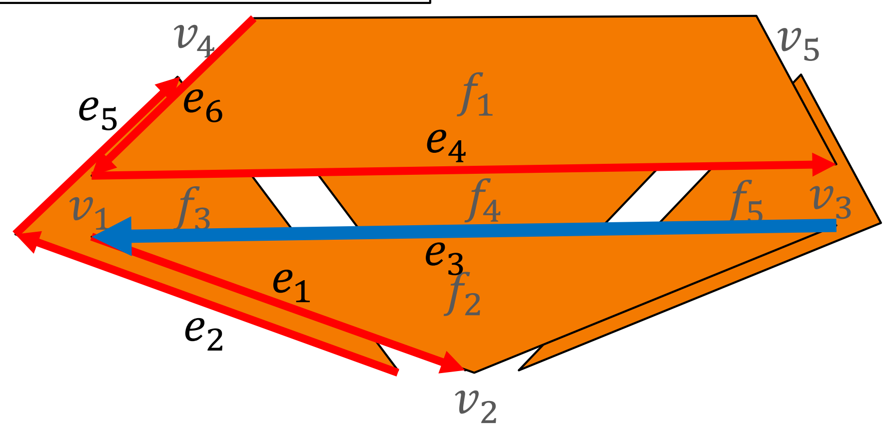

Example:

Find CCW vertices around v_1: v_3, v_4, v_2

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
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Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	...	4	1	1

tion

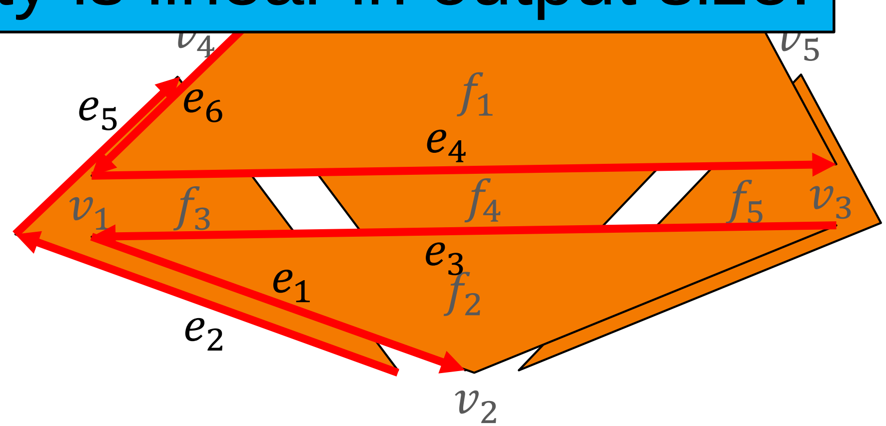
ity querying,

ınd face lists:

Computational complexity is linear in output size.

» Outgoing h.e. index

- **Face Entry**:
  » h.e. index
- **Half-Edge Entry**:



<u>Example</u>:

Find CCW vertices around v_1: v_3, v_4, v_2

(Oriented) Mesh Representation

Goal:

Given a face-vertex representation of a mesh (V,F), convert it to a winged-edge representation (_V,_E,_F).

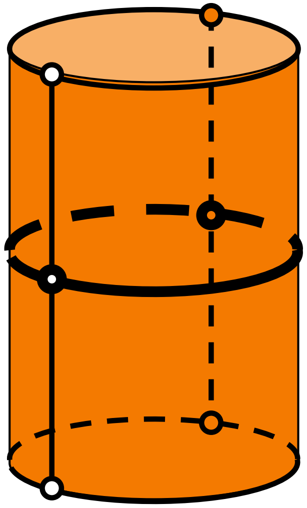
(Oriented) Mesh Representation

Goal:

Given a face-vertex representation of a mesh (V,F), convert it to a winged-edge representation (_V,_E,_F).

Warning:

The following discussion assumes that in a mesh, a (directed) edge is uniquely determined by its starting and ending vertices.



This does not have to be true.

(Oriented) Mesh Representation

```
GenerateHalfEdge( V , F , _V , _E , _F )
    _V.resize( v.size() ) , _F.resize( F.size() )
    for( i=0 ; i<V.size() ; i++ ) _V[i].p = V[i]

    unordered_map< VertexPair , int > eMap
    ConstructEdgeToFaceMap( F, eMap )

    _E.resize( eMap.size() )

    SetHalfEdgeIndices( eMap , _V , _E , _F )
```

Assuming that:

- The VertexPair object defines a hashing function

(Oriented) Mesh Representation

```
ConstructEdgeToFaceMap( F , eMap )
    for( f=0 ; f<F.size() ; f++ )
        for( v=0 ; v<F[f].size() ; v++ )
            VertexPair key( F[f][v] , F[f][v+1] )
            eMap[key] = f
```

Assuming that:

- Indexing is modulo the face size

(Oriented) Mesh Representation

```
SetHalfEdgeIndices( eMap , _V , _E , _F )
    int e = 0
    for( iter i=eMap.begin() ; i!=eMap.end() ; i++ , e++ )
        int v1 = i.key.first , v2 = i.key.second , f = i.value
        _E[e].v = v2 , _E[e].f = f
        _V[v1].he = _F[f].he = i.value = e
    for( f=0 ; f<F.size() ; f++ ) for( v=0 ; v<F[f].size() ; v++ )
            VertexPair  key( F[f][v  ] , F[f][v+1] )
            VertexPair oKey( F[f][v+1] , F[f][v  ] )
            VertexPair nKey( F[f][v+1] , F[f][v+2] )
            _E[ eMap[ key ] ].opposite = eMap[ oKey ]
            _E[ eMap[ key ] ].next = eMap[ nKey ]
            _E[ eMap[ nKey ] ].previous = eMap[ key ]
```