Introduction to Computational Geometry

Michael Kazhdan

(600.459)

What are we studying?

- Algorithms and data-structures for solving geometric problems.
- Following O'Rourke, "Computational Geometry in C (2nd Edition)":
  http://cs.smith.edu/~orourke/books/compgeom.html
  http://cs.smith.edu/~orourke/books/ftp.html

More Specifically

- Polygon Triangulation
- Polygon Partitioning
- Convex Hulls in 2D/3D
- Voronoi Diagrams
- Arrangements
- Search & Intersection
- Motion Planning

Additional Resources

QHull: www.qhull.org/

ANN: www.cs.umd.edu/~mount/ANN/

Triangle: www.cs.cmu.edu/~quake/triangle.html

TetGen: wias-berlin.de/software/tetgen/

CGAL: www.cgal.org/

Syllabus

When: MW @ 1:30 – 2:45

Where: Hodson 316

Web: http://www.cs.jhu.edu/~misha/Spring20/

TA: Tommy Mitchell

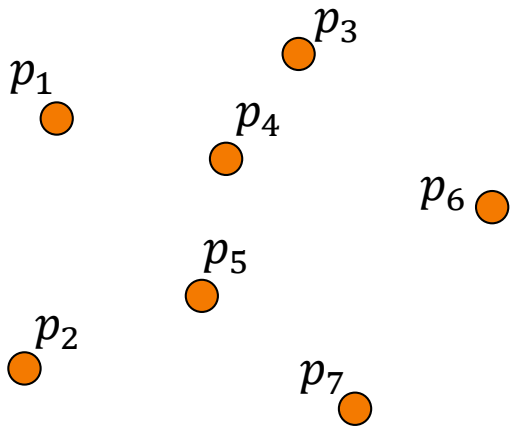
Assignments: Yes

Exams: No

Example: Convex Hulls in 2D

Given: A set $P = \{p_1, \ldots, p_n\} \subset \mathbb{R}^2$.

Output: An ordered subset $CH = \{p_{i_1}, \ldots, p_{i_m}\} \subset P$ describing the convex hull.



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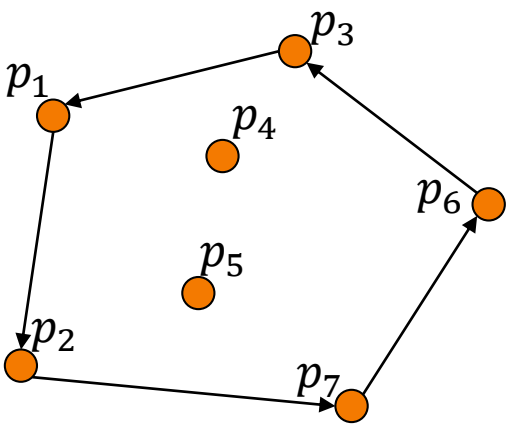
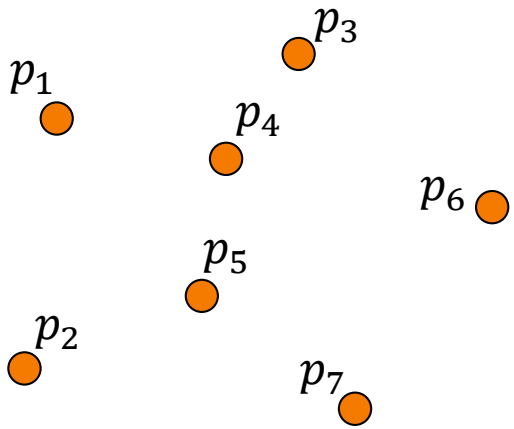


$CH = \{p_1, p_2, p_7, p_6, p_3\}$

Example: Convex Hulls in 2D

Note:

Hull edges have the property that all other points are to the left of the edge.



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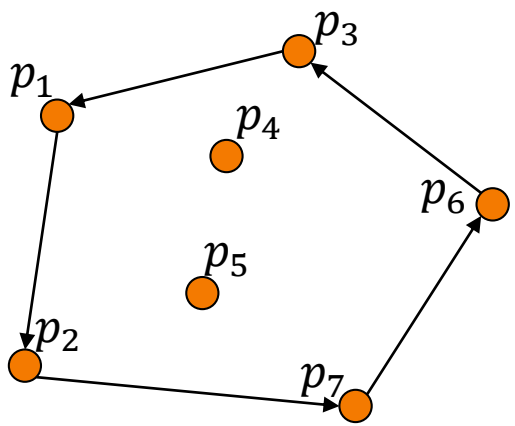

$CH = \{p_1, p_2, p_7, p_6, p_3\}$

Example: Convex Hulls in 2D

Take 1:

1. $E \leftarrow \emptyset$
2. $\forall (p, q) \in P \times P$ with $p \neq q$
3. $\quad hullEdge \leftarrow true$
4. $\quad \forall r \in P$ with $r \neq p$ and $r \neq q$
5. $\quad\quad$ If r is right of $\overrightarrow{pq}$: $hullEdge \leftarrow false$
6. $\quad$ If $hullEdge$: $E \leftarrow E \cup \overrightarrow{pq}$
7. Construct CH from E.

Example: Convex Hulls in 2D

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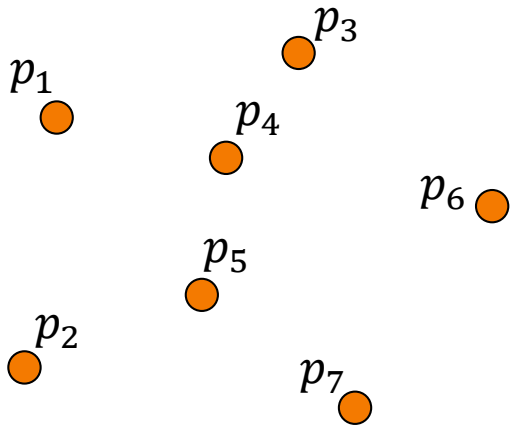
Correctness?
Robustness?
Efficiency?

Example: Convex Hulls in 2D

Note:

Successive edges on the hull make left turns.



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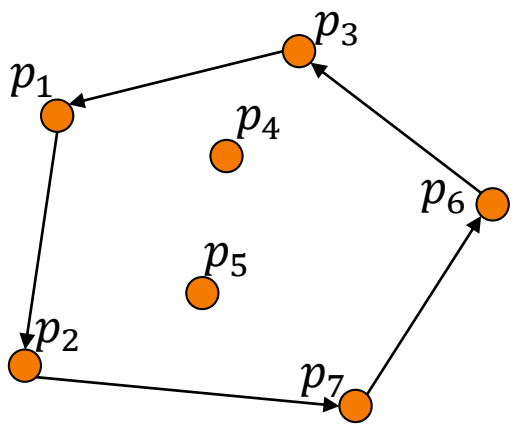

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Example: Convex Hulls in 2D

Take 2: (Graham Scan)

1. Find bottom-most (left-most) point p
2. Sort remaining points in P by slope
3. $CH \leftarrow \{p, p_1\}$
4. For $i \in \{2, \ldots, n-1\}$
5. $\quad CH \leftarrow CH \cup \{p_i\}$
6. $\quad$ While last three points don't make a left turn
7. $\quad\quad$ Delete the middle of the last three points

Correctness?
Robustness?
Efficiency?

For Next Class (Wednesday)

Chapter 1: Polygon Triangulation