Intro. to Geometry Processing 600.756

http://www.cs.jhu.edu/~misha/Spring17a/

Assignment 2

✓Computed mean/Gaussian/principal curvatures!

✘Did not compute the principal directions correctly

- As a sanity check, validate that the principal directions are orthogonal

Assignment 3 (smoothing)

- Smooth a 3D mesh using the gradient-domain formulation:

$$\tilde{c}_\alpha = \underset{c \in \mathbb{R}^n}{\operatorname{argmin}} \|c - c_\alpha\|^2 + \varepsilon \|\nabla c\|^2$$

  - $\{c_\alpha\}_{\alpha \in \{x,y,z\}}$ are the input x-,y-,z-coordinates
  - $\{\tilde{c}_\alpha\}_{\alpha \in \{x,y,z\}}$ are the output x-,y-,z-coordinates
  - ε is the smoothing weight
  - n is the number of vertices

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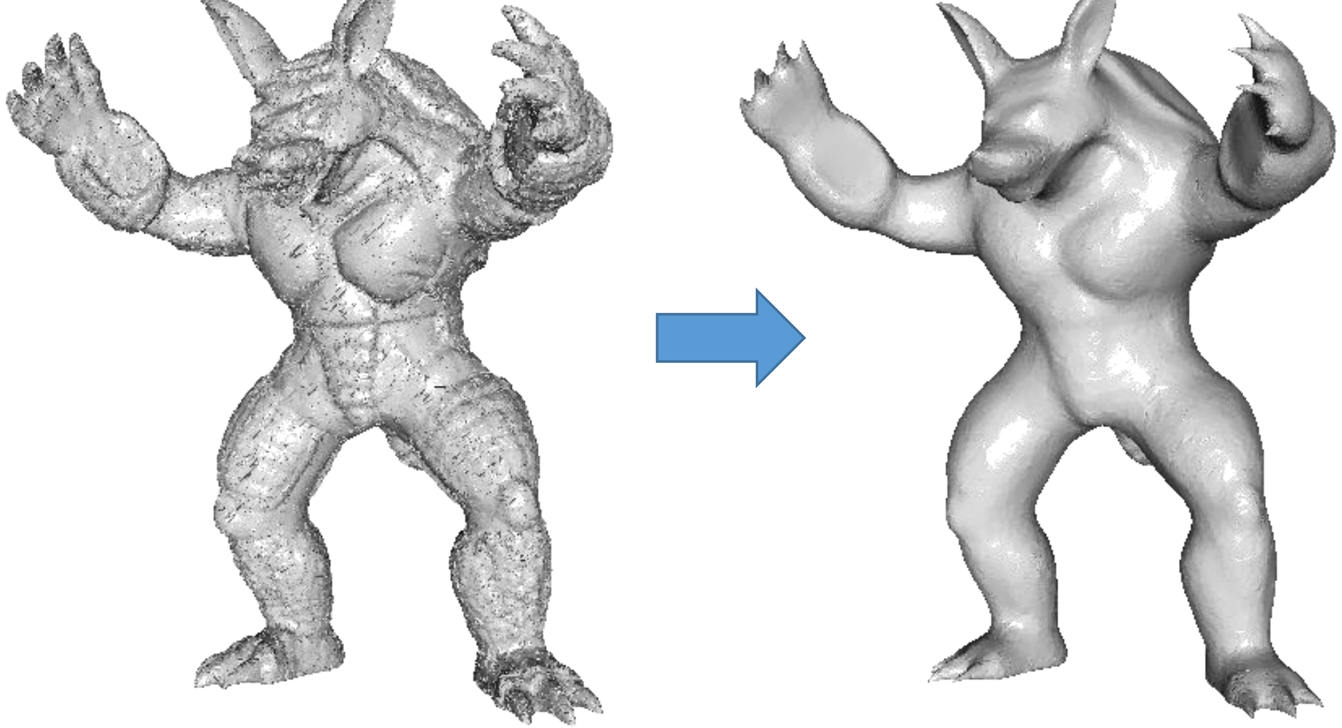

```
smooth --out o.ply --smooth 1e-4 --in armadillo.bad.ply
```

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Recall:

- If we set the M and S to be the stiffness matrices and c to be the input vector of coefficients, then the solution $\tilde{c}$ is the minimizer of:

$$\begin{aligned} E(\tilde{c}) &= (c - \tilde{c})^t \cdot M \cdot (c - \tilde{c}) + \varepsilon \cdot \tilde{c}^t \cdot S \cdot \tilde{c} \\ &= \tilde{c}^t \cdot (M + \varepsilon \cdot S) \cdot \tilde{c} - 2\tilde{c}^t \cdot M \cdot c + c^t \cdot M \cdot c \end{aligned}$$

- Taking the gradient with respect to $\tilde{c}$ and setting to zero, we get:

$$\begin{aligned} 0 &= 2(M + \varepsilon \cdot S) \cdot \tilde{c} - 2M \cdot c \\ \tilde{c} &= (M + \varepsilon \cdot S)^{-1} \cdot M \cdot c \end{aligned}$$

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Use the Eigen package:

- Eigen::VectorXd
  - Vectors (with double values)
- Eigen::SparseMatrix< double >
  - Sparse matrices
- Eigen::Triplet< double >
  - A triplet of values storing the column/row index and the associated matrix entry
- Eigen::SimplicialLLt< Eigen::SparseMatrix< double > >
  - Solver for sparse symmetric positive definite linear systems

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Use the Eigen package:

- Note that standard algebraic operators are defined for matrices and vectors:

```
double s;
Eigen::SparseMatrix< double > A , B , C;
Eigen::VectorXd x , b;
…
C = A + B * s;
b = C * x
```

Assignment 3 (smoothing)

- Smooth a 3D mesh using the gradient-domain formulation:

$$\tilde{c}_\alpha = \underset{c\in\mathbb{R}^n}{\operatorname{argmin}} \|c - c_\alpha\|^2 + \varepsilon\|\nabla c\|^2$$

1. Normalize the input so that it has unit area

   (The energy is scale-dependent so normalize so that ε is consistent)

Assignment 3 (smoothing)

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$$\tilde{c}_\alpha = \underset{c\in\mathbb{R}^n}{\operatorname{argmin}} \|c - c_\alpha\|^2 + \varepsilon\|\nabla c\|^2$$

1. Normalize the input so that it has unit area
2. Compute the mass and stiffness matrices
   - Set the entries using the `Eigen::SparseMatrix::setFromTriplets` method which takes a `std::vector< Eigen::Triplet >` object
     - If the same entry index pair appears multiple times in the `std::vector`, the corresponding entries are summed

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1. Normalize the input so that it has unit area
2. Compute the mass and stiffness matrices
3. Compute the right-hand-side for each coefficient

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1. Normalize the input so that it has unit area
2. Compute the mass and stiffness matrices
3. Compute the right-hand-side for each coefficient
4. Solve the linear system to get the new coefficient values
   - The `Eigen::SimplicialLLt` constructor takes the matrix to invert
   - The `Eigen::SimplicialLLt::solve` method takes a RHS `Eigen::VectorXd` as its argument and returns the solution

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--smooth:

What happens as the smoothing value gets larger?

--iters:

What happens if you run multiple smoothing iterations, updating the mass and stiffness matrices at each iteration using the new geometry?

--cmcf:

What happens if you run multiple smoothing iterations, but only update the mass matrix at each iteration using the new geometry (keeping the stiffness from the first iteration)?