Lecture 7

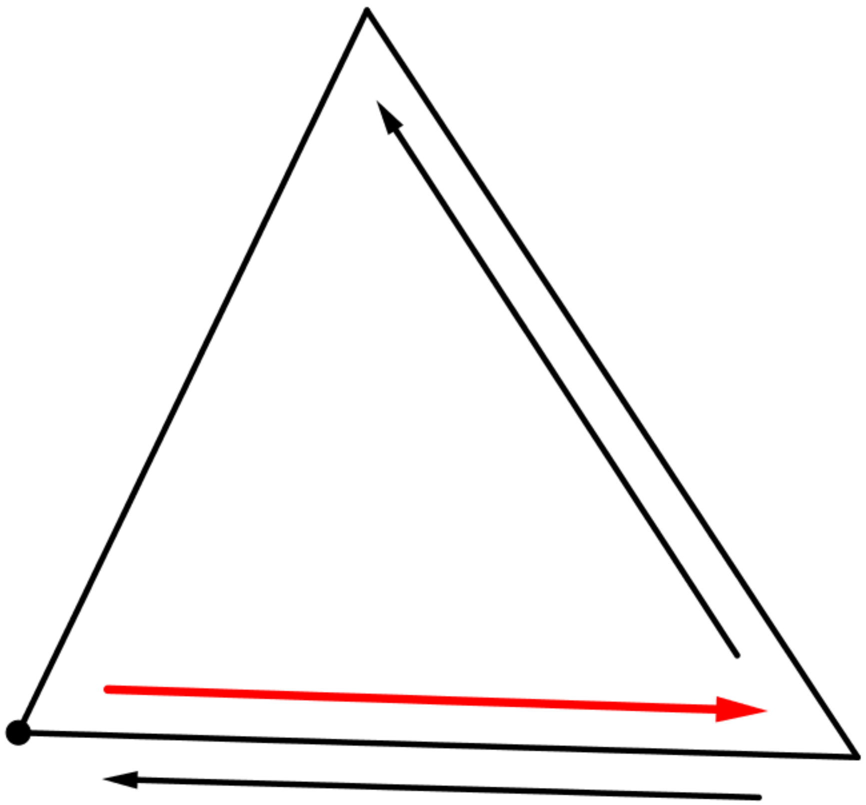
Introduction to Geometry Processing

Spring 2017

Johns Hopkins University

Mesh

Data Representations



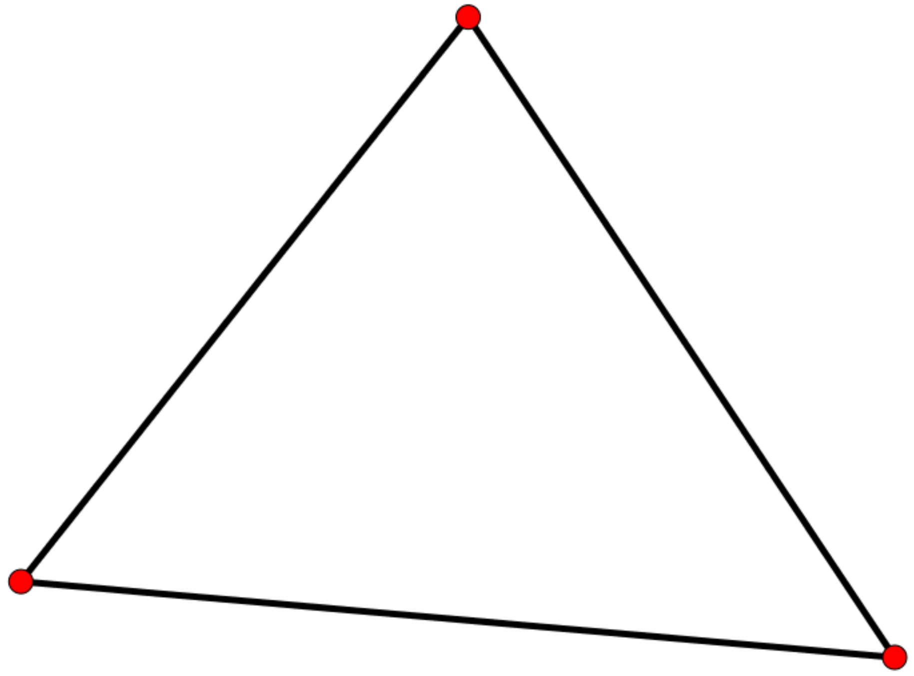
Half Edges

- List of vertices:
  - Pointer to a half edge sourced at the vertex.
- List of triangles
  - Pointer to half edge
- List of half edges:
  - Pointer to next half edge
  - Pointer to opposite half edge
  - Pointer to the incident face
  - Pointer to source vertex

What is this useful for?
Fast local queries!!

Mesh

Data Representations

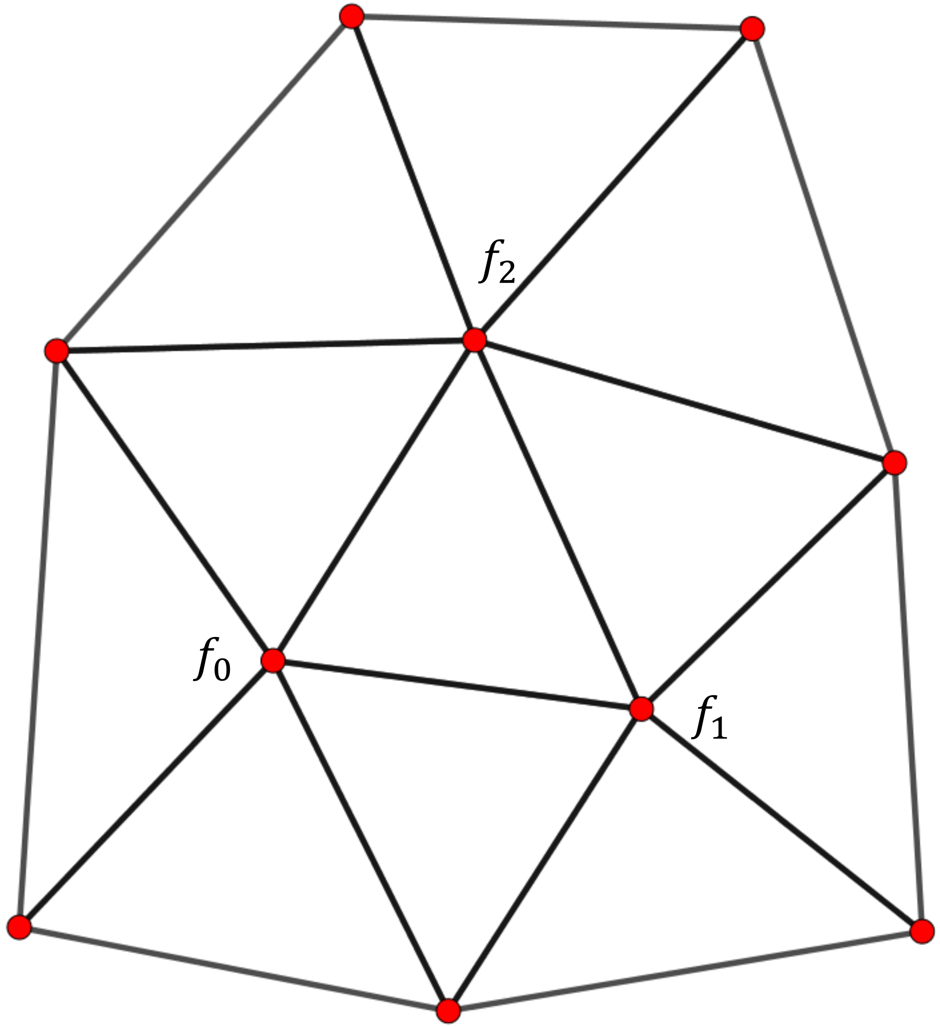


Triangle's Soup

- List of vertices
- List of triangles
    - Pointer to three vertices

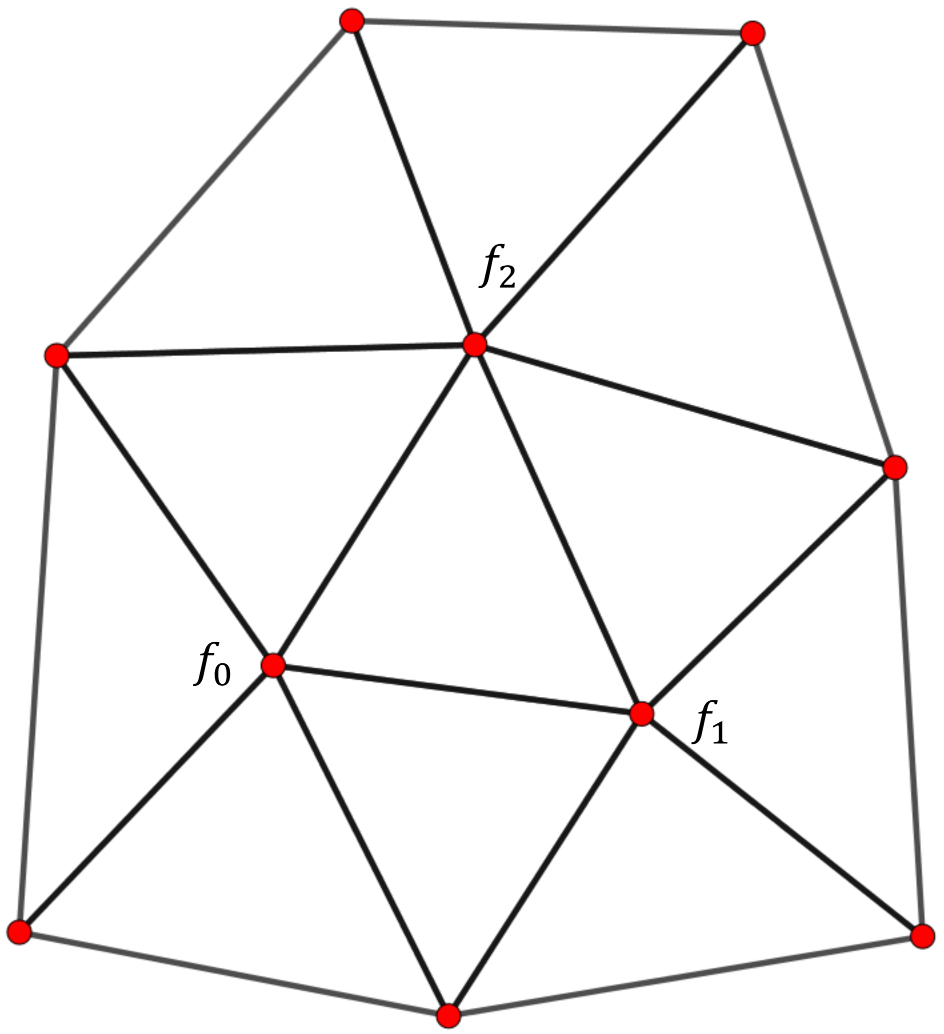
What is this useful for?
Global computations!!

Functions on meshes



Defined at vertices, affine linear at faces

Functions on meshes



Integration is done per triangle element:

$$\int_T f := \sum_{t_i \in T} \int_{t_i} f$$

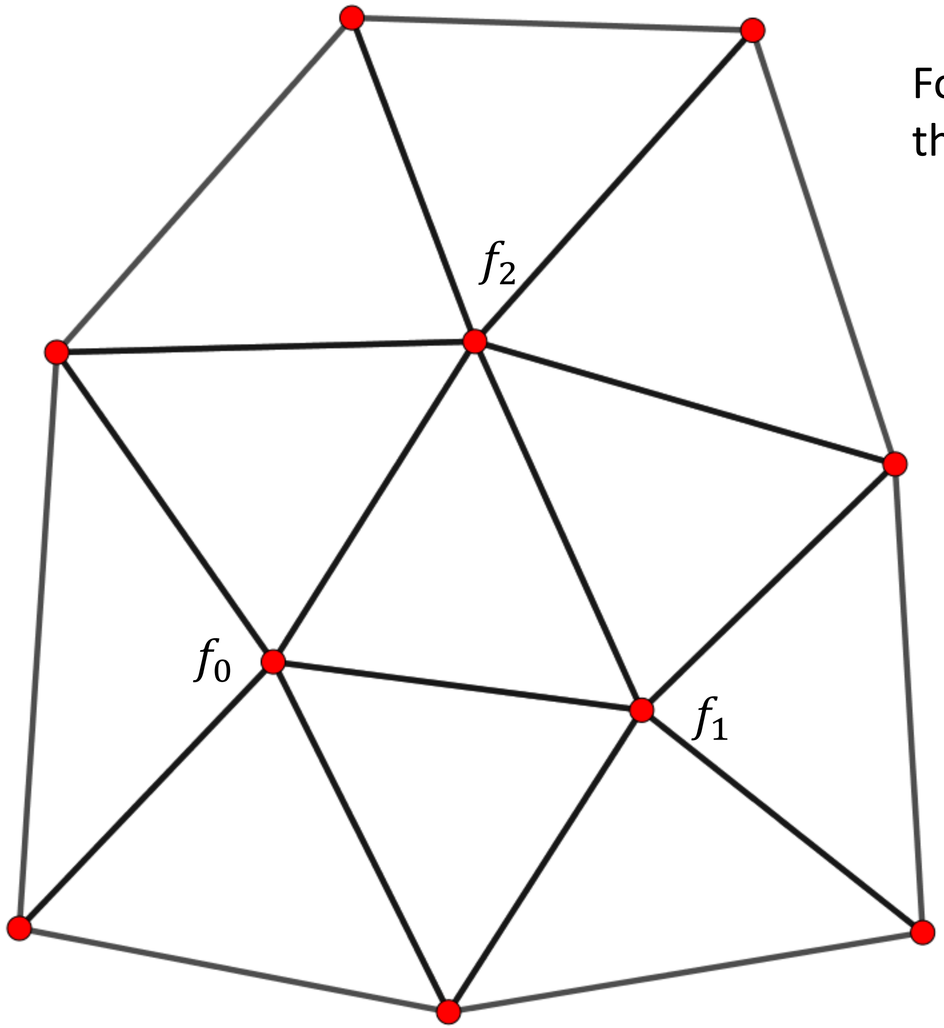
Exercise:

Given a triangle $t \subset R^3$, with vertices v_0, v_1, v_2, and $f: t \to R$, affine linear function with $f(v_0) = f_0, f(v_1) = f_1, f(v_2) = f_2$

Compute:

- $\int_t f$ in terms of the triangle area A_t and f_0, f_1, f_2
- $\int_t f^2$ in terms of the triangle area A_t and f_0, f_1, f_2

Functions on meshes



For each vertex i define B_i, the piecewise affine linear function, that has value 1 at vertex i and 0 at the other vertices.

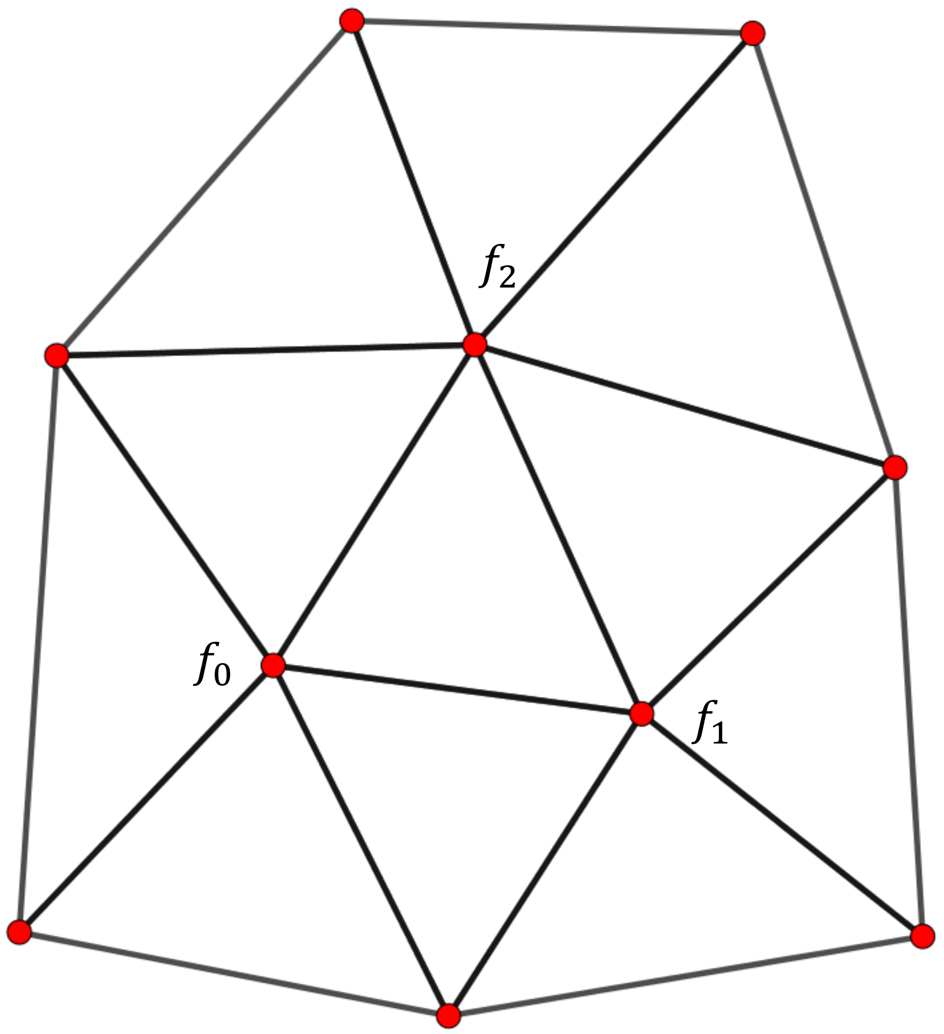
We can represent $f = \sum f_i B_i$

Given a triangle $t \subset R^3$, with vertices v_0, v_1, v_2,

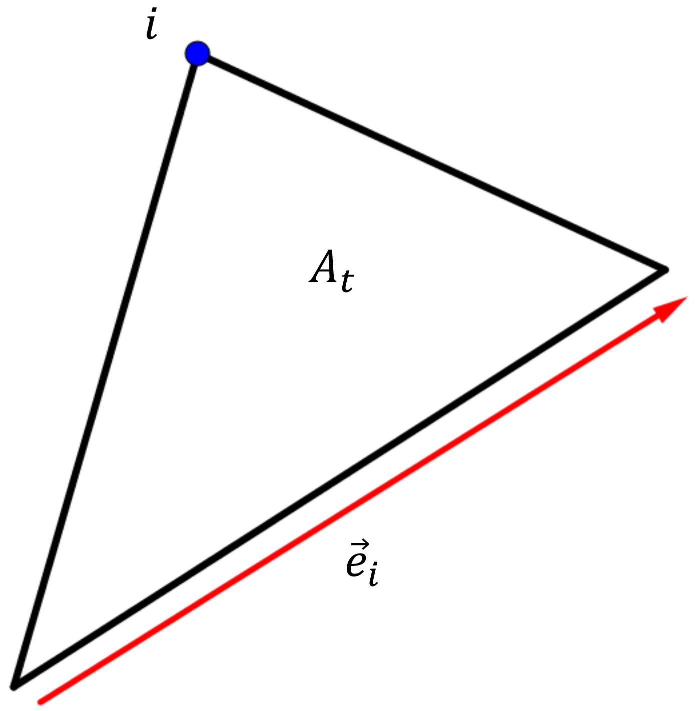
Compute

- $\int_t B_i B_j$ for $i, j \in \{0,1,2\}$ in terms of the triangle area.

Functions on meshes

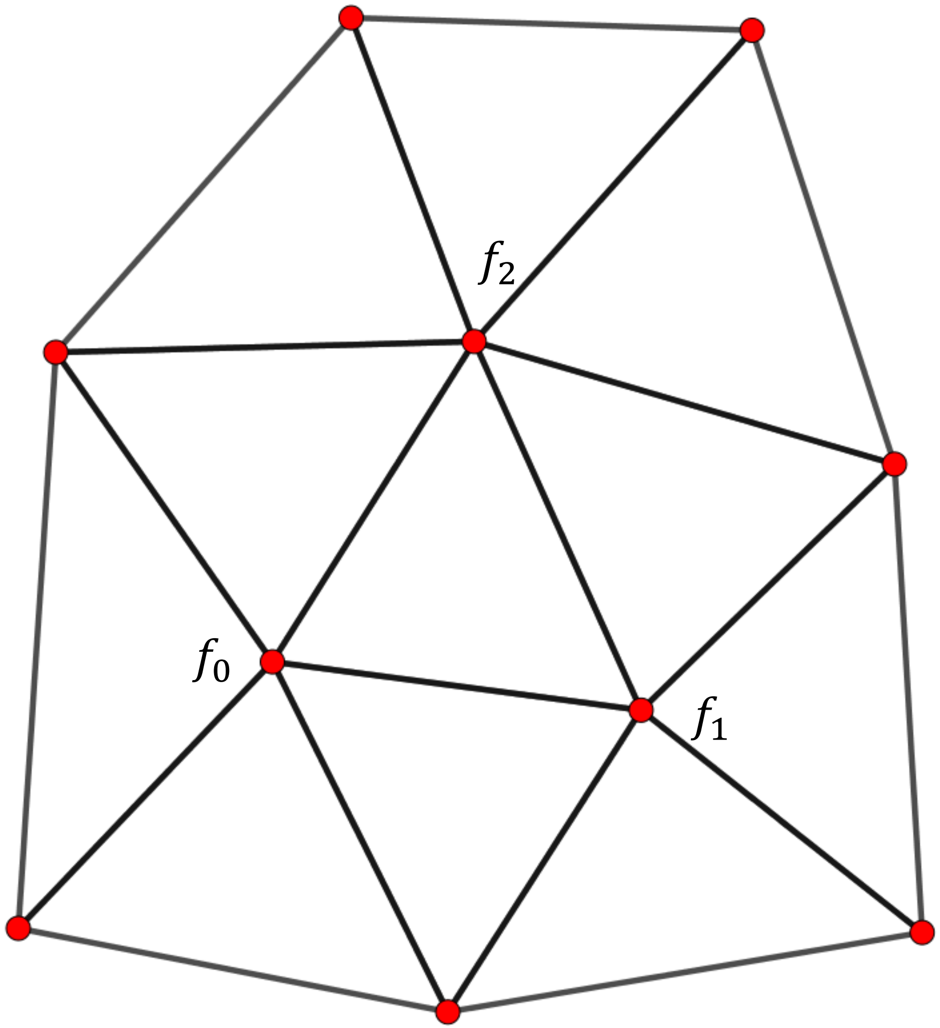


The gradient of a piecewise affine linear function is piecewise constant

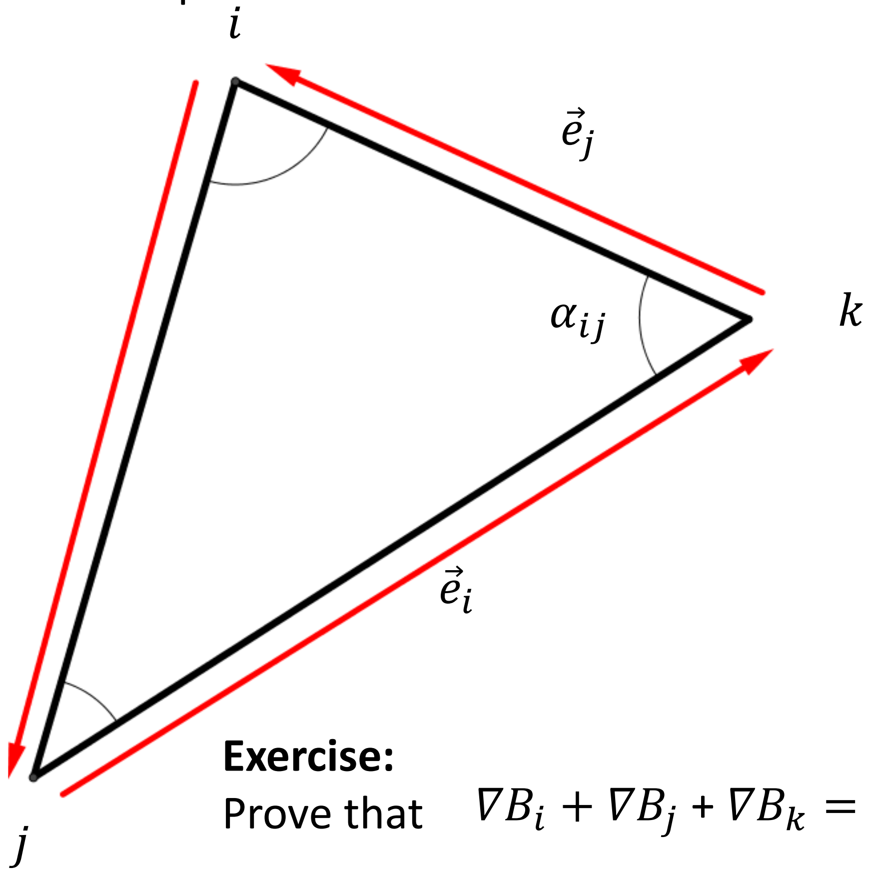


Exercise:
What is the direction of ∇B_i? What is its magnitude?
Find an expression for ∇B_i in terms of the opposite edge $\vec{e}_i$ and the triangle area A_t.

Functions on meshes



The gradient of a piecewise affine linear function is piecewise constant



Exercise:

Prove that $\nabla B_i + \nabla B_j + \nabla B_k = 0$

Prove that $\int_t \langle \nabla B_i, \nabla B_j \rangle = -\frac{\cot(\alpha_{ij})}{2}$