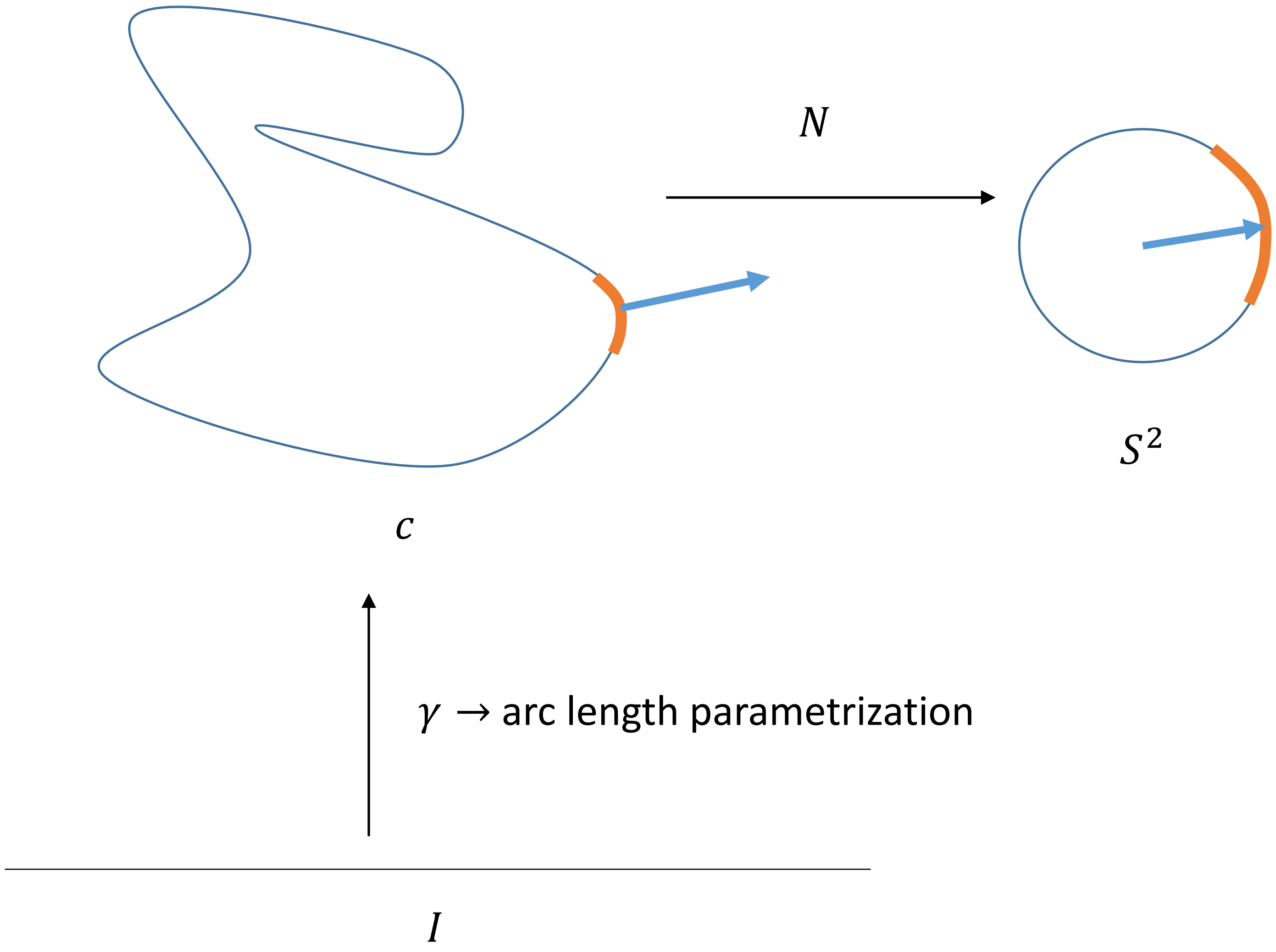
Lecture 5

Introduction to Geometry Processing

Spring 2017

Johns Hopkins University

Gauss-Bonnet Theorem (Simple Closed Curve)

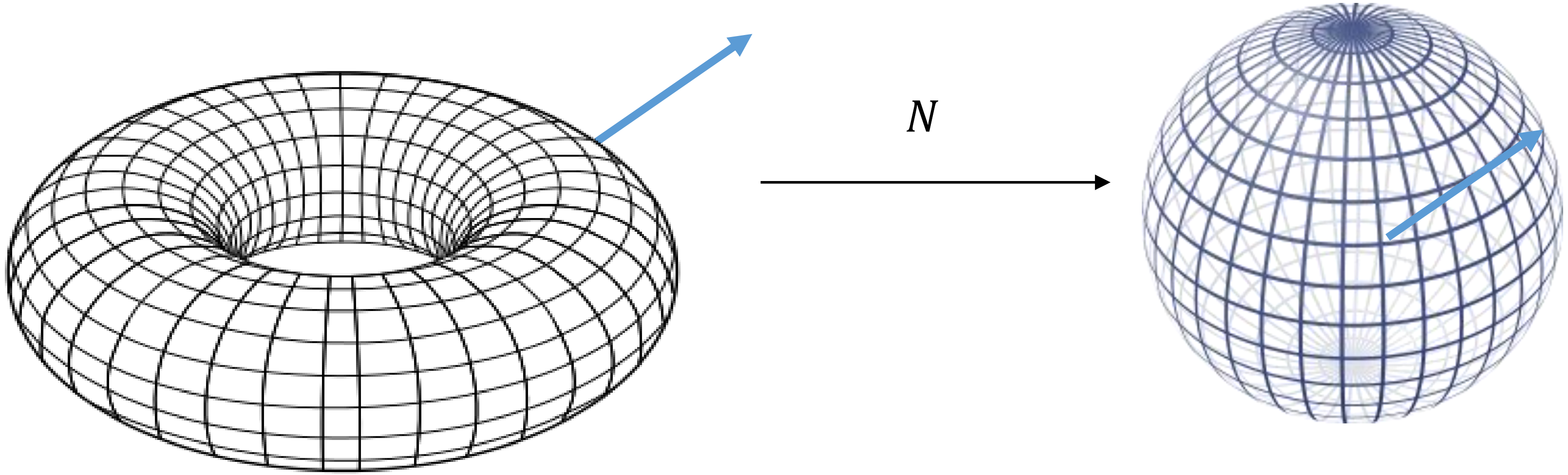


$$\int_c k = ?$$

$$\int_I \langle (N \circ \gamma)', \gamma' \rangle \, dt$$

$$2\pi$$

Gauss-Bonnet Theorem(Genus 1 Surface)

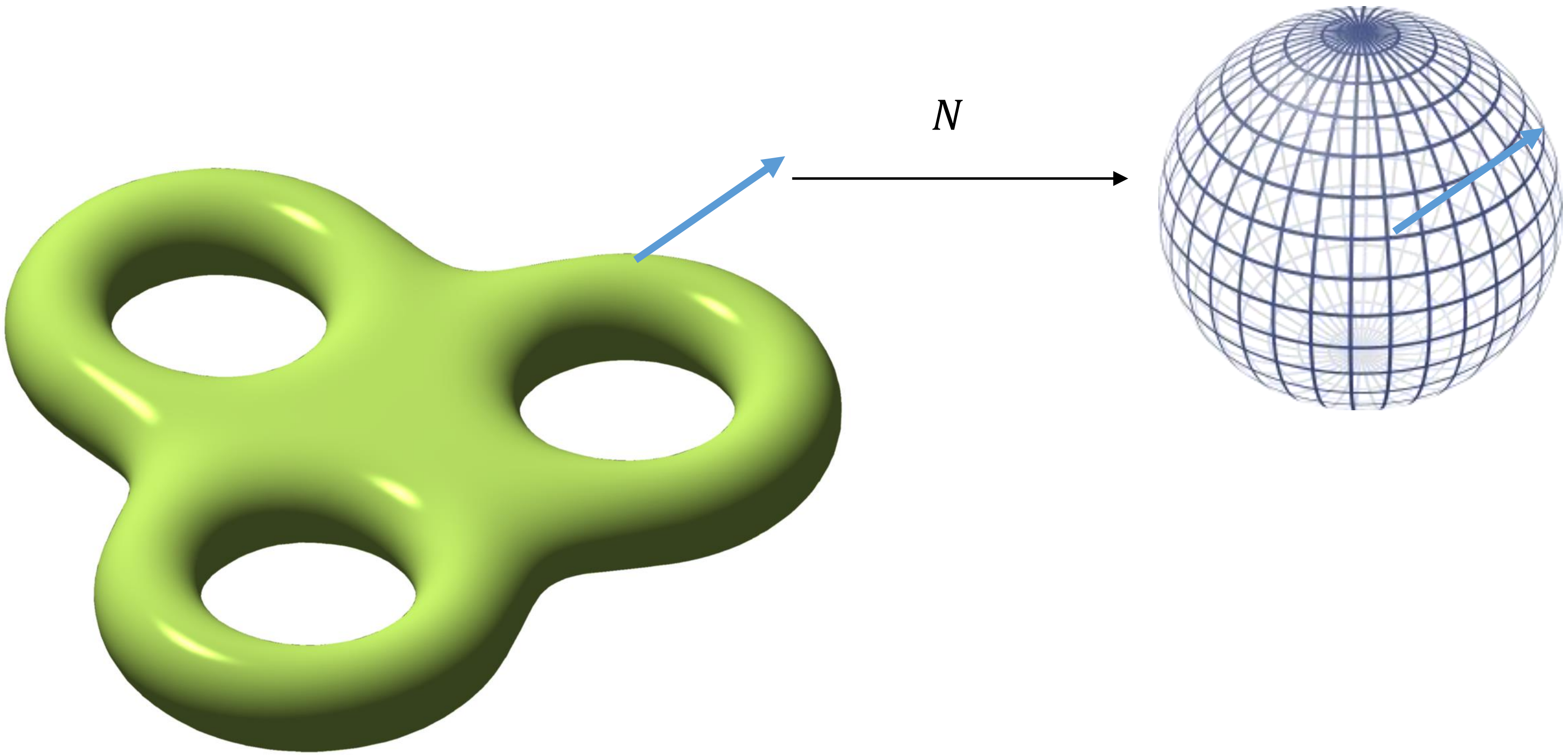


$$\int_S k_G = ?$$

$$\int_S \det(dN)$$

$$0$$

Gauss-Bonnet Theorem(Genus 3 Surface)



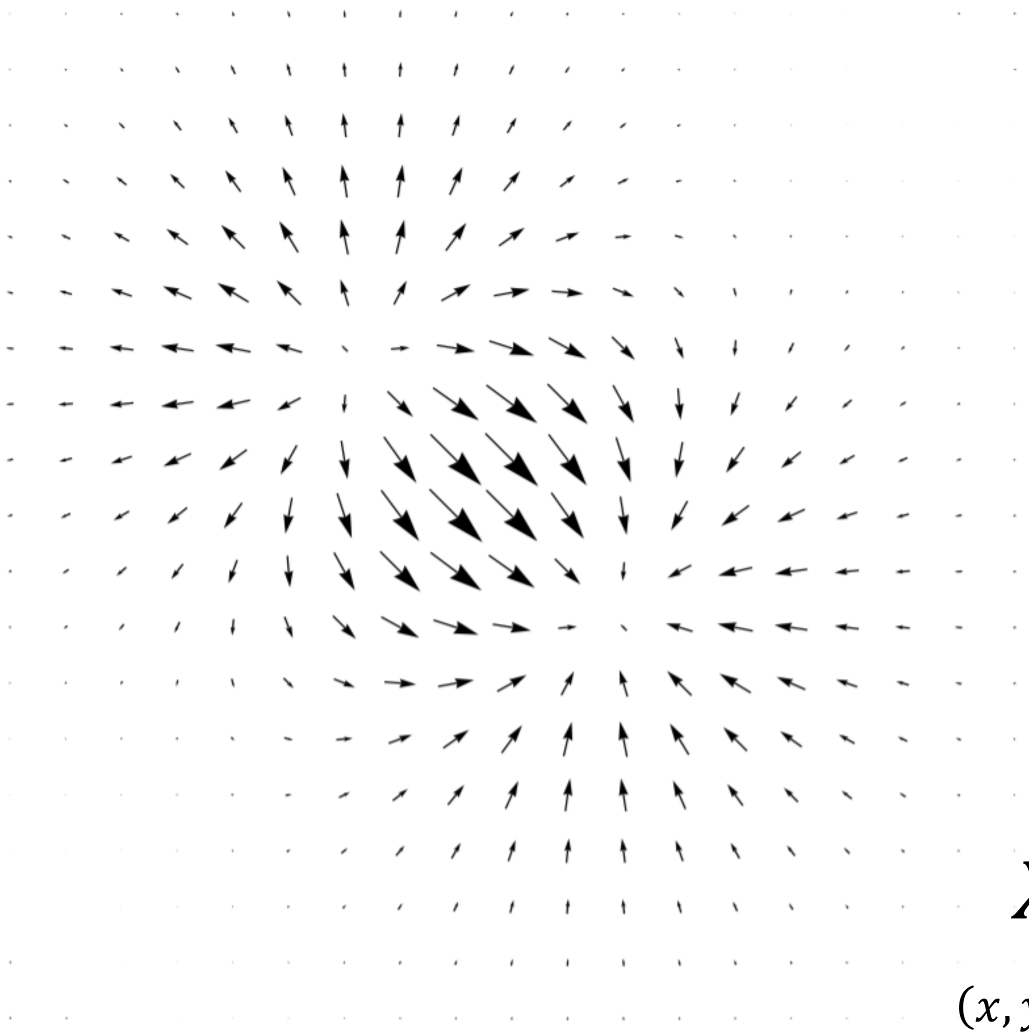
$$\int_S k_G = ?$$

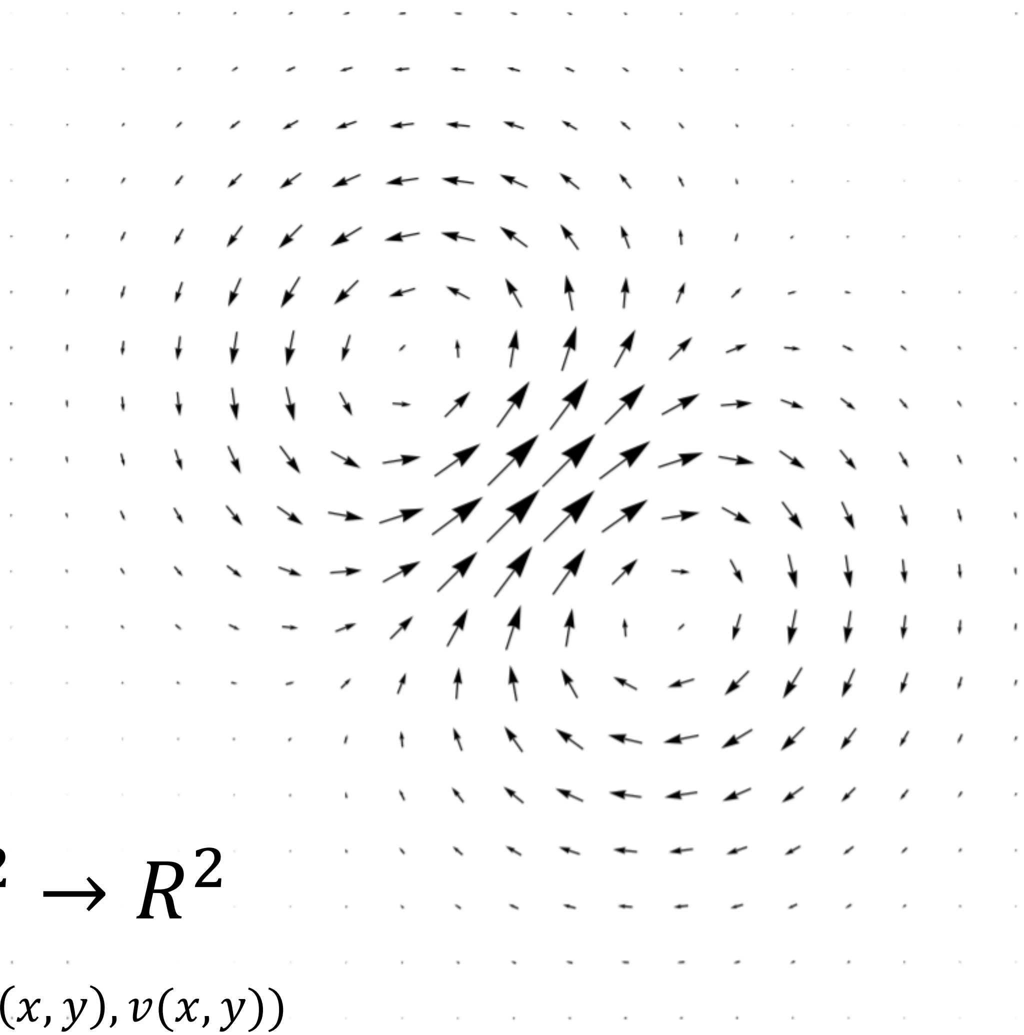
$$\int_S \det(dN)$$

$$-8\pi$$

In general, $\int_S k_G = 2\pi(2-2g)$, where g is the number of holes (also called genus)!

Vector Fields



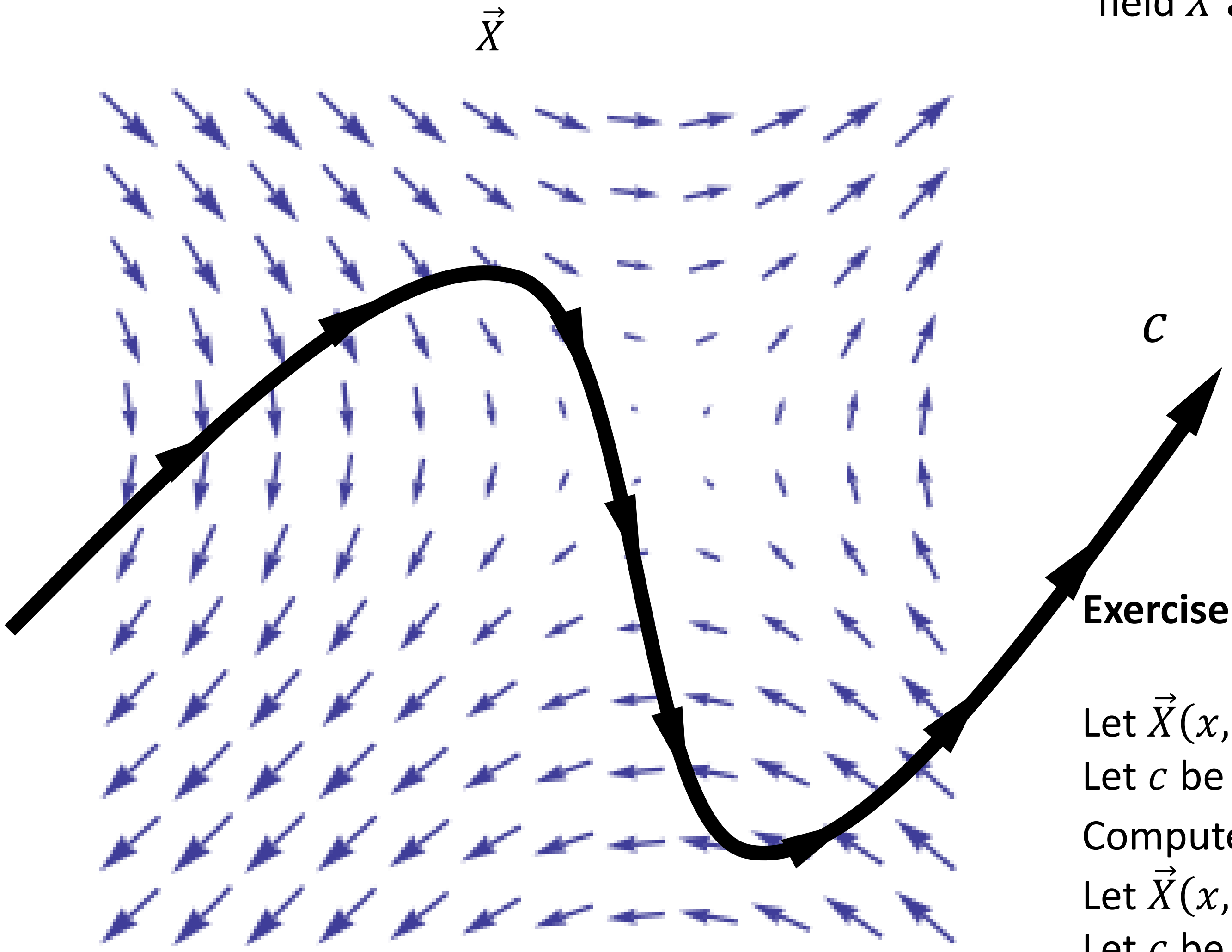


$$\vec{X}: R^2 \rightarrow R^2$$

$$(x, y) \rightarrow (u(x, y), v(x, y))$$

Line Integral

Given any parametrization $\gamma: I \rightarrow c,$ the line intergral of field $\vec{X}$ along c is given by:



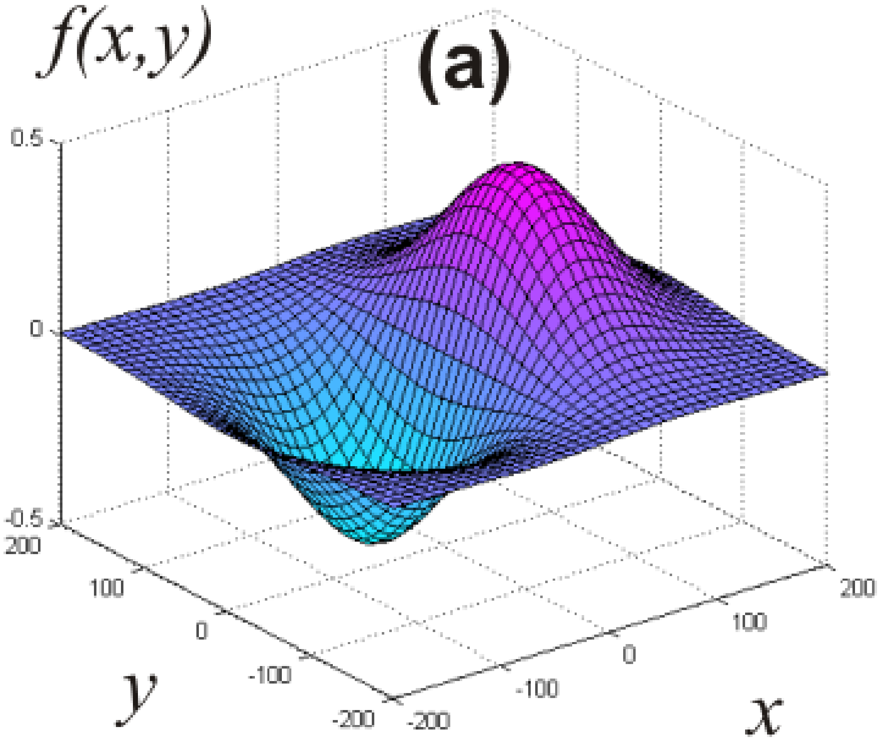
$$\int_c \vec{X} \cdot ds := \int_I \langle \gamma'(t), \vec{X}(\gamma(t)) \rangle \, dt$$

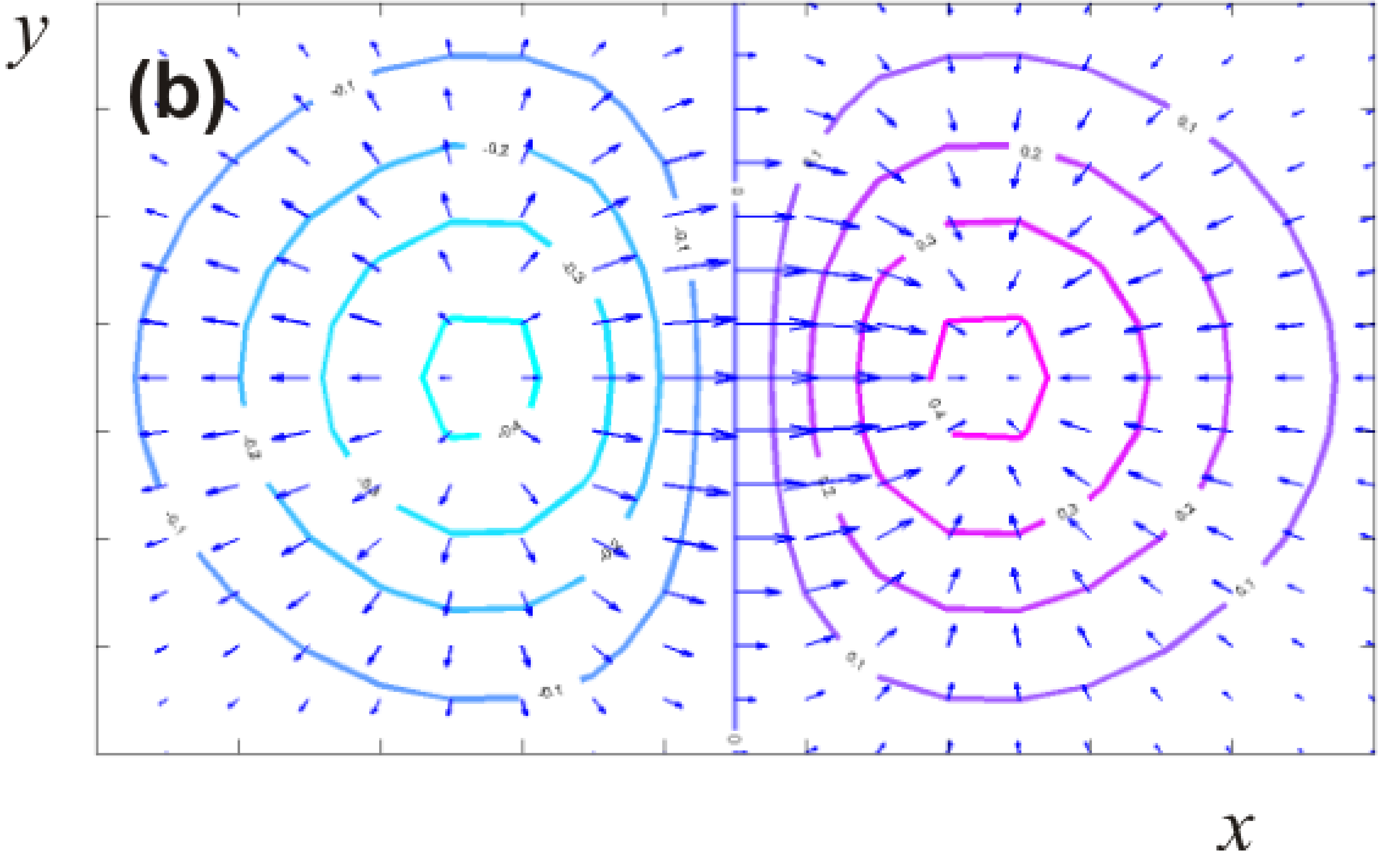
Exercise

Let $\vec{X}(x, y) = (2x + y, x)$
Let c be the unit circle traversed in anti-clockwise orientation.
Compute the line integral of $\vec{X}$ along c.
Let $\vec{X}(x, y) = (-(2y + x), y)$
Let c be the unit circle traversed in anti-clockwise orientation.
Compute the line integral of $\vec{X}$ along c.

Gradient field

$$\vec{X} = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

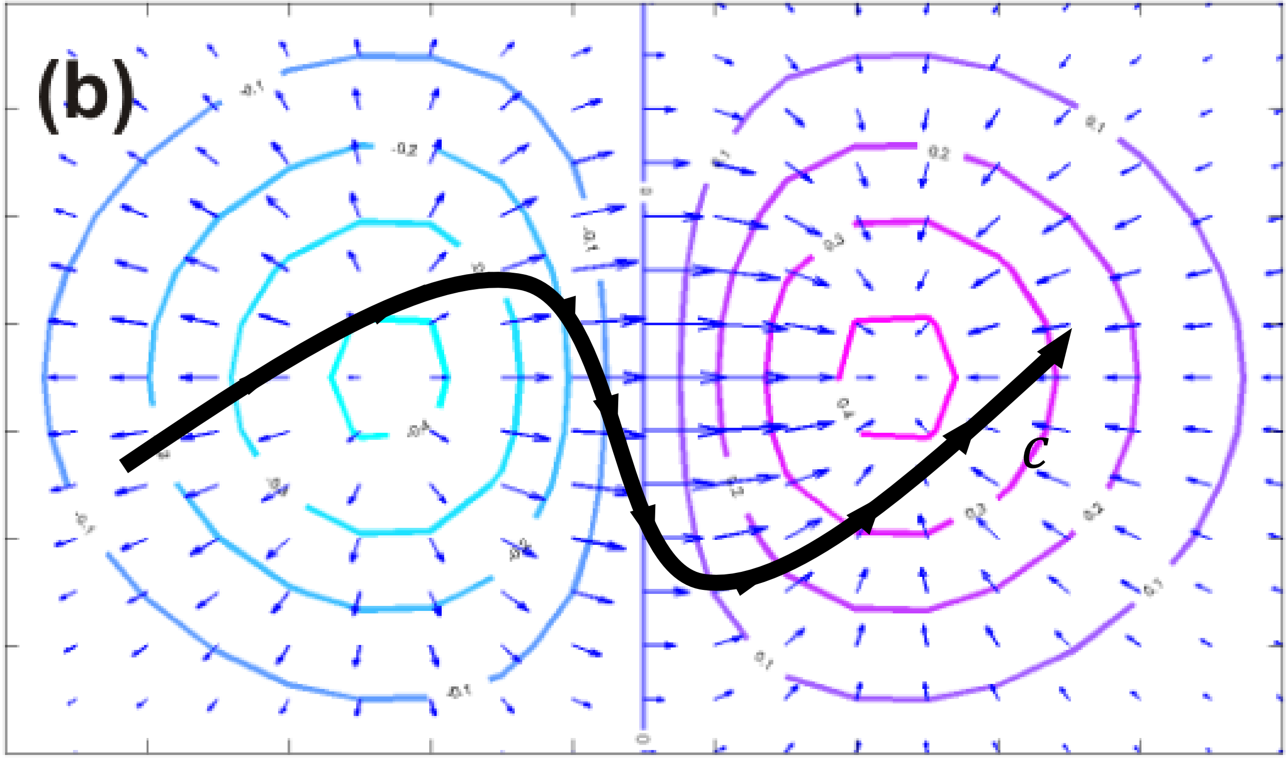




Wikipedia

Gradient field

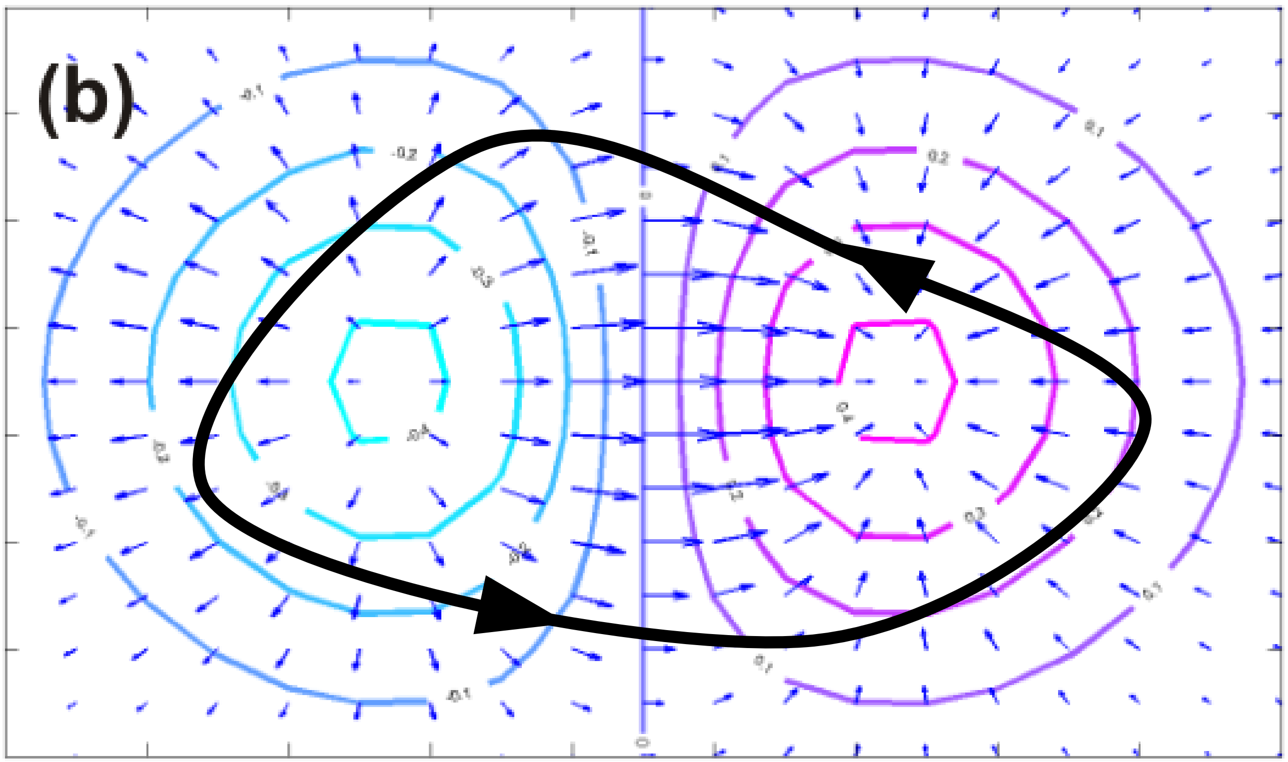
Given a curve c from point p to q and $\vec{X} = \nabla f$. What can you say about the line integral of $\vec{X}$ along c ?



$$\int_c \nabla f \cdot ds = f(q) - f(p)$$

Gradient field

Given a closed curve c and $\vec{X} = \nabla f$. What can you say about the line integral of $\vec{X}$ along c ?



$$\int_c \nabla f \cdot ds = 0$$

Can you find $f(x, y)$ such that $\vec{X} = \nabla f = (2x + y, x)$?