Convex Hulls (3D)

O'Rourke, Chapter 4

Outline

- Polyhedra
  - Polytopes
  - Euler Characteristic
- (Oriented) Mesh Representation

Polyhedra

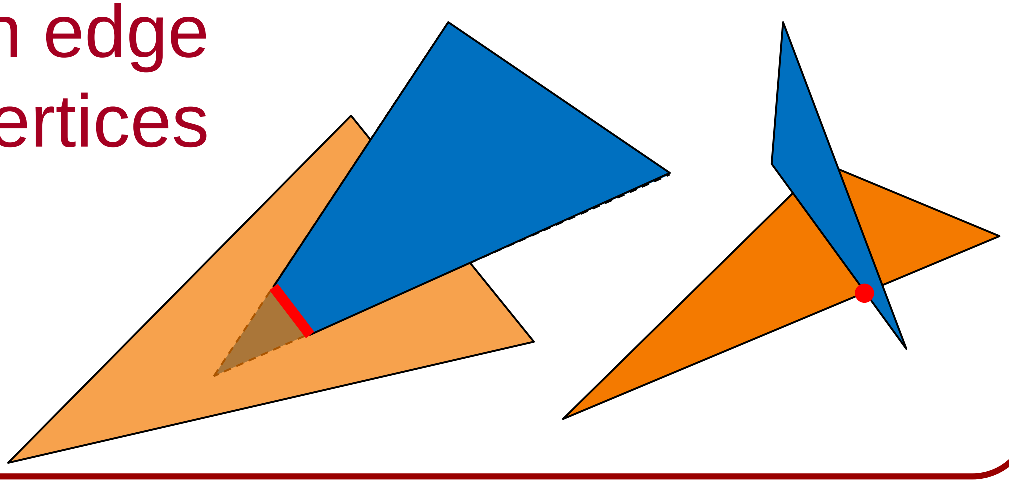
Definition:

A *polyhedron* is a solid region in 3D space whose boundary is made up of planar polygonal faces comprising a connected 2D manifold.

Polyhedra

The boundary of a polyhedron can be expressed as a combination of vertices, edges, and faces:

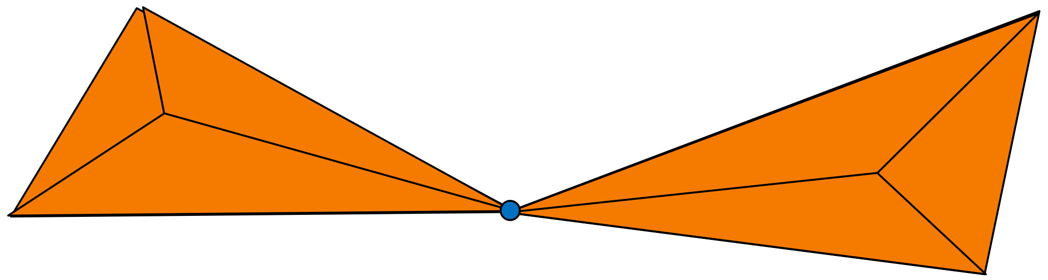
- Intersections are proper:
  - Elements don't overlap, or
  - They share a single vertex, or
  - They share an edge and the two vertices



Polyhedra

The boundary of a polyhedron can be expressed as a combination of vertices, edges, and faces:

- Intersections are proper
- Locally manifold:
  - Edges around a vertex can be sorted to match their incidence on faces.

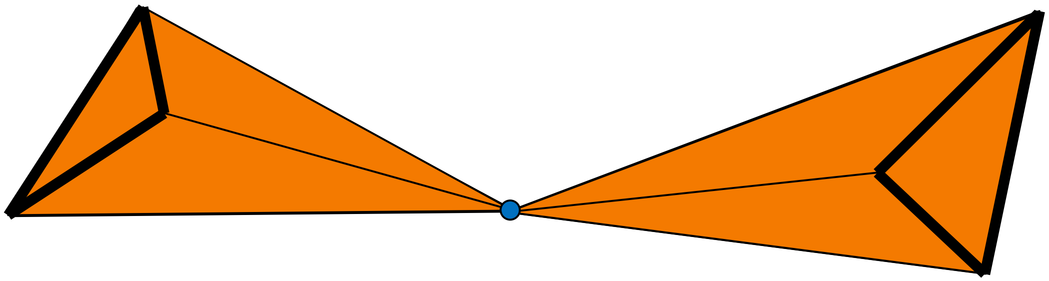


Polyhedra

The boundary of a polyhedron can be expressed as a combination of vertices, edges, and faces:

- Intersections are proper
- Locally manifold:

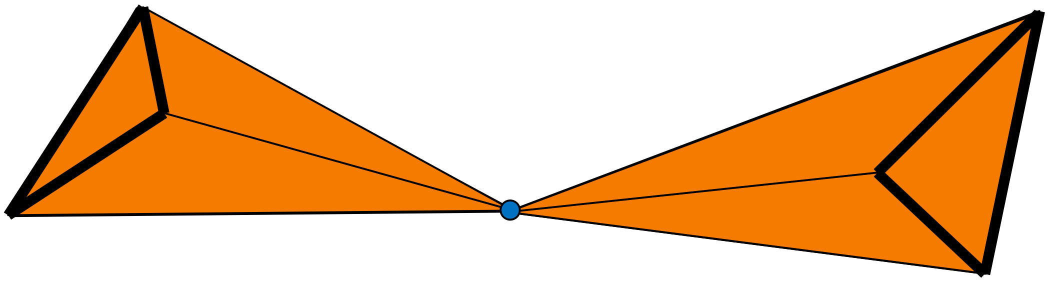
Alternatively, the subgraph of the dual obtained by restricting to the adjacent faces (the link) is connected.

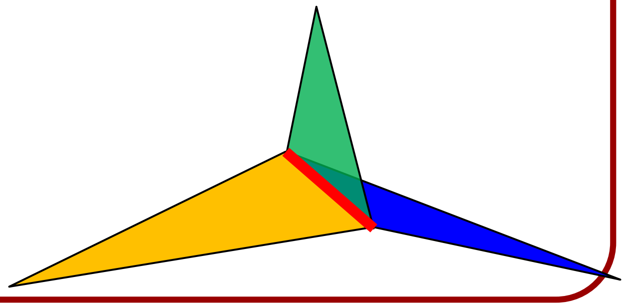


Polyhedra

The boundary of a polyhedron can be expressed as a combination of vertices, edges, and faces:

- Intersections are proper
- Locally manifold:
  - Edges around a vertex can be sorted to match their incidence on faces.
  - Exactly two faces meet at each edge.





Polyhedra

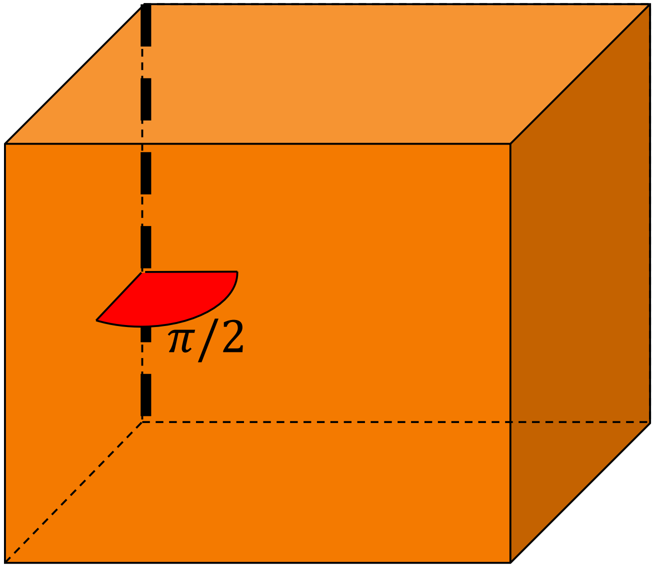
The boundary of a polyhedron can be expressed as a combination of vertices, edges, and faces:

- Intersections are proper
- Locally manifold
- Globally connected

Definition

<u>Definition</u>:

Given an edge on a polyhedron, the *dihedral angle* of the edge is the internal angle between the two adjacent faces.

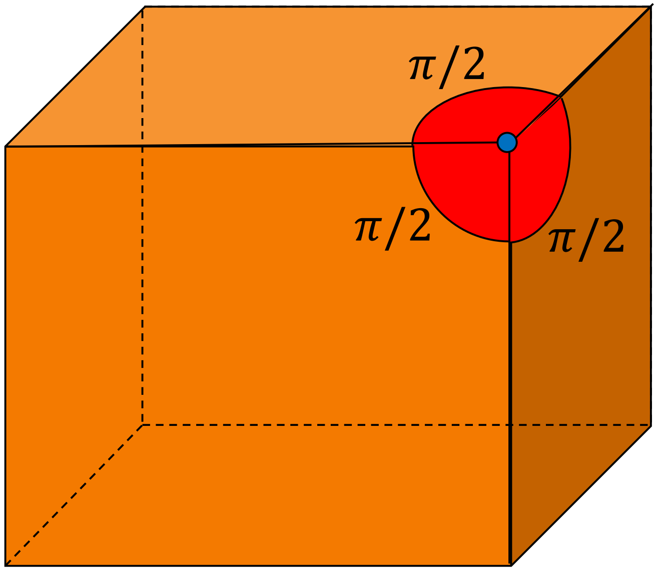


<u>Aside</u>:
The dihedral angle is a discrete measure of mean curvature.

Definition

<u>Definition</u>:

Given a vertex on a polyhedron, the *deficit angle* at the vertex is 2π minus the sum of angles around the vertex.



$\Rightarrow \pi/2$

<u>Aside</u>:
The deficit angle is a discrete measure of Gauss curvature.

Polytopes

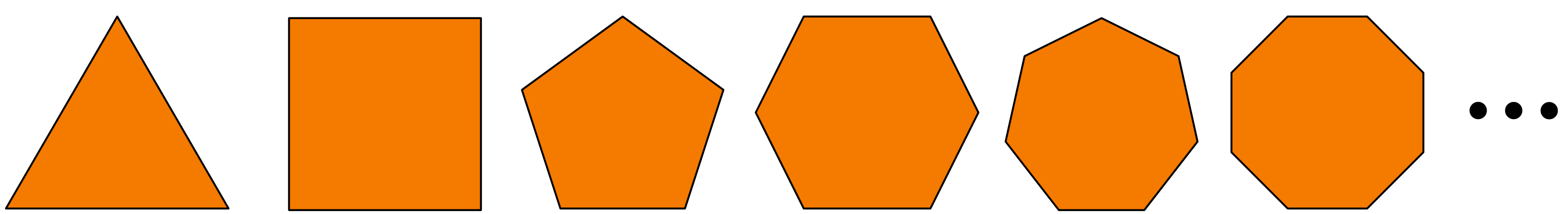
A convex polyhedron is a *polytope*:

- Non-negative mean curvature:
  All dihedral angles are less than or equal to π.
  (Necessary and sufficient.)
- Non-negative Gaussian curvature:
  Sum of angles around a vertex is at most 2π.
  (Necessary but not sufficient).

Platonic Solids

Definition:

A *regular polygon* is a polygon with equal sides and equal angles.



Platonic Solids

Definition:

A *regular polygon* is a polygon with equal sides and equal angles.

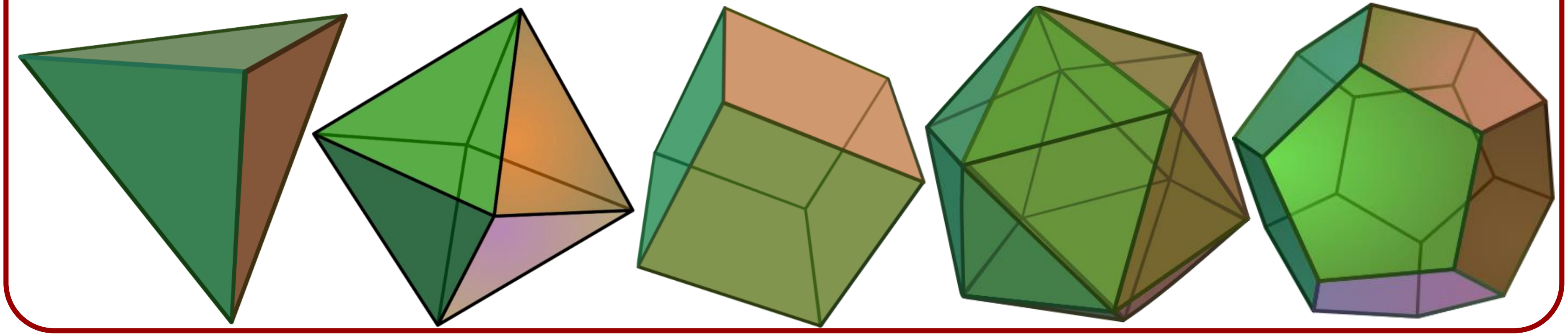
A *regular polyhedron* is a convex polyhedron, with all faces congruent regular polygons and vertices having the same valence.

Platonic Solids

Claim:

The five platonic solids are the only regular polyhedra.

[Images courtesy of Wikipedia]



Platonic Solids

Proof:

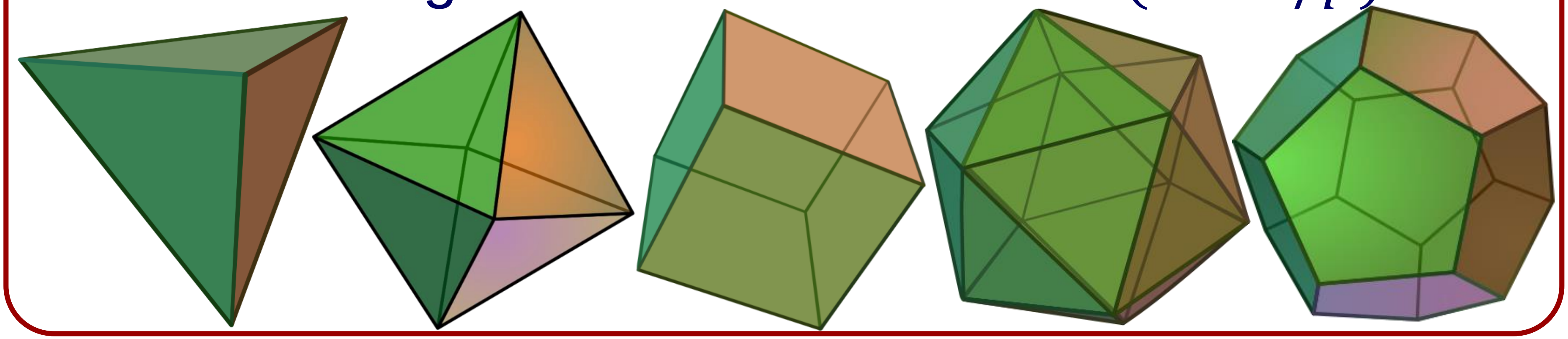
Assume each face is p-sided:

⇒ The sum of angles in a face is $\pi(p-2)$

⇒ The angle at each vertex is $\pi(1-2/p)$

Assume each vertex has valence v:

⇒ The angle-sum at a vertex is $v\pi\,(1-2/p)$



Platonic Solids

Proof:

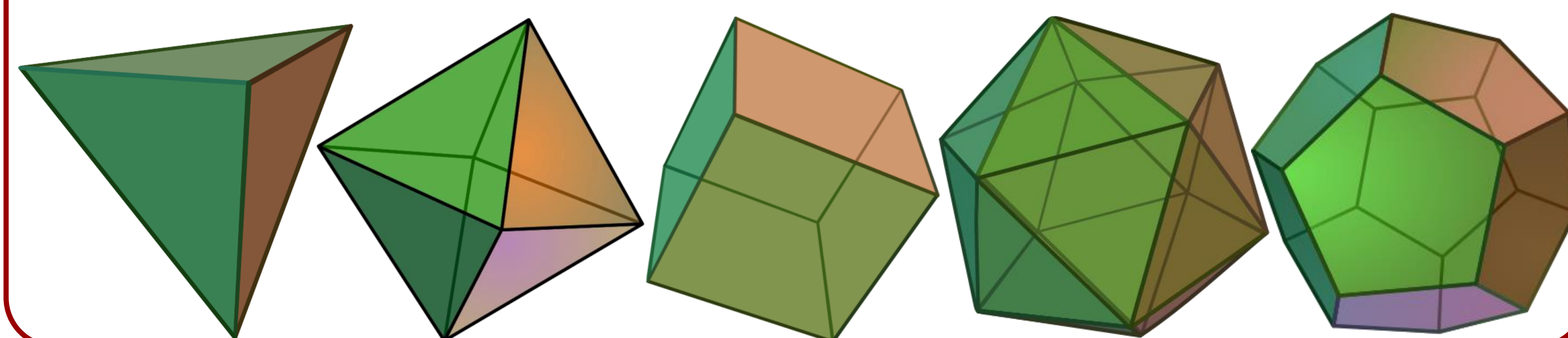
Since the polyhedron is convex:

$$v\pi(1-2/p) < 2\pi \iff v(1-2/p) < 2$$

$$\iff v(p-2) < 2p$$

$$\iff vp - 2v - 2p < 0$$

$$\iff (p-2)(v-2) - 4 < 0$$



Platonic Solids

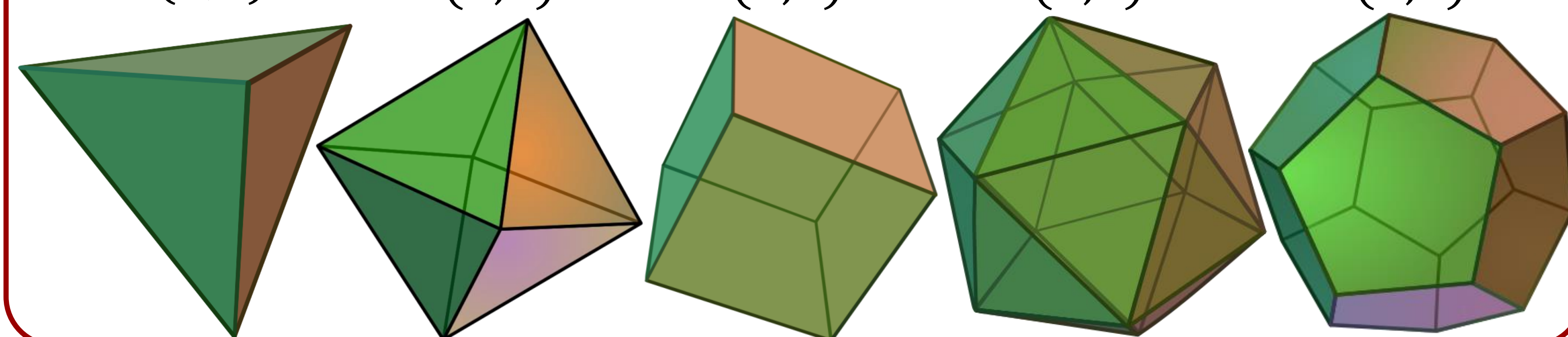
Proof:

Since the polyhedron is convex:

$$(p-2)(v-2)-4<0$$

Since $p, v \geq 3$, valid options are (p, v):

(3,3) (3,4) (4,3) (3,5) (5,3)



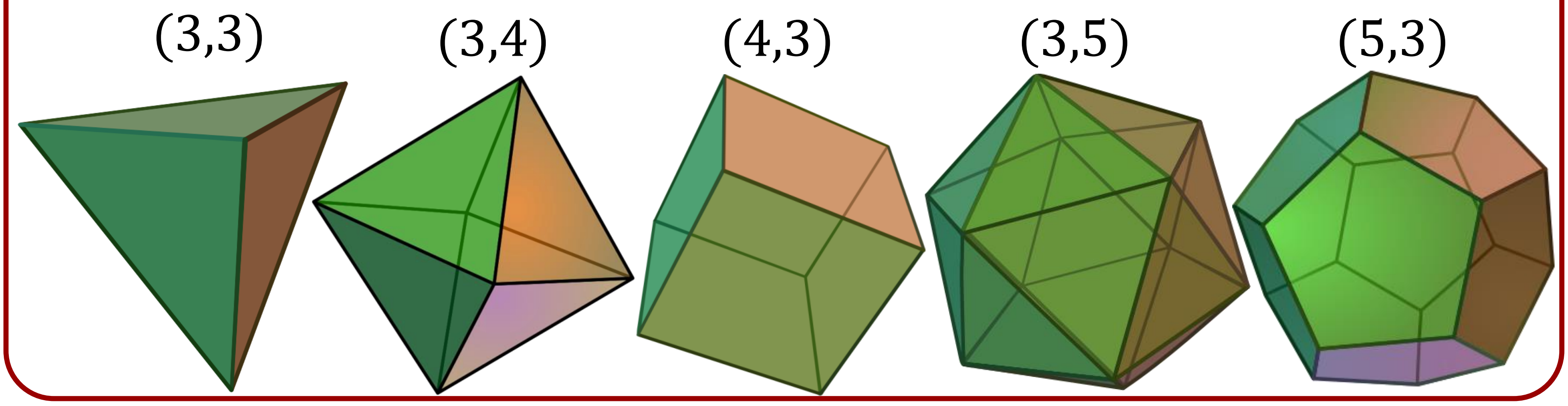
Platonic Solids

The platonic solids come in dual pairs, where one solid is obtained from the other by replacing faces with vertices:

Cube ↔ Octahedron

Icosahedron ↔ Dodecahedron

Tetrahedron ↔ Tetrahedron



Topological Polyhedra

The boundary of a polyhedron can be expressed as a combination of vertices, edges, and faces:

- Intersections are proper
- Locally manifold
- Globally connected

(Intersections are proper: Geometric; Locally manifold, Globally connected: Topological)

Topological Polyhedra

If we ignore the vertex positions, we get a combinatorial structure composed of faces (cells), edges, and vertices.*

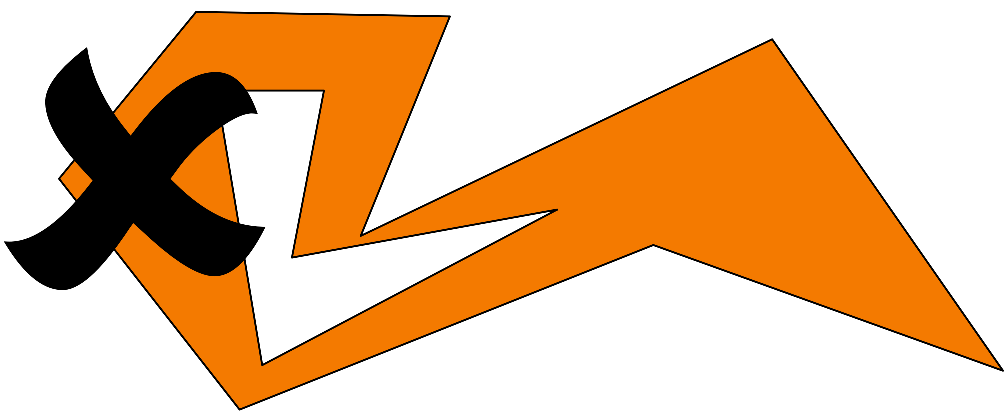
[Nivoliers and Levy, 2013]

*These are CW complexes. (And, if faces are triangles, these are simplicial complexes).

Topological Polyhedra

Properties (CW Complex):

- Faces intersect at edges and vertices.
- Edges are topologically line segments and intersect at vertices.
- Interiors of faces have disk-topology and the boundary is a polygon made up of edges.



Topological Polyhedra

Properties (Manifold):

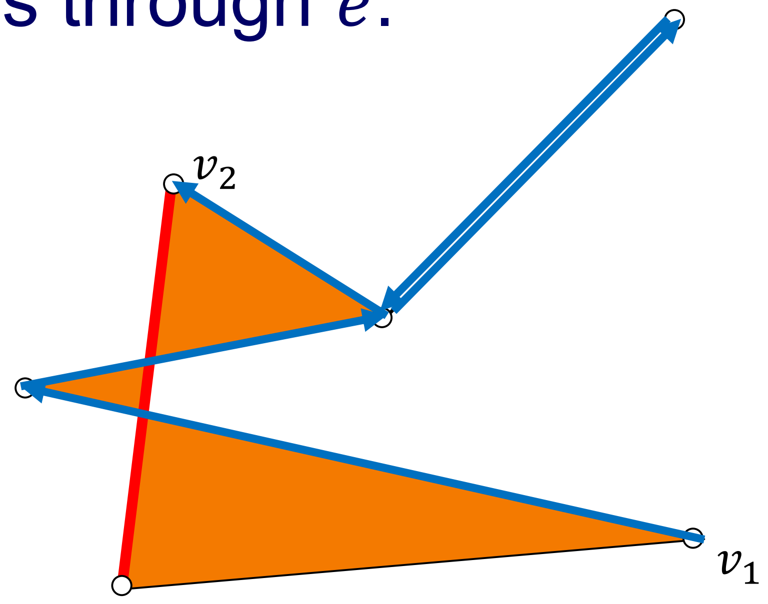
- Each vertex is on the boundary of some edge.
- Each edge is on the boundary of some face.
- An edge is on the boundary of two faces.
- Edges around a vertex can be sorted.

Topological Polyhedra

Note:

Given a topological polygon P, and given an edge $e \in P$ that only occurs once on P:

For any vertices $v_1, v_2 \in P$ there is a path from v_1 to v_2 that doesn't pass through e.



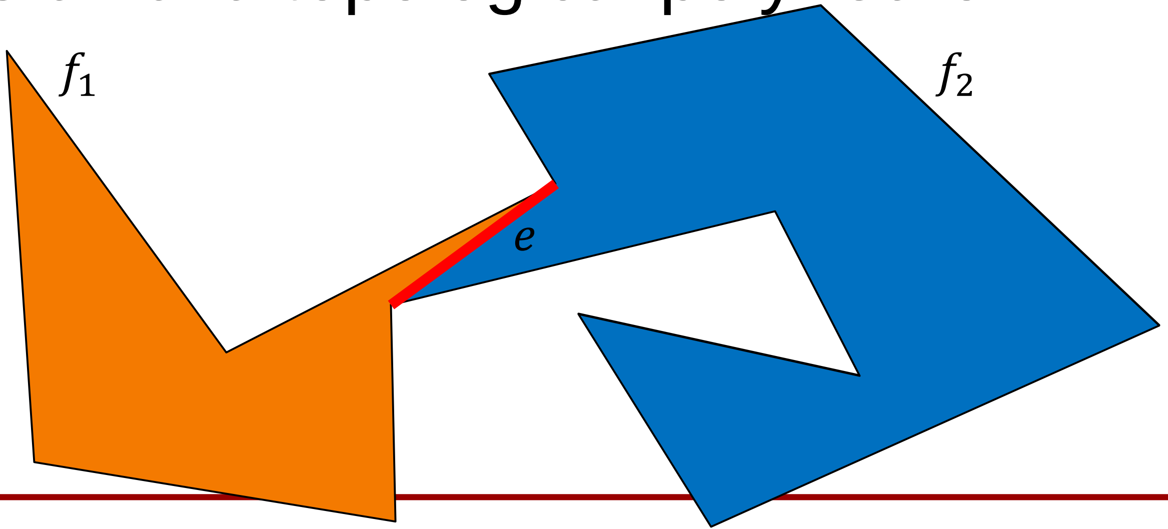
Topological Polyhedra

Claim:

If f_1 and f_2 are distinct faces of a topological polyhedron which share an edge e, then:

- replacing f_1 and f_2 with $f_1 \cup f_2$, and
- removing e from the edge list,

we still have a valid topological polyhedron.



Topological Polyhedra

Proof (CW Complex):

The edges/vertices of $f_1 \cup f_2$ are in the complex (since e is not on the boundary).

*Since the intersection $f_1 \cap f_2$ is connected and the interiors of f_1 and f_2 have disk-topology, the interior of $f_1 \cup f_2$ also has disk-topology.

*This is just a sketch of the proof.

Topological Polyhedra

<u>Proof (CW Complex)</u>:

The boundary of $f_1 \cup f_2$ is connected.

- Let $v \in e$ be an end-point.
- For $v_1, v_2 \in f_1 \cup f_2$, there is a curve connecting v to each v_i that does not contain the edge e.
- Concatenating the two curves we connect v_1 to v_2 along the boundary of $f_1 \cup f_2$.

Topological Polyhedra

Proof (Manifold):

The smaller polyhedron still passes through all the vertices.

The edge e is removed and all other edges remain adjacent to a face.

Topological Polyhedra

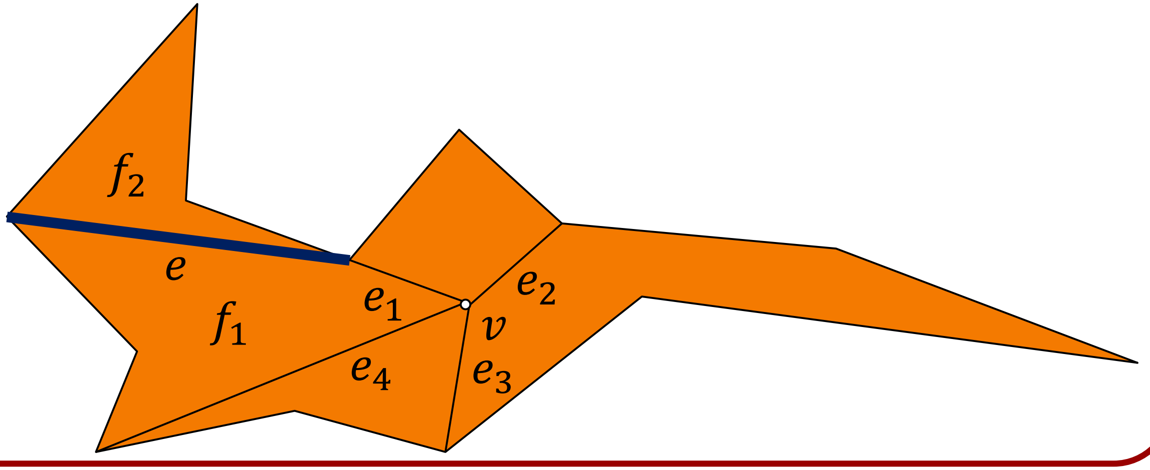
Proof (Manifold Edges):

The old edges still have only two faces on them (or one face twice).

Topological Polyhedra

Proof (Manifold Vertices):

If $v \notin e$, we can use the old edge ordering.



Topological Polyhedra

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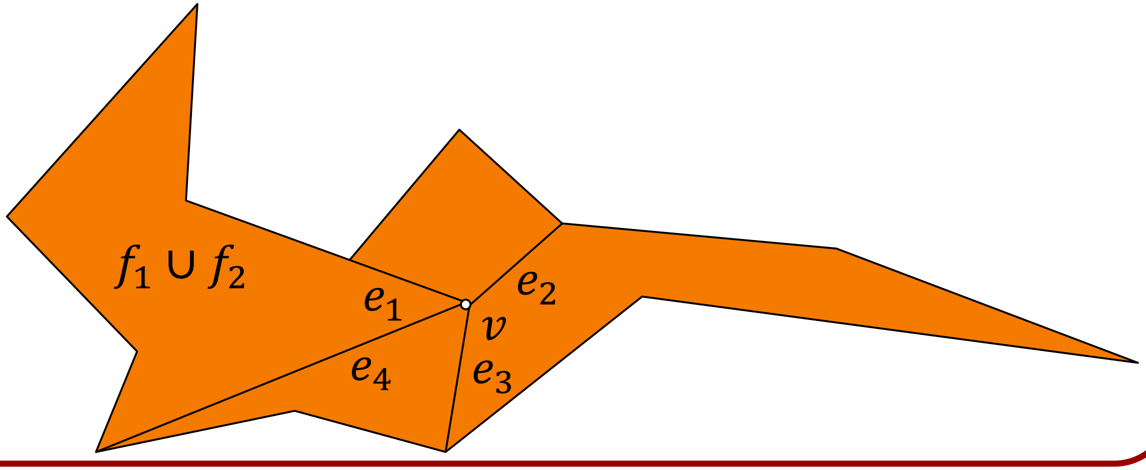


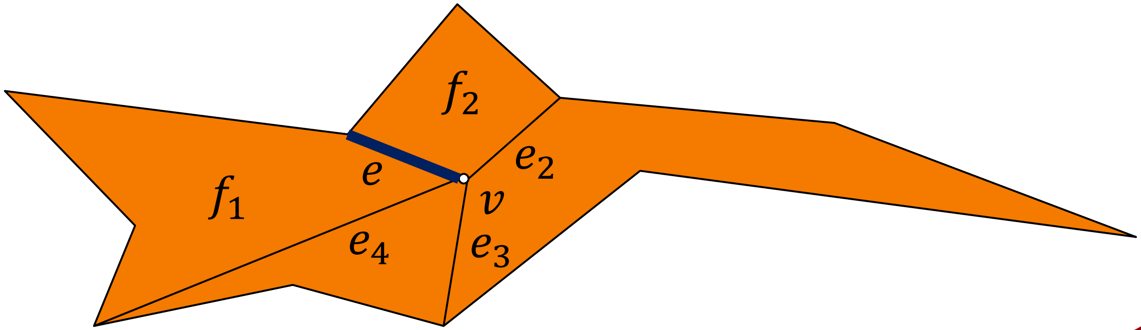
Topological Polyhedra

<u>Proof (Manifold Vertices)</u>:

If $v \notin e$, we can use the old edge ordering.

If $v \in e$ let $\{e_1, e_2, \ldots, e_k\}$ be the old ordered edges around v, shifted so that $e_1 = e$.

Then e_k and e_2 are consecutive edges on $f_1 \cup f_2$ so $\{e_2, \ldots, e_k\}$ is a valid ordering.



Topological Polyhedra

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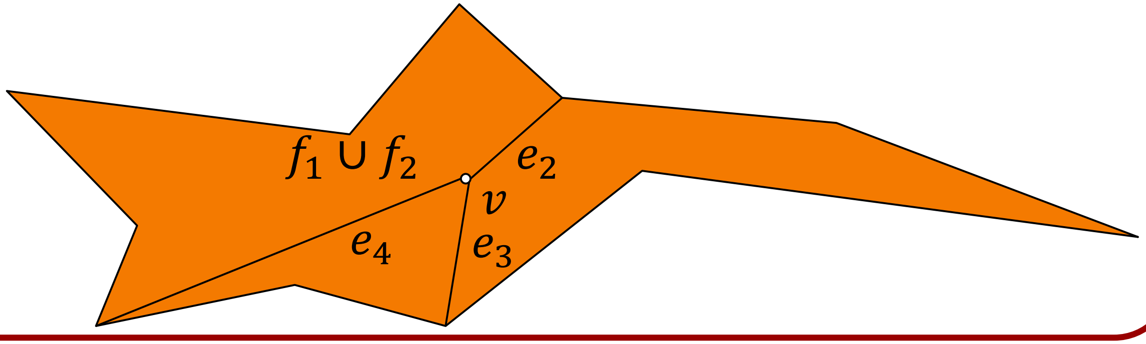
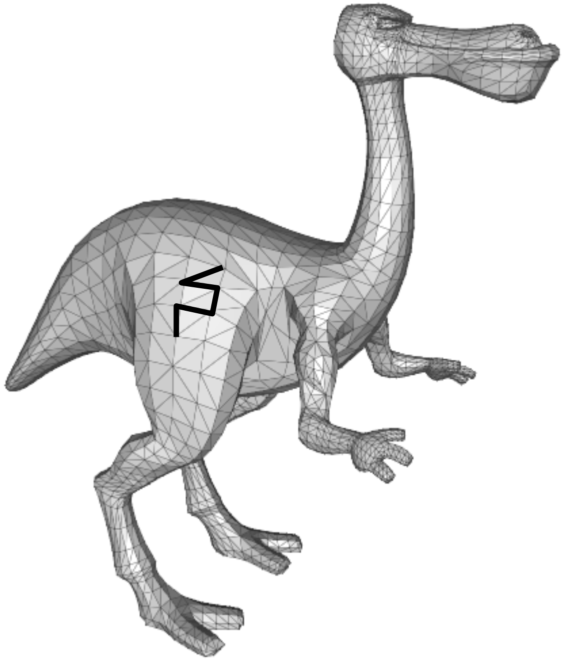


Curves

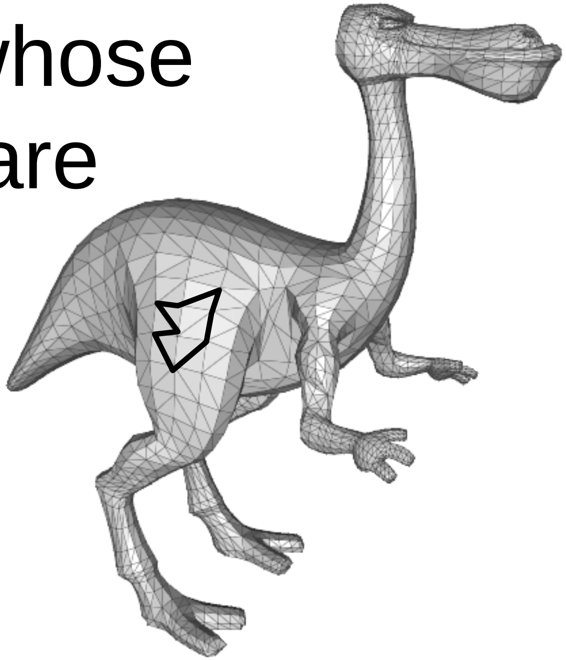
A (connected) *curve* on a topological polyhedron is a list of edges such that the ending vertex of one edge is the starting vertex of the next.



Curves

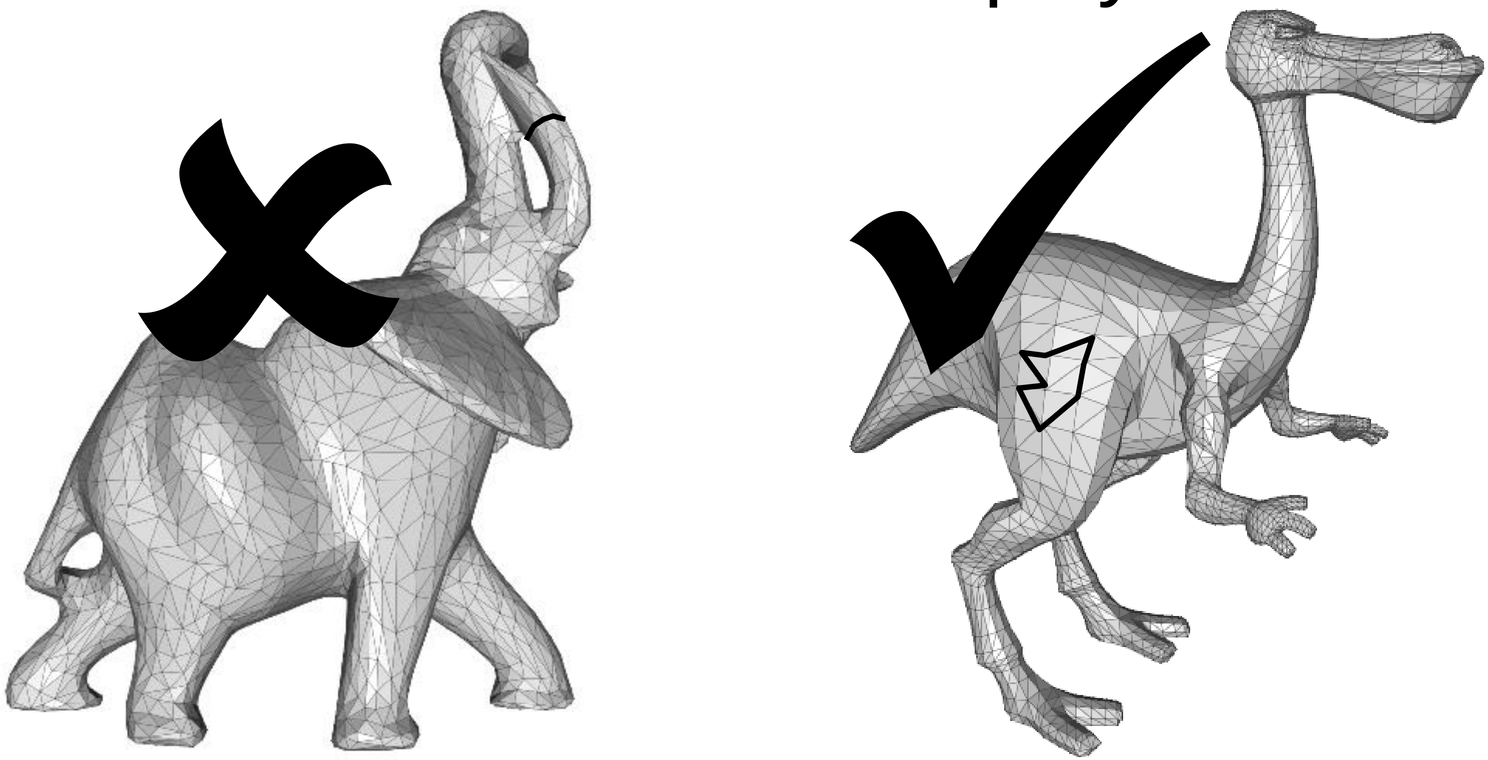
A (connected) *curve* on a topological polyhedron is a list of edges such that the ending vertex of one edge is the starting vertex of the next.

A *closed curve* is a curve whose starting and ending points are the same.



Genus-0 Polyhedra

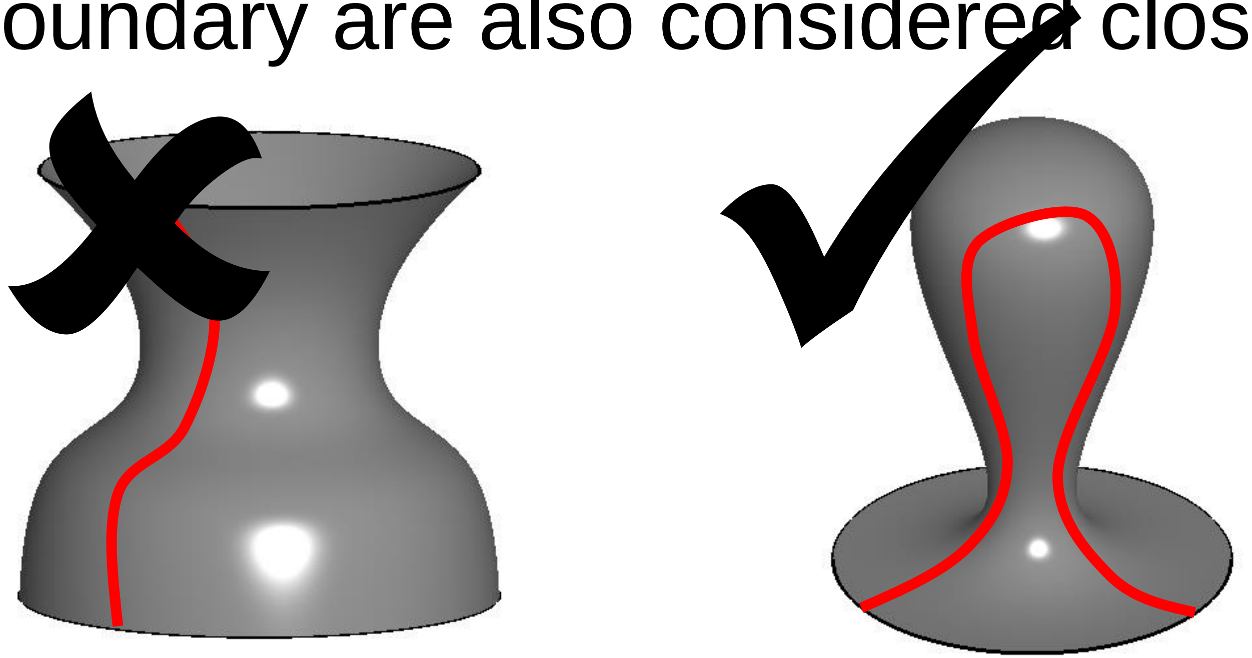
A polyhedron is *genus-0* (or *simply connected*) if every non-trivial closed curve disconnects the faces of the polyhedron.



Genus-0 Polyhedra

Aside:

The definition can be extended to surfaces with boundary if we curves that start and end the boundary are also considered closed.



Genus-0 Polyhedra

Equivalently, given a topological polyhedron P we can define the dual graph $P^* = (V^*, E^*)$.

$\Rightarrow$ A curve $C \subset E$ corresponds to a set of dual edges $C^* \subset E^*$ of the dual.

$\Rightarrow$ P is genus-0 if removing C^* disconnects P^*.

Genus-0 Polyhedra

1. There is a continuous map from a polytope to a sphere.
   (e.g. Put the center of mass at the origin and normalize the positions.)

2. By the Jordan Curve Theorem the sphere is genus-zero.

One Can Show:

⇒ The polytope must also be genus-0.

Euler's Formula

For a genus-0 polyhedron P, the number of vertices, $|V|$, the number of edges, $|E|$, and the number of faces, $|F|$, satisfy:

$$|V| - |E| + |F| = 2$$

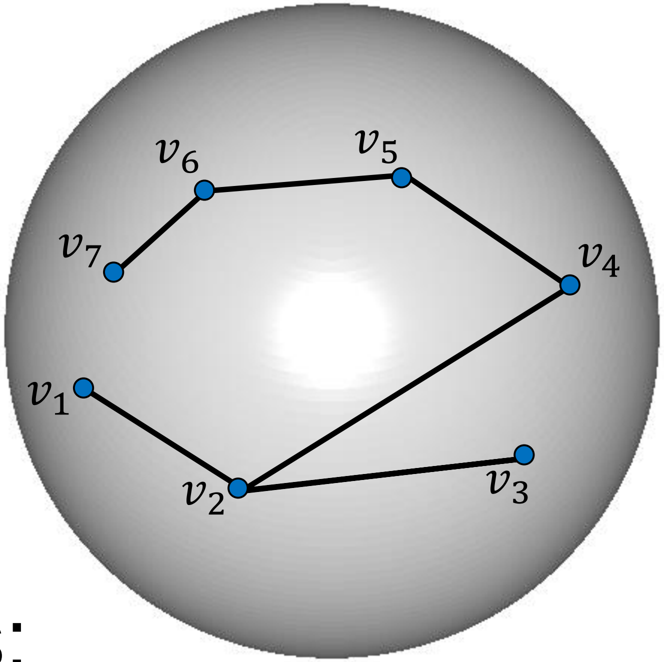
Euler's Formula (by Induction on $|F|$)

Base case: $|F| = 1$

We have:

- $V = \{v_1, \ldots, v_n\}$

*The edges on the boundary of the face form a connected tree (otherwise there is a closed loop and the interior of the face is disconnected).



Then there are $n - 1$ edges:

$$|V| - |E| + |F| = n - (n - 1) + 1 = 2$$

*This is just a sketch of the proof.

Euler's Formula (by Induction on $|F|$)

Induction: Assume true for $|F| = n - 1$

Find $e \in E$ shared by two distinct faces.

If no such e exists, then all faces are adjacent to themselves, which contradicts the assumption that the polyhedron is connected.

Euler's Formula (by Induction on $|F|$)

Induction: Assume true for $|F| = n - 1$

Find $e \in E$ shared by two distinct faces.

Remove e and merge the two adjoining faces.

Claim:

The new polyhedron, P', is still genus-0.

Euler's Formula (by Induction on $|F|$)

Proof (P' is genus-zero):

Let C be a non-trivial curve on P'.

$\Rightarrow$ C is a non-trivial curve on P with $e \notin C$.

$\Rightarrow$ f_1 and f_2 are in the same component.

$\Rightarrow$ C disconnects $f_1 \cup f_2$ from a face g on P.

$\Rightarrow$ C disconnects $f_1 \cup f_2$ from g in P'.

Euler's Formula (by Induction on $|F|$)

Induction: Assume true for $|E| = n - 1$

Find $e \in E$ shared by two distinct faces.

Remove e and merge the two adjoining faces.

P' is genus-0 with $|E| - 1$ edges, $|F| - 1$ faces, and $|V|$ vertices.

By the induction hypothesis we have:

$$|V| - (|E| - 1) + (|F| - 1) = 2$$

$$\Updownarrow$$

$$|V| - |E| + |F| = 2$$

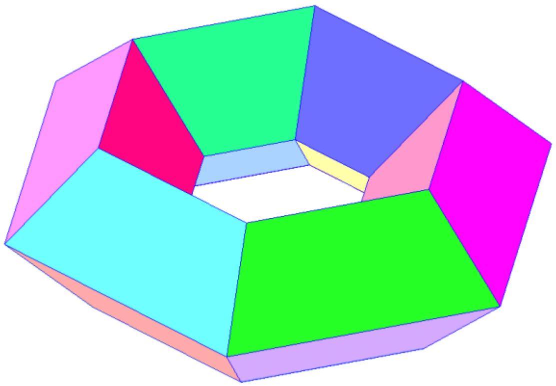
Euler's Formula

$$|V| - |E| + |F| = 2$$

<u>More Generally</u>:

If a polygon mesh is genus-g (g is the number of handles) then:

$$|V| - |E| + |F| = 2 - 2g.$$



$|V| = 24, |E| = 48, |F| = 24$

[Wikipedia: Toroidal Polyhedron]

Euler's Formula

Implication:

The number of faces and edges is linear in the number of vertices.

Euler's Formula

Proof:

Assume all faces are triangles.
(Triangulating only increase $|F|$ and $|E|$.)

Since each edge is shared by two triangles:

$$|E| = 3|F|/2$$

Using Euler's Formula:

$$|V| - |E| + |F| = 2$$

$$\Updownarrow$$

$$|F| = 2|V| - 4 \quad \text{and} \quad |E| = 3|V| - 6$$

Outline

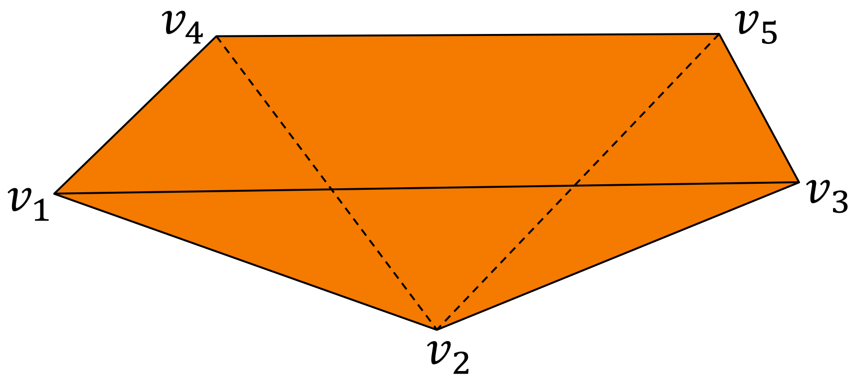
- Polyhedra
- (Oriented) Mesh Representation
  - Face-vertex data-structure
  - Winged-edge data-structure

(Oriented) Mesh Representation

Face-Vertex Lists:

Most often (e.g. ply, obj, etc. formats) polygon meshes are represented using vertex and face lists:

- **Vertex Entry**: (x, y, z) coordinates.
- **Face Entry**: Count and CCW indices of the vertices.



(Oriented) Mesh Representation

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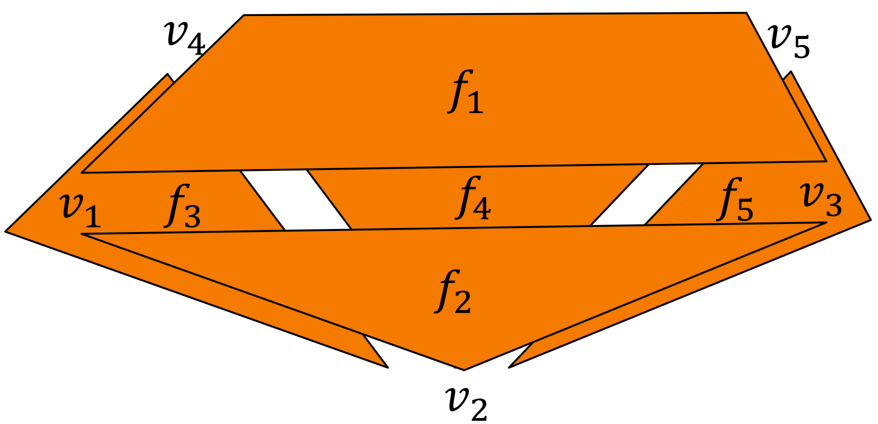
Vertex List

Id	x	y	z
1	-1	-1	0
2	0	0	-1
3	1	-1	0
4	-1	1	0
5	1	1	-1

Face List

Id	#	Indices			
1	4	1	3	5	4
2	3	1	2	3	
3	3	4	2	1	
4	3	5	2	4	
5	3	3	2	5	



(Oriented) Mesh Representation

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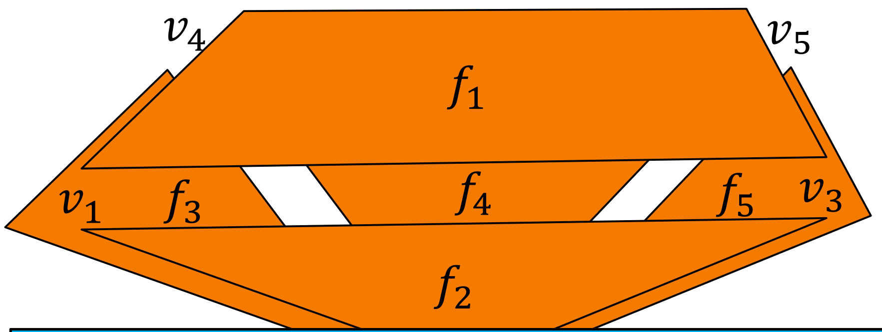
3	1	-1	0
4	-1	1	0
5	1	1	-1

Face List

Id	#	Indices			
1	4	1	3	5	4
2	3	1	2	3	
3	3	4	2	1	
4	3	5	2	4	
5	3	3	2	5	



Limitation:

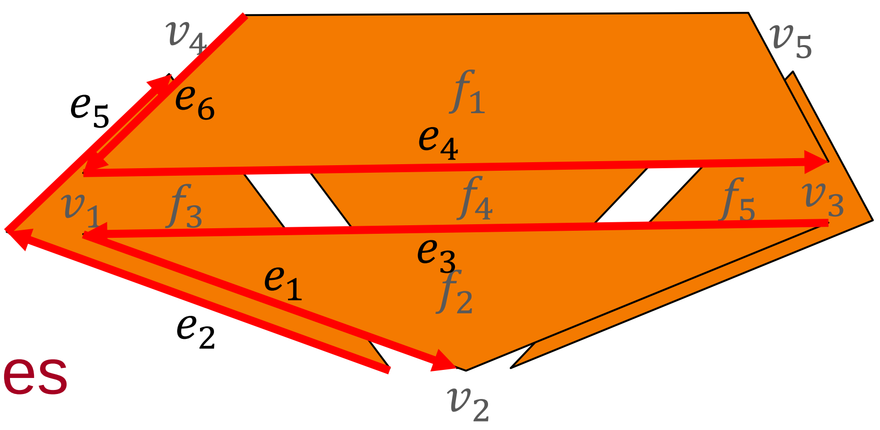
- Variable sized rows
- No explicit connectivity

(Oriented) Mesh Representation

Winged-Edge List:

Common representation for connectivity querying, represented using vertex, half-edge, and face lists:

- **Vertex Entry**:
  - » (x, y, z) coordinates
  - » Outgoing h.e. index
- **Face Entry**:
  - » h.e. index
- **Half-Edge Entry**:
  - » in/out wing h.e. indices
  - » opposite h.e. index
  - » end vertex index
  - » face index



Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Mesh Representation

e List:

presentation for connectivity querying,
using vertex, half-edge, and face lists:

- **Vertex Entry**:
  - (x, y, z) coordinates
  - Outgoing h.e. index
- **Face Entry**:
  - h.e. index
- **Half-Edge Entry**:
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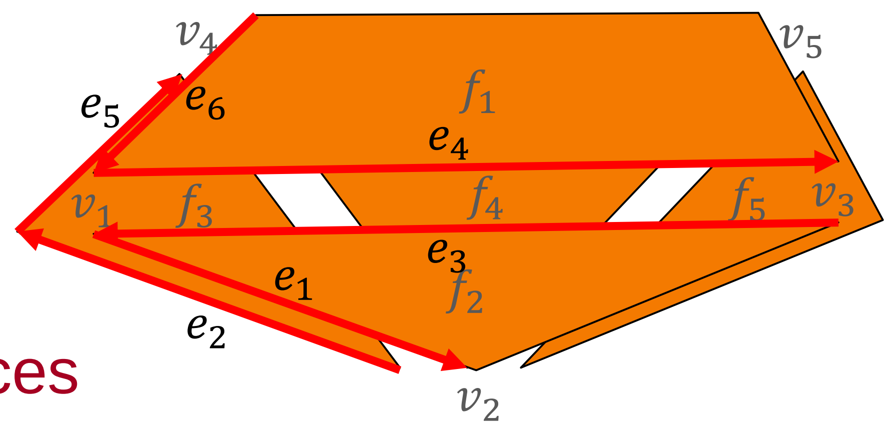
  - face index



Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

n Representation

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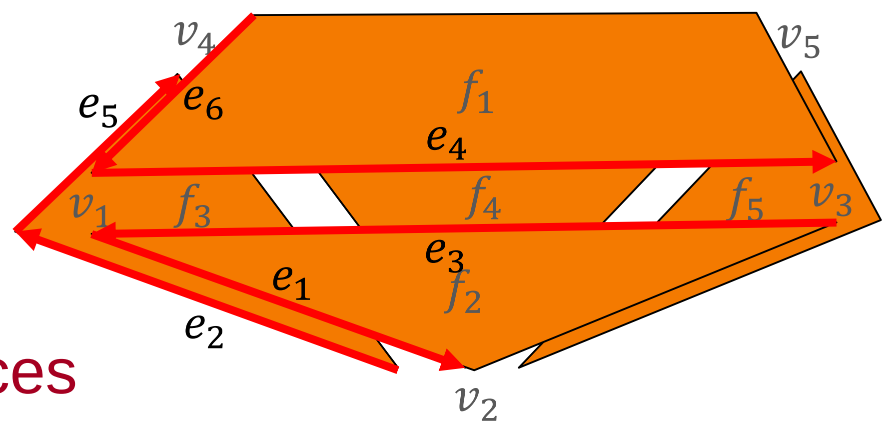

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	...	4	1	1
…					

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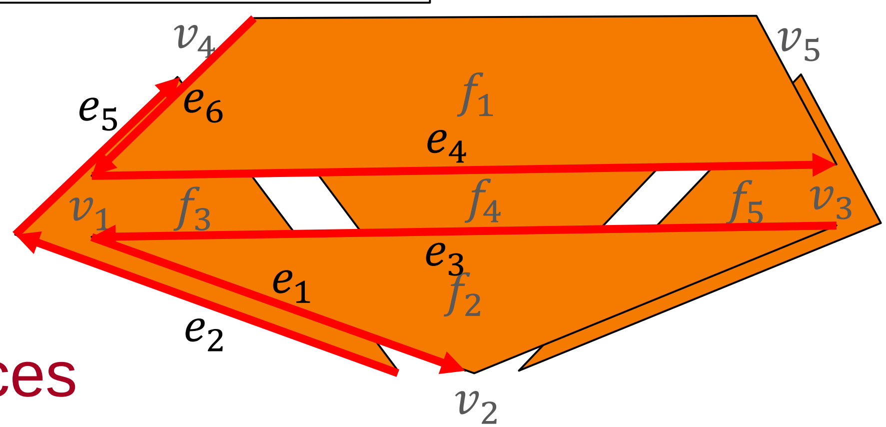
  - » opposite h.e. index
  - » end vertex index
  - » face index



Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

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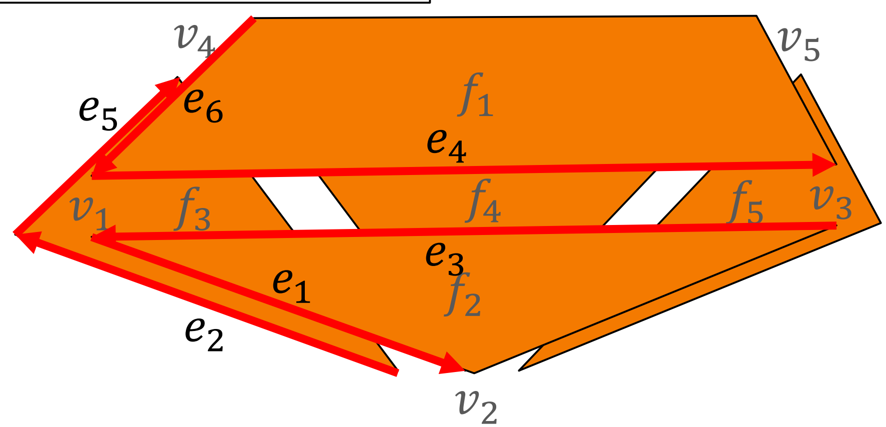
…nd face lists:

  - » Outgoing h.e. index
- ○ **Face Entry**:
  - » h.e. index
- ○ **Half-Edge Entry**:



Example:

Find CCW vertices around v_1:

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

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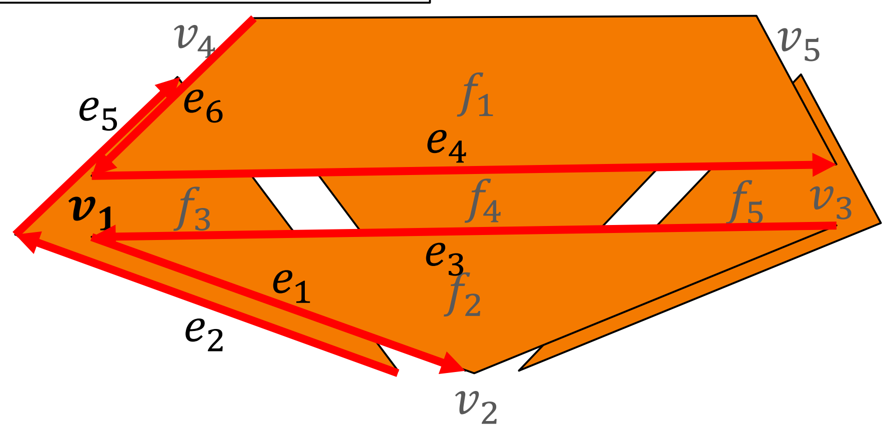
» Outgoing h.e. index

- **Face Entry**:
  » h.e. index
- **Half-Edge Entry**:



Example:

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Vertex List

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1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	**3**	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

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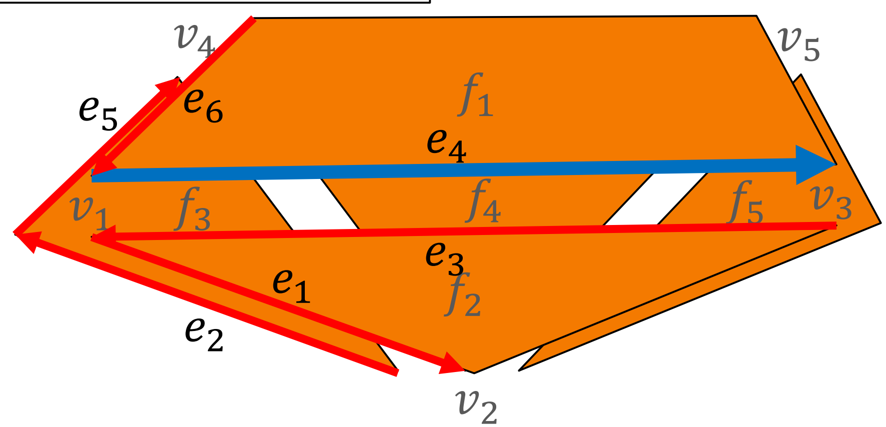
   » Outgoing h.e. index

- **Face Entry**:
   » h.e. index
- **Half-Edge Entry**:



<u>Example</u>:

Find CCW vertices around v_1: v_3

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nd face lists:

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	...	4	1	1
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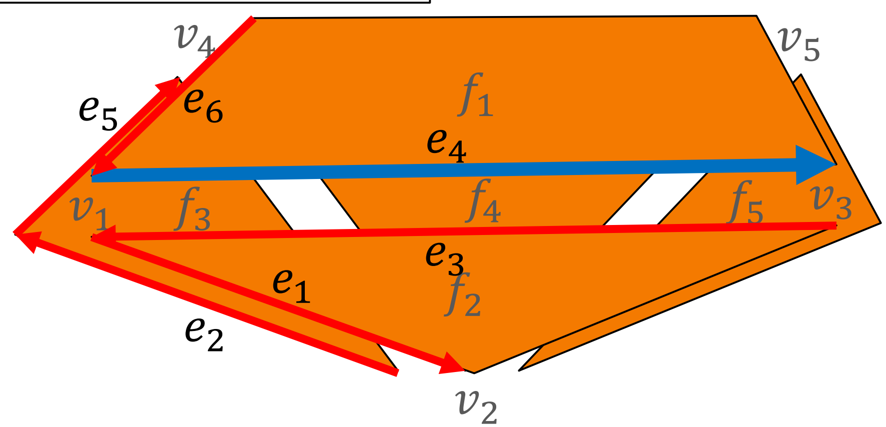
- » Outgoing h.e. index
- ◦ **Face Entry**:
  - » h.e. index
- ◦ **Half-Edge Entry**:



<u>Example</u>:

Find CCW vertices around v_1: v_3

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

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- » Outgoing h.e. index
- ○ **Face Entry**:
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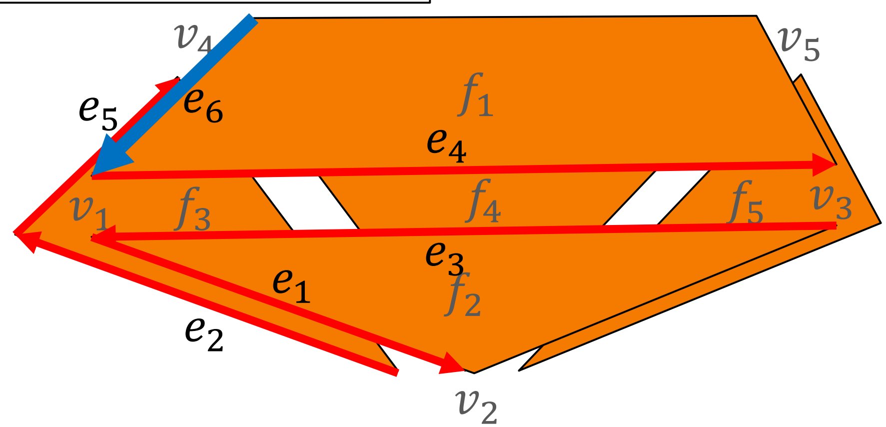
- ○ **Half-Edge Entry**:



<u>Example</u>:

Find CCW vertices around v_1: v_3

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	**4**	3
6	5	…	4	1	1
…					

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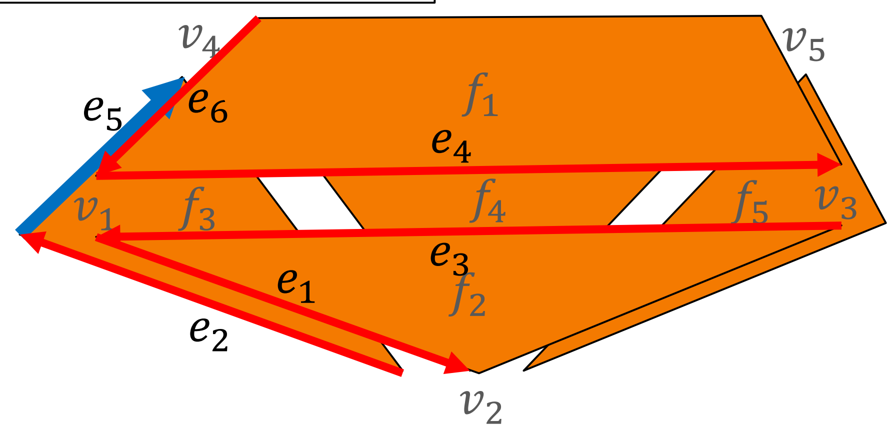
  - » Outgoing h.e. index
- ◦ **Face Entry**:
  - » h.e. index
- ◦ **Half-Edge Entry**:



Example:

Find CCW vertices around v_1: v_3, v_4

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	...	4	1	1
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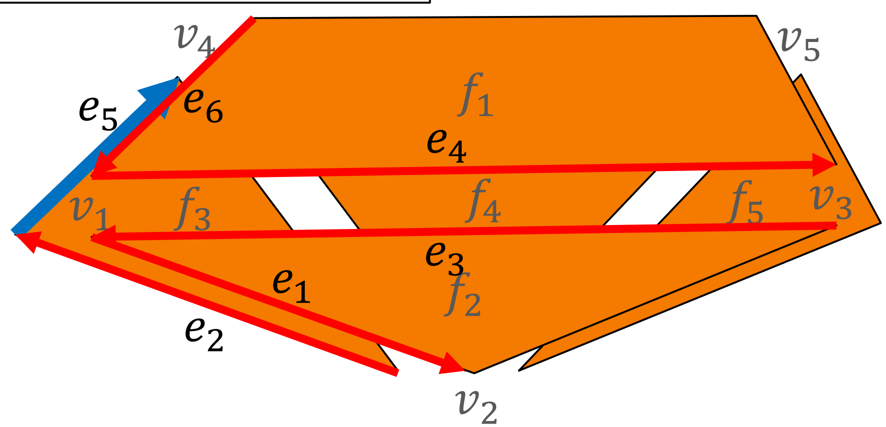
» Outgoing h.e. index

◦ **Face Entry**:

» h.e. index

◦ **Half-Edge Entry**:



<u>Example</u>:

Find CCW vertices around v_1: v_3, v_4

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

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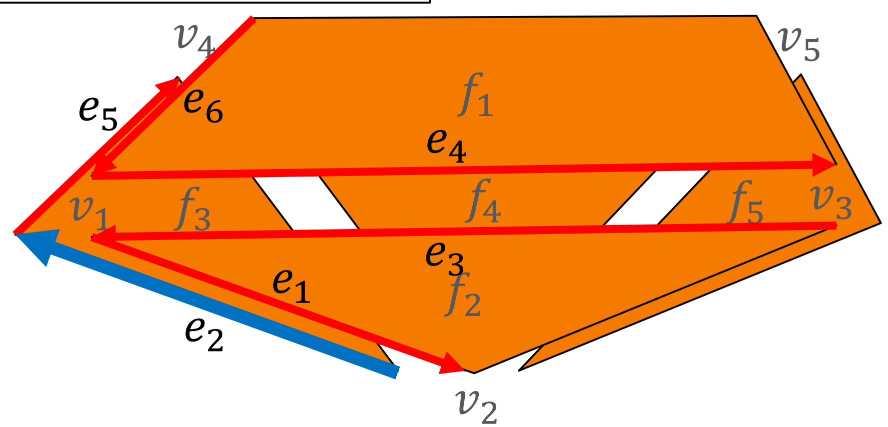
» Outgoing h.e. index

- **Face Entry**:
  » h.e. index
- **Half-Edge Entry**:



Example:

Find CCW vertices around v_1: v_3, v_4

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	**2**	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
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- » Outgoing h.e. index
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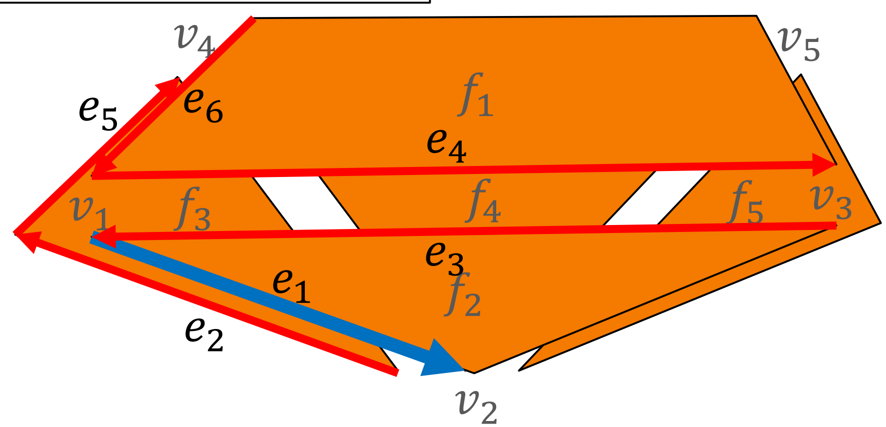
- ◦ **Half-Edge Entry**:



<u>Example</u>:

Find CCW vertices around v_1: v_3, v_4, v_2

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	**3**	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	…	4	1	1
…					

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   » Outgoing h.e. index
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   » h.e. index
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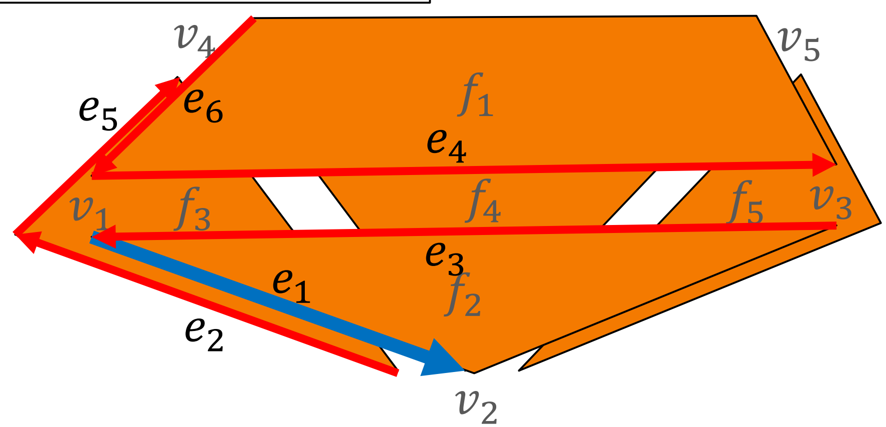

<u>Example</u>:

Find CCW vertices around v_1: v_3, v_4, v_2

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Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
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4	3	6	…	3	1
5	6	2	…	4	3
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…					

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- » Outgoing h.e. index
- ○ **Face Entry**:
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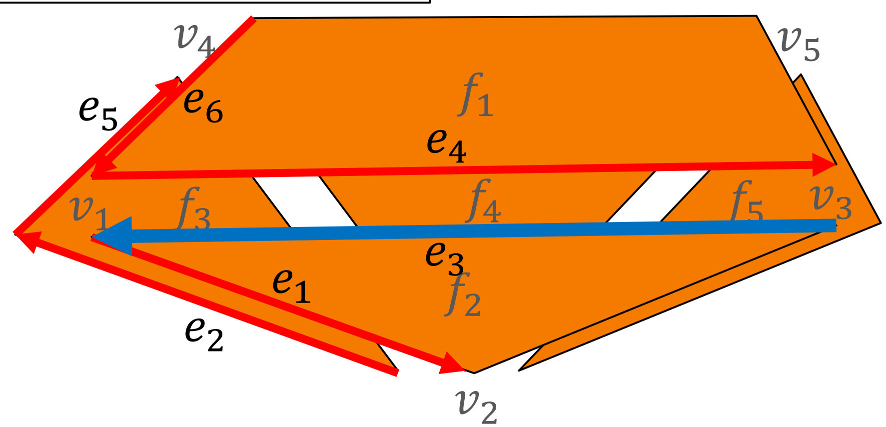
- ○ **Half-Edge Entry**:



<u>Example</u>:

Find CCW vertices around v_1: v_3, v_4, v_2

Vertex List

Id	x	y	z	h
1	-1	-1	0	4
2	0	0	-1	2
3	1	-1	0	3
4	-1	1	0	6
5	1	1	-1	…

Face List

Id	h
1	4
2	3
3	5
4	…
5	…

Half-Edge List

Id	o	w_i	w_o	v	f
1	2	3	…	2	2
2	1	…	5	1	3
3	4	…	1	1	2
4	3	6	…	3	1
5	6	2	…	4	3
6	5	...	4	1	1

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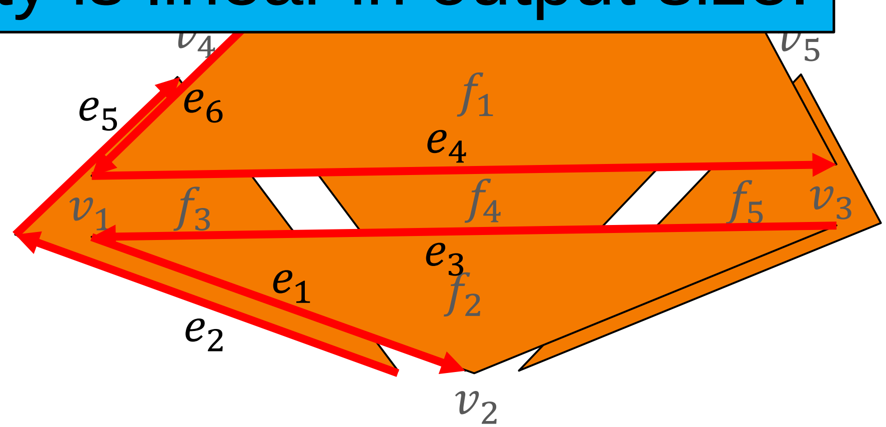
…nd face lists:

Computational complexity is linear in output size.

- » Outgoing h.e. index
- ◦ **Face Entry**:
  - » h.e. index
- ◦ **Half-Edge Entry**:



Example:

Find CCW vertices around v_1: v_3, v_4, v_2

(Oriented) Mesh Representation

GenerateHalfEdge(V , F , _V , _E , _F)

- _V.resize(v.size()) , _F.resize(F.size())
- for(i=0 ; i<V.size() ; i++) _V[i].p = V[i].p
- unordered_map< long long , int > fMap
- ConstructFaceMap(F, fMap)
- _E.resize(fMap.size())
- SetVertexAndFaceIndices(fMap , _V , _E , _F)
- SetHalfEdges(fMap , F , _E)

(Oriented) Mesh Representation

ConstructFaceMap(F , fMap)

- `for( f=0 ; f<F.size() ; f++ )`
  - `for( v=0 ; v<F[f].size() ; v++ )`
    - `long long key = F[f][v]<<32 | F[f][v+1]`
    - `fMap[key] = f`

Assuming that:

- Indexing is modulo the face size
- We don't lose precision due to casting/shifting.

(Oriented) Mesh Representation

SetVertexAndFaceIndices(fMap , _V , _E , _F)

- int count = 0
- for(iter i=fMap.begin() ; i!=fMap.end() ; i++)
  - int v = i.key>>32 , f = i.value
  - _E[count].v = v , _E[count].f = f
  - _V[v].he = _F[f].he = i.value = count++

Note that the values of the face map are over-written with the edge indices.

(Oriented) Mesh Representation

SetHalfEdges(fMap , F , _E)

- for(f=0 ; f<F.size() ; f++)
  - for(v=0 ; v<F[f].size() ; v++)
    - long long key = F[f][v]<<32 | F[f][v+1]
    - long long oKey = F[f][v+1]<<32 | F[f][v]
    - long long nKey = F[f][v+1]<<32 | F[f][v+2]
    - E[fMap[key]].o = fMap[oKey]
    - E[fMap[key]].w2 = fMap[nKey]
    - E[fMap[nKey]].w1 = fMap[key]