



Radiosity

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(601.457/657)

Overview

- Ray Tracing Revisited
- Radiosity





Ray Casting

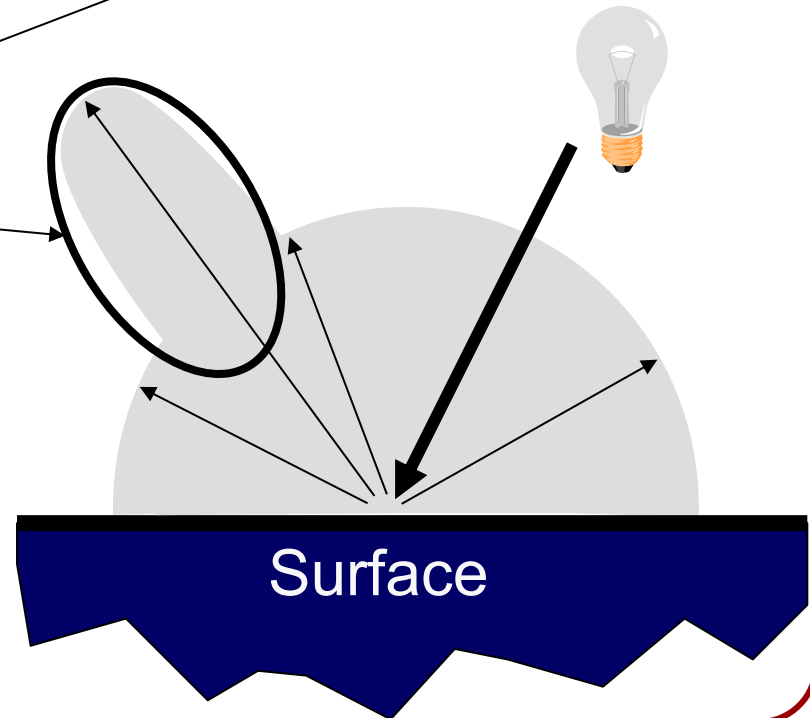
Ray tracing is based on the Phong lighting model:

Surfaces reflect light non-uniformly, with stronger reflection in the specular direction:

$$I = I_E + K_A \cdot I_L^A + \sum_L \left(\langle \vec{N}, \vec{L} \rangle + K_S \cdot \langle \vec{V}, \vec{R}(\vec{L}) \rangle^{K_n} \right) \cdot I_L$$

Specular Contribution

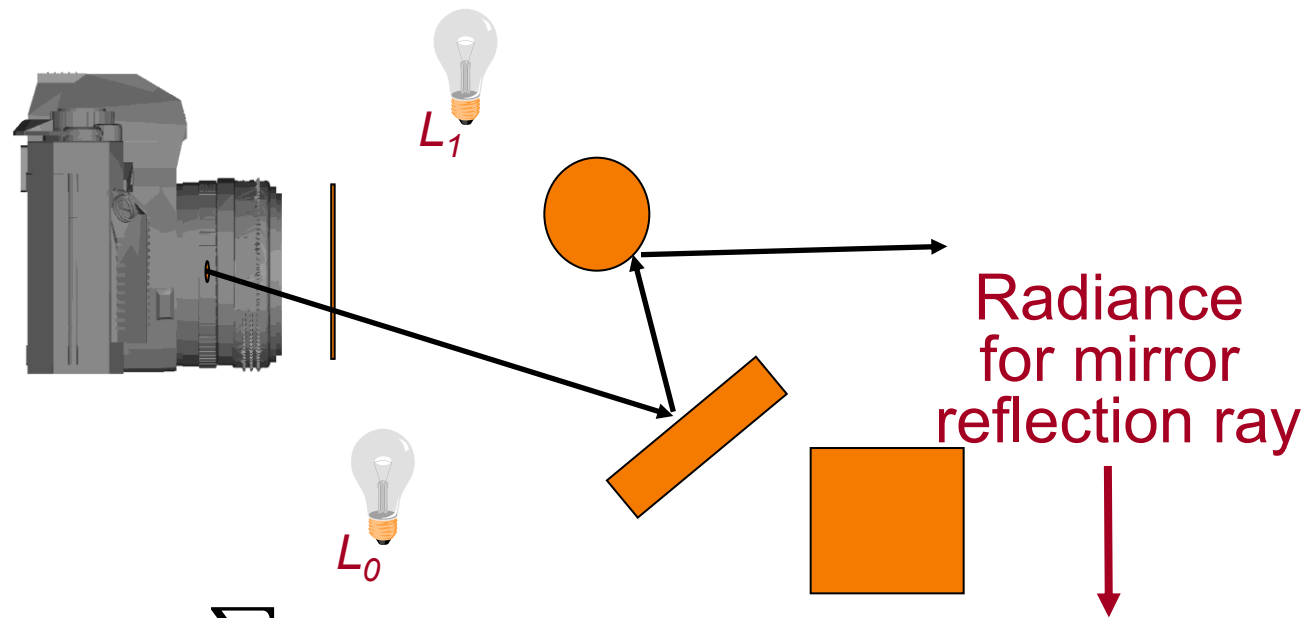
Specular Lobe





Ray Tracing

Ray tracing is based on the Phong lighting model:
For this reason, we only cast secondary rays in the reflected direction – to maximize the contribution to the lighting computation.



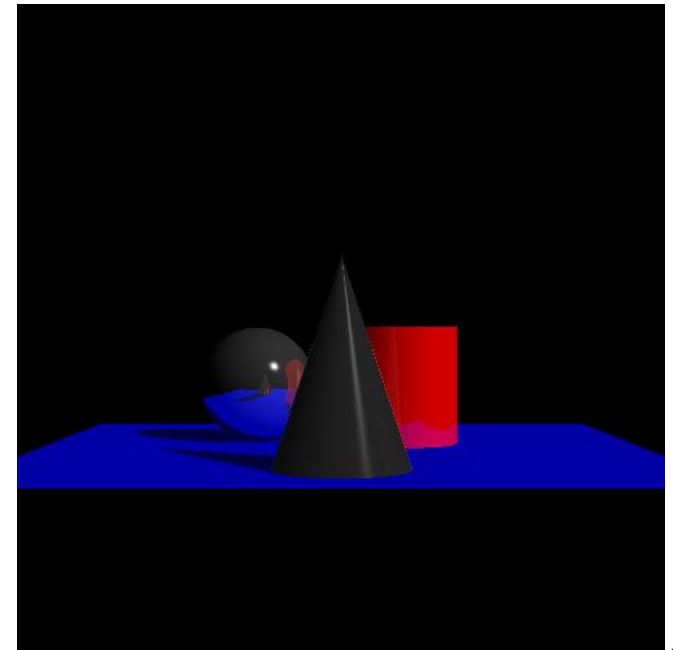
$$I = I_E + K_A \cdot I_L^A + \sum_L [(\langle \vec{N}, \vec{L} \rangle + K_S \cdot \langle \vec{V}, \vec{R}(\vec{L}) \rangle^{K_n}) \cdot I_L + K_S \cdot I_R]$$

Ray Tracing



Properties:

- ✓ Good at capturing the specular properties of materials

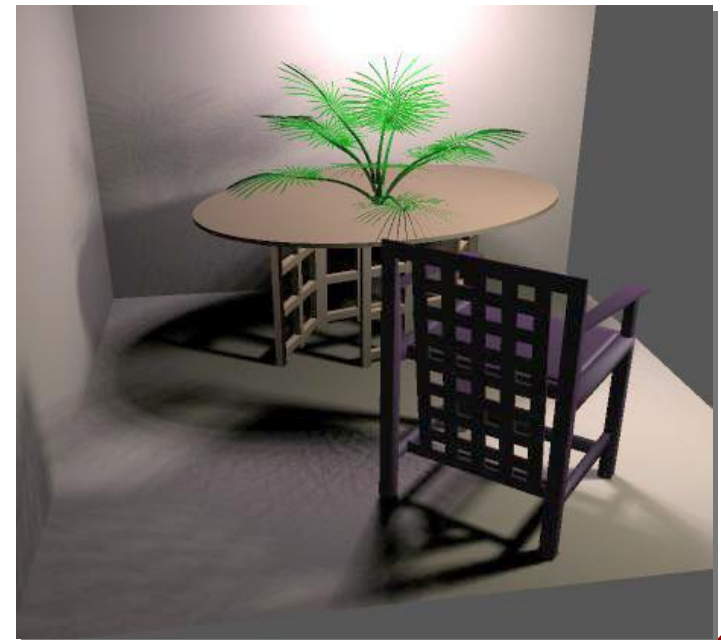




Ray Tracing

Properties:

- ✓ Good at capturing the specular properties of materials
- ✗ Difficult to support soft shadows from area lights
- ✗ Difficult to support caustics
- ✗ Need the ambient term as a hack for the global illumination



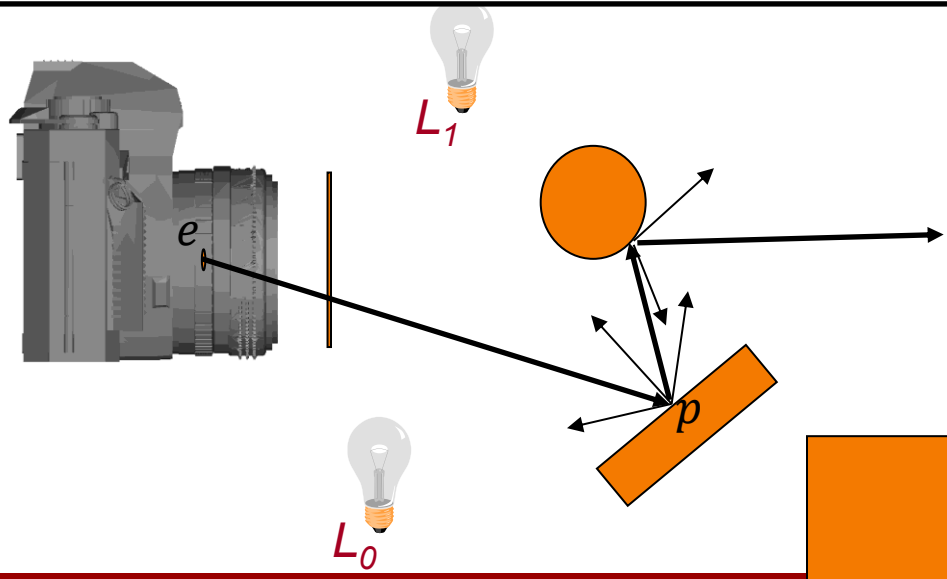


Lighting

What do we really want to compute?

The accumulation of light coming in from **all** directions, **modulated** by how much the light is reflected in/from that direction.

Use Monte-Carlo integration with *importance sampling* to generate more reflected rays in directions that contribute more strongly.



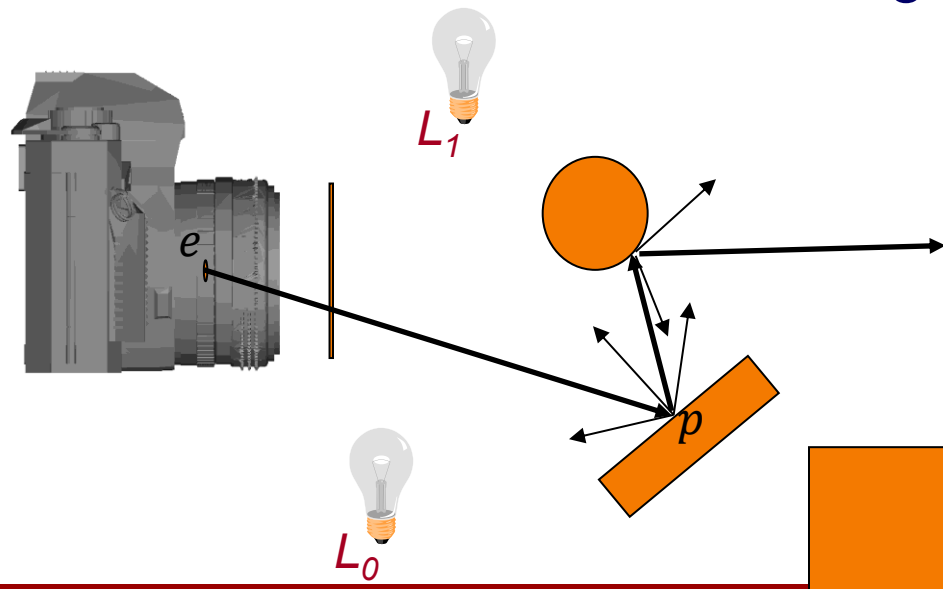


Lighting

What do we really want to compute?

The brightness/intensity reaching the camera (e) from a point in the scene (p) is the sum of:

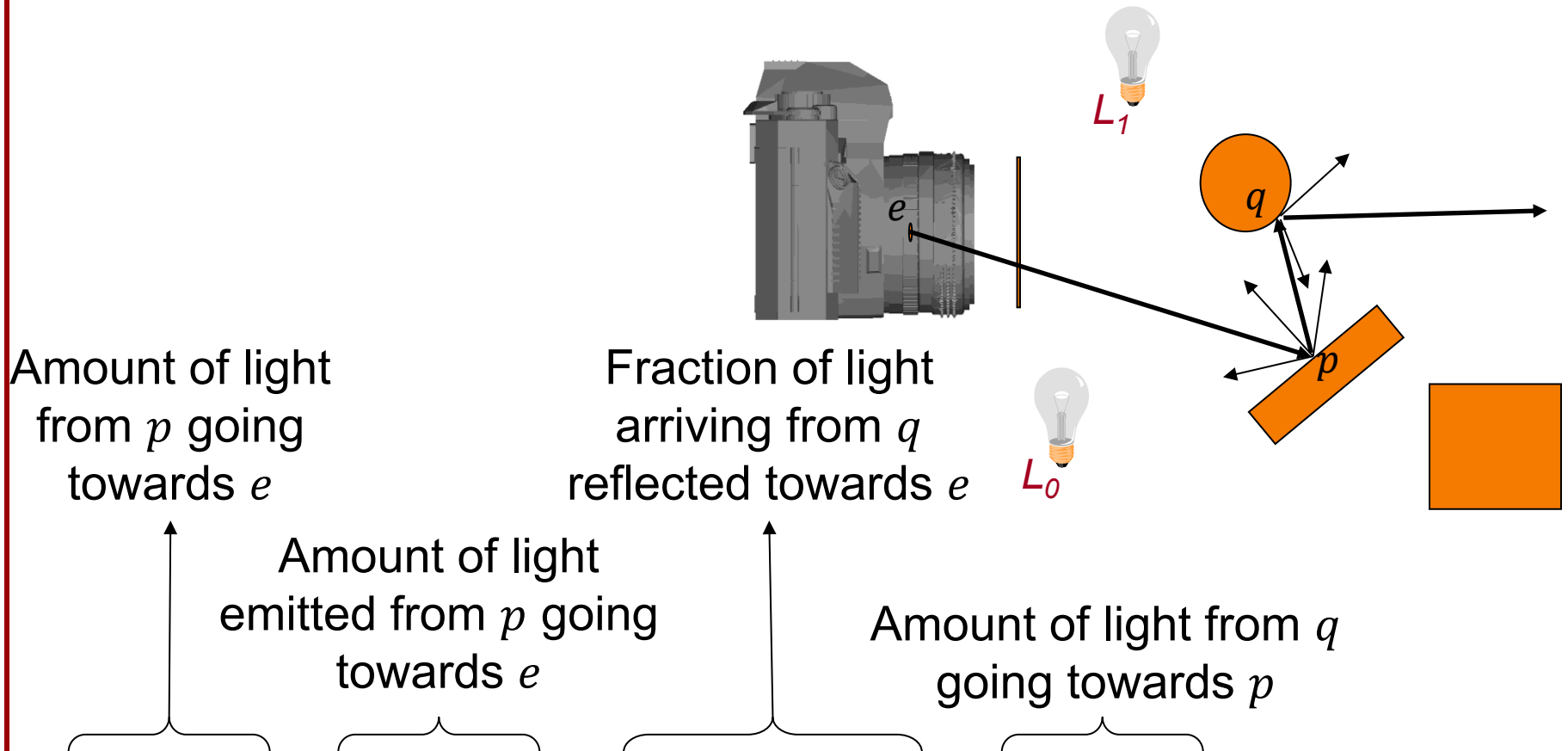
1. The light emitted from p , to e , and
2. The light coming from **all** scene points scaled by the extent to which it is reflected through p to e .





Lighting

What do we really want to compute?



$$B(p \rightarrow e) = E(p \rightarrow e) + \int_{\Omega} F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p) dq$$



Lighting

Challenge:

The integral needs to be estimated precisely to capture discontinuities.

The function is recursive since the amount of light entering a point depends on the amount leaving it.



Jensen

$$B(p \rightarrow e) = E(p \rightarrow e) + \int_{\Omega} F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p) dq$$



Ray-Tracing

Specular simplifying assumption:

Only reflect lights from the reflected ray direction:

$$F_r(q \rightarrow p \rightarrow e) = \begin{cases} K_s(p) & q = \text{Isect}(p, \text{Ref}(p \rightarrow e)) \\ 0 & \text{otherwise} \end{cases}$$

($\text{Isect}(p, \text{Ref}(p \rightarrow e))$) is the first intersection of the ray in the reflected view direction)

$$B(p \rightarrow e) = E(p \rightarrow e) + \int_{\Omega} F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p) dq$$



$$B(p \rightarrow e) = E(p \rightarrow e) + K_s(p) \cdot B(I(p, \text{Ref}(p \rightarrow e)) \rightarrow p)$$



Radiosity

Lambertian simplifying assumption:

The *apparent* brightness of a patch of surface is constant independent of the view direction.

$$B(p \rightarrow e) = E(p \rightarrow e) + \int_{\Omega} F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p) dq$$



$$B(p) = E(p) + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) dq$$



Radiosity

Lambertian simplifying assumption:

The *apparent* brightness of a patch of surface is constant independent of the view direction.

Emitters appear equally bright from all directions

Q: What does this imply about the directional dependence of the intensity?

Emission $\neq 0$

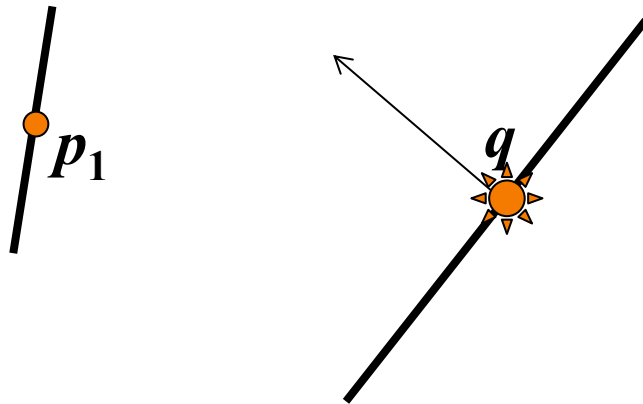


$$B(p) = \boxed{E(p)} + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) dq$$



Lambertian Emitters

Given an emitter at point q , the *apparent* brightness point p is independent of its orientation w.r.t. to q .



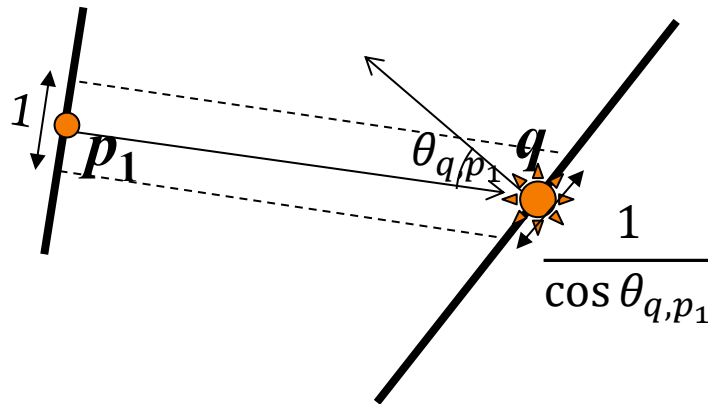
$$B(p) = \boxed{E(p)} + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) dq$$



Lambertian Emitters

Given an emitter at point q , the apparent brightness point p is independent of its orientation w.r.t. to q .

A unit-area patch about p “sees” more of q ’s surface as $\theta_{q,p}$ is more grazing.



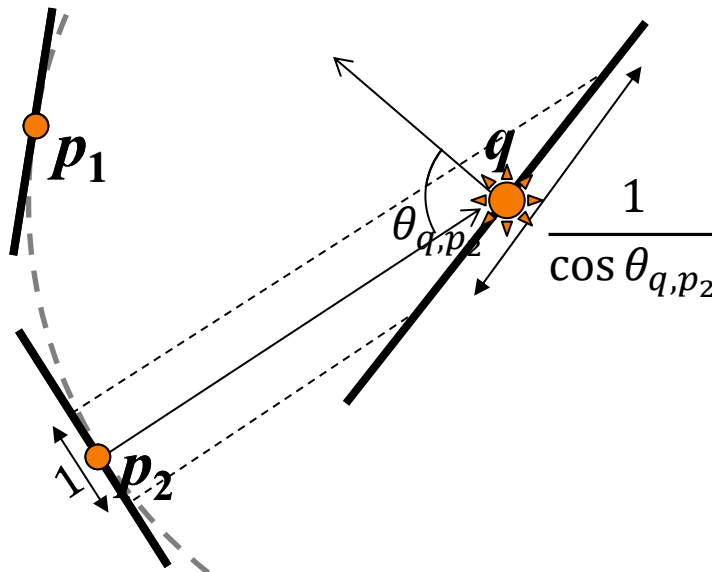
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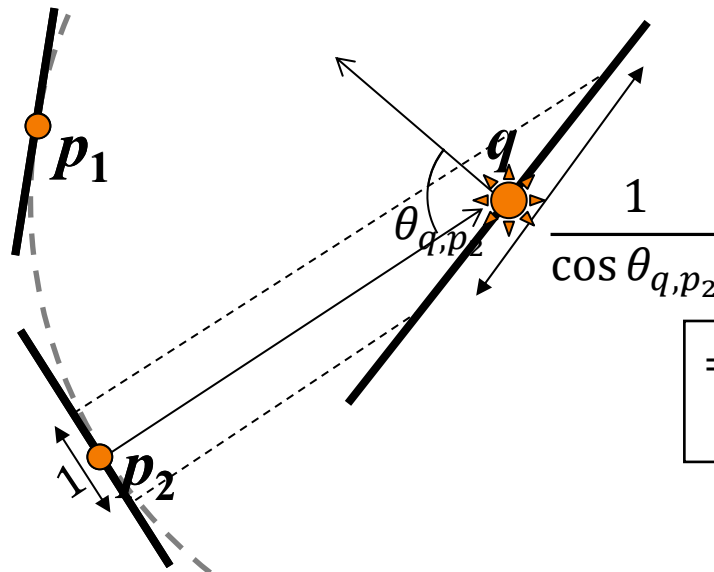
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Lambertian Emitters

Given an emitter at point q , the apparent brightness point p is independent of its orientation w.r.t. to q .

A unit-area patch about p “sees” more of q ’s surface as $\theta_{q,p}$ is more grazing.



$\Rightarrow$ A unit-area patch about p receives light from a patch of area $1/\cos \theta_{q,p}$ about q .

$\Rightarrow$ The intensity emitted from a patch of about q in direction p falls off as $\cos \theta_{q,p}$.

$$B(p) = \boxed{E(p)} + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) dq$$



Radiosity

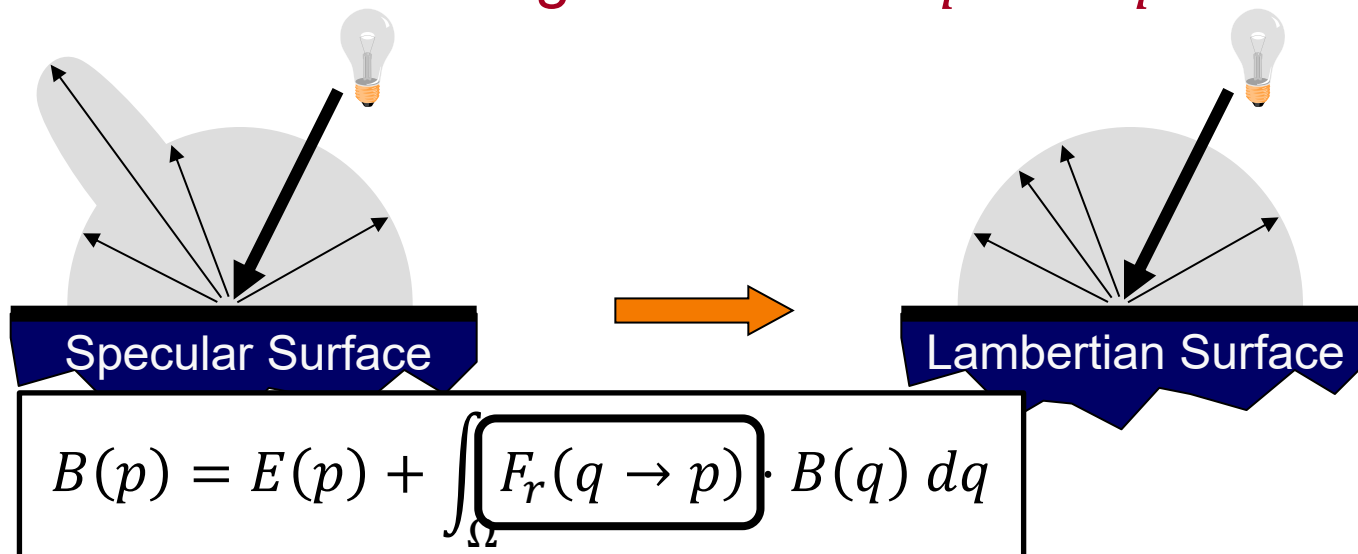
Lambertian simplifying assumption:

The *apparent* brightness of a patch of surface is constant independent of the view direction.

Emitters appear equally bright from all directions

Reflectors appear equally bright from all directions

Q: What does this imply about how reflection depends on the relative geometries at p and q ?





Lambertian Reflectors

How does the amount of light going from q reflected through p depend on:

1. The direction to p relative to the orientation at q ,
2. The direction to q relative to the orientation at p ,
3. The distance between p and q ?

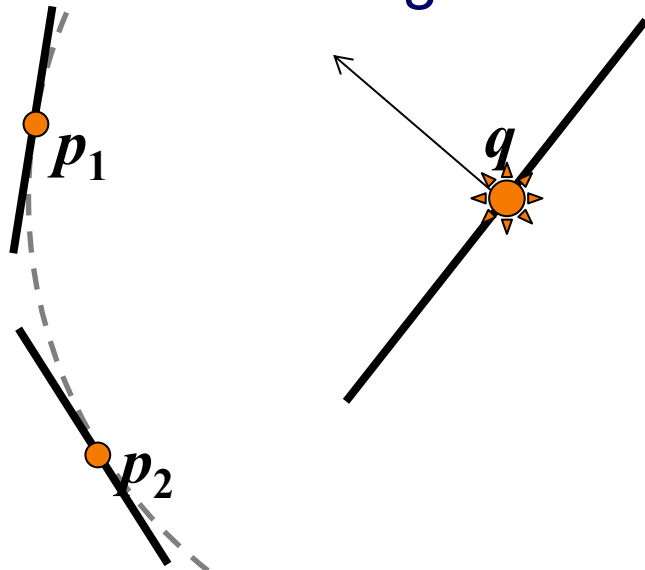
$$B(p) = E(p) + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) dq$$



Lambertian Reflectors (1)

Treating q as an emitter, the light emitted from q in direction p falls off as $\cos \theta_{q,p}$ – with $\theta_{q,p}$ the angle between the normal at q and direction to p .

⇒ Reflected brightness at p falls off as $\cos \theta_{q,p} = \frac{\langle n_q, p-q \rangle}{|p-q|}$



$$B(p) = E(p) + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) dq$$

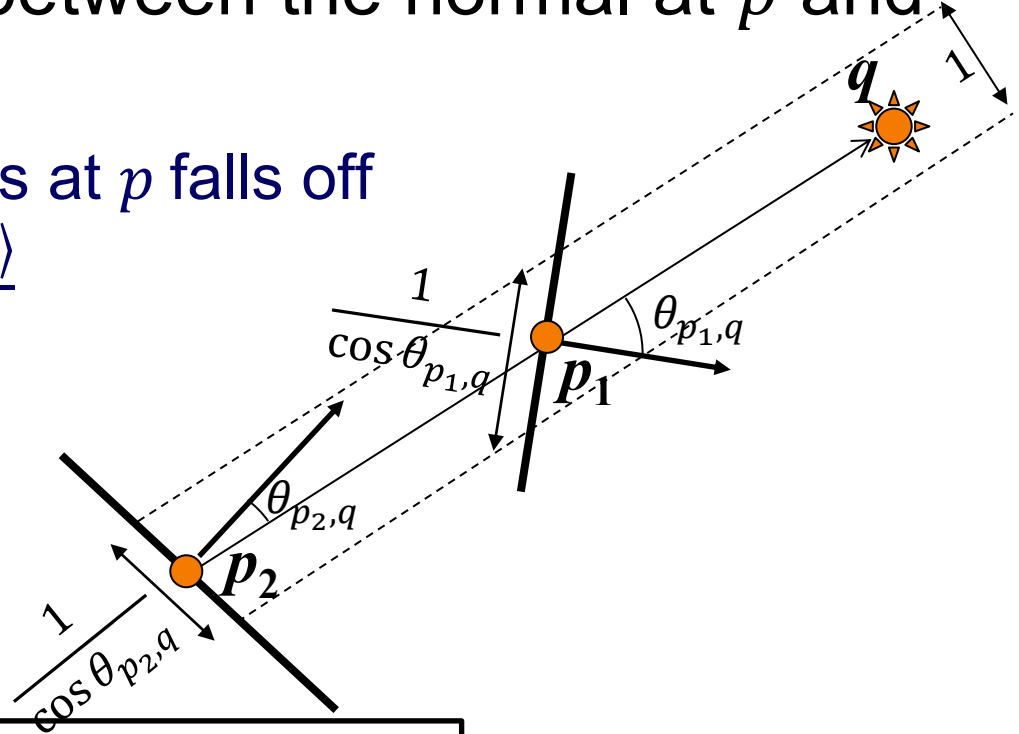


Lambertian Reflectors (2)

A beam of unit cross-sectional leaving q towards p , will spread out across a patch of area $1 / \cos \theta_{p,q}$ at p – with $\theta_{p,q}$ the angle between the normal at p and direction to q .

⇒ Reflected brightness at p falls off

$$\text{as } \cos \theta_{p,q} = \frac{\langle n_{p,q-p} \rangle}{|q-p|}$$

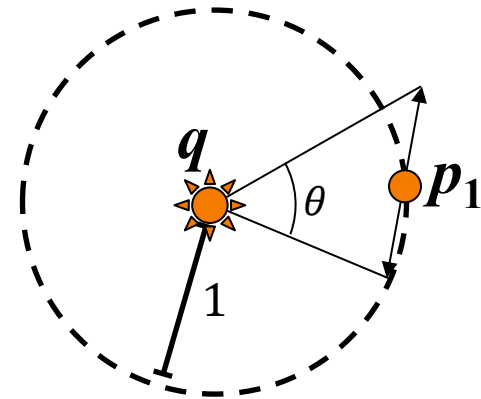


$$B(p) = E(p) + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) dq$$



Lambertian Reflectors (3)

The apparent brightness at p is proportional to the subtended spherical angle by a unit area patch at q .



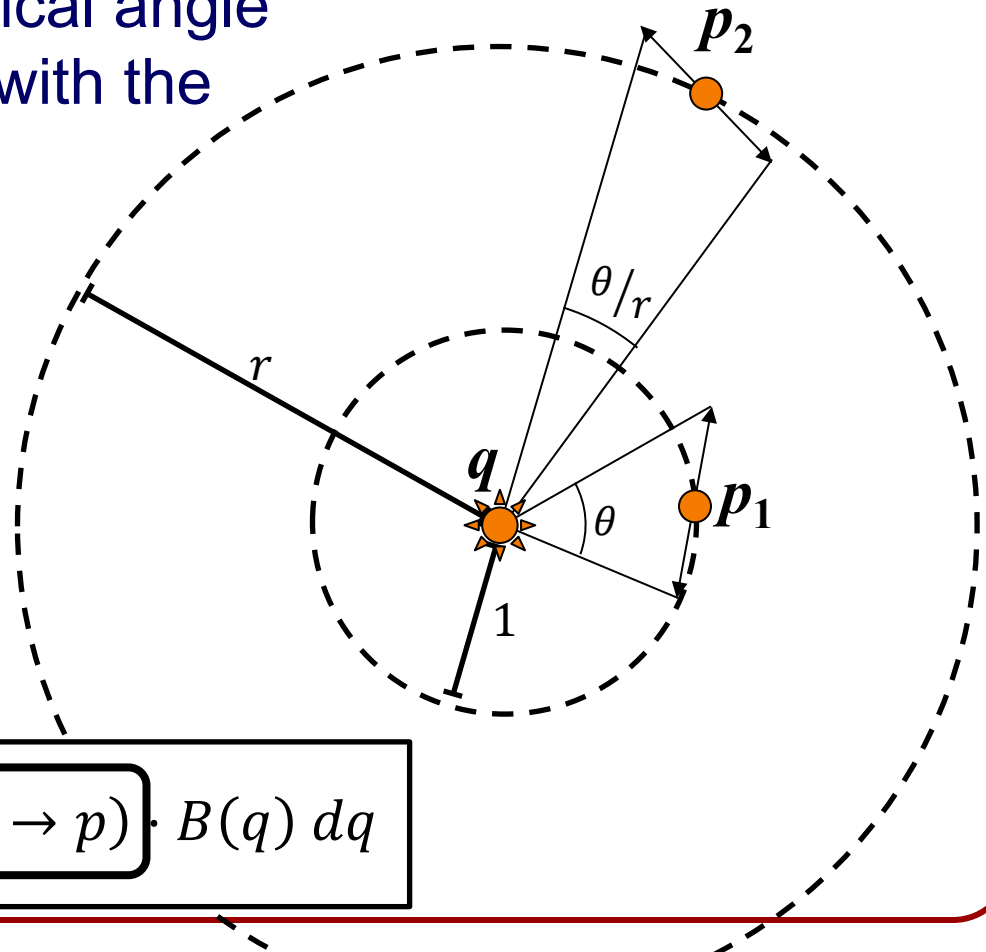
$$B(p) = E(p) + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) dq$$



Lambertian Reflectors (3)

The apparent brightness at p is proportional to the subtended spherical angle by a unit area patch at q .

- ⇒ The subtended spherical angle falls off quadratically with the distance of p from q .
- ⇒ Perceived brightness decays as the square of the distance.



$$B(p) = E(p) + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) dq$$

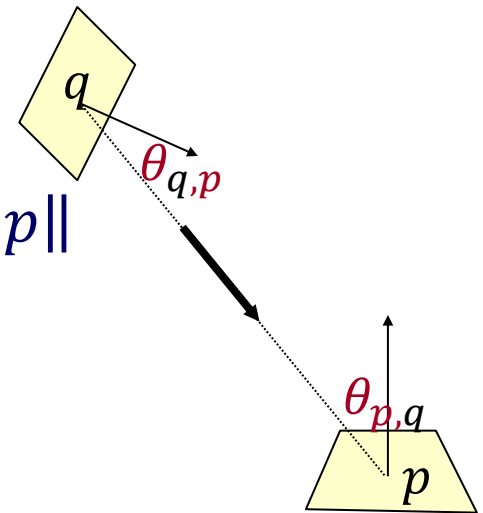


Lambertian Reflectors

⇒ The fraction of light from q that is reflected off p is determined by:

- The angle: $\theta_{q,p}$
- The angle: $\theta_{p,q}$
- The square distance from q to p : $\|q - p\|^2$

$$F_r(q \rightarrow p) = \frac{\cos \theta_{q,p} \cdot \cos \theta_{p,q}}{\|q - p\|^2}$$



$$B(p) = E(p) + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) dq$$

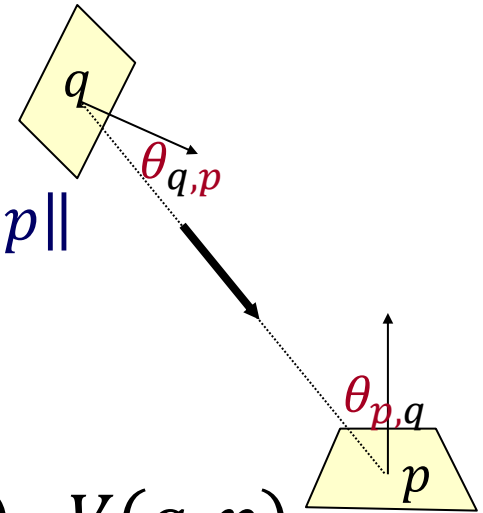


Lambertian Reflectors

⇒ The fraction of light from q that is reflected off p is determined by:

- The angle: $\theta_{q,p}$
- The angle: $\theta_{p,q}$
- The square distance from q to p : $\|q - p\|^2$
- The visibility of p from q : $V(q, p)$
- The albedo at p : $\rho(p)$

$$F_r(q \rightarrow p) = \frac{\cos \theta_{q,p} \cdot \cos \theta_{p,q}}{\|q - p\|^2} \cdot \rho(p) \cdot V(q, p)$$



$$B(p) = E(p) + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) \, dq$$



Radiosity

Lambertian simplifying assumption:

The *apparent* brightness of a patch of surface is constant independent of the view direction.

Emitters appear equally bright from all directions

Reflectors appear equally bright from all directions

$$B(p) = E(p) + \rho(p) \int_{\Omega} V(q, p) \cdot \frac{\cos \theta_{q,p} \cdot \cos \theta_{p,q}}{\|q - p\|^2} \cdot B(q) dq$$

The radiosity equation



Radiosity

Approximate the integral by decomposing surfaces into patches and doing a discrete summation:

$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} \cdot B_j$$

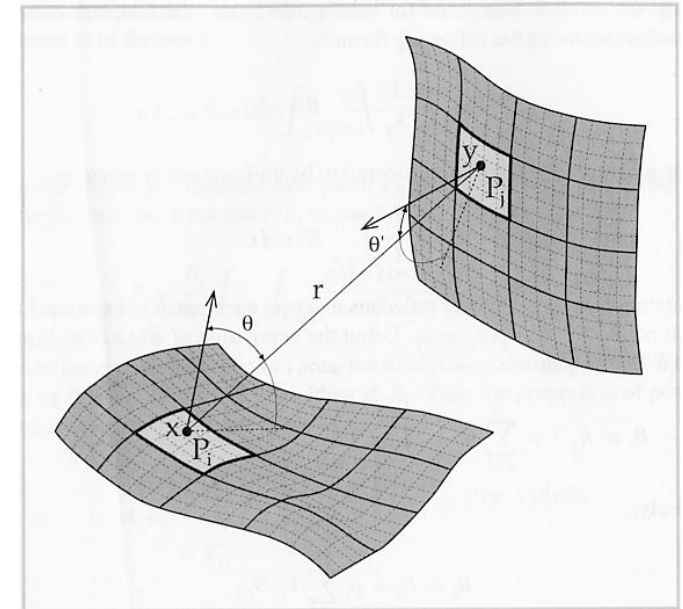
Form Factor

For patch i :

B_i : Total brightness

E_i : Total emissivity

ρ_i : Albedo



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$$B(p) = E(p) + \rho(p) \int_{\Omega} V(q, p) \cdot \frac{\cos \theta_{q,p} \cdot \cos \theta_{p,q}}{\|q - p\|^2} \cdot B(q) dq$$

The radiosity equation



Form Factor

Approximate the integral by decomposing surfaces into patches and doing a discrete summation:

$$B_i = E_i + \rho_i \sum_{j=1}^n \boxed{F_{ij}} \cdot B_j$$

The **form factor** $0 \leq F_{ij} \leq 1$ is the proportion of the power leaving patch P_j , received by patch P_i :

- Symmetry/Reciprocity: $A_j F_{ij} = A_i F_{ji}$
- Positivity: $F_{ij} \geq 0$
- Definiteness: $F_{ii} = 0$ unless the patch is concave
- Partition of unity: $\sum_i F_{ij} = 1$



Radiosity

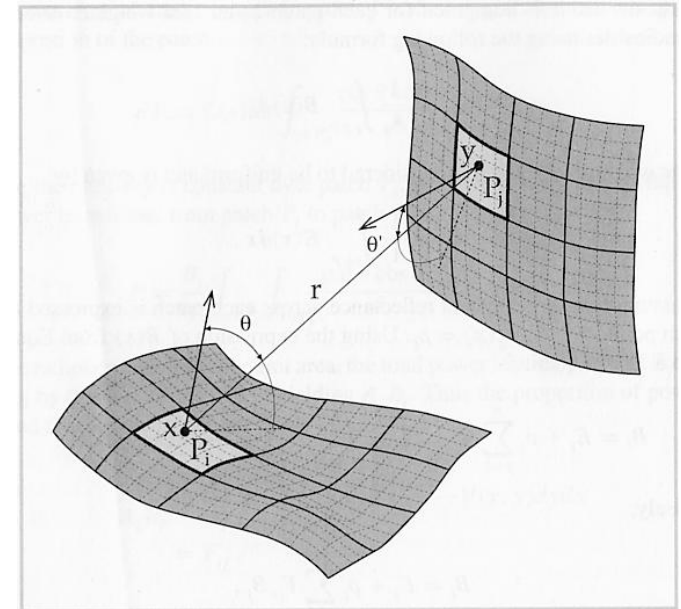
Approximate the integral by decomposing surfaces into patches and doing a discrete summation:

$$B_i = E_i + \rho_i \sum_{j=1}^n \boxed{F_{ij}} \cdot B_j$$

This amounts to solving a **linear** system of equations

E_i , ρ_i , and F_{ij} are given

B_i are the unknowns.



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Radiosity

Re-ordering terms in the equation gives:

$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} \cdot B_j$$

$\Downarrow$

$$E_i = B_i - \rho_i \sum_{j=1}^n F_{ij} \cdot B_j$$

$\Downarrow$

$$\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix} = \begin{pmatrix} 1 - \rho_1 \cdot F_{11} & -\rho_1 \cdot F_{21} & \cdots & -\rho_1 \cdot F_{n1} \\ -\rho_2 \cdot F_{12} & 1 - \rho_2 \cdot F_{22} & \cdots & -\rho_2 \cdot F_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n \cdot F_{1n} & -\rho_n \cdot F_{2n} & \cdots & 1 - \rho_n \cdot F_{nn} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix}$$



Solving the System of Equations

Challenges:

- Size of matrix
- Cost of computing form factors

$$\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix} = \begin{pmatrix} 1 - \rho_1 \cdot F_{11} & -\rho_1 \cdot F_{21} & \cdots & -\rho_1 \cdot F_{n1} \\ -\rho_2 \cdot F_{12} & 1 - \rho_2 \cdot F_{22} & \cdots & -\rho_2 \cdot F_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n \cdot F_{1n} & -\rho_n \cdot F_{2n} & \cdots & 1 - \rho_n \cdot F_{nn} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix}$$

e = **A** · **b**



Solving the System of Equations

Solution methods:

- ~~Invert the matrix $O(n^3)$~~
- Gathering methods – $O(n^2)$
- Shooting methods – $< O(n^2)$

$$\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix} = \begin{pmatrix} 1 - \rho_1 \cdot F_{11} & -\rho_1 \cdot F_{21} & \cdots & -\rho_1 \cdot F_{n1} \\ -\rho_2 \cdot F_{12} & 1 - \rho_2 \cdot F_{22} & \cdots & -\rho_2 \cdot F_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n \cdot F_{1n} & -\rho_n \cdot F_{2n} & \cdots & 1 - \rho_n \cdot F_{nn} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix}$$

e = **A** · **b**



Gathering Iteration

Initialization:

For each patch P_i , initialize its radiosity to its emissivity:

$$B_i = E_i$$

Iteration:

At each iteration, update the values of each of the B_i based on the values of all the other B_j :

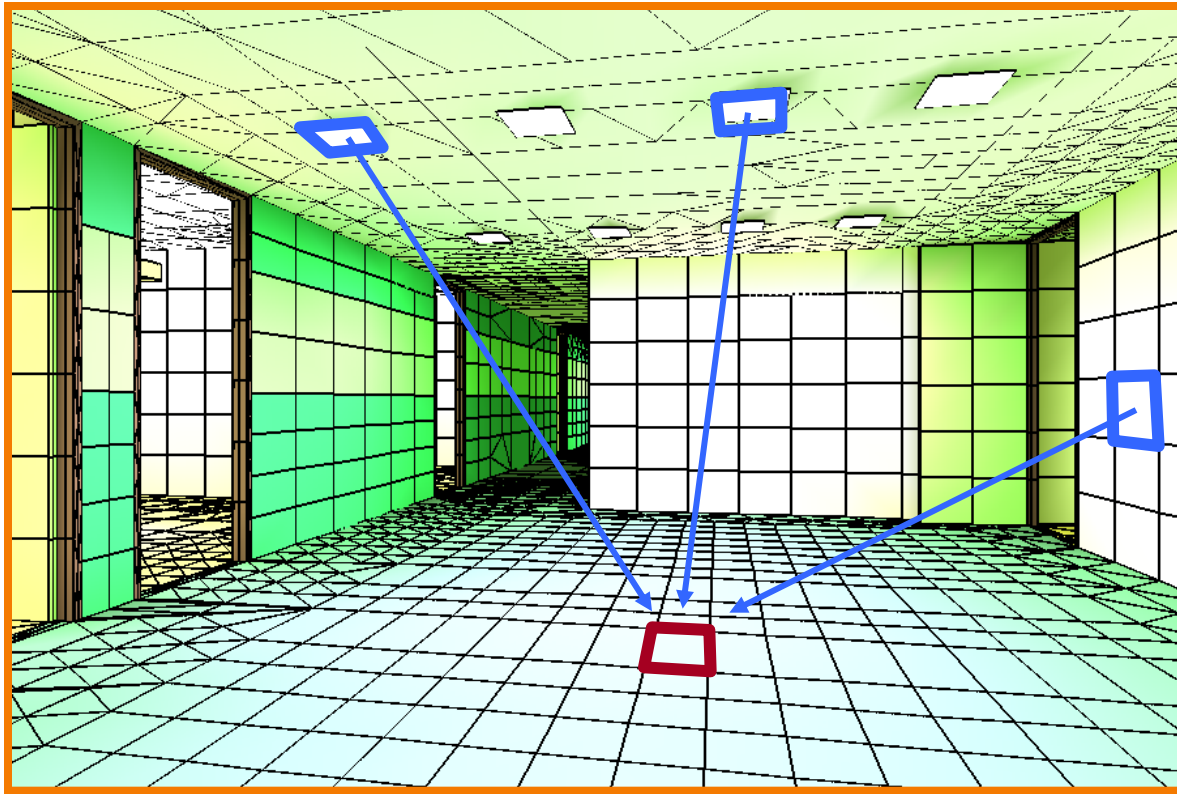
$$B_i = E_i + \rho_i \sum_{j \neq i} F_{ij} \cdot B_j$$



Gathering Iteration

Geometric interpretation

Iteratively gather radiosity from elements

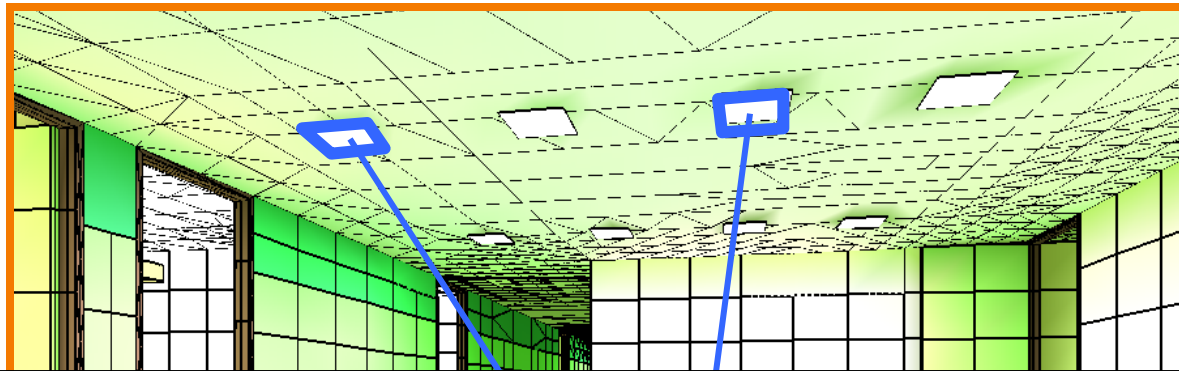




Gathering Iteration

Geometric interpretation

Iteratively gather radiosity from elements



This simulates how light distributes through the scene after we “turn the emitters on”.

Limitation:

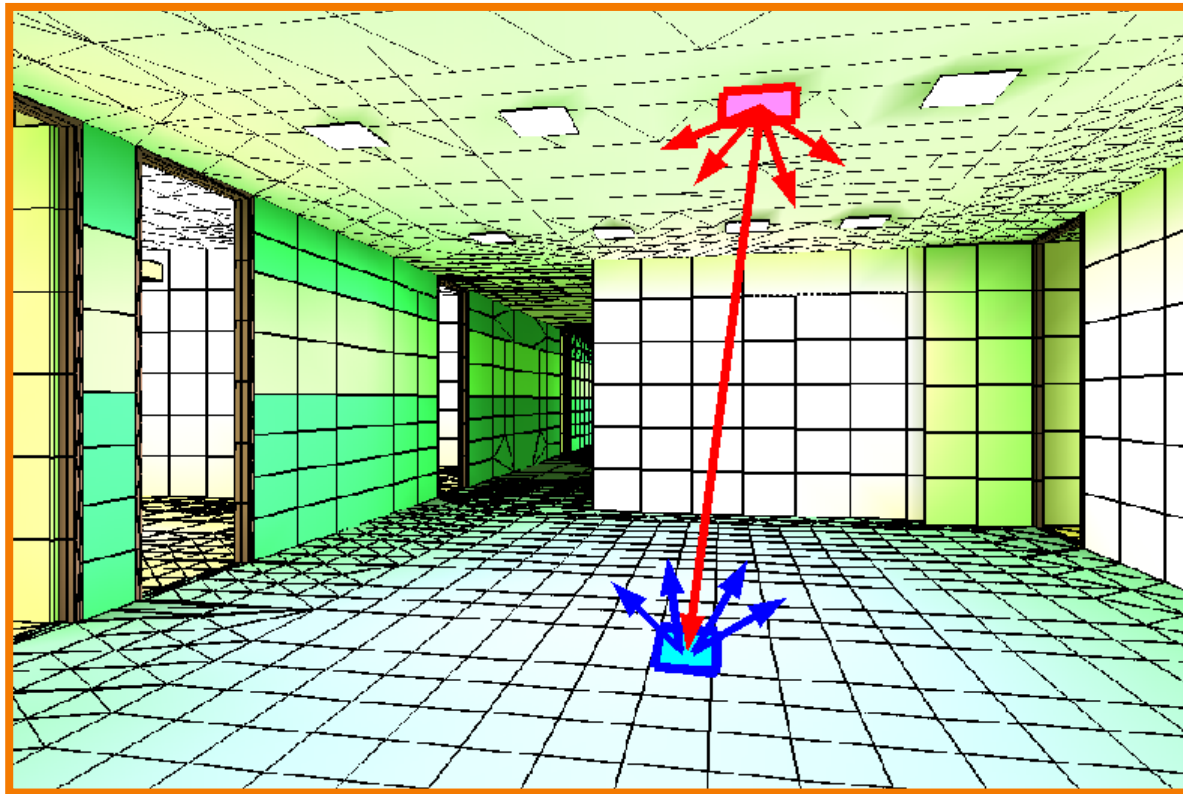
Can spend a lot of time gathering radiosity from patches that don’t contribute much.



Shooting Iteration

Geometric interpretation:

Iteratively shoot “unshot” radiosity from elements
Select shooters in order of unshot radiosity





Summary

If we could, we would compute the lighting by recursively reflecting secondary rays in all directions to compute the brightness of a single point.

Ray-Tracing:

Assume that surfaces are specular and only bounce in a single (specular) direction.

Radiosity:

Assume that surfaces are Lambertian so that they reflect light in the same way in all directions.

Reality:

Surfaces reflect in all directions, but not uniformly.