Signal Processing
From Images to Surfaces

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Benedict Brown, Szymon Rusinkiewicz, Hugues Hoppe

Estimating the Laplace-Beltrami Operator by Restricting 3D Functions. [Chuang et al., 2009]
Fast Mean-Curvature Flow via Finite Elements Tracking. [Chuang et al., 2011]
Interactive and Anisotropic Geometry Processing Using the Screened Poisson Equation. [Chuang et al., 2011]
Unconditionally Stable Shock Filters for Image and Geometry Processing. [Prada et al., 2015]
Motion Graphs for Unstructured Textured Meshes. [Prada et al., 2016]

Goal

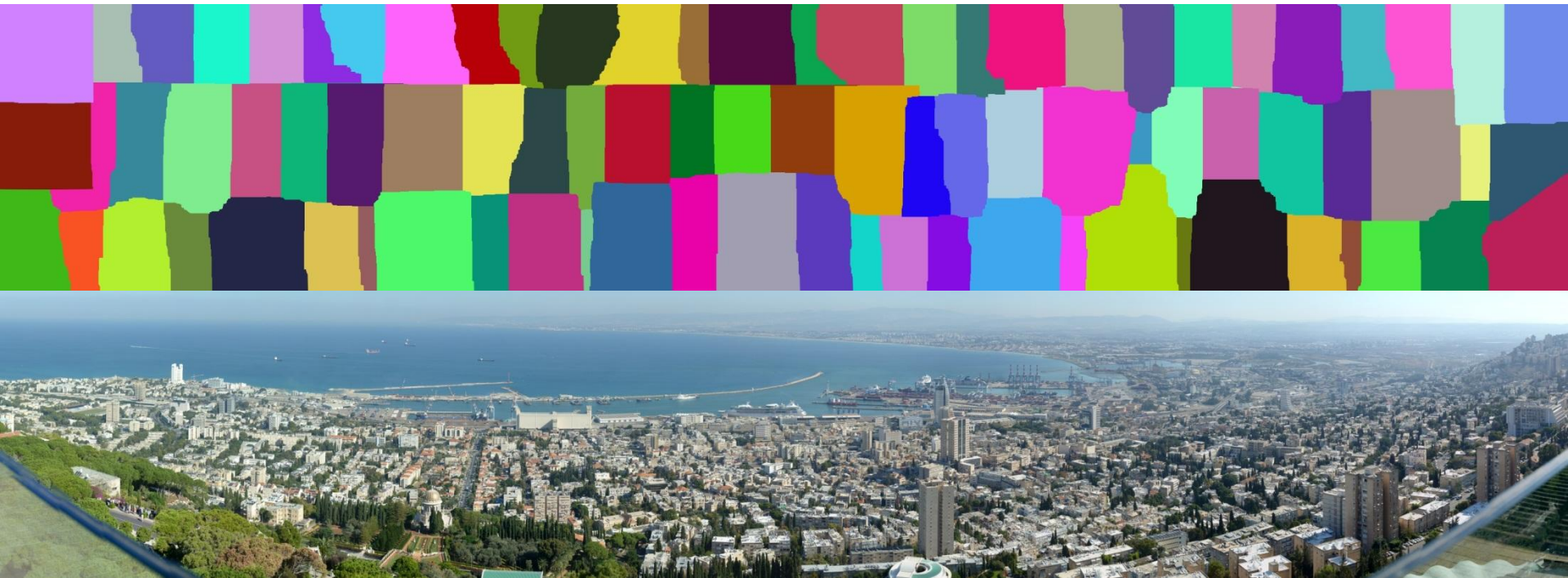Extend image-processing techniques to surfaces:

1. Gradient Domain

Goal

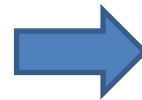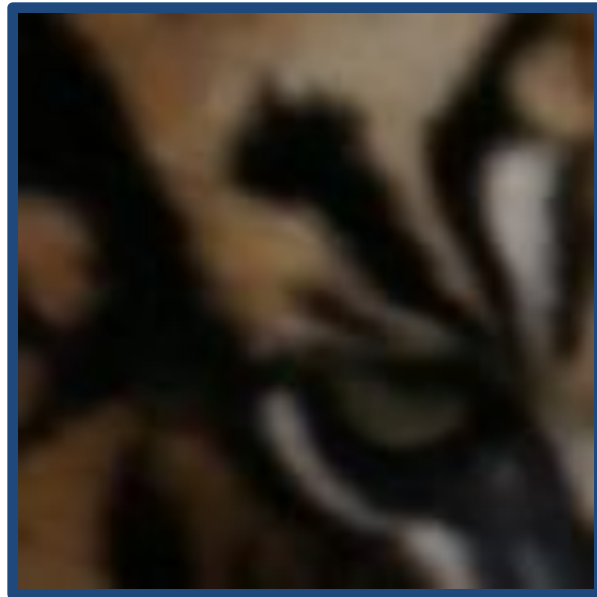Extend image-processing techniques to surfaces:

1. Gradient Domain

Goal

Extend image-processing techniques to surfaces:

1. Gradient Domain
2. Shock Filters

Goal

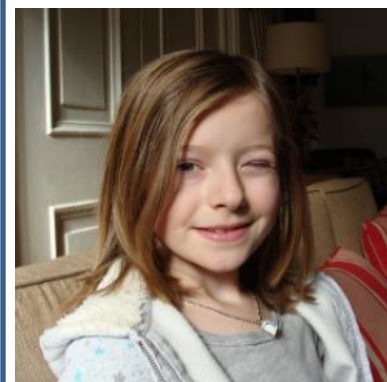Extend image-processing techniques to surfaces:

1. Gradient Domain
2. Shock Filters
3. Optical Flow

Source   Optical Flow Interpolation   Target

Moving Gradients: A Path Based Method for Plausible Image Interpolation. [Mahajan et al., 2009]

Outline

Motivation

Processing Tools

- **Screened Poisson Equation**
- **Flow Fields/Lines**

Extensions to Signals on Surfaces

Conclusion

Image-Processing Tools

1. Screened Poisson Equation:

Given a 2D domain Ω, a function g, and a vector field $\vec{v}$, *solve for the function f minimizing*:

$$E(f) = \int_\Omega \underbrace{\|f - g\|^2}_{\text{value-fitting}} + \underbrace{\alpha\|\nabla f - \vec{v}\|^2}_{\text{gradient-fitting}}$$

Image-Processing Tools

1. Screened Poisson Equation:

Given a 2D domain Ω, a function g, and a vector field $\vec{v}$, *solve for the function f minimizing*:

$$E(f) = \int_\Omega \|f - g\|^2 + \alpha\|\nabla f - \vec{v}\|^2$$

$$\Downarrow$$

$$(\mathbf{1} - \alpha \cdot \mathbf{\Delta})f = \mathbf{1} \cdot g - \alpha \cdot \mathbf{div}(\vec{v})$$

Image-Processing Tools

2a. Flow Fields/Lines:

Given a 2D domain Ω and a vector field $\vec{v}$, a *flow-line* of $\vec{v}$ is a curve γ_p such that:

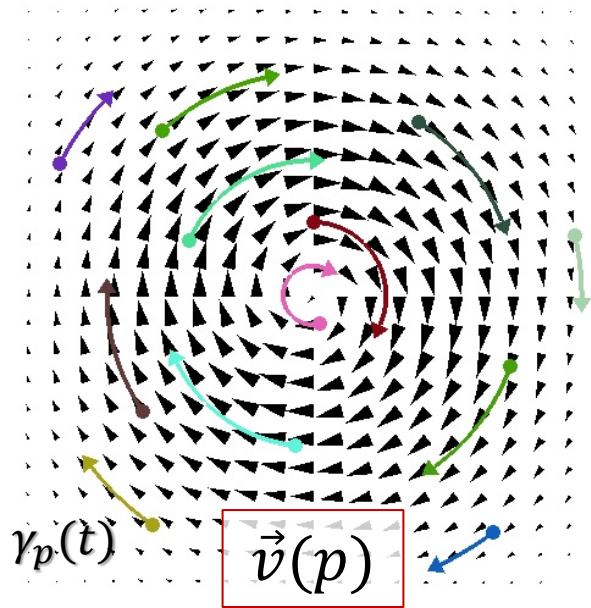$$\gamma_p(0) = p \quad \text{and} \quad \gamma_p'(t) = \vec{v}\left(\gamma_p(t)\right).$$

Image-Processing Tools

2b. Flow Fields/Lines:

Given a 2D domain Ω and a vector field $\vec{v}$, the *advection* of a function f along $\vec{v}$ is the function:

$$[\text{Adv}_{\vec{v}}(f)](p) = f\left(\gamma_p(-1)\right).$$
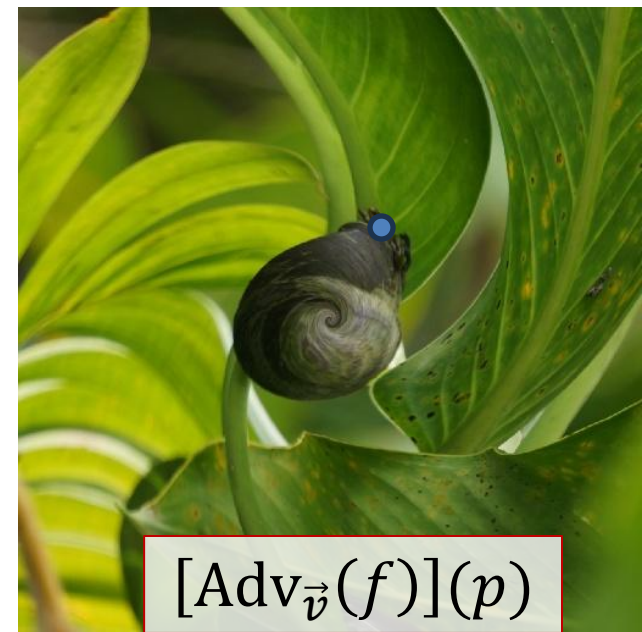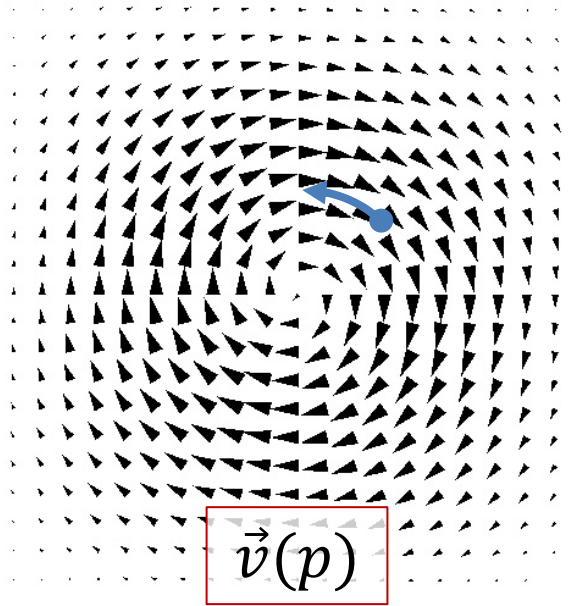

$f(p)$

$\vec{v}(p)$

$[\text{Adv}_{\vec{v}}(f)](p)$

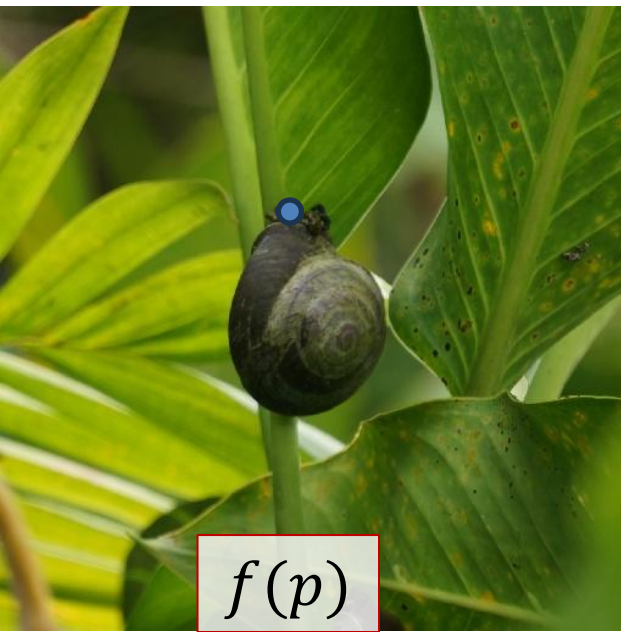# Image-Processing Tools

2c.   <u>Flow Fields/Lines</u>:

Given..., for small t we have:

$$[\text{Adv}_{t \cdot \vec{v}}(f)](p) - f(p) \approx f(p - t\vec{v}) - f(p)$$
$$\approx -t \cdot \langle \nabla f(p), \vec{v} \rangle$$

$f(p)$

$\vec{v}(p)$
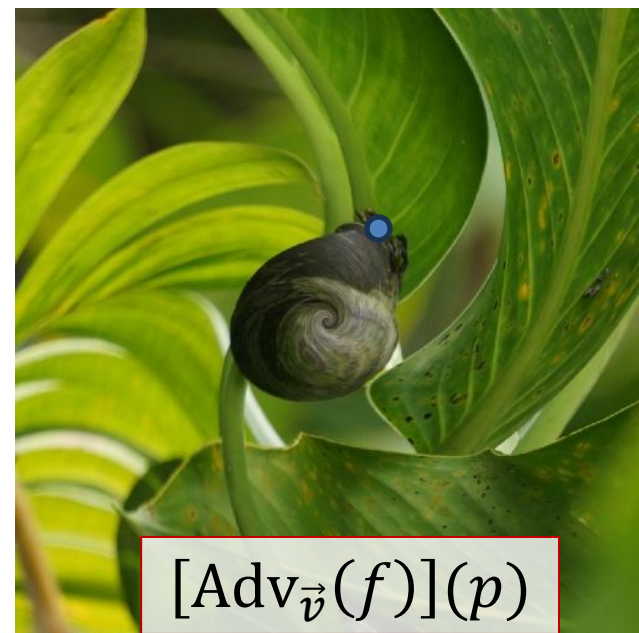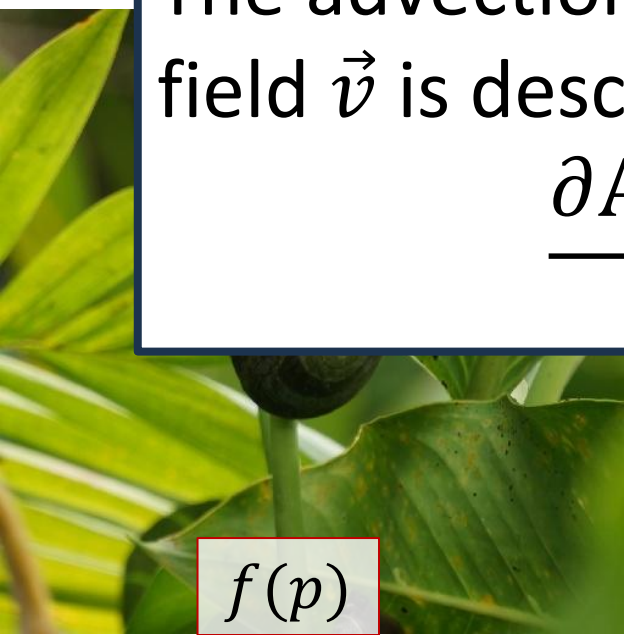
$[\text{Adv}_{\vec{v}}(f)](p)$

Image-Processing Tools

2c. Flow Fields/Lines:

Given…, for small t we have:
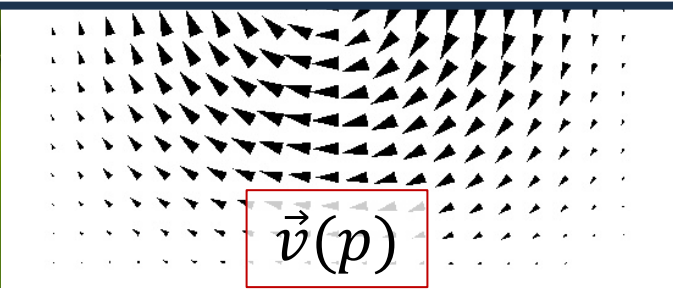
$$[\text{Adv}_{t \cdot \vec{v}}(f)](p) - f(p) \approx f(p - t\vec{v}) - f(p)$$
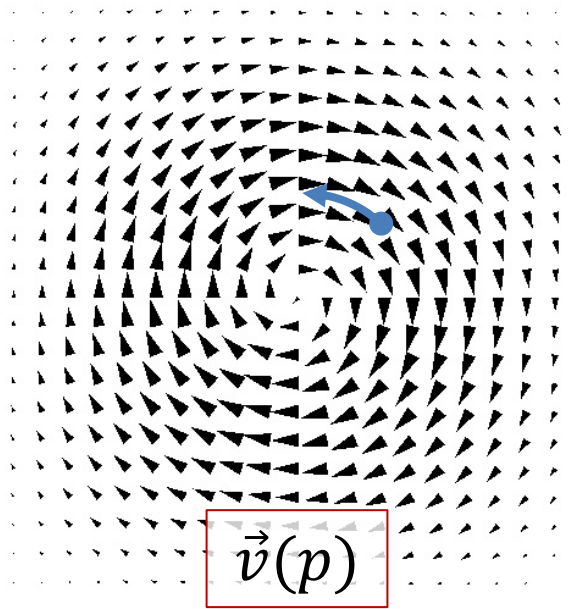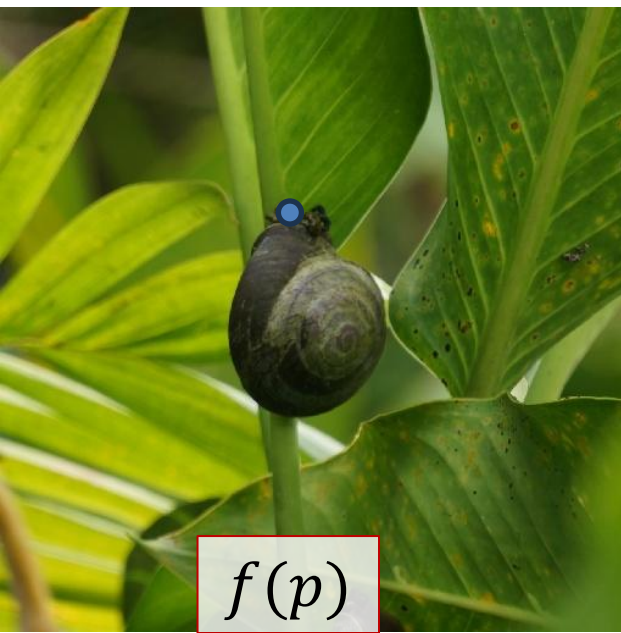$$\approx -t \cdot \langle \nabla f(p), \vec{v} \rangle$$

The advection of the signal f along the vector field $\vec{v}$ is described by the PDE:
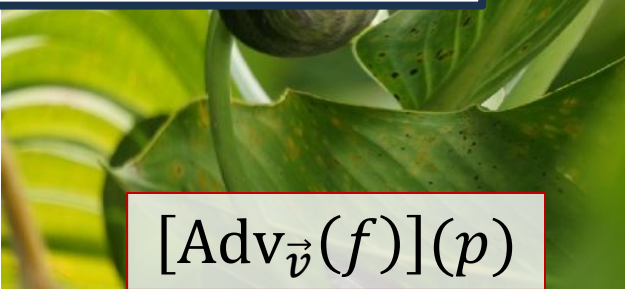
$$\frac{\partial \text{Adv}_{t \cdot \vec{v}}(f)}{\partial t} = -\langle \nabla f, \vec{v} \rangle$$

$f(p)$

$\vec{v}(p)$

$[\text{Adv}_{\vec{v}}(f)](p)$

Geometry-Processing Tools
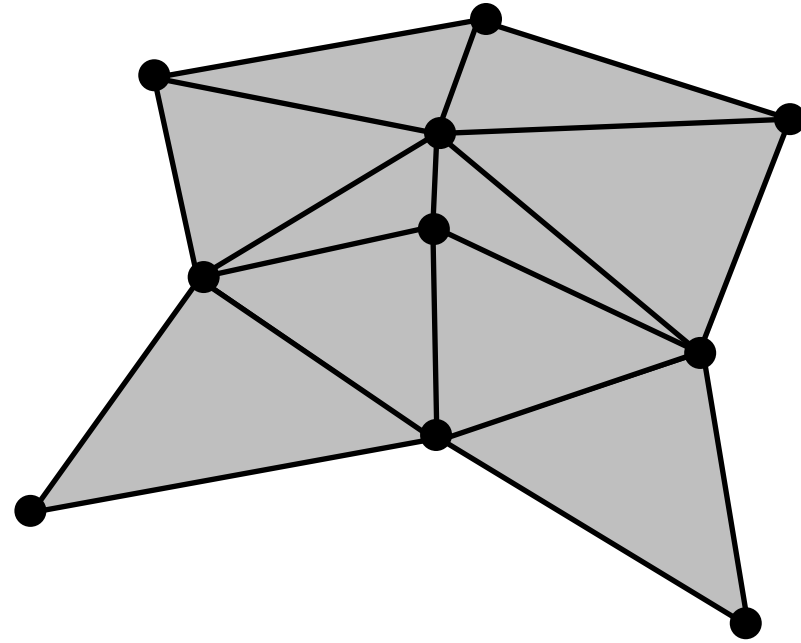
1. Screened Poisson Equation:
$$(\mathbf{1} - \alpha \cdot \mathbf{\Delta})f = \mathbf{1} \cdot g - \alpha \cdot \mathbf{div}(\vec{v})$$

On a mesh:

Geometry-Processing Tools

1. <u>Screened Poisson Equation</u>:
$$(\mathbf{1} - \alpha \cdot \mathbf{\Delta})f = \mathbf{1} \cdot g - \alpha \cdot \mathbf{div}(\vec{v})$$

On a mesh:

$f, g \rightarrow$ maps from vertices to real values

Geometry-Processing Tools

1. Screened Poisson Equation:
$$(\mathbf{1} - \alpha \cdot \mathbf{\Delta})f = \mathbf{1} \cdot g - \alpha \cdot \mathbf{div}\,\boxed{\vec{v}}$$

On a mesh:

$f, g \rightarrow$ maps from vertices to real values

$\vec{v} \rightarrow$ a map from triangles to tangent vectors

Geometry-Processing Tools

1. Screened Poisson Equation:
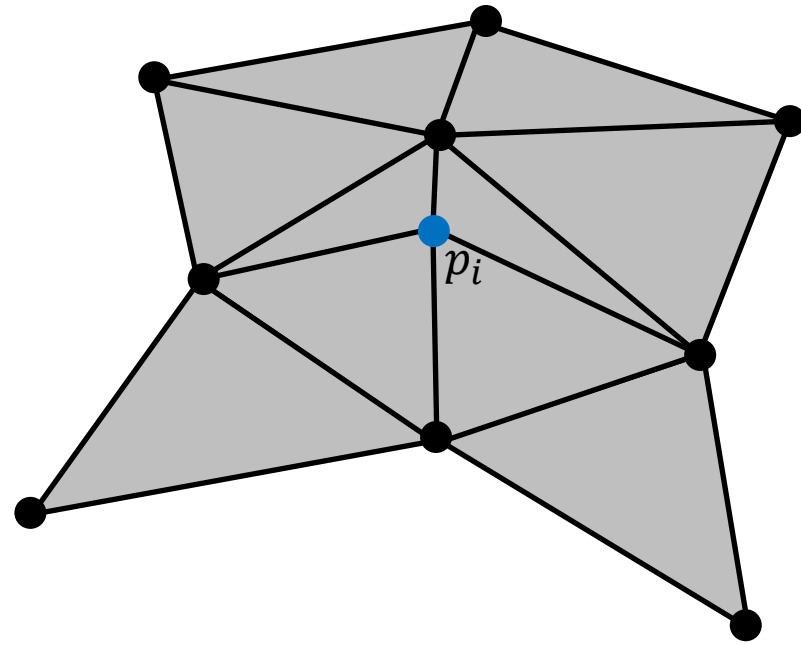 $$(\mathbf{1} - \alpha \cdot \boxed{\mathbf{\Delta}})f = \mathbf{1} \cdot g - \alpha \cdot \mathbf{div}(\vec{v})$$

On a mesh:

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$\vec{v} \rightarrow$ a map from triangles to tangent vectors

$\mathbf{\Delta} \rightarrow$ the cotan. Laplacian matrix

Geometry-Processing Tools

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$\mathbf{1} \rightarrow$ the mass-matrix
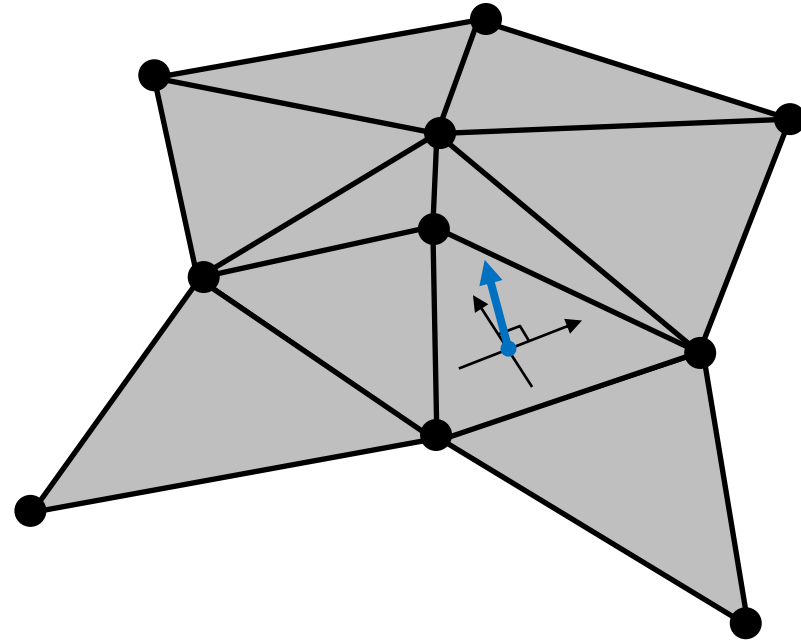
Geometry-Processing Tools
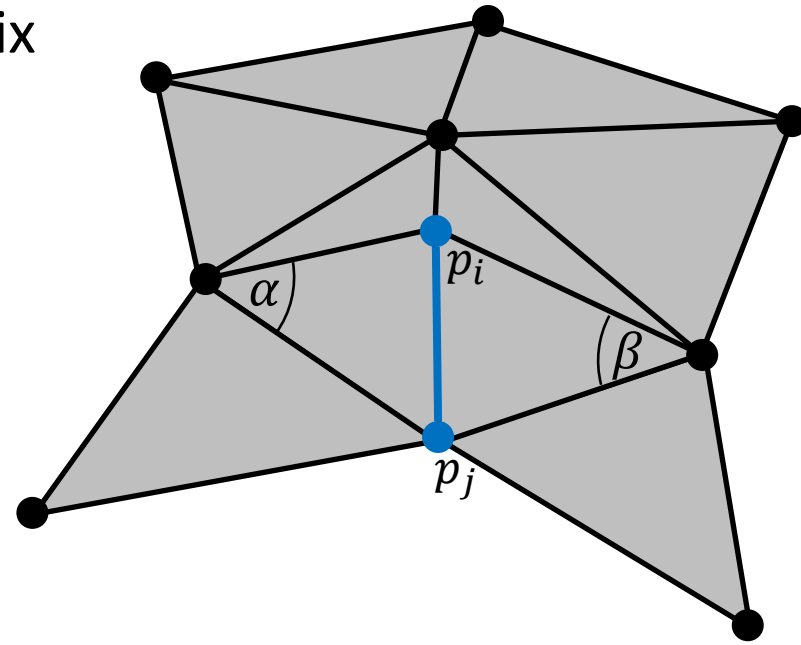
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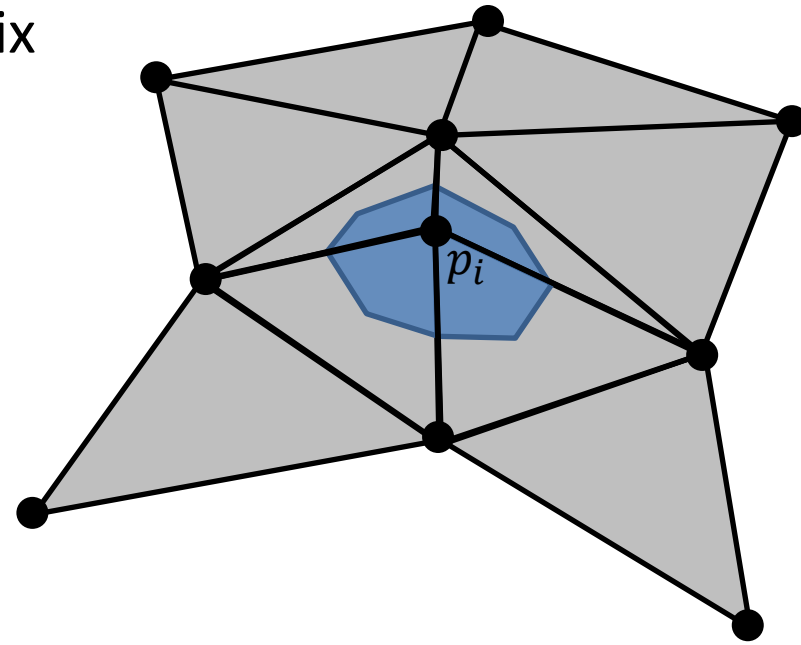
On a mesh:

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$\mathbf{\Delta} \rightarrow$ the cotan. Laplacian matrix

$\mathbf{1} \rightarrow$ the mass-matrix

$\mathbf{div} \rightarrow \mathbf{\nabla}^\top \cdot \mathbf{\Lambda}$:

$\mathbf{\Lambda}$: diagonal with triangle areas

$\mathbf{\nabla}$: the gradient operator

Geometry-Processing Tools

2. Flow Fields/Lines:

$$\gamma_p(0) = p \quad \text{and} \quad \gamma_p'(t) = \vec{v}\left(\gamma_p(t)\right)$$

Iteratively:

Sample the flow field at p.

Take a small step in a straight line along the flow direction.

Geometry-Processing Tools

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$$\gamma_p(0) = p \quad \text{and} \quad \gamma'_p(t) = \vec{v}\left(\gamma_p(t)\right)$$

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$\vec{v}(p)$
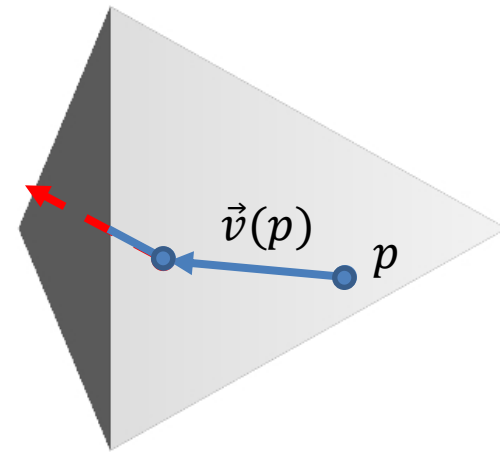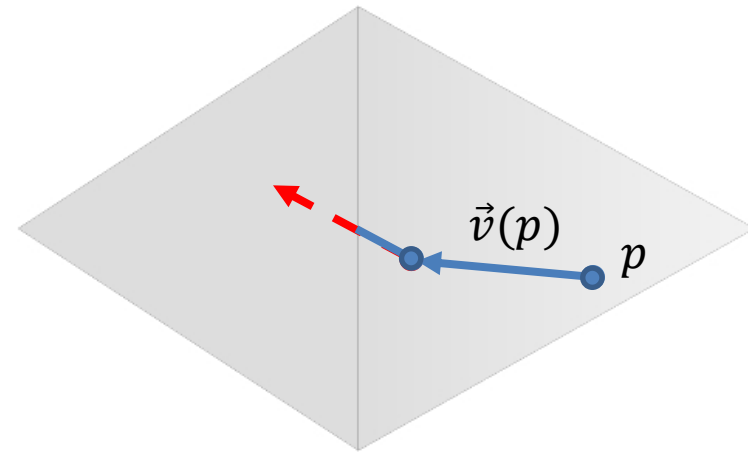
p

Geometry-Processing Tools

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Sample the flow field at p.

Take a small step in a straight
line along the flow direction.

Outline

Motivation

Tools of the Trade

Extensions to Signals on Surfaces

    – **Gradient Domain**   **[Poisson]**

    – Shock Filters   [Advection]

    – Optical Flow   [Poisson + Advection]

Conclusion

Gradient Domain (Stitching)

Different exposures $\Rightarrow$ Seams in the panorama

Gradient Domain (Stitching)

Copy interior gradients into $\vec{v}$

Set seam-crossing gradients to zero

$$f^{out} = \operatorname*{argmin}_{f:\Omega\to\mathbb{R}} \int \|\nabla f - \vec{v}\|^2 dp$$

Image(s) courtesy of Uyttendaele

Gradient Domain (Stitching)

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Gradient Domain (Sharpening)

Fit input colors: $g = f^{in}$

Scale (amplify) input gradients: $\vec{v} = \beta \cdot \nabla f^{in}$

$$f^{out} = \underset{f:\Omega\to\mathbb{R}}{\mathrm{argmin}} \int_{\Omega} \underbrace{\left\| f - f^{in} \right\|^2}_{\text{value-fitting}} + \alpha \underbrace{\left\| \nabla f - \beta \cdot \nabla f^{in} \right\|^2}_{\text{gradient-fitting}}$$

$\beta > 1$

Fourier Analysis of the 2D Screened Poisson Equation for Gradient Domain Problems. [Bhat *et al.* 2008]

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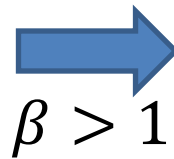

$\beta < 1$ $\beta > 1$

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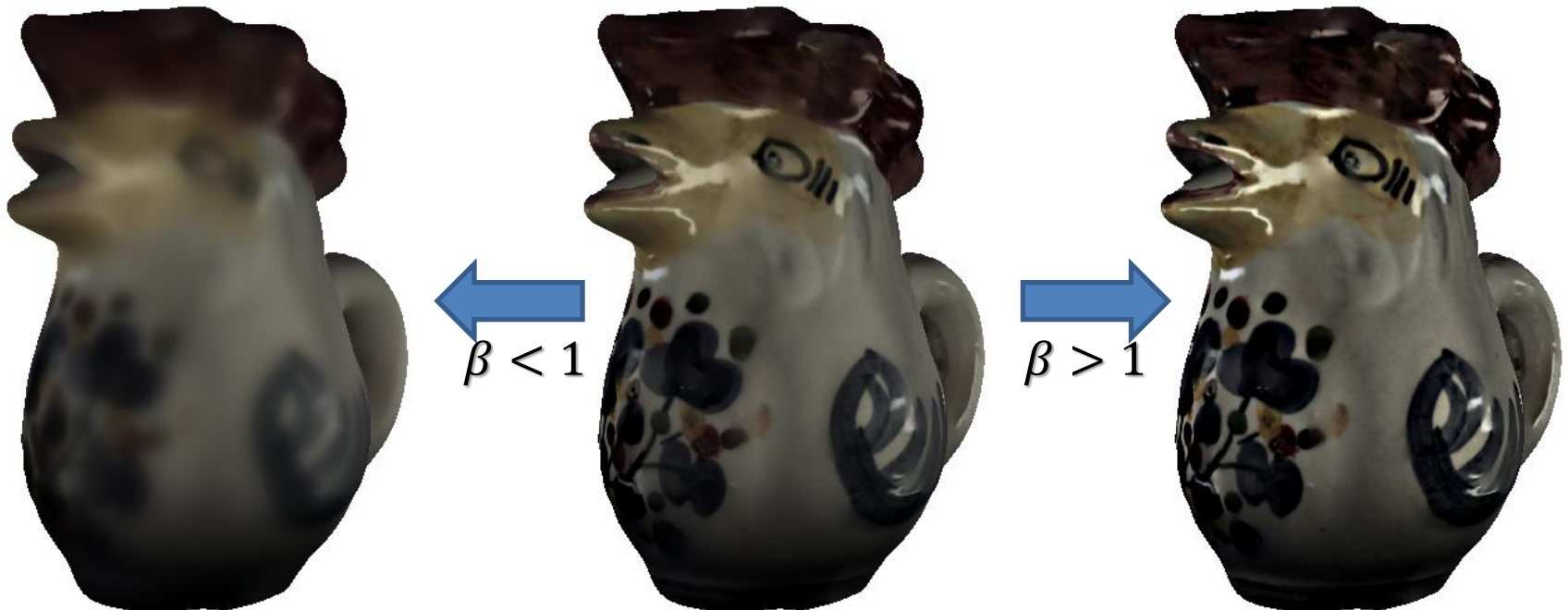

$\beta < 1$ $\qquad$ $\beta > 1$

Setting f^{in} to the <u>positions</u> of the vertices in 3D

Outline

Motivation

Tools of the Trade

Extensions to Signals on Surfaces

    – Gradient Domain   [Poisson]

    – **Shock Filters**   **[Advection]**

    – Optical Flow   [Poisson + Advection]

Conclusion

Shock Filters

[Osher and Rudin, 1990]:

Progressively sharpen a signal so that:

Extrema preserved

Zero derivative → value fixed

Edges pronounced

Concave up → value decreases

Concave down → value increases
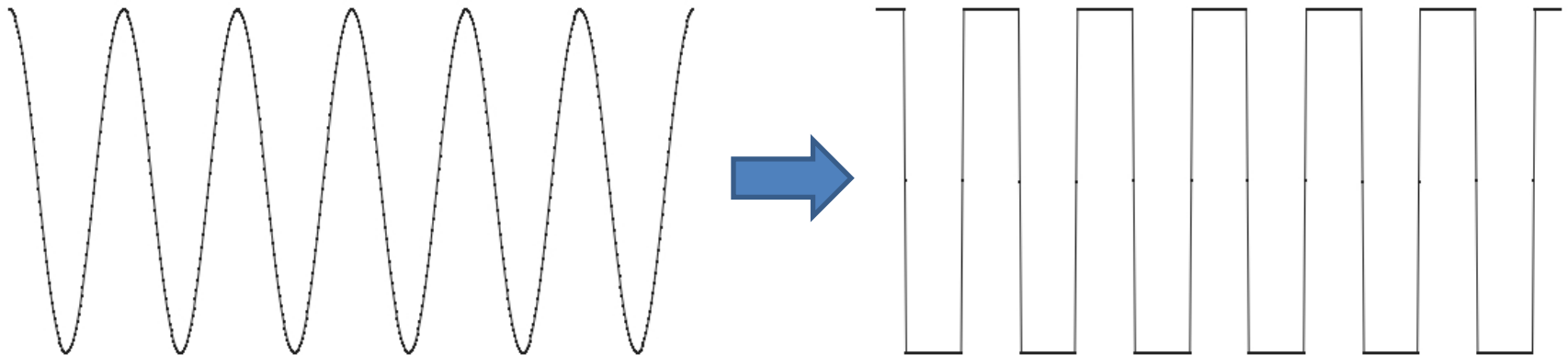
Shock Filters

<u>[Osher and Rudin, 1990]</u>:

Progressively sharpen a signal so that:

Extrema preserved → $\mathcal{F}$ vanishes with the gradient

Edges pronounced → $\mathcal{G}$ gives the convexity/concavity
along the gradient direction

[Osher and Rudin, 1990] Solve the PDE by carefully adapting step-sizes.

$$\frac{df}{dt} = \mathcal{F}(f) \cdot \mathcal{G}(f)$$

$$\mathcal{F}(f) = \|\nabla f\|^2$$

$$\mathcal{G}(f) = -\frac{\partial^2 f}{\partial(\nabla f / \|\nabla f\|)^2}$$ (Second derivative in the gradient direction)

Shock Filters

Method of Characteristics:

We can re-write the PDE:

$$\frac{df}{dt} = \mathcal{F}(f) \cdot \mathcal{G}(f)$$

$$= -\langle \nabla f, H_f \cdot \nabla f \rangle$$

This describes the advection of f along the flow:

$$\vec{v} = H_f \cdot \nabla f$$

$$\mathcal{F}(f) = \|\nabla f\|^2$$

$$\mathcal{G}(f) = -\frac{\partial^2 f}{\partial (\nabla f / \|\nabla f\|)^2} = -\frac{1}{\|\nabla f\|^2} \langle \nabla f, H_f \cdot \nabla f \rangle$$

Shock Filters

Method of Characteristics:

We can re-write the PDE:

$$\frac{df}{dt} = \mathcal{F}(f) \cdot \mathcal{G}(f)$$
$$= -\langle \nabla f, H_f \cdot \nabla f \rangle$$

This describes the advection of f along the flow:

$$\vec{v} = H_f \cdot \nabla f = \frac{1}{2} \nabla \|\nabla f\|^2$$

$$\mathcal{F}(f) = \|\nabla f\|^2$$

$$\mathcal{G}(f) = -\frac{\partial^2 f}{\partial (\nabla f / \|\nabla f\|)^2} = -\frac{1}{\|\nabla f\|^2} \langle \nabla f, H_f \cdot \nabla f \rangle$$

Shock Filters

f

Sharpen f by advecting along the flow:
$$\vec{v} = \frac{1}{2} \nabla \|\nabla f\|^2$$

Shock Filters

Sharpen f by advecting along the flow:

$$\vec{v} = \frac{1}{2}\nabla P \quad \text{w/} \quad P = \|\nabla f\|^2$$

f

$P = \|\nabla f\|^2$

$\vec{v} = \nabla P / 2$

$\underline{\text{ShockAdvect}}(\ f\ ,\ t\ )$

1. $P \leftarrow \|\nabla f\|^2$ // edge potential

2. $\vec{v} \leftarrow \frac{1}{2}\nabla P$ // flow field

3. return Advect(f , $\vec{v}$, t)

Shock Filters

Sharpen f by advecting along the flow:
$$\vec{v} = \frac{1}{2}\nabla P \quad \text{w/} \quad P = \|\nabla f\|^2$$

Values are transported:

From regions where gradient has small magnitude to regions where the gradient has large magnitude

$\Leftrightarrow$ From regions where the signal is (nearly) constant towards edges

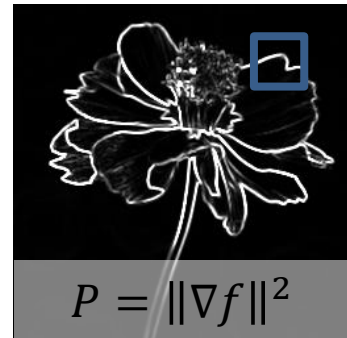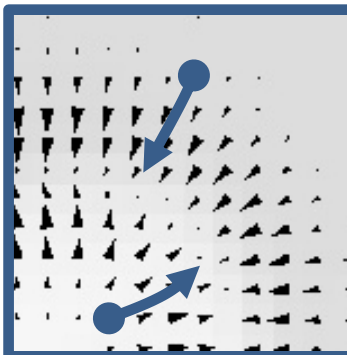$\Rightarrow$ "Piecewise constant" image with input extrema pushed to the edges.
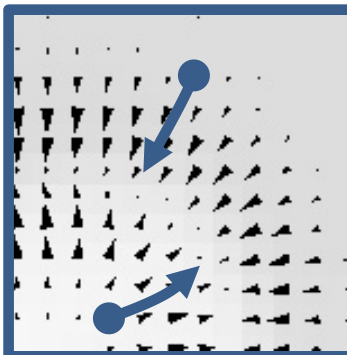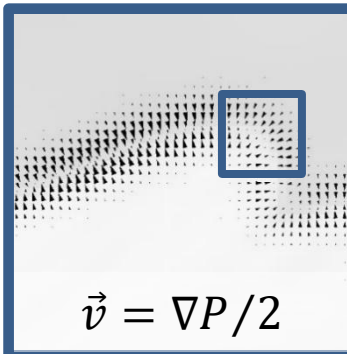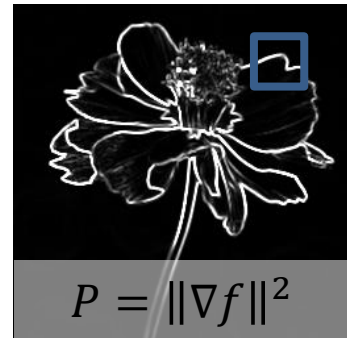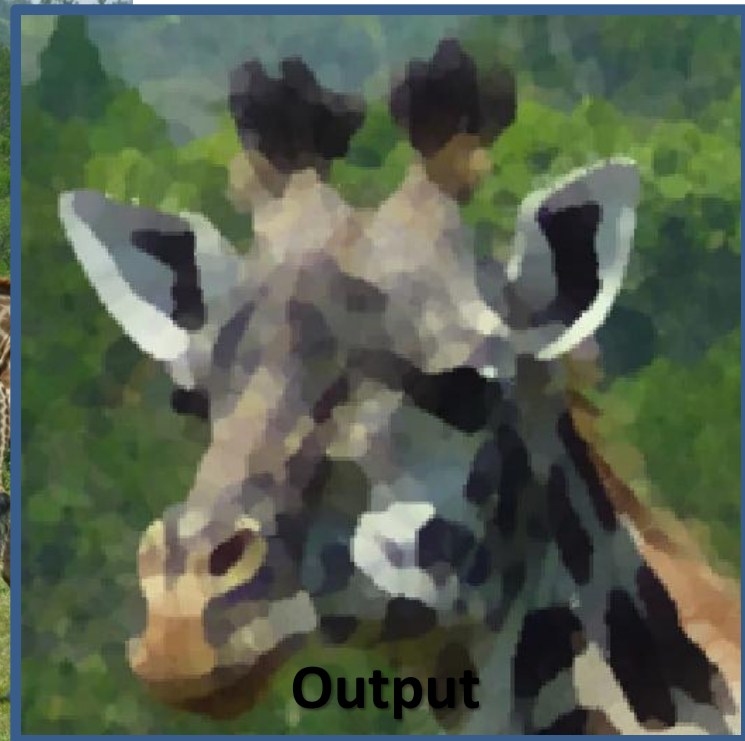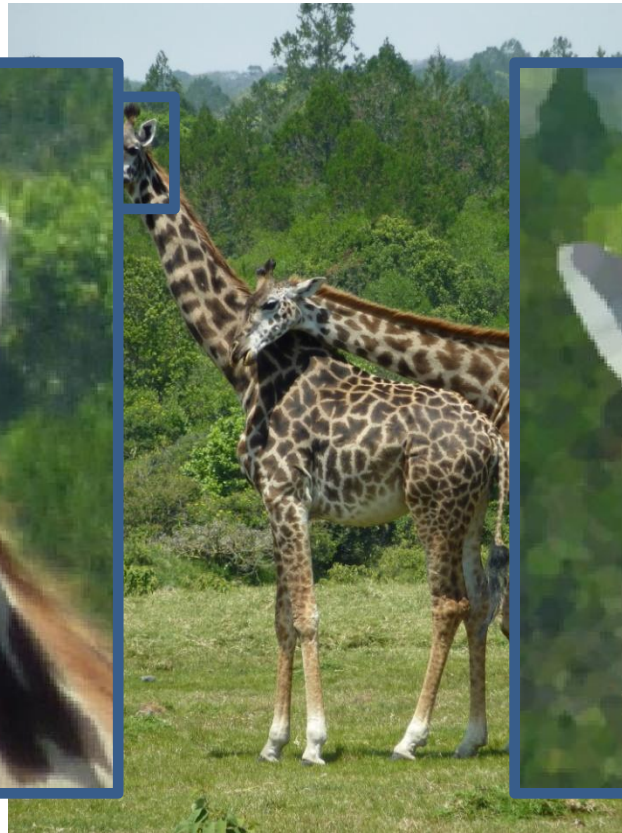
f

$P = \|\nabla f\|^2$

$\vec{v} = \nabla P/2$

Shock Filters

Sharpen f by advecting along the flow:

$$\vec{v} = \frac{1}{2}\nabla P \quad \text{w/} \quad P = \|\nabla f\|^2$$

Input

Output

Shock Filters

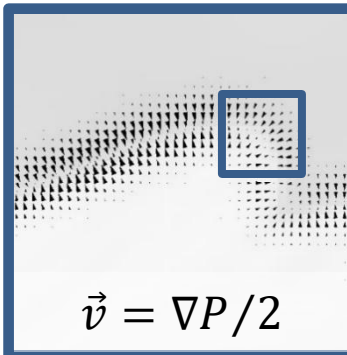Sharpen f by advecting along the flow:

$$\vec{v} = \frac{1}{2} \nabla P \quad \text{w/} \quad P = \|\nabla f\|^2$$

Input

Output

Shock Filters

Sharpen f by advecting along the flow:

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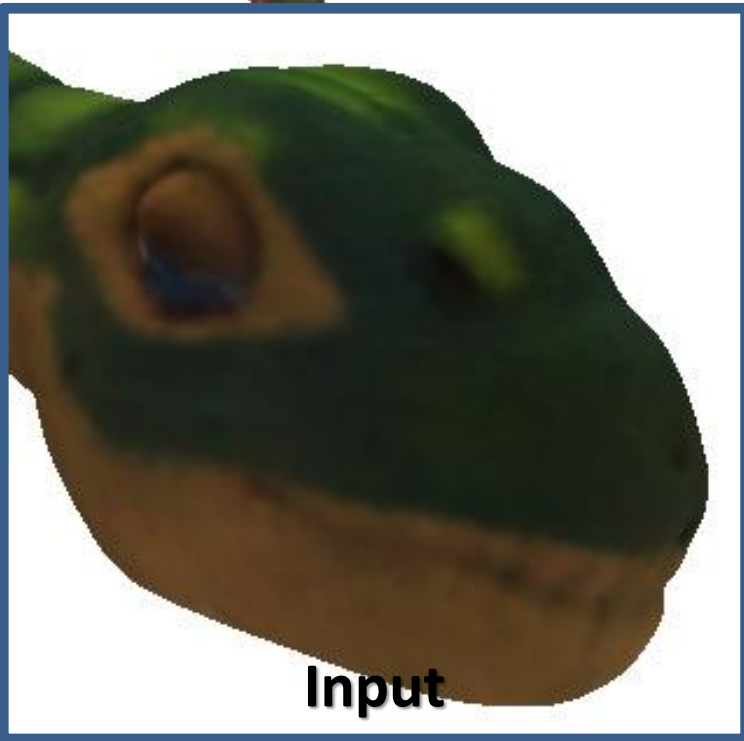
Setting f to the <u>normals</u> of the vertices

Outline

Motivation

Tools of the Trade

Extensions to Signals on Surfaces

    – Gradient Domain   [Poisson]

    – Shock Filters   [Advection]

    – **Optical Flow**   **[Poisson + Advection]**

Conclusion

Optical Flow

<u>Video Textures</u>:

Given a video, generate "a continuous, infinitely varying stream of video images".

Video Textures. [Schödl, Szeliski, Salesin, and Essa, 2000]

Optical Flow

Extend video textures [Schödl et al., 2000] to 3D:

Optical Flow

Extend video textures [Schödl et al., 2000] to 3D:

- Identify similar windows in the video

- Interpolate geometries

- Interpolate textures

Optical Flow

Image Interpolation:

Target

Optical Flow

Image Interpolation (Linear):

Linear interpolation causes ghosting.
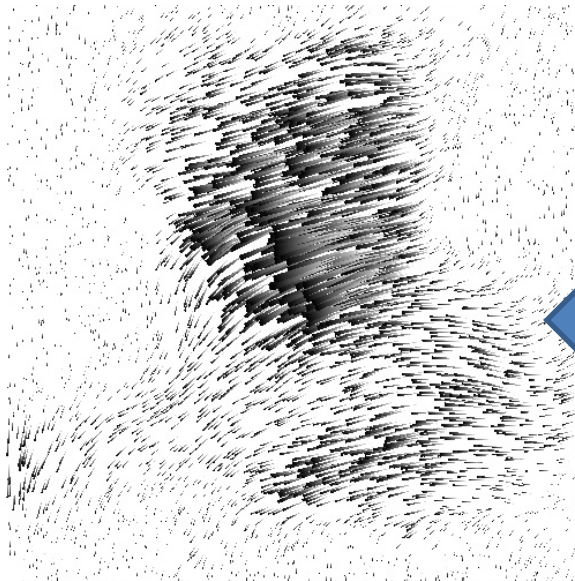

Target

Linear Blend

Optical Flow

Image Interpolation (Advected):

Estimate optical flow field.

Optical Flow Field   Target   Linear Blend

Optical Flow

Image Interpolation (Advected):

Estimate optical flow field.

Advect forward/backward and blend.

Optical Flow Blend   Target   Linear Blend

Optical Flow

<u>Brightness Constancy [Lucas and Kanade, 1981]</u>:

Solve for $\vec{v}$ that advects the source/target towards each other by minimizing:

$$E(\vec{v}) = \|\mathrm{Adv}_{-\vec{v}}(f^t) - \mathrm{Adv}_{\vec{v}}(f^s)\|^2$$
$$\approx \|(f^t - \langle \nabla f^t, -\vec{v} \rangle) - (f^s - \langle \nabla f^s, \vec{v} \rangle)\|^2$$
$$= \|(f^t - f^s) + \langle \nabla(f^t + f^s), \vec{v} \rangle\|^2$$

<u>Estimate $\vec{v}$ hierarchically (coarse-to-fine)</u>:

Advance the source/target along $\vec{v}$

Solve for the correcting flow

Incorporate the correcting flow into $\vec{v}$

Advance to the next level of the hierarchy

Optical Flow

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Smooth signals/vector-fields are implicitly <u>mandated</u> by working in a space that does not have high-frequencies.

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Optical Flow

Scale Space Formulation:

Smooth solutions are explicitly <u>encouraged</u> by:

Smoothing the source and target at each level:

$$E\left(\tilde{f}^{s/t}\right) = \underbrace{\left\|\tilde{f}^{s/t} - f^{s/t}\right\|^2}_{\text{fitting term}} + \underbrace{\frac{\alpha}{4^l}\left\|\nabla\tilde{f}^{s/t}\right\|^2}_{\substack{\text{level-weighted} \\ \text{smoothness term}}}$$

Optical Flow

Scale Space Formulation:

Smooth solutions are explicitly encouraged by:

Smoothing the source and target at each level:

$$E\big(\tilde{f}^{s/t}\big) = \big\|\tilde{f}^{s/t} - f^{s/t}\big\|^2 + \frac{\alpha}{4^l}\big\|\nabla\tilde{f}^{s/t}\big\|^2$$

Incorporating a smoothness term in the energy:

$$E(\vec{v}) = \big\|\mathrm{Adv}_{\vec{v}}\big(\tilde{f}^s\big) - \mathrm{Adv}_{-\vec{v}}\big(\tilde{f}^t\big)\big\|^2 + \frac{\alpha}{4^l}\|\nabla\vec{v}\|^2$$

brightness constancy term   level-weighted smoothness term

Optical Flow

Scale Space Formulation:

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$$E\big(\tilde{f}^{s/t}\big) = \big\|\tilde{f}^{s/t} - f^{s/t}\big\|^2 + \frac{\alpha}{4^l}\big\|\nabla\tilde{f}^{s/t}\big\|^2$$

Incorporating a smoothness term in the energy:

$$E(\vec{v}) = \big\|\text{Adv}_{\vec{v}}\big(\tilde{f}^s\big) - \text{Adv}_{-\vec{v}}\big(\tilde{f}^t\big)\big\|^2 + \frac{\alpha}{4^l}\|\nabla\vec{v}\|^2$$

Solve two Poisson equations per level.*

*In the second, the Laplacian is the vector-field (Hodge) Laplacian.

Optical Flow

Texture Interpolation:

Source   Target   Synthesized

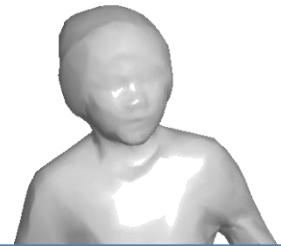# Optical Flow

Texture Interpolation:

Source   Target   Synthesized
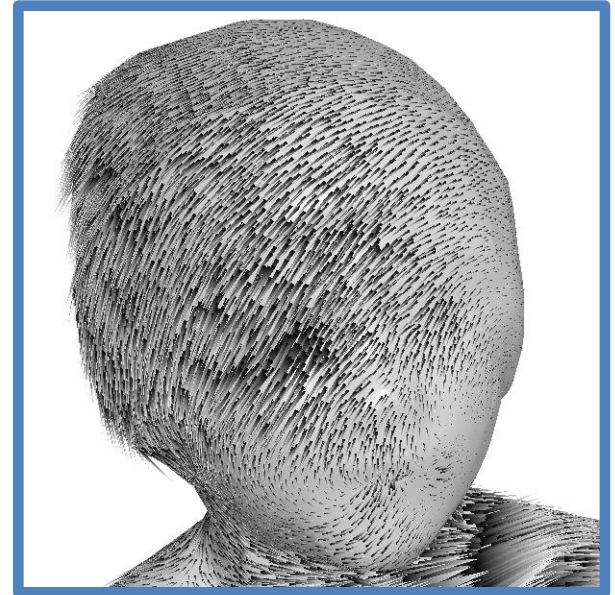
Optical Flow

Texture Interpolation:

Source   Target   Synthesized

Optical Flow

Optical Flow Field

Texture Interpolation:

Source

Target

Synthesized

Optical Flow

Texture Interpolation:

Linear Blend

Optical Flow Blend

Outline

Motivation

Tools of the Trade

Extensions to Signals on Surfaces

Conclusion

Conclusion

Extend fundamental image-processing operators to the context of surfaces
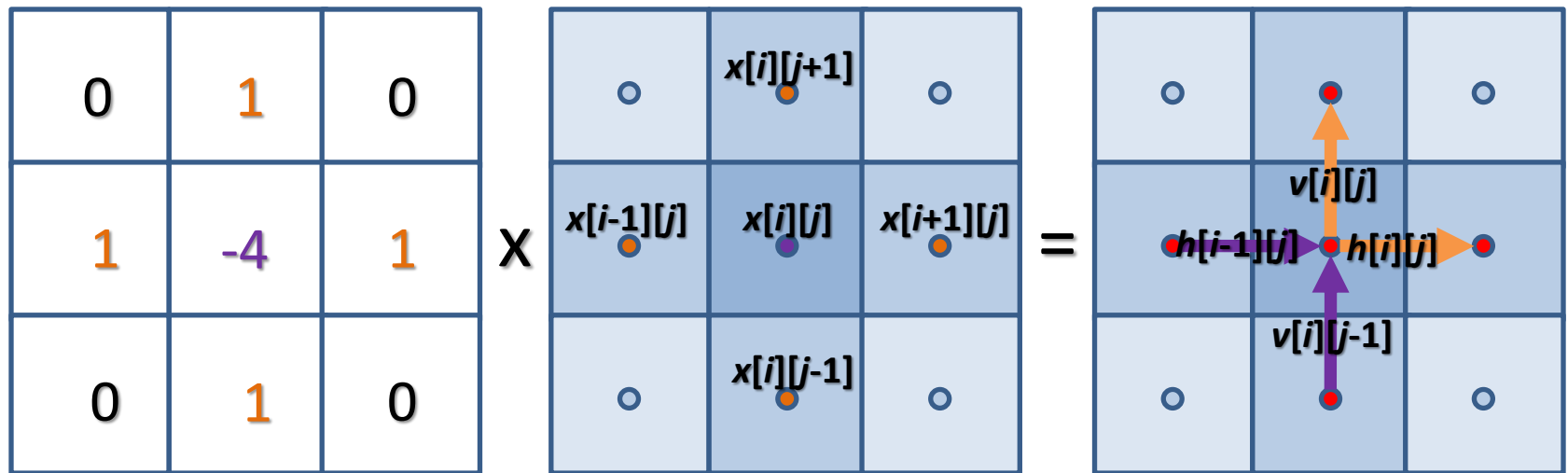[Differential Operators / Flows]

Much of the heavy lifting has already been done for us

Well-established image-processing techniques carry over
[Gradient Domain / Shock Filters / Optical Flow]

Conclusion

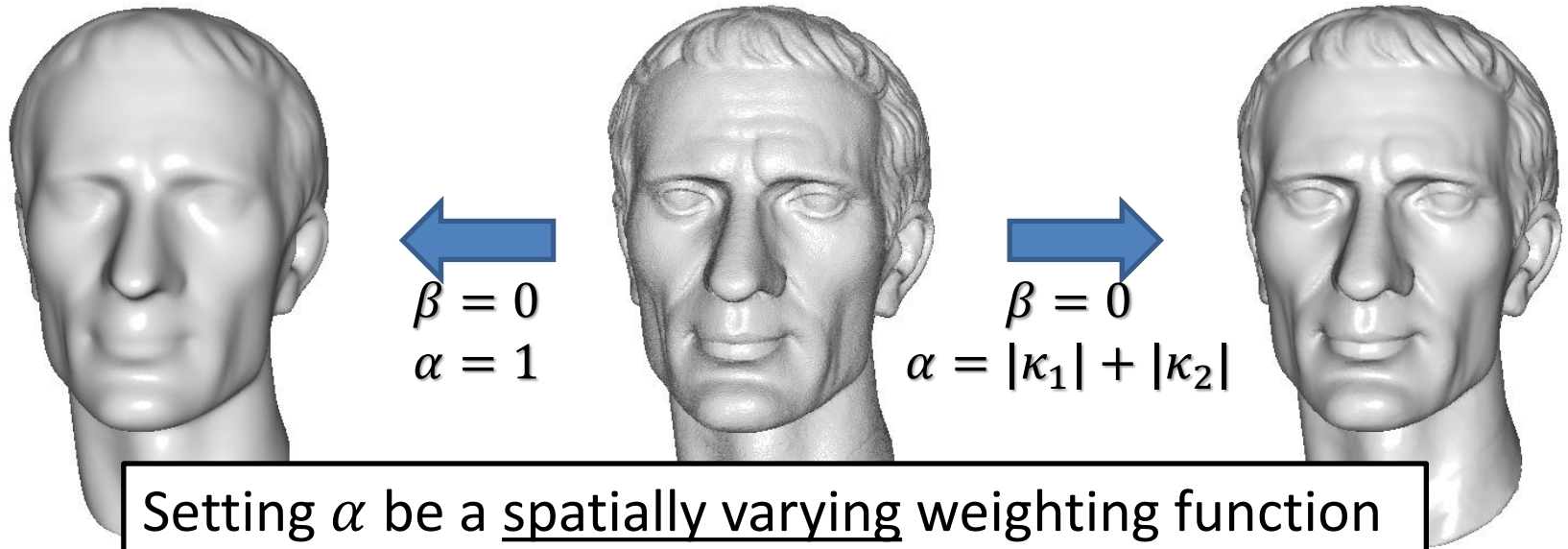Working with images makes simple things easier.

Fixed stencil Laplacian / Gradient / Divergence
Parallelization / Out-of-Core Streaming
Fast Fourier Transform

Conclusion

Working with images makes simple things easier.

Working with meshes makes hard things easier.

$$f^{out} = \underset{f:\Omega\to\mathbb{R}}{\mathrm{argmin}} \int_\Omega \alpha\|f - f^{in}\|^2 + \|\nabla f - \beta \cdot \nabla f^{in}\|^2$$

$\beta = 0$
$\alpha = 1$

$\beta = 0$
$\alpha = |\kappa_1| + |\kappa_2|$

Setting α be a <u>spatially varying</u> weighting function

Conclusion

Working with images makes simple things easier.

$\beta = 0$
$\alpha = 1$

$\beta = 0$
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Setting α be a spatially varying weighting function