Surface Reconstruction

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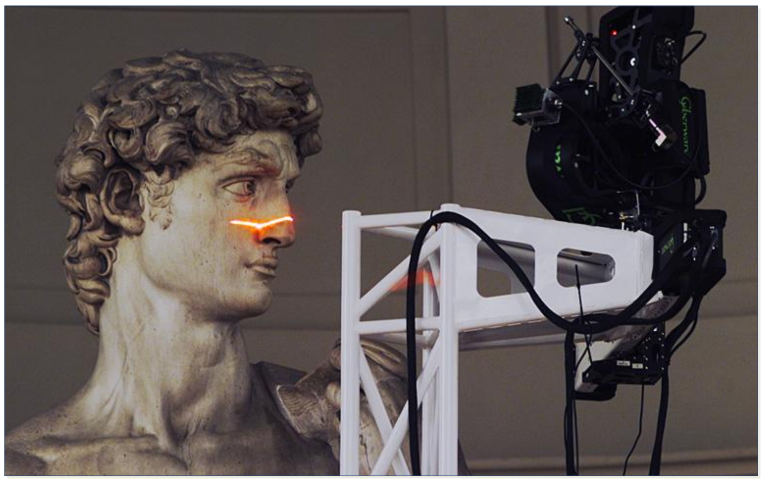
(601.457/657)

Motivation

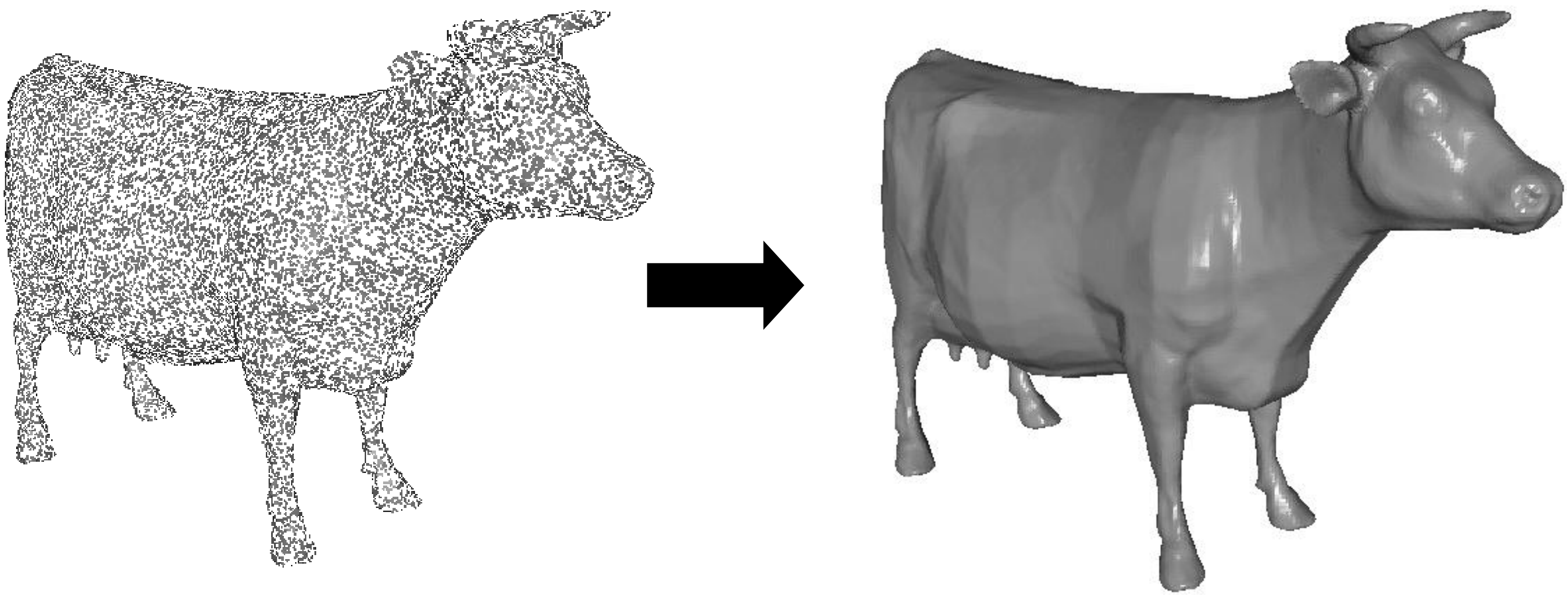
3D Scanners are ubiquitous (and cheap)



[Images courtesy of Rusinkiewicz, Strecha, createdigitalmotion.com, and NextEngine]

Motivation

Merged scans typically consist of un/semi-structured sets of points that need to be connected into a single (water-tight) model.



Related Work

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…/… [403]

[List courtesy of Alliez]

Related Work

Classification:

- Approach:
  - Computational Geometry
  - Implicit Surfaces
- Input:
  - Structured vs. Unstructured
  - Oriented vs. Unoriented
- Output:
  - Water-tight vs. Surface with Boundary

Related Work

Classification:

- Computational Geometry (Unoriented Points)
  - Use input to partition space
  - Use a subset of the partition to define the shape
- Implicit Surfaces (Oriented Points)
  - Fit implicit function to the input
  - Extract iso-surface

Outline

Introduction

Preliminaries

- Convex Hulls
- Delaunay Triangulations
- Voronoi Diagrams
- Medial Axes

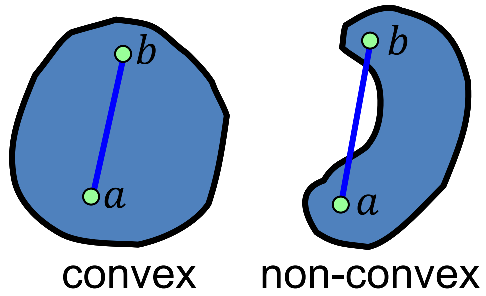
A sampling of methods

Why is reconstruction hard?

Computational Geometry

Convex Hulls:

A set S is *convex* if for any two points $a, b \in S$, the line segment between a and b is also in S.



[Notes courtesy of Alliez]

Computational Geometry

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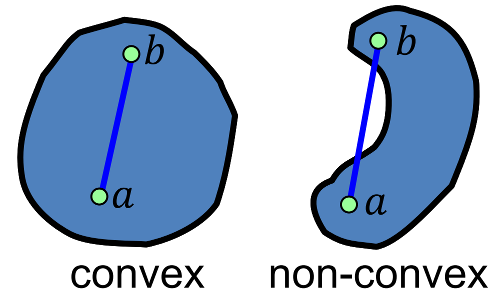
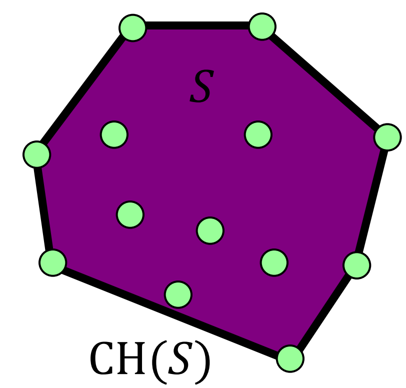


The *convex hull* of a set of points is the smallest convex set containing S.



[Notes courtesy of Alliez]

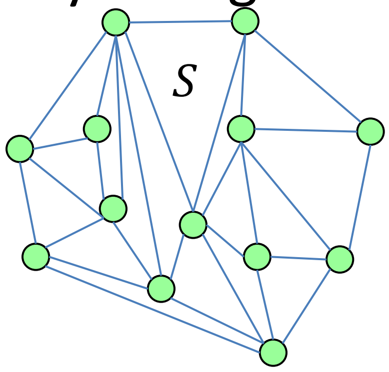
Computational Geometry

Triangulation:

A *triangulation* of a set of sites/points S a decomposition of the convex hull of the points into triangles, whose vertex set is the set of sites/points.

There are many ways to triangulate the set S.

Not all are equally "good" (e.g. can have skinny triangles with small angles)



[Notes courtesy of Alliez]

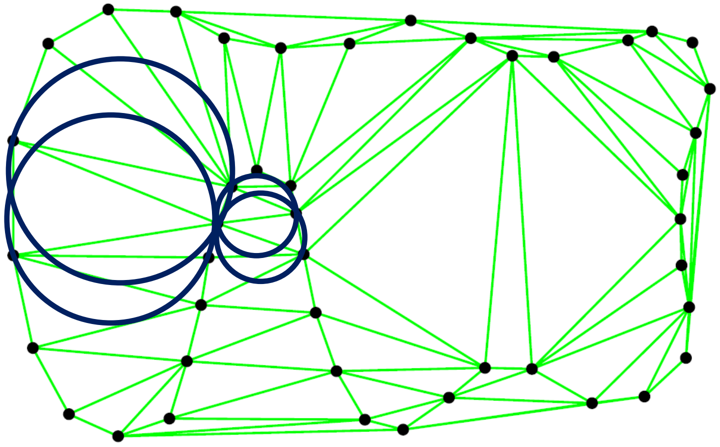
Computational Geometry

Delaunay Triangulation:

A *Delaunay Triangulation* of a set of sites S is a triangulation of S such that the circumscribing circle of any triangle contains no other site in S*.

Compactness Property:

This triangulation maximizes the minimum angle.



[*Assuming general position]

[Notes courtesy of Alliez]

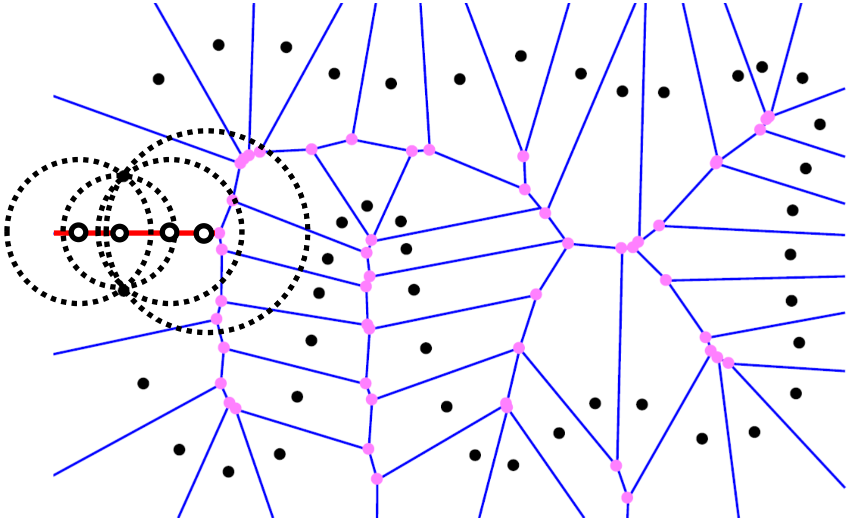
Computational Geometry

<u>Voronoi Diagrams</u>:

The *Voronoi Diagram* of S is a partition of space into regions $\mathrm{VD}(s)$, with $s \in S$, such that all points in $\mathrm{VD}(s)$ are closer to s than to any other site.

Edges are equidistant from the two sites in the incident cells.

For each edge point there is an empty circle, centered at the point, only touching the sites in the two incident cells.



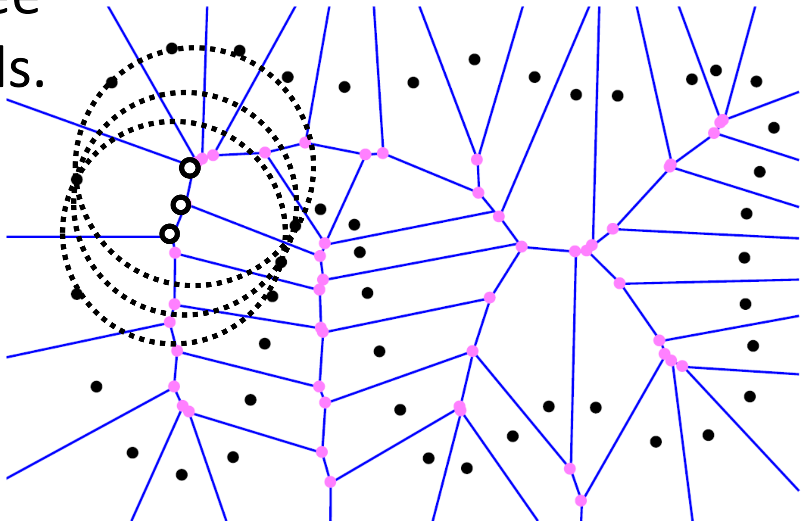
[Notes courtesy of Alliez]

Computational Geometry

Voronoi Diagrams:

The *Voronoi Diagram* of S is a partition of space into regions $\mathrm{VD}(s)$, with $s \in S$, such that all points in $\mathrm{VD}(s)$ are closer to s than to any other site.

- Vertices are equidistant from three (or more) sites in the incident cells.
- For a vertex, can draw an empty circle, centered at the vertex, that just touches the sites in the three (or more) incident cells.



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Computational Geometry

Voronoi Diagrams:

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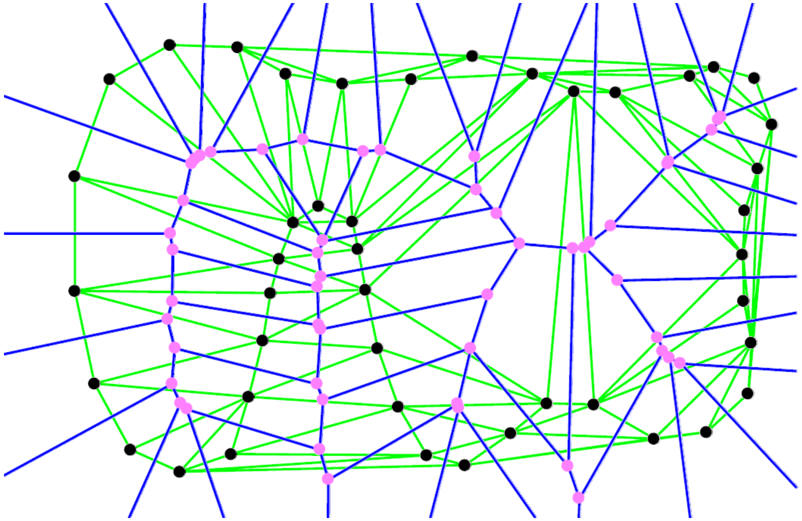
Duality:

Each Voronoi vertex is in one-to-one correspondence with a Delaunay triangle.

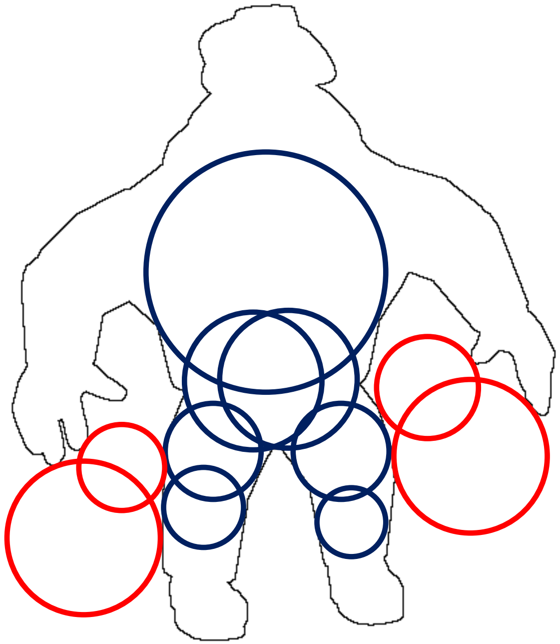


[Notes courtesy of Alliez]

Computational Geometry

<u>Medial Axis</u>:

For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.

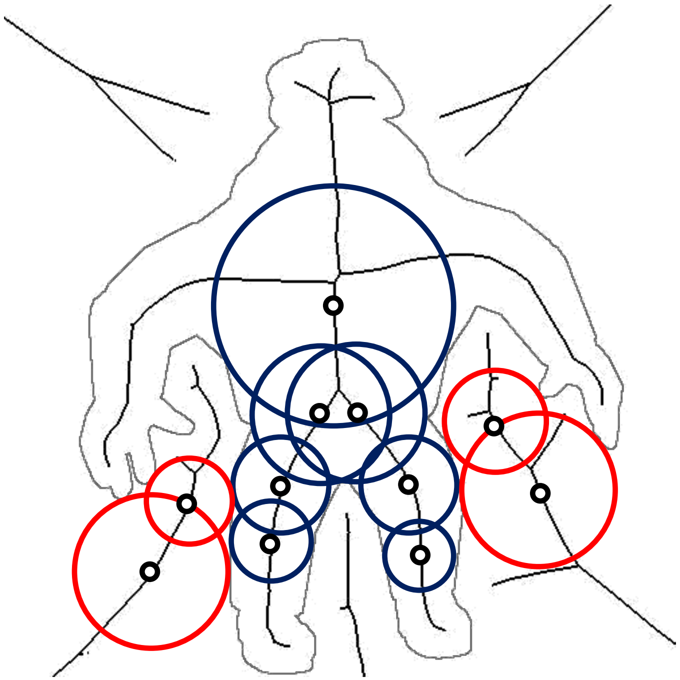


Computational Geometry

Medial Axis:

For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.

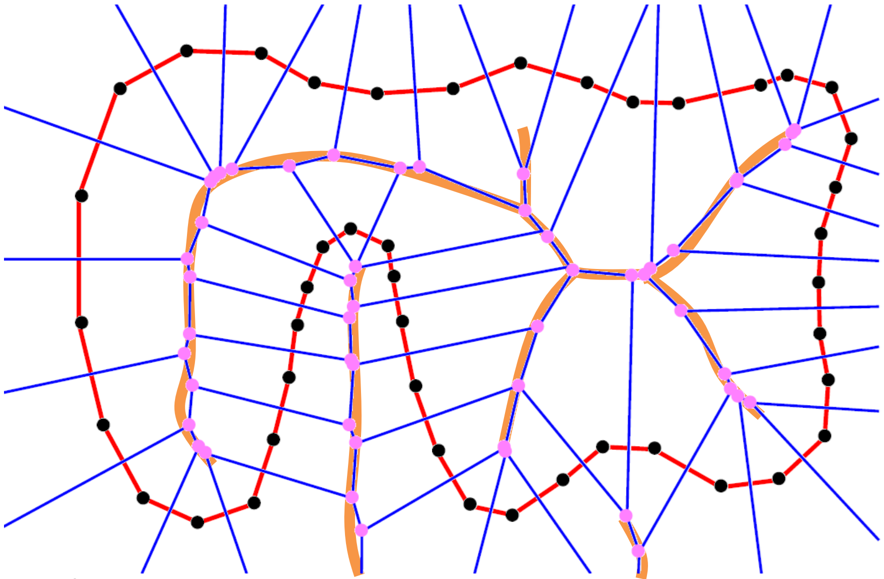
The centers of all such balls make up the *medial axis/skeleton*.



Computational Geometry

Observation in 2D*:

For a reasonable point sample, the medial axis is well-sampled by the Voronoi vertices.



*In 3D, this is true for a subset of the Voronoi vertices – the *poles*.

Outline

- Introduction
- Preliminaries
- A sampling of methods
  - **Space Partitioning**
  - Crust
  - … from Unorganized Points
  - Poisson Reconstruction
- Why is reconstruction hard?

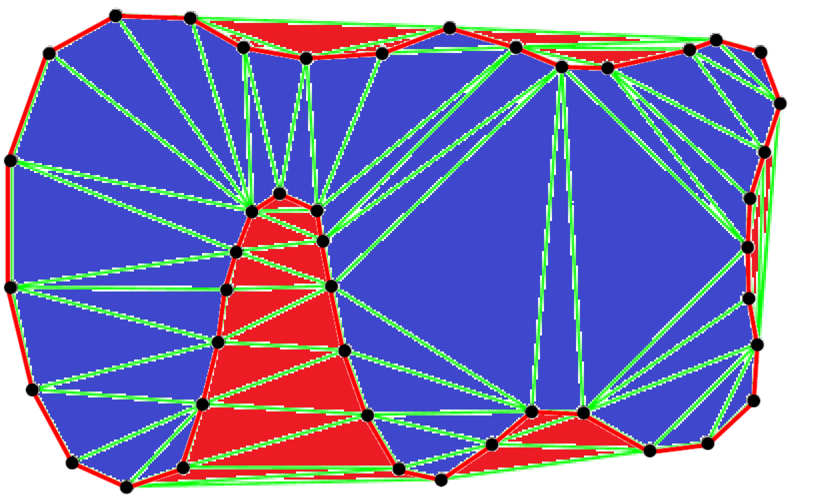
Computational Geometry (Space Partitioning, Crust)

Implicit Surfaces (… from Unorganized Points, Poisson Reconstruction)

Space Partitioning

Given a set of points, we can construct the Delaunay triangulation.

⇒ **If** we could label each triangle as inside/outside, then the surface of interest is the set of edges that lie between inside and outside triangles.

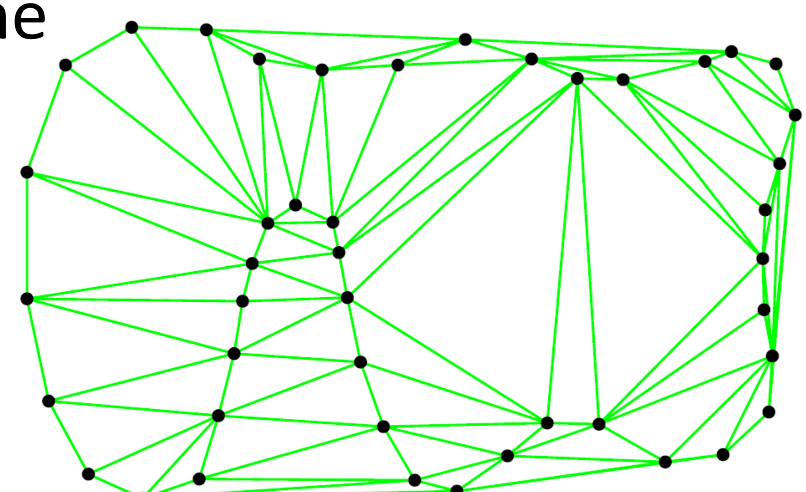


Space Partitioning

Q: How should we assign labels?

A: Spectral Partitioning [Kolluri *et al*. 2004]

1. **Local**: Assign a weight to each (interior) edge indicating if the two triangles should have the same label.
2. **Global**: Evenly partition the triangles, minimizing the sum of the weights along partitioning edges.



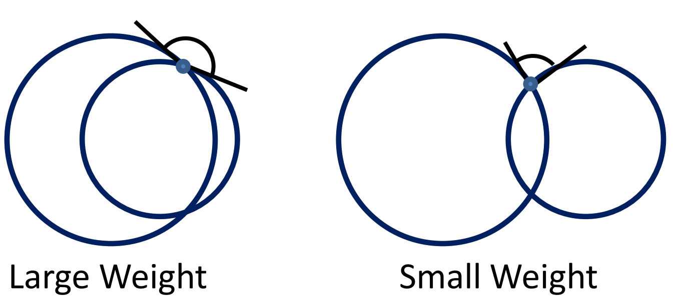
Space Partitioning

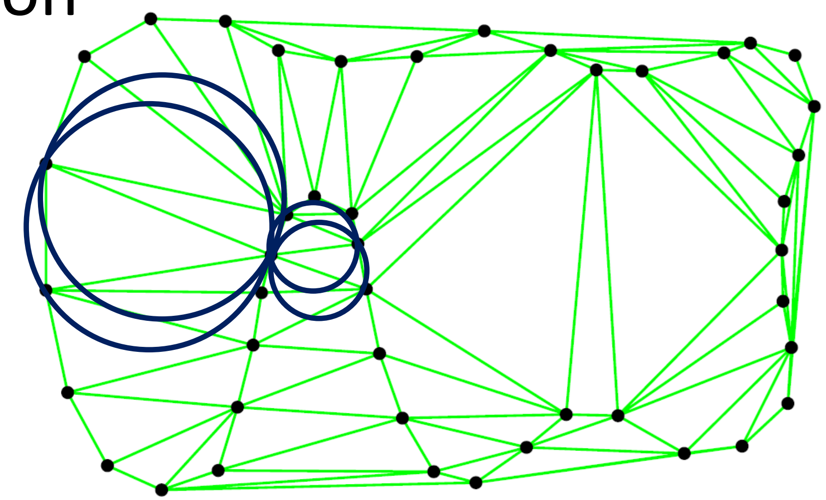
[Local] Assign Edge Weights:

Q: When are triangles on opposite sides of an edge likely to have the same label?

A: If the triangles are on the same side, their circumscribing circles intersect deeply.

⇒ Use the angle of intersection to set the weight.





Outline

Introduction

Preliminaries

A sampling of methods

- Space Partitioning
- **Crust**
- … from Unorganized Points
- Poisson Reconstruction

Computational Geometry

Implicit Surfaces

Why is reconstruction hard?

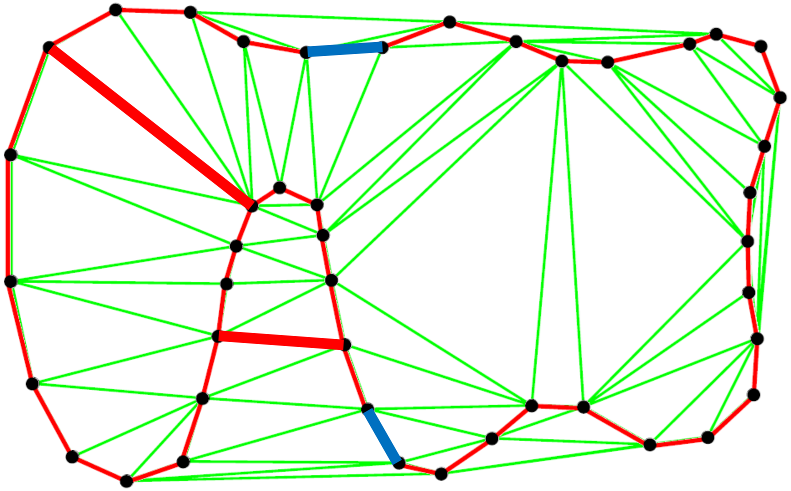
Crust [Amenta *et al*. 1998]

If we consider the Delaunay Triangulation of a point set sampling a curve, the curve should be (approximately) a subset of the Delaunay edges.

Q: How do we determine which edges to keep?

A: Two types of edges:

1. Those connecting adjacent points on the curve
2. Those traversing.

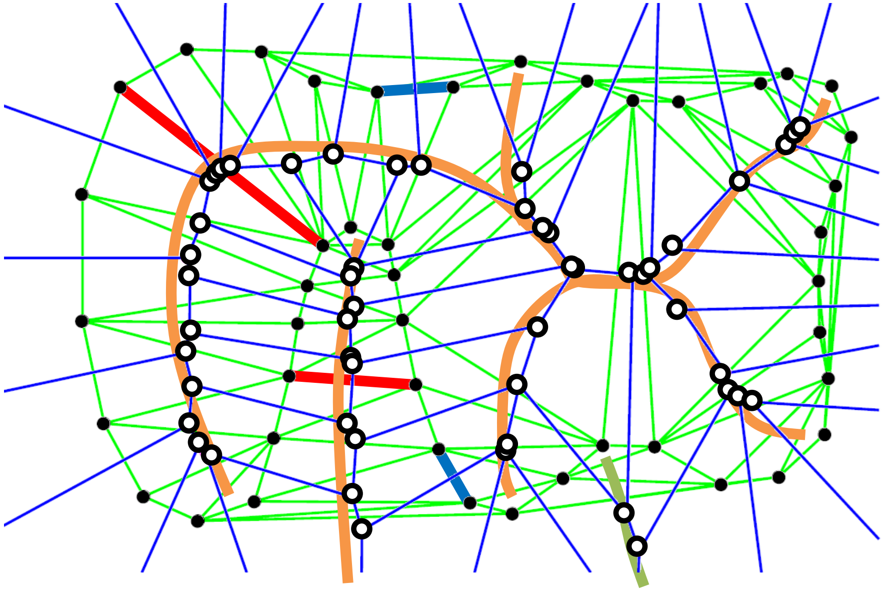


Discard those that traverse.

Crust [Amenta *et al*. 1998]

Observation:

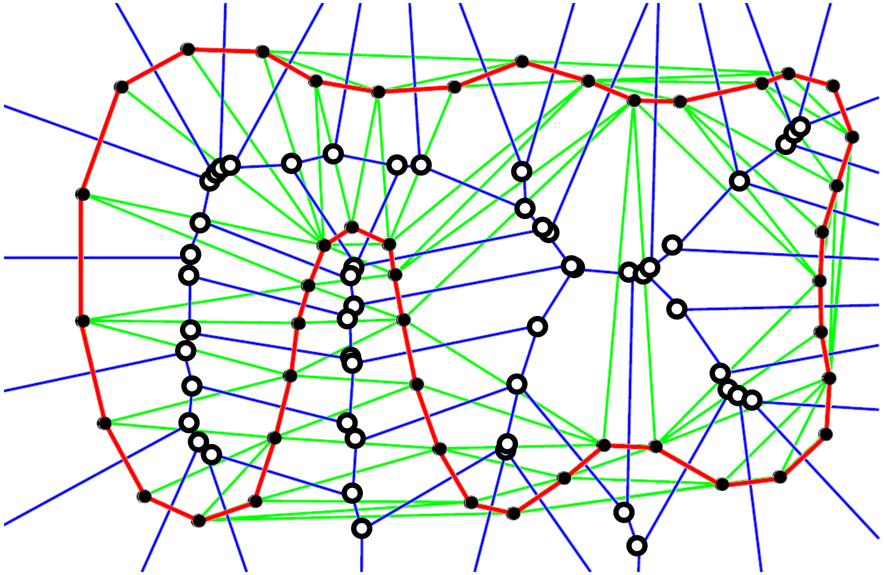
- Edges that traverse must cross the medial axis.
  - Though we don't know the medial axis, we can sample it with the Voronoi vertices.
  - Edges that traverse must be near the Voronoi vertices.
  - An edge does not traverse if we can escribe it in a circle that is empty of Voronoi vertices.



Crust [Amenta *et al*. 1998]

Algorithm:

1. Compute the Delaunay triangulation.
2. Compute the Voronoi vertices
3. Keep all edges for which there is an escribing circle that is empty of Voronoi vertices.



Note:
As opposed to the previous method, it is not obvious that this will generate a closed, manifold curve/surface.

Outline

Introduction

Preliminaries

A sampling of methods

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- **… from Unorganized Points**
- Poisson Reconstruction

Computational Geometry

Implicit Surfaces

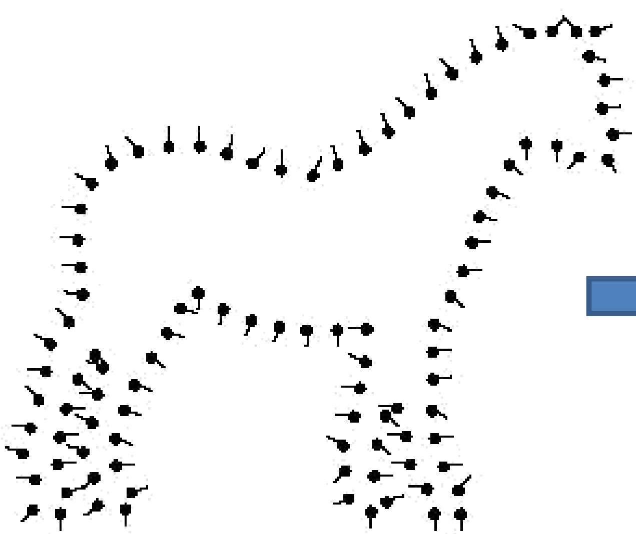
Why is reconstruction hard?

Implicit Surface Reconstruction

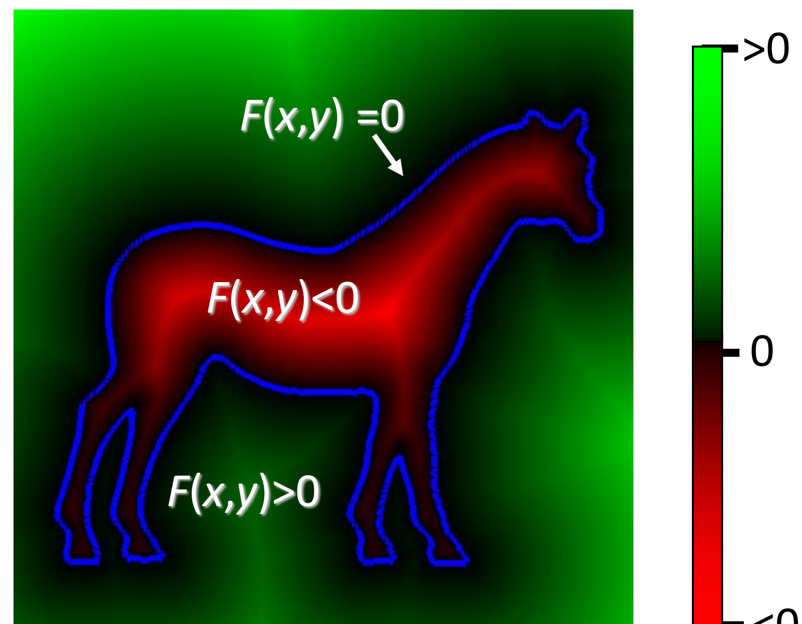
<u>Key Idea</u>:

Use the point samples to define a function whose value at each sample positions is zero.

Extract the zero level-set. [Lorensen and Cline, 1987]



Sample Points



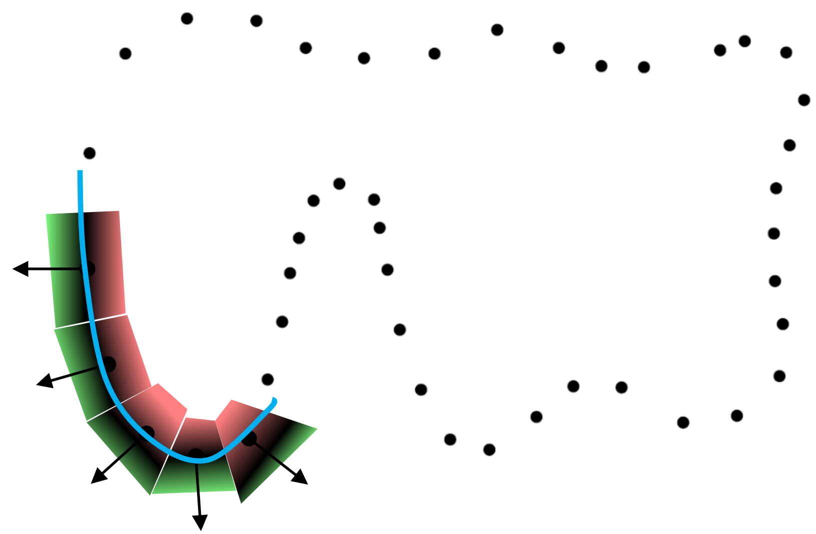
$F(x, y) = 0$

… Unorganized Points [Hoppe *et al*. 1992]

Compute a *truncated signed distance function* by using the sample normals to define a **local** linear approximation to the function.

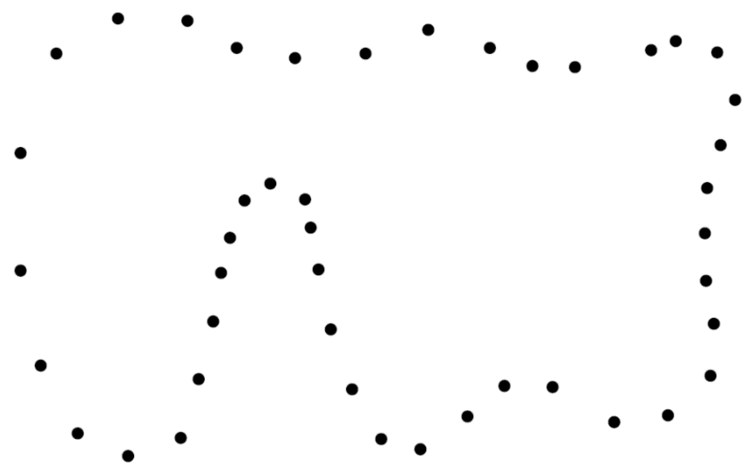
Blend the linear approximations.

Extract the zero level-set (where defined).



… Unorganized Points [Hoppe *et al*. 1992]

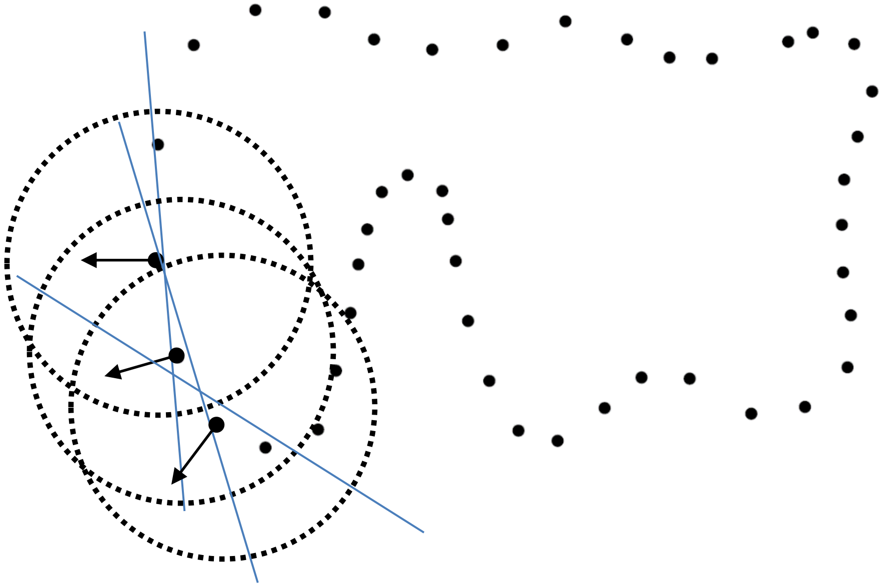
Q: How do we get the normals?



… Unorganized Points [Hoppe *et al*. 1992]

Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.



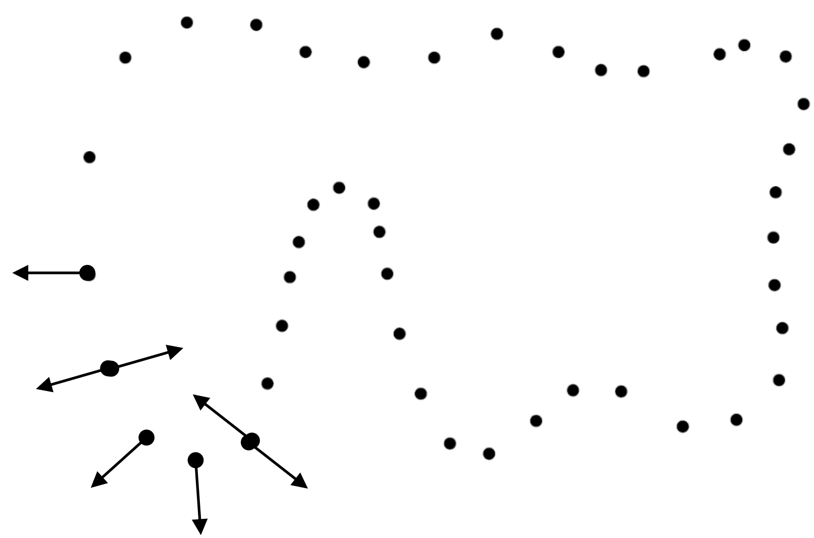
… Unorganized Points [Hoppe *et al*. 1992]

Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.

This doesn’t guarantee a consistent orientation!

For the orientation to be consistent, neighboring points should point in the same direction.

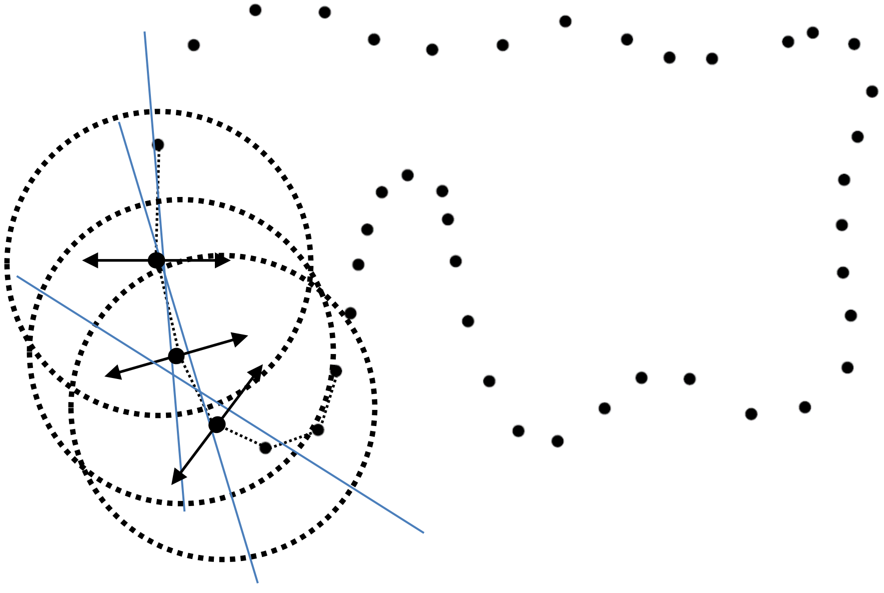


… Unorganized Points [Hoppe *et al*. 1992]

Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.

A2: Build a (Euclidian) minimal spanning tree
and propagate the orientation from a root.



… Unorganized Points [Hoppe *et al*. 1992]

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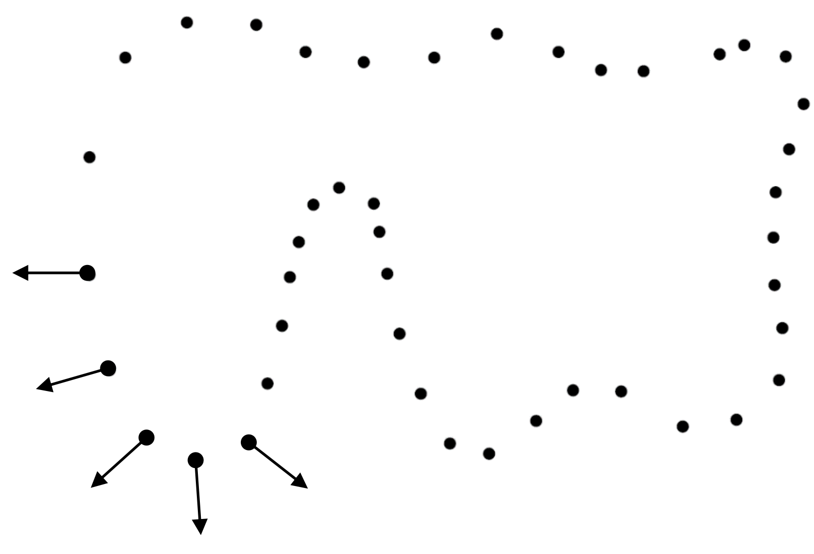

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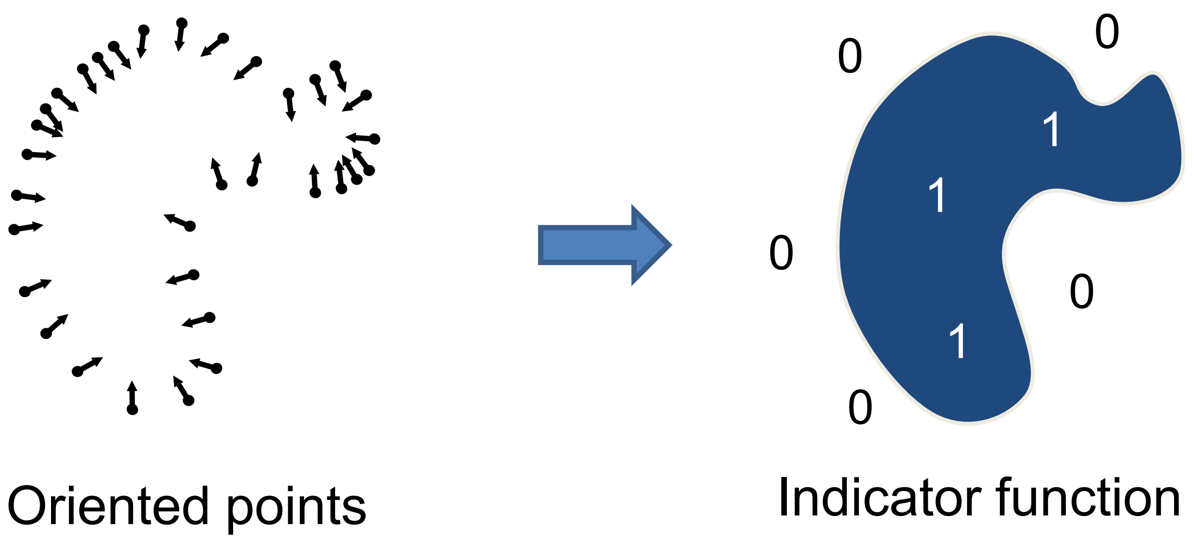
Computational Geometry (Space Partitioning, Crust)

Implicit Surfaces (… from Unorganized Points, Poisson Reconstruction)

Poisson Reconstruction [Kazhdan *et al*. 2006]

Reconstruct the *indicator function* of the surface and then extract the boundary.

Q: How to fit a function to the samples?

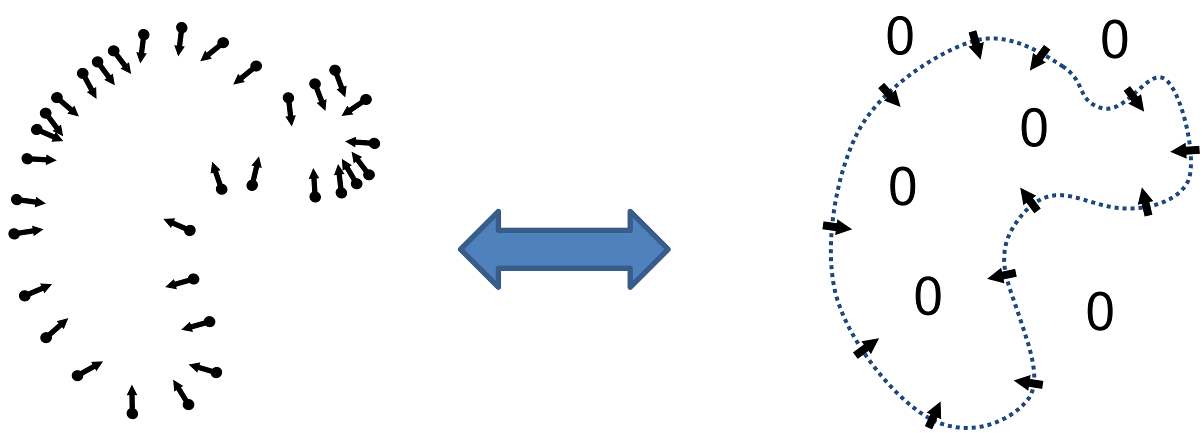


Poisson Reconstruction [Kazhdan *et al*. 2006]

Reconstruct the *indicator function* of the surface and then extract the boundary.

Q: How to fit a function to the samples?

A: Normals are samples of function's gradients.



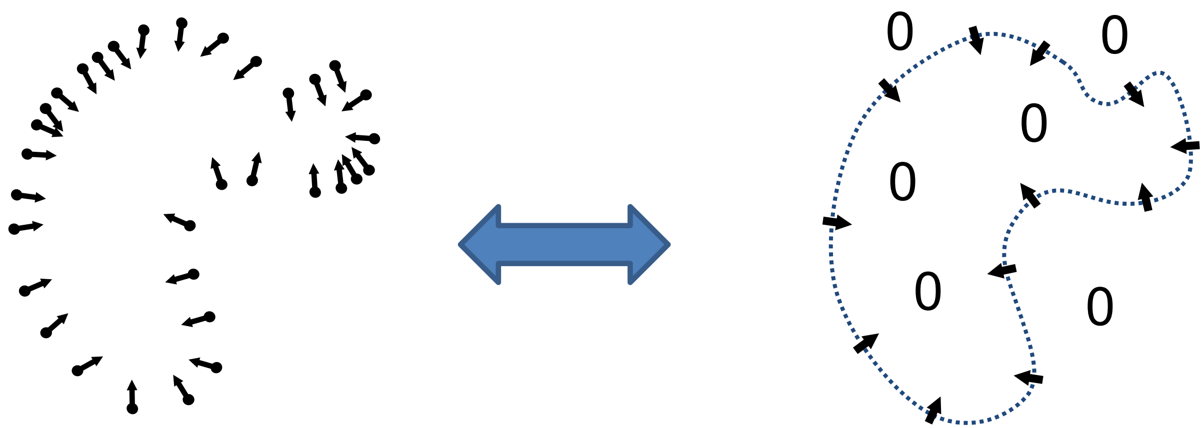
Oriented points Indicator gradient

Poisson Reconstruction [Kazhdan *et al*. 2006]

To fit a function F to the gradients $\vec{V}$ solve:

$$\nabla F = \vec{V}$$

- ✘ This is an over-constrained problem, so there is (usually) no solution.



Oriented points — Indicator gradient

Poisson Reconstruction [Kazhdan *et al*. 2006]

To fit a function F to the gradients $\vec{V}$ solve:

$$\nabla F = \vec{V}$$

- ✘ This is an over-constrained problem, so there is (usually) no solution.
- ✓ Solve for the best (least-squares) solution:

$$\arg\min_{F} \left\| \nabla F - \vec{V} \right\|^2$$

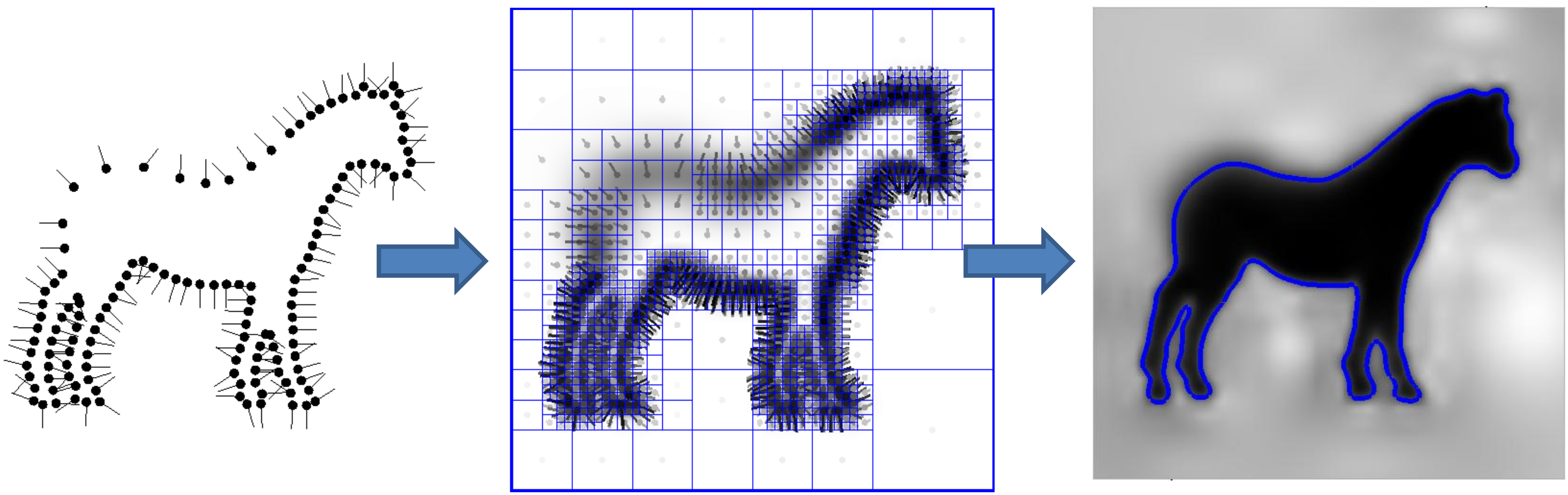
⇒ Taking the divergence, this becomes:

$$\nabla \cdot \left(\nabla F - \vec{V} \right) = 0 \quad \Leftrightarrow \quad \Delta F = \nabla \cdot \vec{V}$$

Poisson Reconstruction [Kazhdan *et al*. 2006]

Algorithm:

1. Transform samples into a vector field.
2. Fit a scalar-field to the gradients.
3. Extract the level-set.



Outline

Introduction

Preliminaries

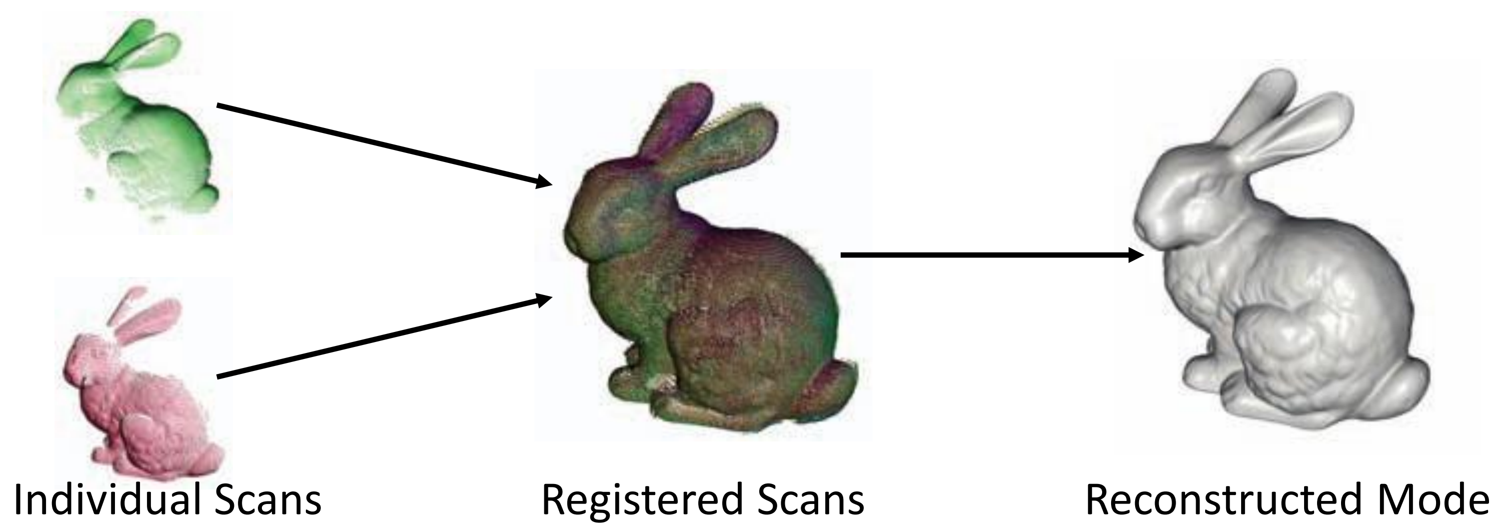
A sampling of methods

Why is reconstruction hard?

Why is Reconstruction Hard?

The point-set is often the result of:

- Scanning
- Registering
- Etc.

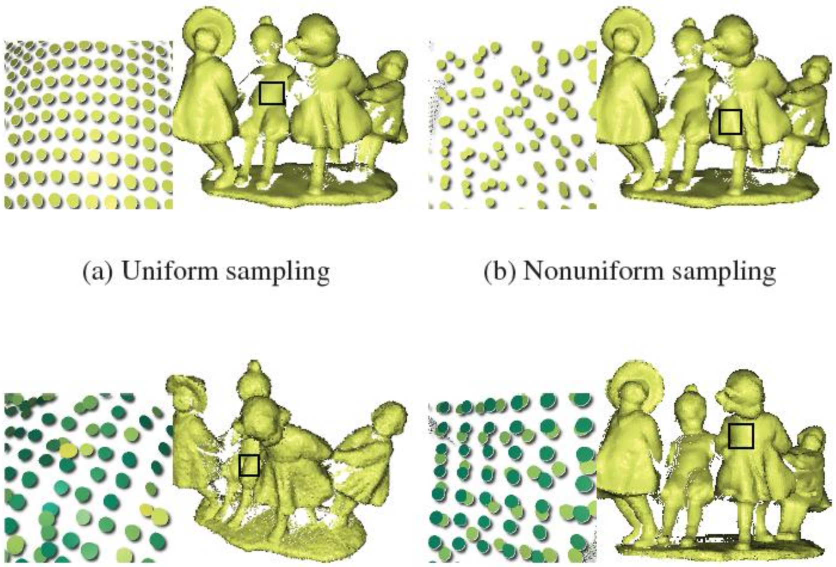


[Image courtesy of Bolitho]

Why is Reconstruction Hard?

Susceptible to:

- Scanning
  - Nonuniform sampling
  - Grazing angles
  - Scanner noise
  - Imprecise estimates
- Registering
  - Misalignment
  - Non-linear camera model



(a) Uniform sampling (b) Nonuniform sampling

(c) Noisy data (d) Misaligned scans

[Image courtesy of Berger *et al*.]

Practical Concerns

Performance in the presence of bad data

Interpolating vs. approximating

Efficiency (space and time) of reconstruction

Quality guarantees

Manifold / water-tight

Incorporation of prior knowledge