Radiosity

Michael Kazhdan

(601.457/657)

Overview

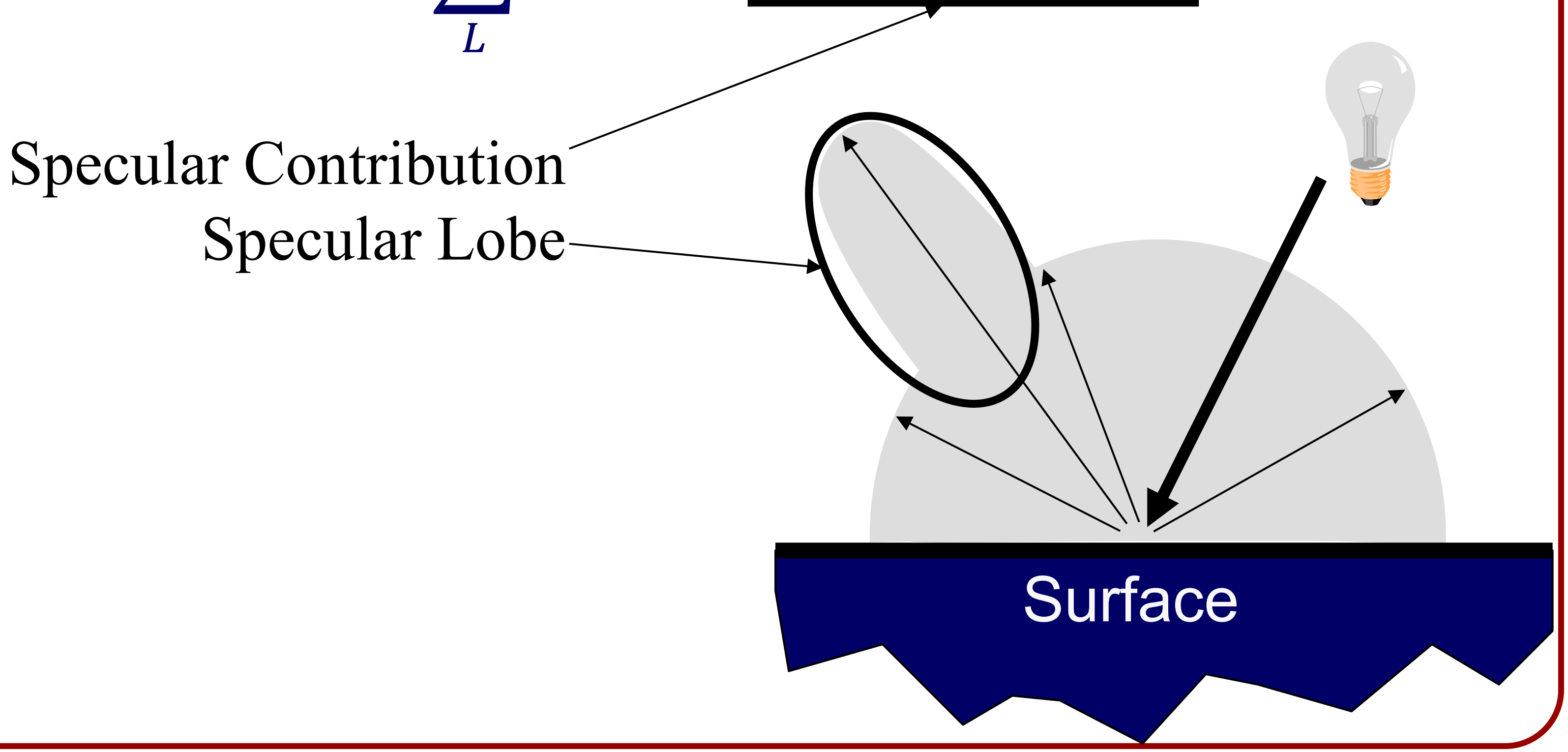
- Ray Tracing Revisited
- Radiosity

Ray Casting

Ray tracing is based on the Phong lighting model:

A surface reflects light non-uniformly, with stronger reflection in the specular direction:

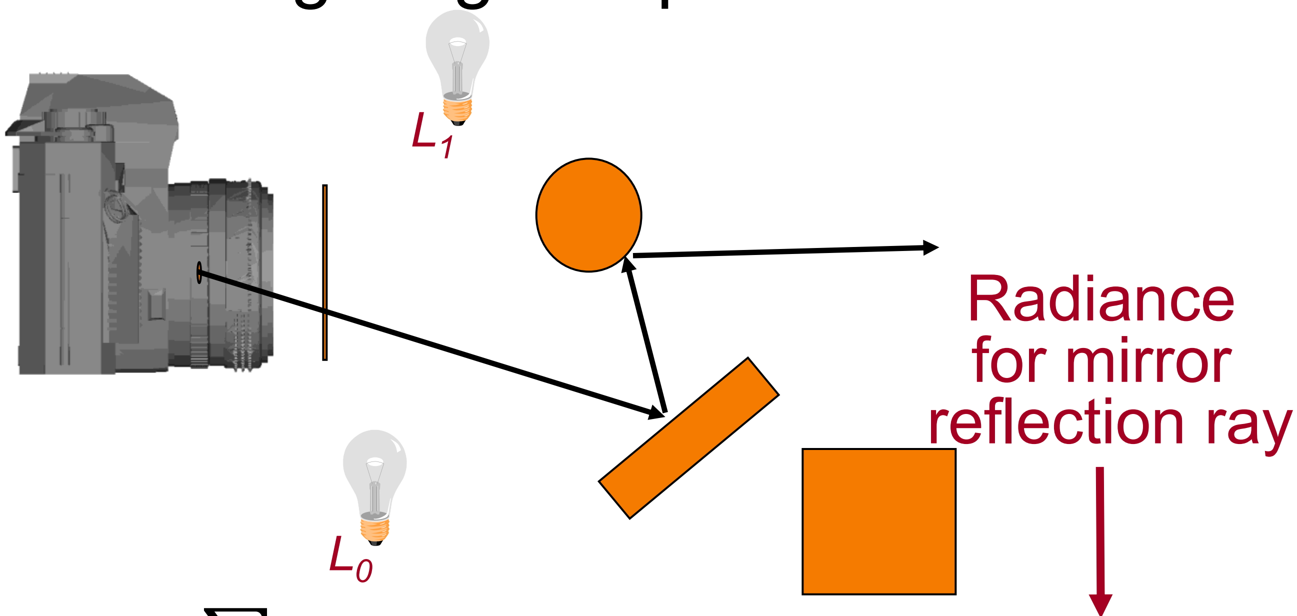
$$I = I_E + K_A \cdot I_L^A + \sum_L \left(\langle \vec{N}, \vec{L} \rangle + \boxed{K_S \cdot \langle \vec{V}, \vec{R}(\vec{L}) \rangle^{K_n}} \right) \cdot I_L$$



Ray Tracing

Ray tracing is based on the Phong lighting model:

For the same reason, we only cast secondary rays in the reflected direction – to maximize the contribution to the lighting computation.

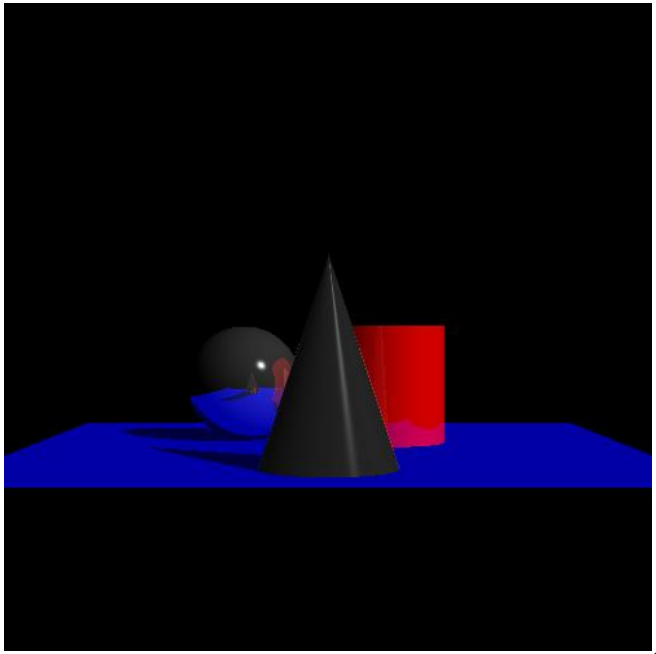


$$I = I_E + K_A \cdot I_L^A + \sum_L \left[\left(\langle \vec{N}, \vec{L} \rangle + K_S \cdot \langle \vec{V}, \vec{R}(\vec{L}) \rangle^{K_n} \right) \cdot I_L + K_s \cdot I_R \right]$$

Ray Tracing

Properties:

- ✓ Good at capturing the specular properties of materials



Ray Tracing

Properties:

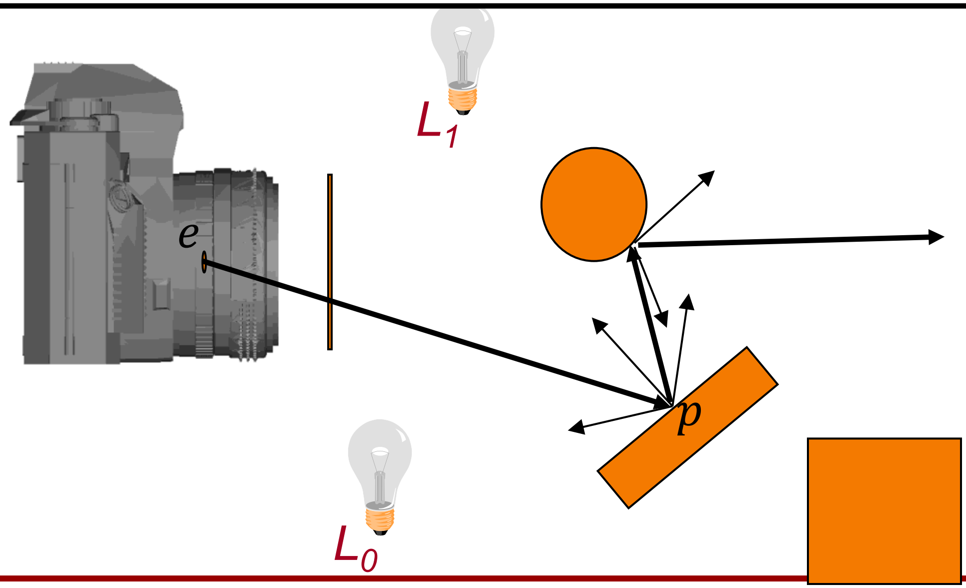
- ✓ Good at capturing the specular properties of materials
- ✗ Difficult to support soft shadows from area lights
- ✗ Difficult to support caustics
- ✗ Need the ambient term as a hack for the global illumination

Lighting

What do we really want to compute?

The accumulation of light coming in from **all** directions, **modulated** by how much the light is reflected in/from that direction.

In practice, use Monte-Carlo integration with importance sampling to generate more reflected rays in directions that contribute more strongly.

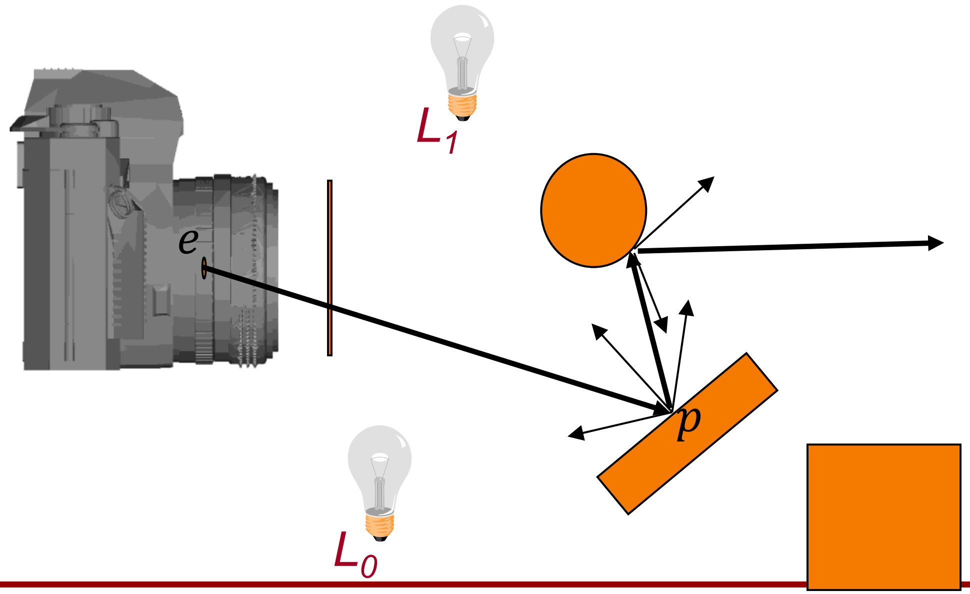


Lighting

What do we really want to compute?

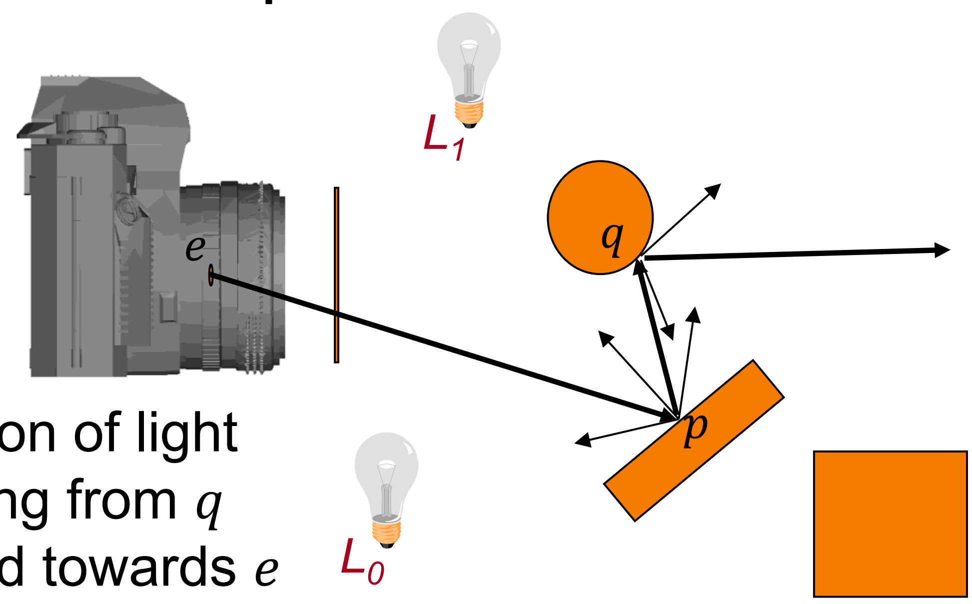
The brightness/intensity reaching the camera (e) from a point in the scene (p) is the sum of:

1. The light emitted from p, to e, and
2. The light emanating from **all** scene points scaled by the extent to which it is reflected through p to e.



Lighting

What do we really want to compute?



Amount of light from p going towards e

Fraction of light arriving from q reflected towards e

Amount of light emitted from p going towards e

Amount of light from q going towards p

$$B(p \to e) = E(p \to e) + \int_\Omega F_r(q \to p \to e) \cdot B(q \to p)\, dq$$

Lighting

Challenge:

- The integral needs to be estimated precisely to capture discontinuities.
- The function is recursive since the amount of light entering a point depends on the amount leaving it.

Jensen

$$B(p \to e) = E(p \to e) + \int_{\Omega} F_r(q \to p \to e) \cdot B(q \to p)\, dq$$

Ray-Tracing

Specular assumption:

Only reflect lights from the reflected ray direction:

$$F_r(q \rightarrow p \rightarrow e) = \begin{cases} K_s(p) & q = I\big(p, \mathrm{Ref}(p \rightarrow e)\big) \\ 0 & \text{otherwise} \end{cases}$$

$$B(p \rightarrow e) = E(p \rightarrow e) + \int_\Omega F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p)\, dq$$

$$B(p \rightarrow e) = E(p \rightarrow e) + K_s(p) \cdot B\Big(I\big(p, \mathrm{Ref}(p \rightarrow e)\big) \rightarrow p\Big)$$

$I\big(p, \mathrm{Ref}(p \rightarrow e)\big)$ is the first intersection of the ray in the reflected view direction.

Radiosity

Lambertian assumption:

The apparent brightness a patch of surface is constant independent of the view direction.

$$B(p \to e) = E(p \to e) + \int_\Omega F_r(q \to p \to e) \cdot B(q \to p)\, dq$$

$$B(p) = E(p) + \int_\Omega F_r(q \to p) \cdot B(q)\, dq$$

Radiosity

<u>Lambertian assumption</u>:

The apparent brightness a patch of surface is constant independent of the view direction.

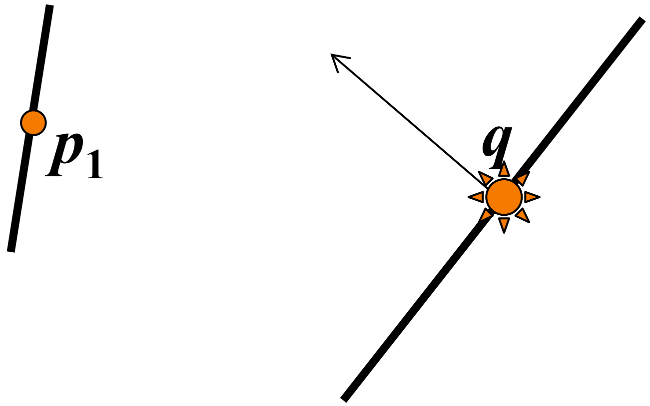
» **Emitters appear equally bright from all directions**

$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q)\, dq$$

Lambertian Emitters

Given an emitter at point q, the apparent brightness point p is independent of its orientation w.r.t. to q.

By assumption, the apparent brightness of q is independent of the view direction of p.



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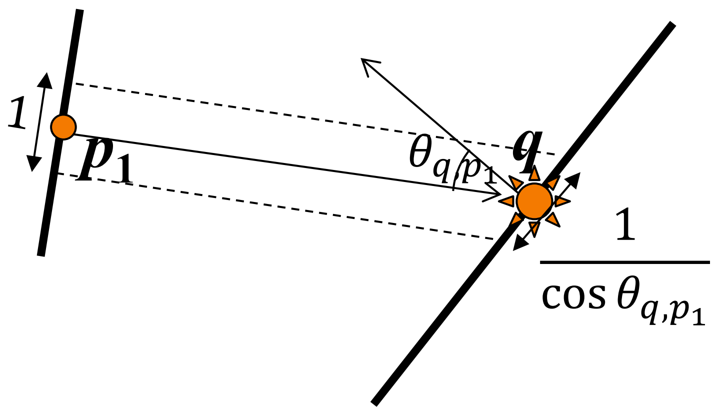

By assumption, the apparent brightness of q is independent of the view direction of p.

A unit-area patch about p "sees" more of q's surface as $\theta_{q,p}$ is more grazing.

$$B(p) = E(p) + \int_\Omega F_r(q \to p) \cdot B(q)\, dq$$

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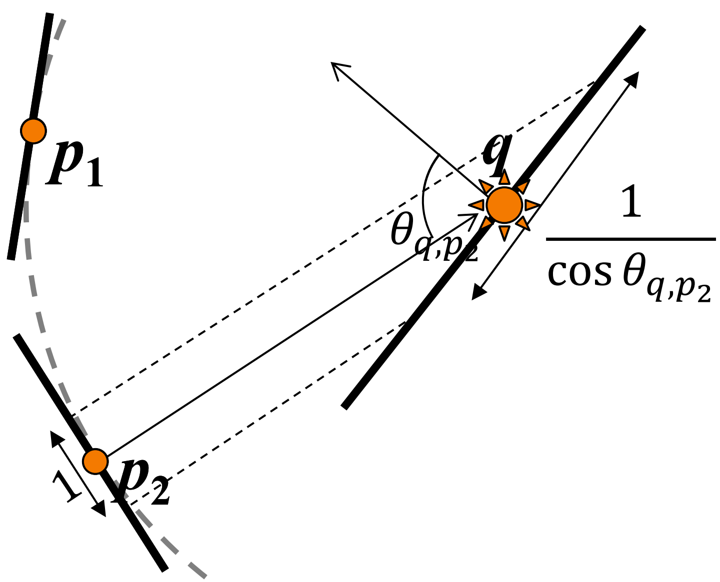


$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q)\, dq$$

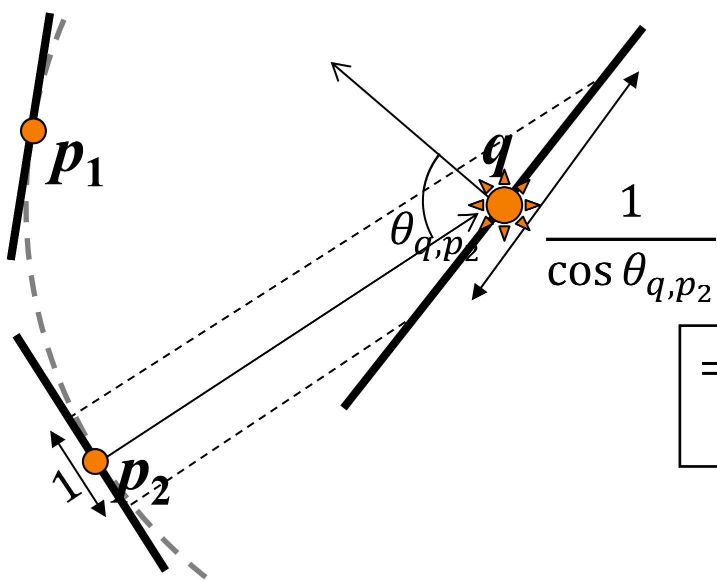
Lambertian Emitters

Given an emitter at point q, the apparent brightness point p is independent of its orientation w.r.t. to q.

By assumption, the apparent brightness of q is independent of the view direction of p.

A unit-area patch about p “sees” more of q’s surface as $\theta_{q,p}$ is more grazing.

$\Rightarrow$ A unit-area patch about p receives light from a patch of area $1/\cos\theta_{q,p}$ about q.



$\Rightarrow$ The intensity emitted from a patch of about q in direction p falls off as $\cos\theta_{q,p}$.

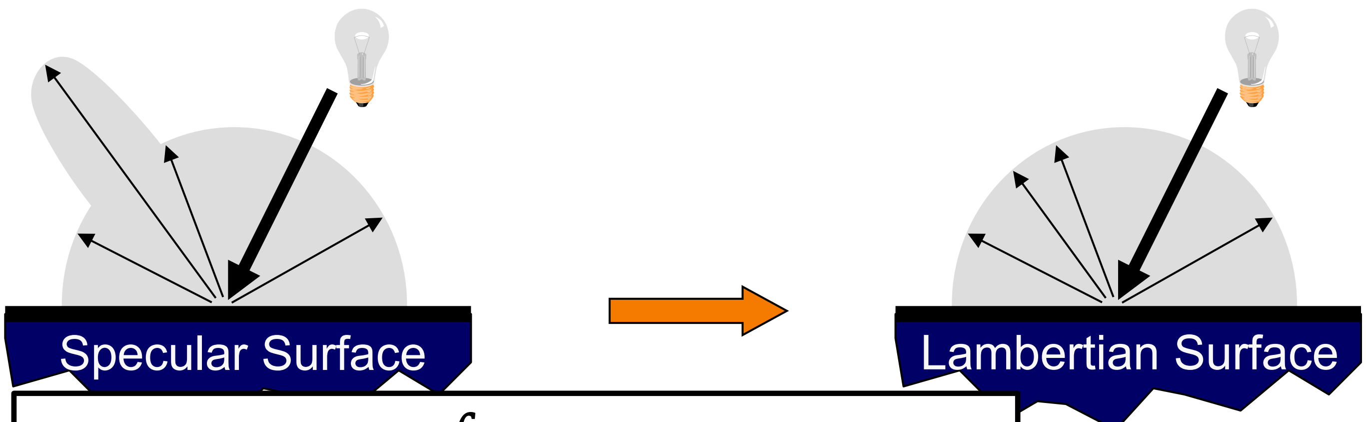
$$B(p) = E(p) + \int_\Omega F_r(q \to p) \cdot B(q)\, dq$$

Radiosity

Lambertian assumption:

The apparent brightness a patch of surface is constant independent of the view direction.

- » Emitters appear equally bright from all directions
- » **Reflectors appear equally bright from all directions**



$$B(p) = E(p) + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q)\, dq$$

Lambertian Reflectors

How does the amount of light going from q reflected through p depend on:

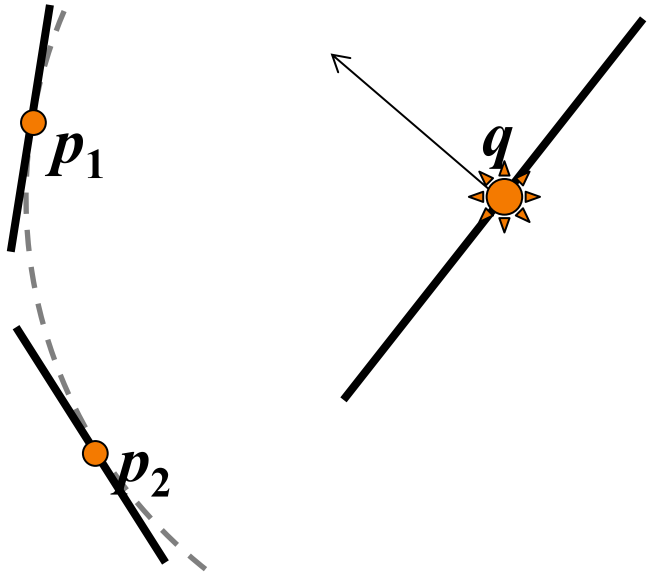
1. The direction to p relative to the orientation at q,
2. The direction to q relative to the orientation at p,
3. The distance between p and q?

$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q)\, dq$$

Lambertian Reflectors (1)

Treating q as an emitter, the light emitted from q in direction p falls off as $\cos\theta_{q,p}$ – with $\theta_{q,p}$ the angle between the normal at q and direction to p.

$\Rightarrow$ Reflected brightness at p falls off as $\cos\theta_{q,p}$

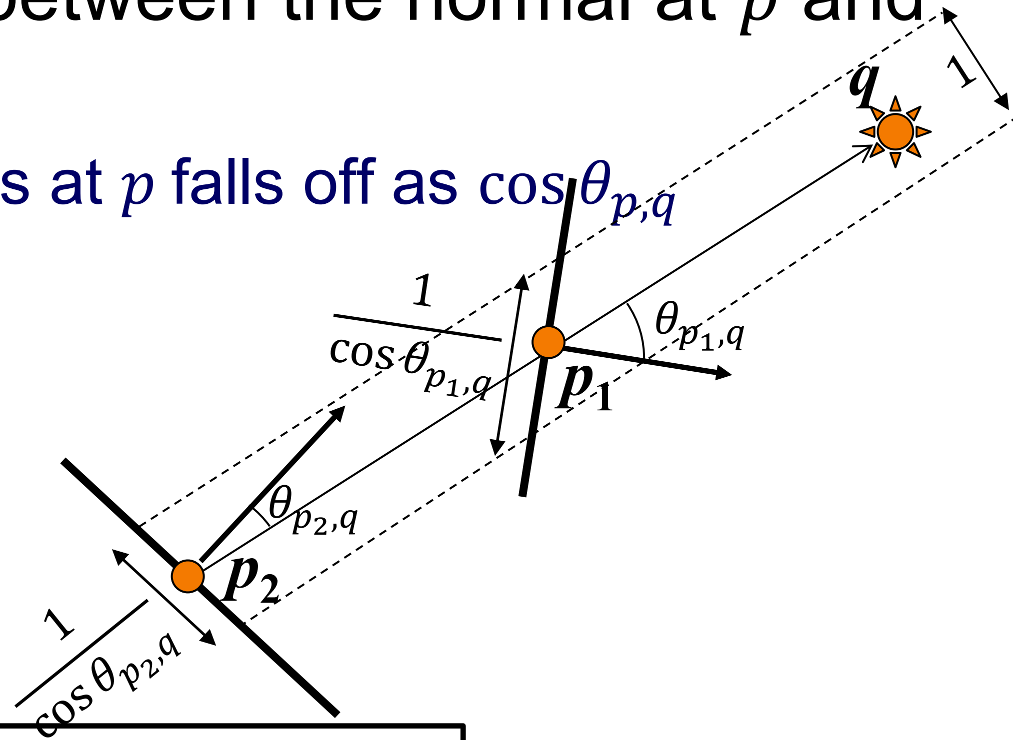


$$B(p) = E(p) + \int_\Omega F_r(q \to p) \cdot B(q)\, dq$$

Lambertian Reflectors (2)

A beam of unit cross-sectional leaving q towards p, will spread out across a patch of area $1/\cos\theta_{p,q}$ at p – with $\theta_{p,q}$ the angle between the normal at p and direction to q.

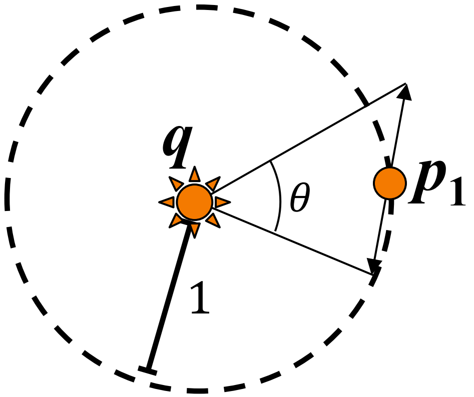
⇒ Reflected brightness at p falls off as $\cos\theta_{p,q}$



$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q)\, dq$$

Lambertian Reflectors (3)

The apparent brightness at p is proportional to the subtended spherical angle by a unit area patch at p.

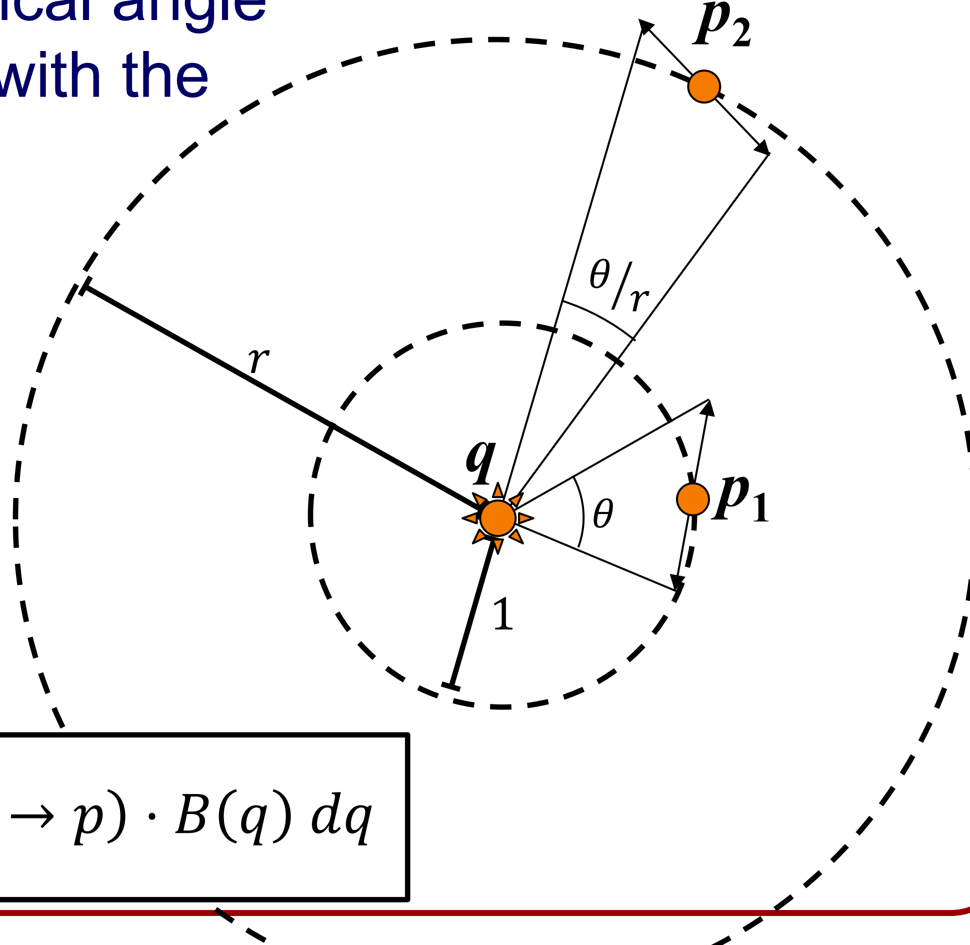


$$B(p) = E(p) + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q)\, dq$$

Lambertian Reflectors (3)

The apparent brightness at p is proportional to the subtended spherical angle by a unit area patch at p.

- ⇒ The subtended spherical angle falls off quadratically with the distance of p from q.
- ⇒ Perceived brightness decays as the square of the distance.

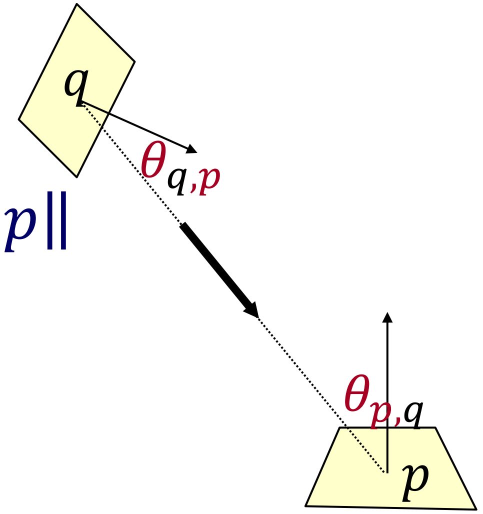


$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q)\, dq$$

Lambertian Reflectors

⇒ The fraction of light from q that is reflected off p is determined by:

- The angle: $\theta_{q,p}$
- The angle: $\theta_{p,q}$
- The square distance from q to p: $\|q - p\|$



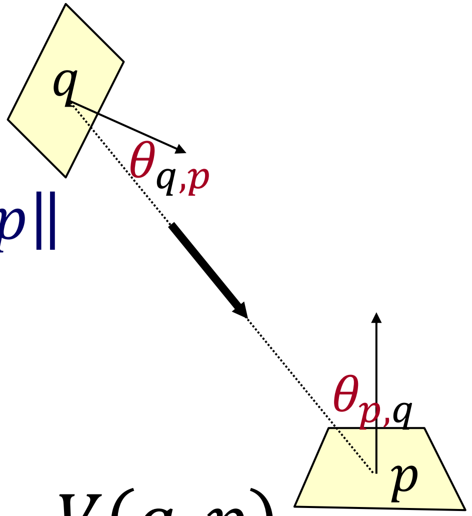
$$F_r(q \to p) = \frac{\cos\theta_{q,p} \cdot \cos\theta_{p,q}}{\|q - p\|^2}$$

$$B(p) = E(p) + \int_{\Omega} F_r(q \to p) \cdot B(q)\, dq$$

Lambertian Reflectors

⇒ The fraction of light from q that is reflected off p is determined by:

- The angle: $\theta_{q,p}$
- The angle: $\theta_{p,q}$
- The square distance from q to p: $\|q-p\|$
- The visibility of p from q: $V(q,p)$
- The albedo at p: $\rho(p)$



$$F_r(q \to p) = \frac{\cos\theta_{q,p} \cdot \cos\theta_{p,q}}{\|q-p\|^2} \cdot \rho(p) \cdot V(q,p)$$

$$B(p) = E(p) + \int_\Omega F_r(q \to p) \cdot B(q)\, dq$$

Radiosity

Lambertian assumption:

The apparent brightness a patch of surface is constant independent of the view direction.

- » Emitters appear equally bright from all directions
- » Reflectors appear equally bright from all directions

$$B(p) = E(p) + \rho(p) \int_\Omega V(q,p) \cdot \frac{\cos\theta_{q,p} \cdot \cos\theta_{p,q}}{\|q-p\|^2} \cdot B(q)\, dq$$

The radiosity equation

Radiosity

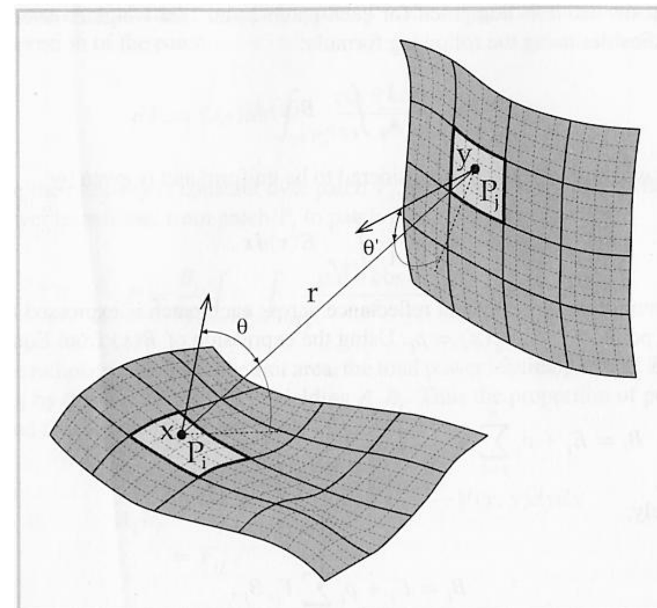
Approximate the integral by decomposing surfaces into patches and doing a discrete summation:

$$B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} \cdot B_j$$

Form Factor

For patch i:

- B_i: Total brightness
- E_i: Total emissivity
- ρ_i: Albedo



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$$B(p) = E(p) + \rho(p) \int_{\Omega} V(q,p) \cdot \frac{\cos\theta_{q,p} \cdot \cos\theta_{p,q}}{\|q-p\|^2} \cdot B(q)\, dq$$

The radiosity equation

Form Factor

Approximate the integral by decomposing surfaces into patches and doing a discrete summation:

$$B_i = E_i + \rho_i \sum_{j=1}^{n} \boxed{F_{ij}} \cdot B_j$$

The **form factor** $0 \leq F_{ij} \leq 1$ is the proportion of the power leaving patch P_j, received by patch P_i:

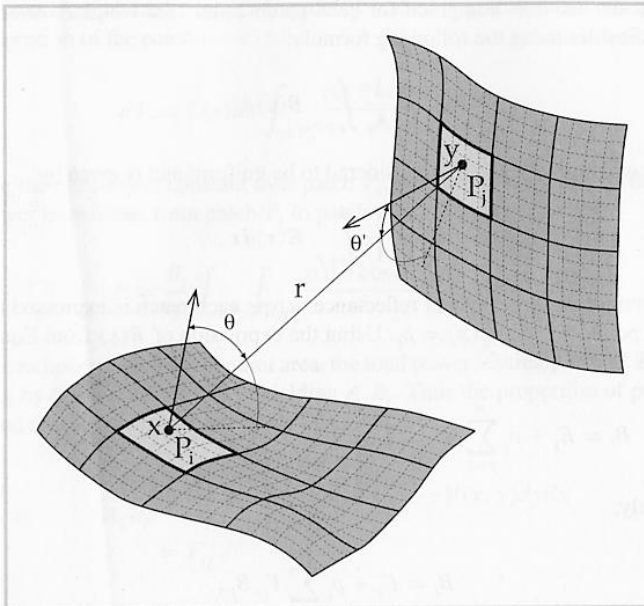
- Symmetry/Reciprocity: $A_j F_{ij} = A_i F_{ji}$
- Definiteness: $F_{ii} = 0$ unless the patch is concave
- Partition of unity: $\sum_i F_{ij} = 1$

Radiosity

Approximate the integral by decomposing surfaces into patches and doing a discrete summation:

$$B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} \cdot B_j$$

Form Factor



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This amounts to solving a **linear** system of equations

- E_i, ρ_i, and F_{ij} are given
- B_i are the unknowns.

Radiosity

Re-ordering terms in the equation gives:

$$B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} \cdot B_j$$

$$\Downarrow$$

$$E_i = B_i - \rho_i \sum_{j=1}^{n} F_{ij} \cdot B_j$$

$$\Downarrow$$

$$\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix} = \begin{pmatrix} 1 - \rho_1 \cdot F_{1,1} & -\rho_1 \cdot F_{2,1} & \cdots & -\rho_1 \cdot F_{n,1} \\ -\rho_2 \cdot F_{1,2} & 1 - \rho_2 \cdot F_{2,2} & \cdots & -\rho_2 \cdot F_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n \cdot F_{1,n} & -\rho_n \cdot F_{2,n} & \cdots & 1 - \rho_n \cdot F_{n,n} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix}$$

Solving the System of Equations

Challenges:

- Size of matrix
- Cost of computing form factors

$$\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix} = \begin{pmatrix} 1-\rho_1\cdot F_{1,1} & -\rho_1\cdot F_{2,1} & \cdots & -\rho_1\cdot F_{n,1} \\ -\rho_2\cdot F_{1,2} & 1-\rho_2\cdot F_{2,2} & \cdots & -\rho_2\cdot F_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n\cdot F_{1,n} & -\rho_n\cdot F_{2,n} & \cdots & 1-\rho_n\cdot F_{n,n} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix}$$

$$\mathbf{e} = \mathbf{A} \cdot \mathbf{b}$$

Solving the System of Equations

Solution methods:

- ~~Invert the matrix – $O(n^3)$~~
- Gathering methods – $O(n^2)$
- Shooting methods – $< O(n^2)$

$$\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix} = \begin{pmatrix} 1-\rho_1 \cdot F_{1,1} & -\rho_1 \cdot F_{2,1} & \cdots & -\rho_1 \cdot F_{n,1} \\ -\rho_2 \cdot F_{1,2} & 1-\rho_2 \cdot F_{2,2} & \cdots & -\rho_2 \cdot F_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n \cdot F_{1,n} & -\rho_n \cdot F_{2,n} & \cdots & 1-\rho_n \cdot F_{n,n} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix}$$

$$\mathbf{e} = \mathbf{A} \cdot \mathbf{b}$$

Gathering Iteration

Initialization:

For each patch P_i, initialize its total radiosity to be equal to its total emissivity:

$$B_i = E_i$$

Iteration:

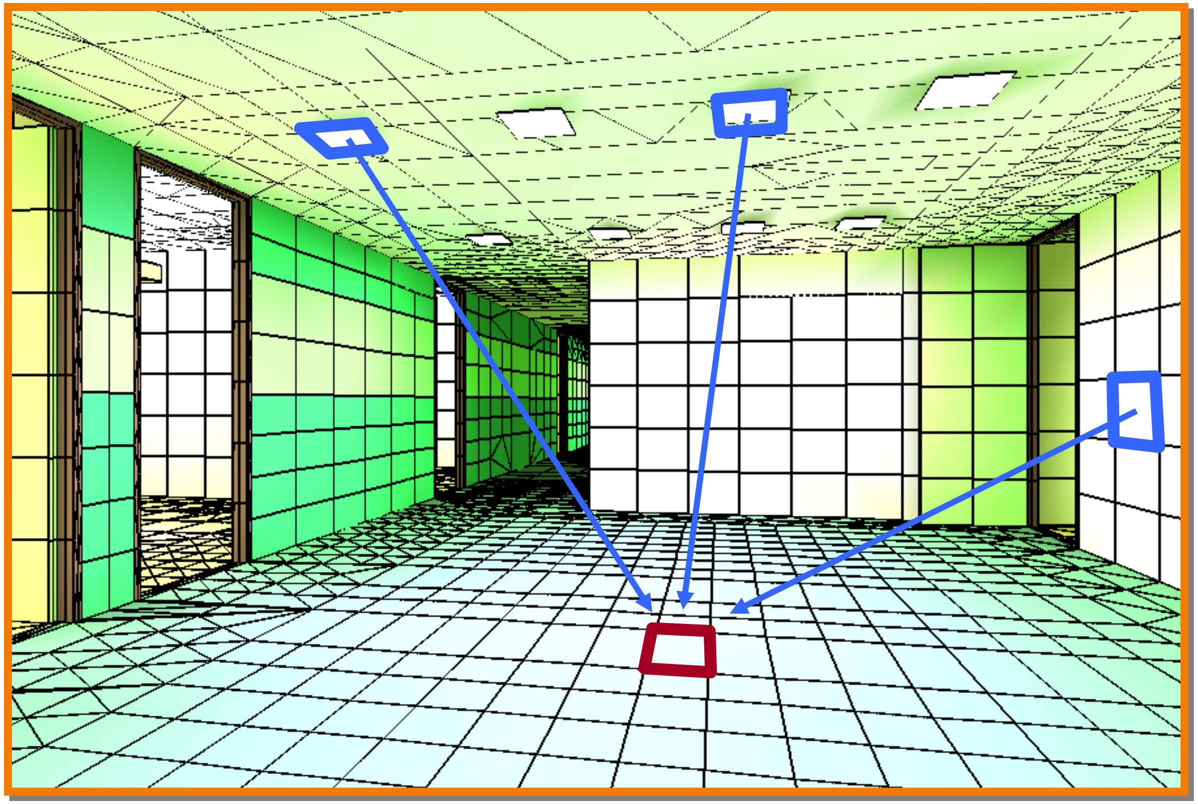
At each iteration, update the values of each of the B_i based on the values of all the other B_j:

$$B_i = E_i + \rho_i \sum_{j \neq i} F_{ij} \cdot B_j$$

Gathering Iteration

Geometric interpretation

Iteratively gather radiosity from elements



Gathering Iteration

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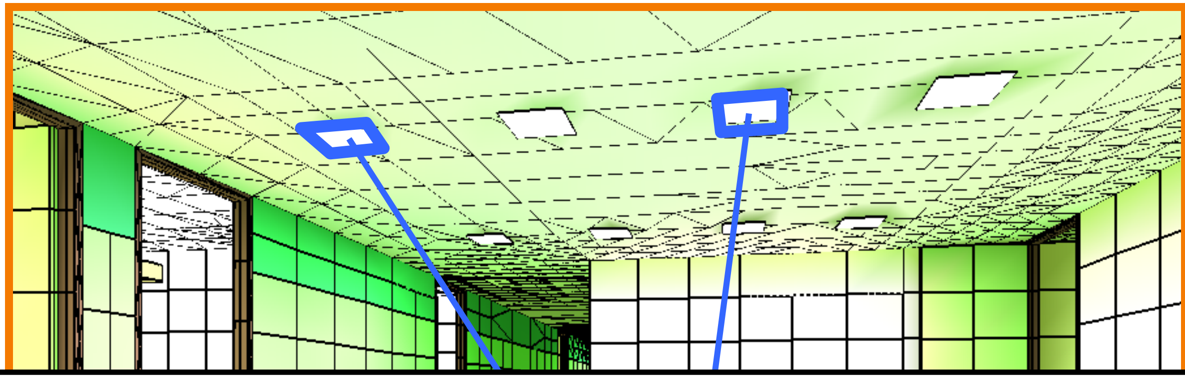


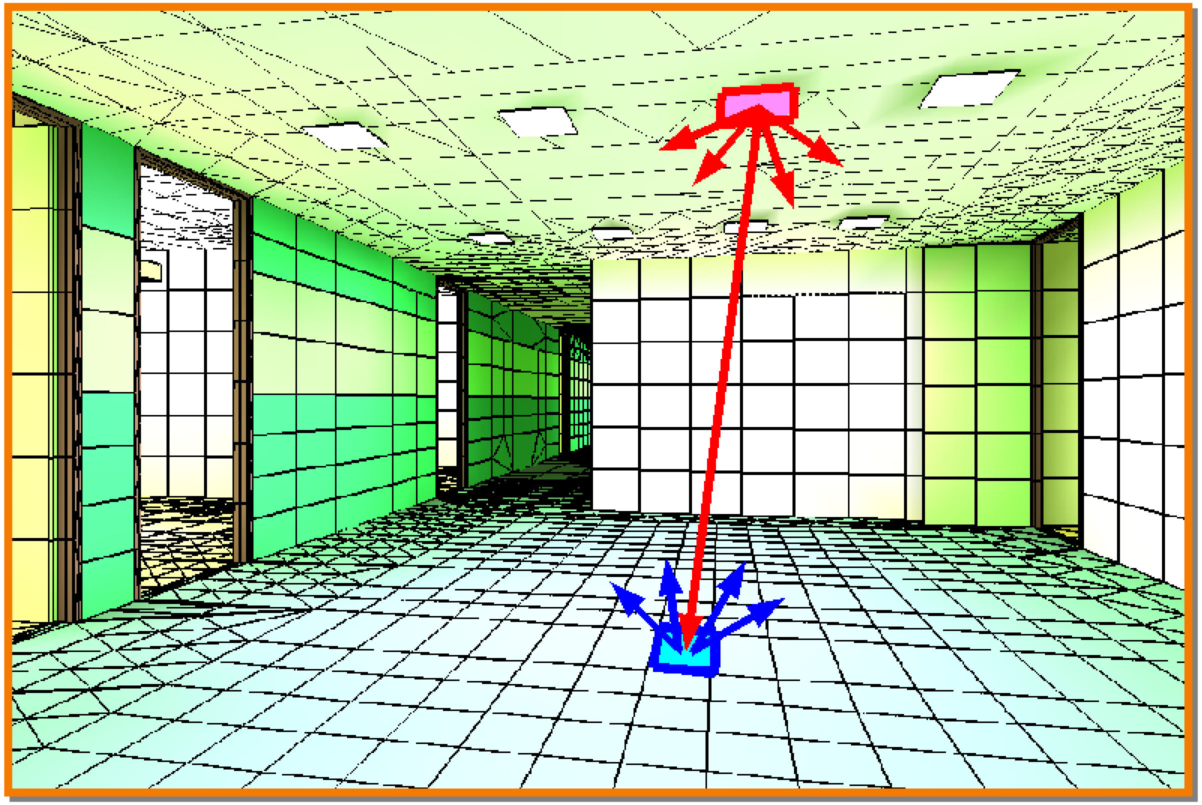
This simulates how light distributes through the scene after we “turn the emitters on”.

Limitation:
Can spend a lot of time gathering radiosity from patches that don’t contribute much.

Shooting Iteration

Geometric interpretation:

- Iteratively shoot “unshot” radiosity from elements
- Select shooters in order of unshot radiosity



Summary

If we could, we would compute the lighting by recursively reflecting secondary rays in all directions to compute the brightness of a single point.

Ray-Tracing:

Assume that surfaces are specular so that you only need to bounce in a single (specular) direction.

Radiosity:

Assume that surfaces are Lambertian so that they reflect light in the same way in all directions.

Reality:

Surfaces reflect in all directions, but not uniformly.