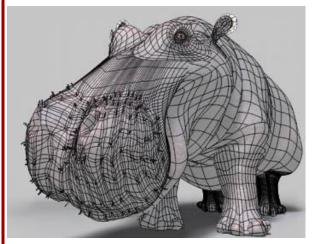


Michael Kazhdan

(601.457/657)









[J. Birn]

We know how to render geometry.

#### How do we get:

- Detailed geometry?
- Detailed colors?
- Shadows?



How should we draw surfaces with complex detail?



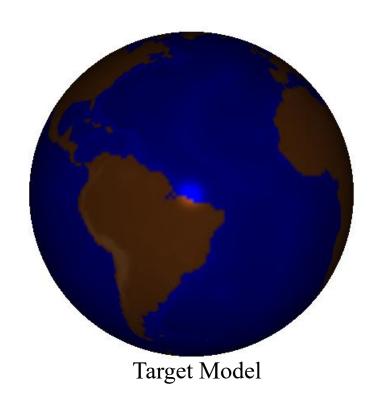
Target Model

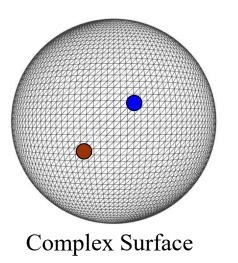


How should we draw surfaces with complex detail?

#### Direct:

 Tessellate finely and assign material properties to each vertex





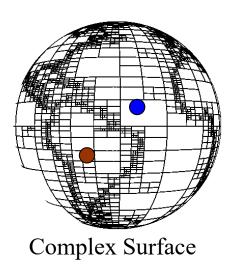


How should we draw surfaces with complex detail?

#### Direct:

 Tessellate adaptively and assign material properties to each vertex



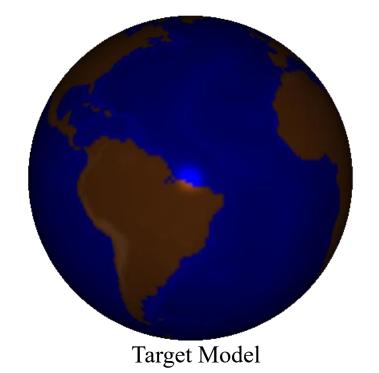




How should we draw surfaces with complex detail?

#### Indirect:

- Use a simple tessellation with an auxiliary texture image.
- Use texture coordinates stored at surface points to look up color values from the texture.

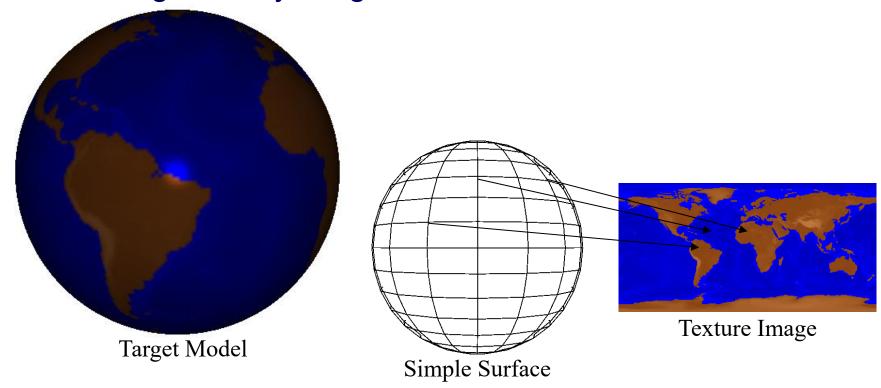


Texture Image
Simple Surface



#### Advantages:

- The 3D model remains simple
- It is easier to design/modify a texture image than it is to design/modify a signal on a surface.



## **Textures (2 dimensions)**



#### <u>Implementation</u>:

Associate a texture coordinate to each vertex v:

$$(s_v, t_v)$$
 with  $(0 \le s_v, t_v \le 1)$ 

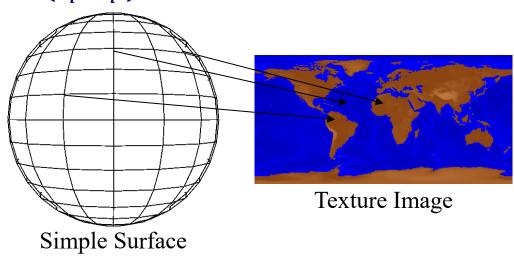
 When rasterizing, interpolate to get the texture coordinate at pixel p:

$$(s_p, t_p)$$

• Sample the texture at  $(s_p, t_p)$  to get the color at p.

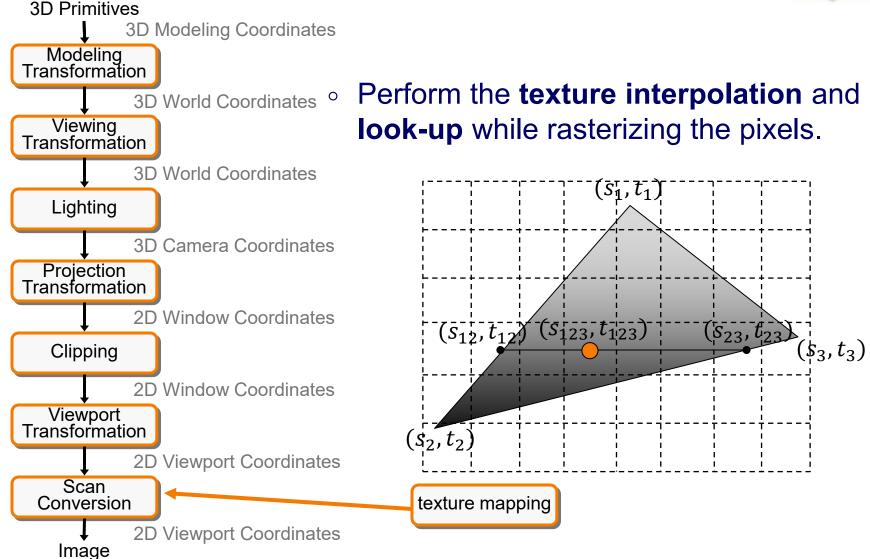
### Terminology:

Texture elements are called *texels* 



### 3D Rendering Pipeline

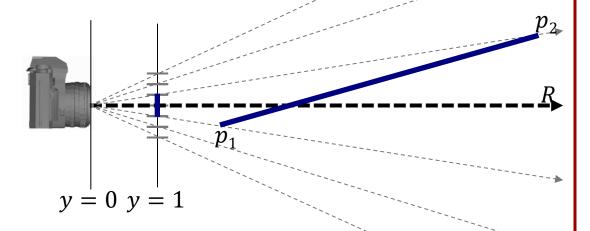




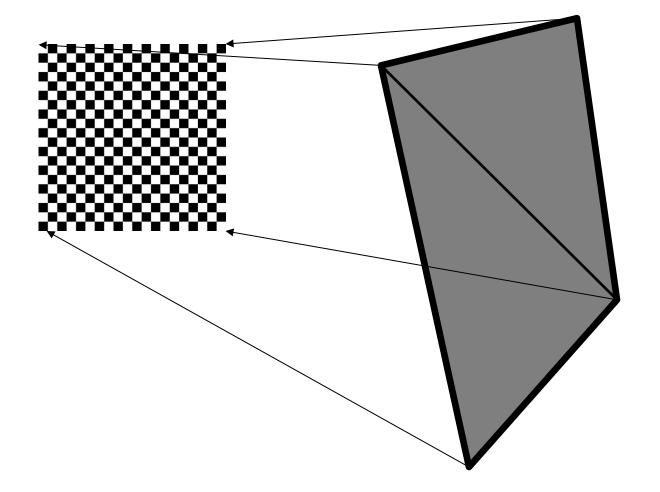


### Recall (Perspective Divide):

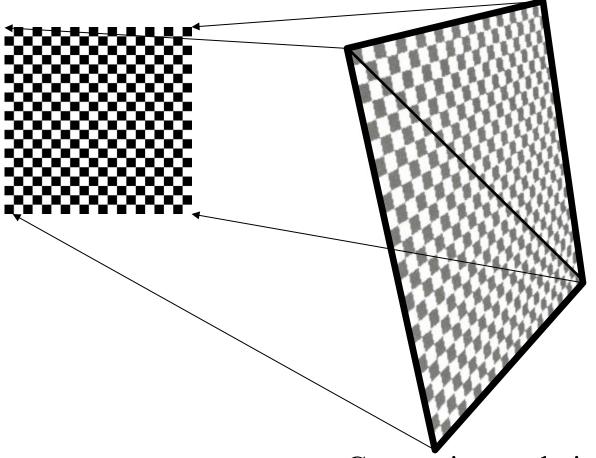
When performing scan-line rasterization and interpolating data from vertices, we need to compute the weights in 3D space.







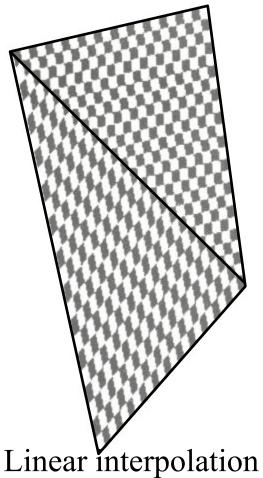




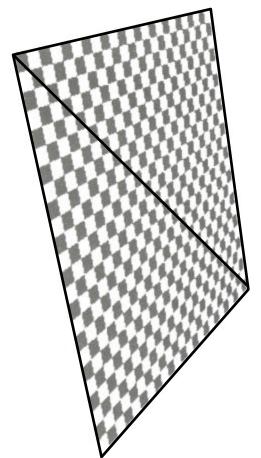
Correct interpolation of texture coordinates with perspective divide

Hill Figure 8.42





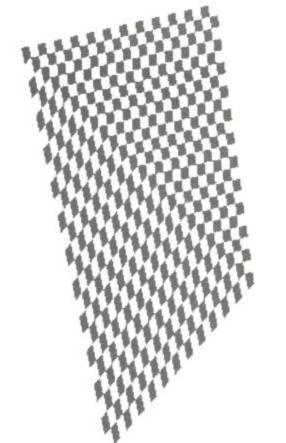
Linear interpolation of texture coordinates w/o perspective divide

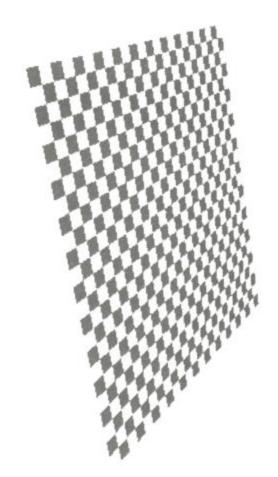


Correct interpolation of texture coordinates w/ perspective divide

Hill Figure 8.42







Linear interpolation of texture coordinates w/o perspective divide

Correct interpolation of texture coordinates w/ perspective divide

Hill Figure 8.42

#### **Overview**



#### Texture mapping methods

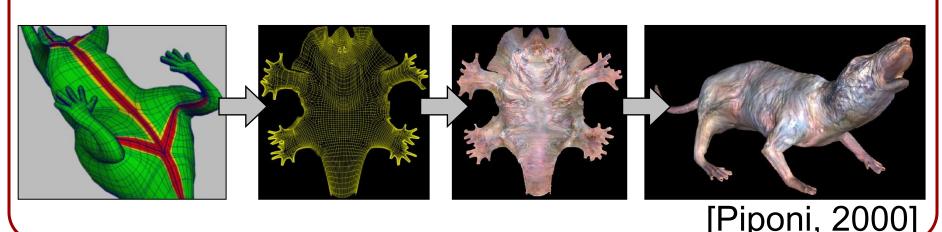
- Parameterization
- Sampling

#### Texture mapping applications

- Modulation textures
- Illumination mapping
- Bump mapping
- Environment mapping
- Shadow maps

# Map to a 2D Domain (w/ Added Cuts)

- Introduce cuts to give the surface a disk topology
- Map the cut surface to the 2D plane
- Assign texture coordinates in the plane
- ✓ Good cut placement can reduce distortion
- Need to ensure cross-seam continuity
- \* Have to contend with distortion

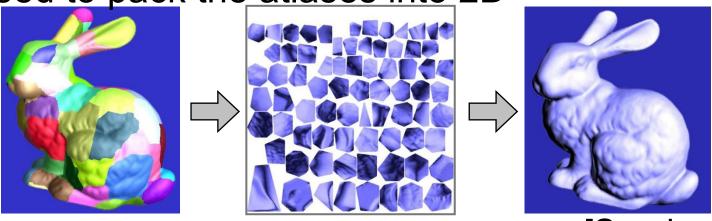


### **Texture Atlases**



- Decompose the surface into multiple charts
- Map each chart to the 2D plane
- Assign texture coordinates in the plane
- ✓ Less distortion in the mapping
- Harder to ensure cross-seam continuity

Need to pack the atlases into 2D



[Sander, 2001]

#### **Overview**



### Texture mapping methods

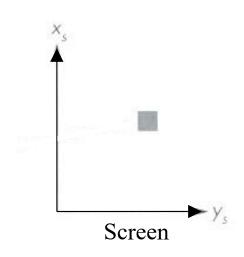
- Parameterization
- Sampling

#### Texture mapping applications

- Modulation textures
- Illumination mapping
- Bump mapping
- Environment mapping
- Shadow maps



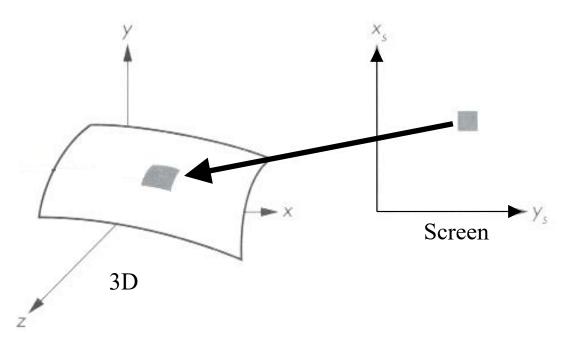
Given pixel on a screen:





#### Given pixel on a screen:

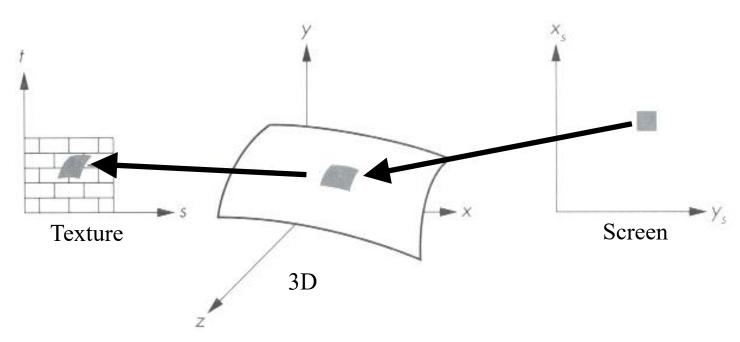
1. Determine the corresponding surface patch





#### Given pixel on a screen:

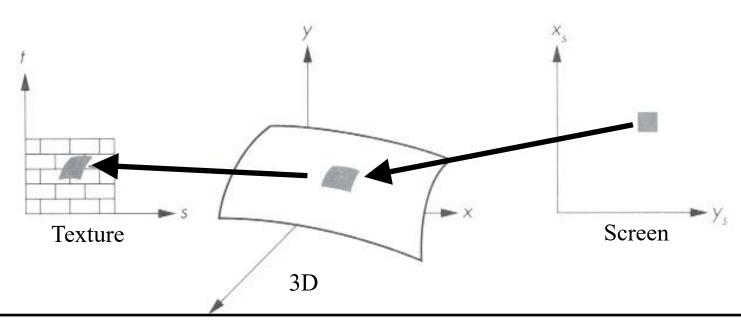
- 1. Determine the corresponding surface patch
- 2. Determine the corresponding texture patch





#### Given pixel on a screen:

- 1. Determine the corresponding surface patch
- 2. Determine the corresponding texture patch

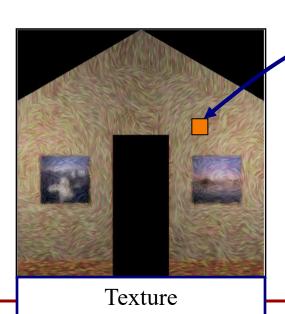


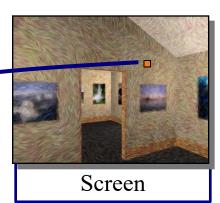
While the true shape of the texture patch mapping to a screen pixel may be hard to compute, we can approximate it using the Jacobian.



#### Given pixel on a screen:

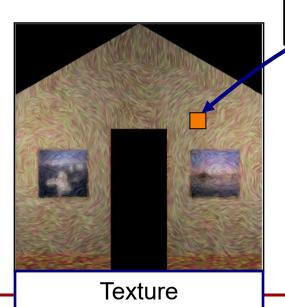
- 1. Determine the corresponding surface patch
- 2. Determine the corresponding texture patch
- 3. Average texel values over the texture patch







- Size of texture patch depends on the deformation
  - Computation is proportional to the pixel footprint
- Can pre-filter images for better performance
  - MIP (Multum In Parvo\*) maps
  - Summed area tables



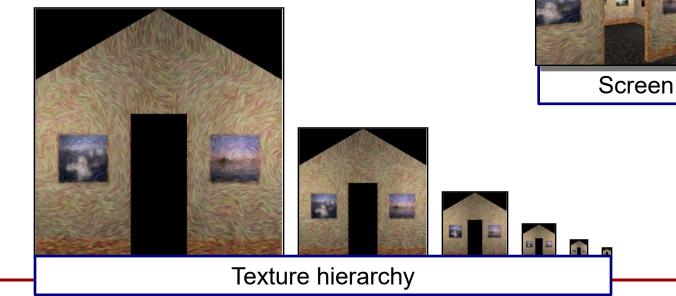
Average over many pixels



\*multum in parvo = much in little



Pre Processing: Compute a hierarchy of successively down-sampled texture images

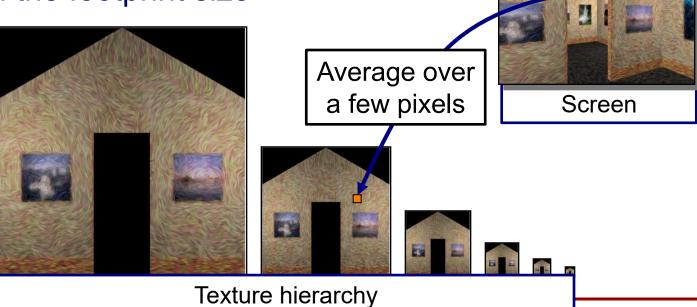




Pre Processing: Compute a hierarchy of successively down-sampled texture images

Run-time: Sample the closest MIP map level(s)

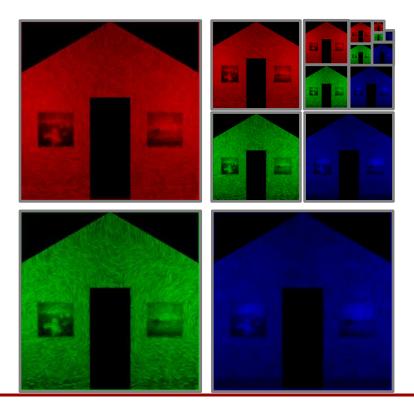
- Easy for hardware
- Computation is constant in the footprint size





Pre Processing: Compute a hierarchy of successively down-sampled texture images

✓ Storage is 4/3× the size of the input image



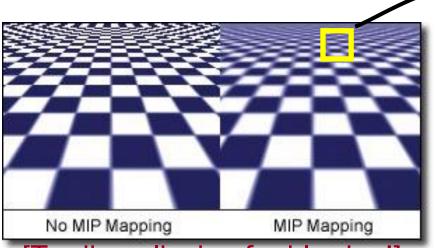


Pre Processing: Compute a hierarchy of successively down-sampled texture images

✓ Storage is 4/3× the size of the input image

Run-time: Sample the closest MIP map level(s)

\* The filtering is isotropic – assumes identical compression along vertical and horizontal directions



[Trading aliasing for blurring!]

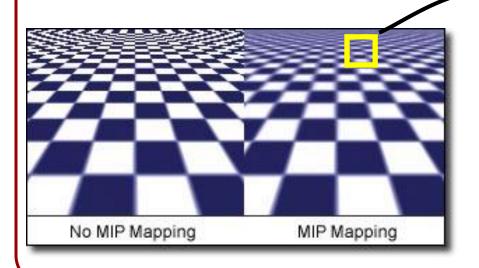
### **Summed-Area Tables**

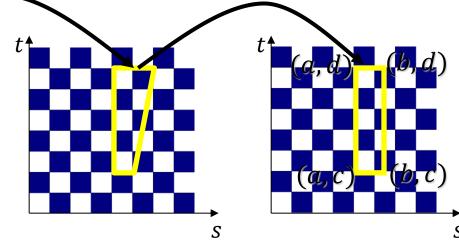


#### Key Idea:

Approximate the summation/integration over an arbitrary region by a summation/integration over an axis-aligned rectangle:

$$Sum([a,b] \times [c,d]) = \int_a^b \int_c^d f(x,y) \, dy \, dx$$



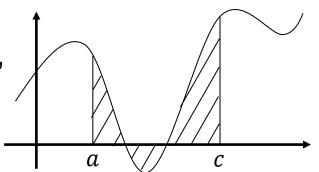




#### **Integration**:

Given a function f(x) and interval [a, c], the integral of f over the interval is:

$$\int_{a}^{c} f(x) dx$$



#### Naïve Approach:

Pre-compute  $S(a,b) \equiv \int_a^b f(x) dx$  in a look-up table and evaluate that.

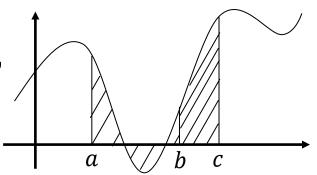
- ✓ Fast (constant time) look up
- $\times$  Replaces 1D function f with 2D function S.



#### Integration:

Given a function f(x) and interval [a, c], the integral of f over the interval is:

$$\int_{a}^{c} f(x) dx$$



#### Recall:

For any point  $b \in [a, c]$  in the interval, we have:

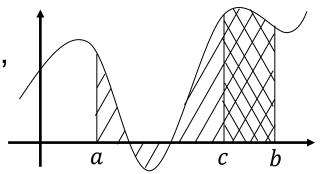
$$\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$$



#### Integration:

Given a function f(x) and interval [a, c], the integral of f over the interval is:

$$\int_{a}^{c} f(x) dx$$



#### Recall:

For any point  $b \in [a, c]$  in the interval, we have:

$$\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$$

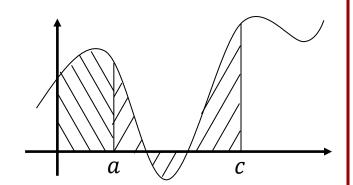
This is true even if b is outside the interval since:

$$\int_{a}^{b} f(x) dx = -\int_{b}^{a} f(x) dx$$



#### Approach:

Replace the integral over an interval, with two variable end-points with the difference between integrals with one variable end-point:



$$\int_{a}^{c} f(x)dx = \int_{0}^{c} f(x)dx - \int_{0}^{a} f(x)dx$$

 $\Rightarrow$  Replace a look-up in the 2D function  $S(a,c) = \int_a^c f(x) dx$  with two look-ups in the 1D function  $S_0(b) = \int_0^b f(x) dx$ 



#### Integration:

In 2D, we can write out the integral of the function f over the rectangle  $[a,b] \times [c,d]$  as:

$$\int_{a}^{b} \int_{c}^{d} f(s,t) dt \, ds$$



#### **Integration**:

In 2D, we can write out the integral of the function f over the rectangle  $[a, b] \times [c, d]$  as:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{a}^{b} \left( \int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$



#### **Integration**:

In 2D, we can write out the integral of the function f over the rectangle  $[a,b] \times [c,d]$  as:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{a}^{b} \left( \int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$

$$= \int_{0}^{b} \left( \int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds - \int_{0}^{d} \left( \int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$



### **Integration**:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{a}^{b} \left( \int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$

$$= \int_{0}^{b} \left( \int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds - \int_{0}^{a} \left( \int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$

$$= \int_{0}^{b} \int_{0}^{d} f(s,t)dt \, ds - \int_{0}^{b} \int_{0}^{c} f(s,t)dt \, ds - \int_{0}^{a} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{a} \int_{0}^{c} f(s,t)dt \, ds$$

Precomputing the 2D function:

$$S_{(0,0)}(x,y) \equiv \int_0^x \int_0^y f(s,t) dt ds$$

lets us evaluate integrals with four look-ups:

$$\int_{a}^{b} \int_{c}^{d} f(s,t) dt ds = S_{(0,0)}(b,d) - S_{(0,0)}(b,c) - S_{(0,0)}(a,d) + S_{(0,0)}(a,c)$$

over the rectangle  $[a,b] \times [c,d]$  as:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{a}^{b} \left( \int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$

$$= \int_{0}^{b} \left( \int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds - \int_{0}^{a} \left( \int_{0}^{d} f(s,t)dt - \int_{0}^{c} f(s,t)dt \right) ds$$

$$= \int_{0}^{b} \int_{0}^{d} f(s,t)dt \, ds - \int_{0}^{b} \int_{0}^{c} f(s,t)dt \, ds - \int_{0}^{a} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{a} \int_{0}^{c} f(s,t)dt \, ds$$



### **Integration**:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{0}^{b} \int_{0}^{d} f(s,t)dt \, ds - \int_{0}^{b} \int_{0}^{c} f(s,t)dt \, ds - \int_{0}^{a} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{a} \int_{0}^{c} f(s,t)dt \, ds$$



### Integration:

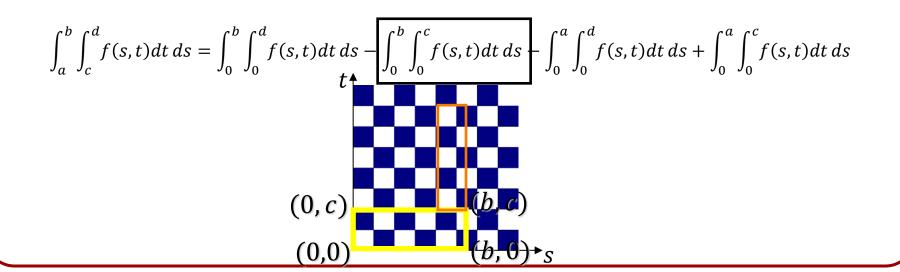
$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{0}^{b} \int_{0}^{d} f(s,t)dt \, ds - \int_{0}^{b} \int_{0}^{c} f(s,t)dt \, ds - \int_{0}^{a} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{a} \int_{0}^{c} f(s,t)dt \, ds$$

$$(0,d)$$

$$(b,d)$$



### Integration:





### **Integration**:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{0}^{b} \int_{0}^{d} f(s,t)dt \, ds - \int_{0}^{b} \int_{0}^{c} f(s,t)dt \, ds - \int_{0}^{a} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{a} \int_{0}^{c} f(s,t)dt \, ds$$

$$(0,d)$$



### **Integration**:

$$\int_{a}^{b} \int_{c}^{d} f(s,t)dt \, ds = \int_{0}^{b} \int_{0}^{d} f(s,t)dt \, ds - \int_{0}^{b} \int_{0}^{c} f(s,t)dt \, ds - \int_{0}^{a} \int_{0}^{d} f(s,t)dt \, ds + \int_{0}^{a} \int_{0}^{c} f(s,t)dt \, ds$$

$$(0,c)$$

$$(0,c)$$



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) dt ds$$

Each summed-area table texel is the sum of all input texels below and to the left

#### **Input image**

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

Each summed-area table texel is the sum of all input texels below and to the left

#### **Input image**

-			
1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

1		



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

Each summed-area table texel is the sum of all input texels below and to the left

#### **Input image**

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

1	3	



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

Each summed-area table texel is the sum of all input texels below and to the left

#### **Input image**

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

1	3	4	



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

Each summed-area table texel is the sum of all input texels below and to the left

#### **Input image**

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

1	3	4	7



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

Each summed-area table texel is the sum of all input texels below and to the left

#### **Input image**

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

5			
1	3	4	7



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

Each summed-area table texel is the sum of all input texels below and to the left

#### **Input image**

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

5	9		
1	3	4	7



Precompute the values of the integral:

$$S_{(0,0)}(a,b) = \int_0^a \int_0^b f(s,t) \, dt \, ds$$

Each summed-area table texel is the sum of all input texels below and to the left

#### **Input image**

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7



### Example:

Compute the average in the rectangle  $[1,3] \times [2,3]$ .

⇒ Compute the sum and divide by the area

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

Input image



#### Example:

Compute the average in the rectangle  $[1,3] \times [2,3]$ .

 $\Rightarrow$  Compute the sum and divide by the area  $Sum([1,3] \times [2,3]) = S_{(0,0)}(3,3)$ 

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

**Input image** 



### Example:

Compute the average in the rectangle  $[1,3] \times [2,3]$ .

⇒ Compute the sum and divide by the area

$$Sum([1,3] \times [2,3]) = S_{(0,0)}(3,3) - S_{(0,0)}(0,3)$$

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

Input image



### Example:

Compute the average in the rectangle  $[1,3] \times [2,3]$ .

⇒ Compute the sum and divide by the area

$$Sum([1,3] \times [2,3]) = S_{(0,0)}(3,3) - S_{(0,0)}(0,3) - S_{(0,0)}(3,1)$$

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

Input image



### Example:

Compute the average in the rectangle  $[1,3] \times [2,3]$ .

⇒ Compute the sum and divide by the area

$$Sum([1,3] \times [2,3]) = S_{(0,0)}(3,3) - S_{(0,0)}(0,3) - S_{(0,0)}(3,1) + S_{(0,0)}(0,1)$$

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

Input image



### Example:

Compute the average in the rectangle  $[1,3] \times [2,3]$ .

⇒ Compute the sum and divide by the area

$$Sum([1,3] \times [2,3]) = S_{(0,0)}(3,3) - S_{(0,0)}(0,3) - S_{(0,0)}(3,1) + S_{(0,0)}(0,1)$$

$$= 26 - 6 - 14 + 5 = 11$$

$$Average([1,3] \times [2,3]) = \frac{Sum([1,3] \times [2,3])}{Area([1,3] \times [2,3])} = \frac{11}{6}$$

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

**Input image** 



### Precompute the values of the integral

- ✓ Constant time averaging, regardless of rectangle size
- ➤ If the input image has values in the range [0,255] (i.e. one byte per channel), the summed area table can have values in the range [0,255 · width · height]

1	2	4	0
0	3	1	1
4	2	0	1
1	2	1	3

6	15	21	26
5	12	14	19
5	9	10	14
1	3	4	7

**Input image** 

### **Overview**



### Texture mapping methods

- Parameterization
- Sampling

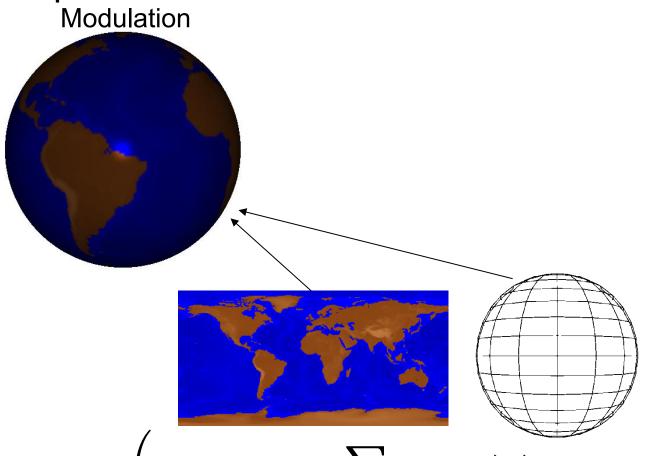
### Texture mapping applications

- Modulation textures
- Illumination mapping
- Bump mapping
- Environment mapping
- Shadow mapping

### **Modulation textures**



Map texture values to scale factor

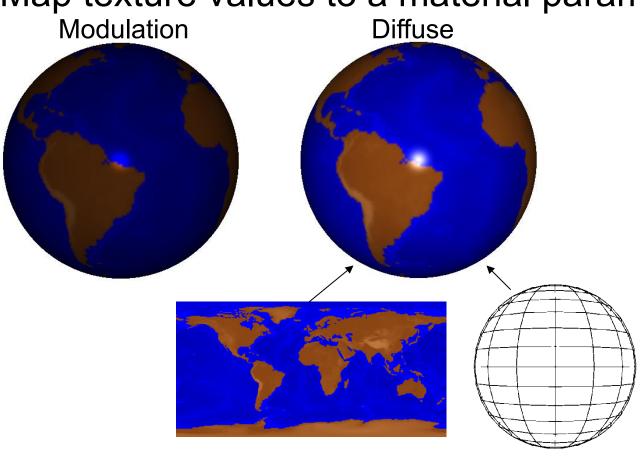


$$I = T(s, t) \left( I_E + K_A \cdot I_L^A + \sum_{l} \left( K_D \cdot \langle \vec{N}, \vec{L} \rangle + K_S \cdot \langle \vec{V}, \vec{R}(\vec{L}) \rangle^{K_n} \right) \cdot I_L \right)$$

### **Illumination Mapping**



Map texture values to a material parameter



$$I = I_E + K_A \cdot I_L^A + \sum_{l} (T(s, t) \langle \vec{N}, \vec{L} \rangle + K_S \cdot \langle \vec{V}, \vec{R}(\vec{L}) \rangle^{K_n}) \cdot I_L$$

# **Illumination Mapping**

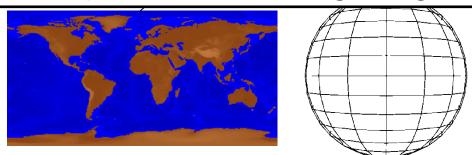


Map texture values to a material parameter

Modulation Diffuse

We need to evaluate the texture at each pixel but can the interpolated lighting values  $\langle \vec{N}, \vec{L} \rangle$  from the corners

This requires the graphics card to separately store the diffuse component of the lighting at each vertex

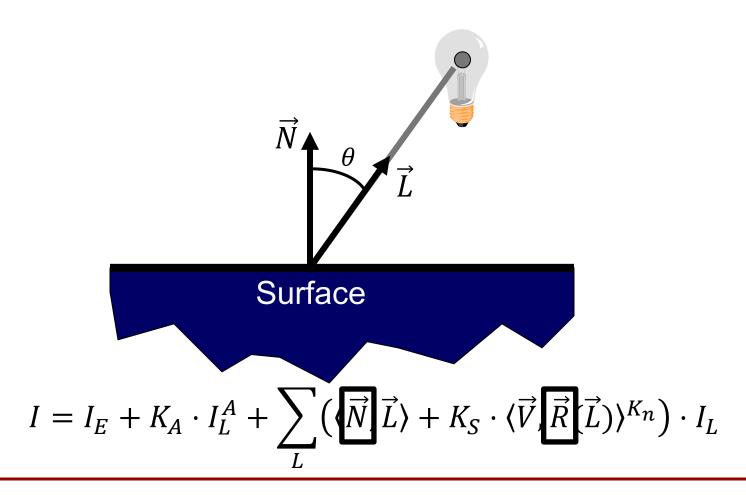


$$I = I_E + K_A \cdot I_L^A + \sum_{l} (T(s, t) \langle \vec{N}, \vec{L} \rangle + K_S \cdot \langle \vec{V}, \vec{R}(\vec{L}) \rangle^{K_n}) \cdot I_L$$

### **Bump Mapping**

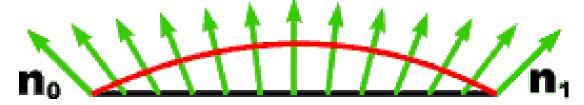


Recall that many parts of our lighting calculation depend on surface normals



### **Bump Mapping**





Phong shading performs per-pixel lighting calculations with the interpolated normal

approximates a smoothly curved surface



Bump maps encode the normals in the texture

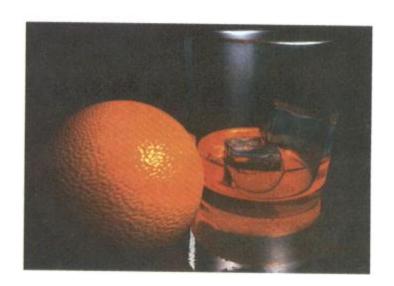
approximates a more complex undulating surface

P. Rheingans

### **Bump Mapping**







Bump mapping does not change object silhouette



#### Goal:

Render shiny surfaces so they the reflect the world.

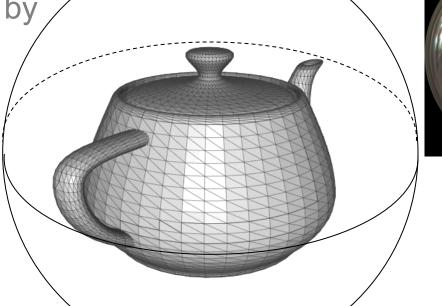


### Goal:

Render shiny surfaces so they the reflect the world.

Pre-compute a map of the surrounding environment

 Set texture coordinates <u>dynamically</u> by reflecting







### Goal:

Render shiny surfaces so they the reflect the world.

Pre-compute a map of the surrounding environment

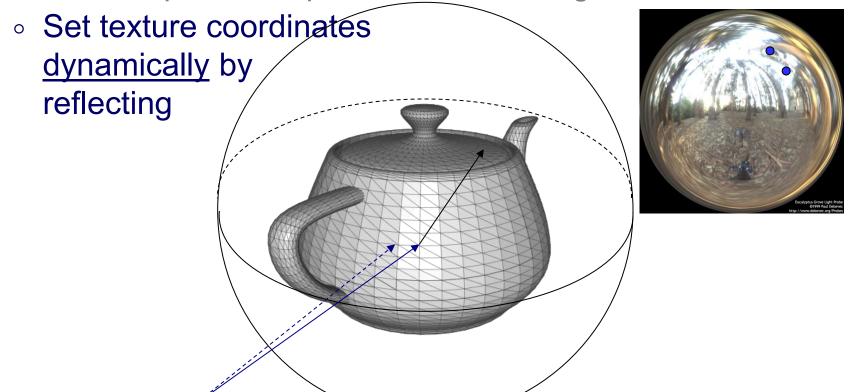
 Set texture coordinates dynamically by reflecting



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### Goal:

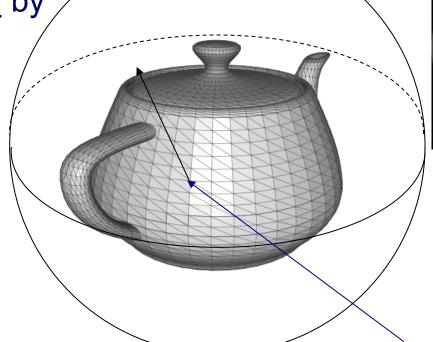
Render shiny surfaces so they the reflect the world.

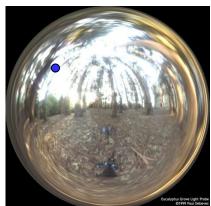
Pre-compute a map of the surrounding environment

 Set texture coordinates dynamically by

reflecting

For the same triangle, changing the position of the camera changes the texture coordinates.







### Goal:

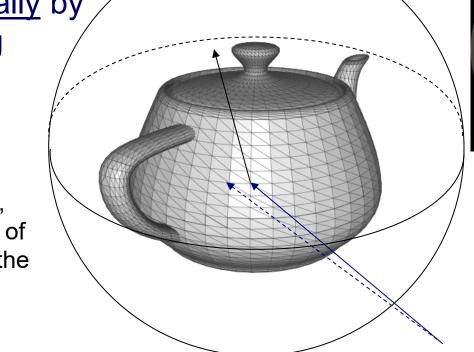
Render shiny surfaces so they the reflect the world.

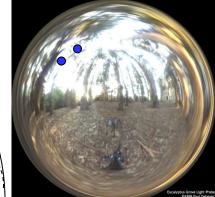
Pre-compute a map of the surrounding environment

Set texture coordinates

dynamically by

reflecting





For the same triangle, changing the position of the camera changes the texture coordinates.



### Goal:

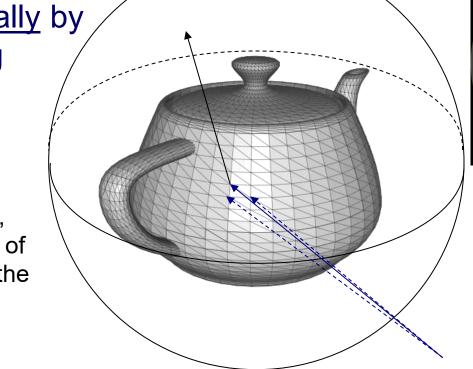
Render shiny surfaces so they the reflect the world.

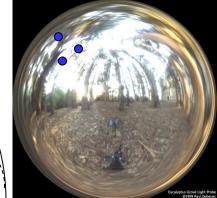
Pre-compute a map of the surrounding environment

Set texture coordinates

dynamically by

reflecting





For the same triangle, changing the position of the camera changes the texture coordinates.



### Goal:

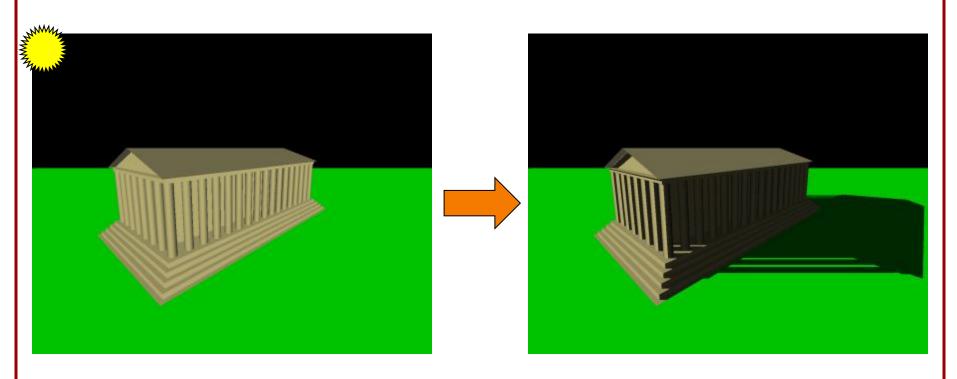
Render shiny surfaces so they the reflect the world.



P. Debevec



Test if surface is in shadow when computing the contribution to the lighting equation.



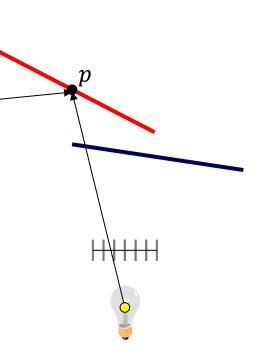
Images courtesy of https://en.wikipedia.org/wiki/Shadow\_mapping



Q: Is a point p, seen by the camera, in shadow with respect to the light?

A: The point is not in shadow if:

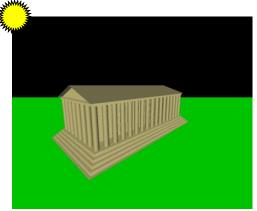
- The light "sees" p.
- ⇒ Rendering the scene from the light's perspective, p's z-coordinate is the value stored in the z-buffer.



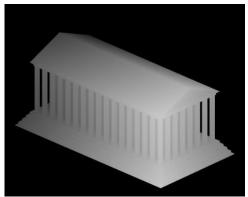


### Algorithm:

- Render the scene from the light's perspective and read back the z-buffer/shadow map.
- For each pixel in the camera view, compute its z-coordinate relative to the light



Camera view

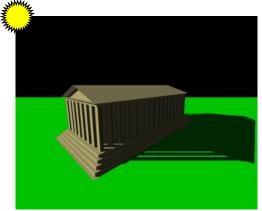


Shadow map

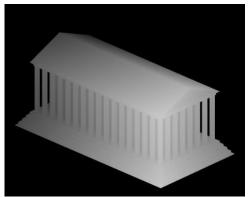


### Algorithm:

- Render the scene from the light's perspective and read back the z-buffer/shadow map.
- For each pixel in the camera view, compute its z-coordinate relative to the light
  - » If it's further back than the value in the shadow map, it's in shadow
  - » Otherwise, it's illuminated



Camera view

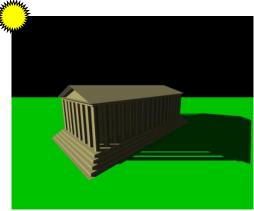


Shadow map

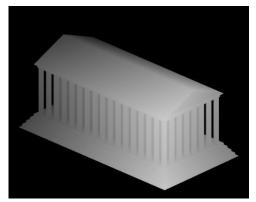


#### Note:

- The projection used for rendering from the light-source depends on the type of light:
  - » Directional → Parallel
  - » Point → Perspective
- Need to use multiple shadow maps if there are multiple lights in the scene



Camera view



Shadow map