

3D Polygon Rendering Pipeline

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(601.457/657)

3D Polygon Rendering



Many applications require *interactive* rendering of 3D polygons with direct illumination



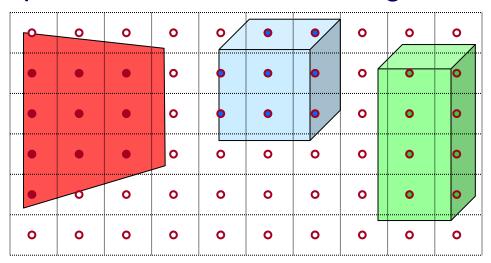
God of War (Santa Monica Studio, 2018)

Ray Casting



For each sample:

- Construct ray <u>from the camera into the scene</u>
- Find first surface intersected by ray through pixel
- Compute color of sample based on surface radiance
- Send 2D pixels into the scene and get color

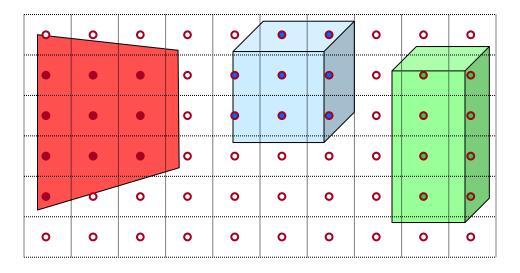


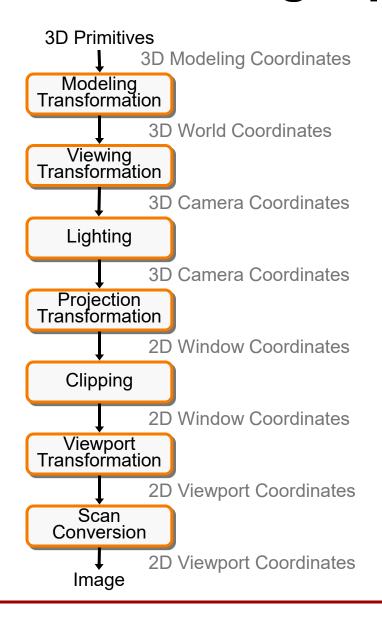
3D Polygon Rendering

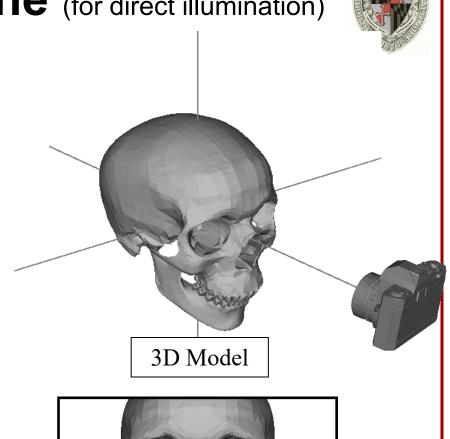


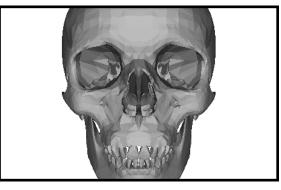
For each primitive:

Send 3D points to the camera and set the pixel color



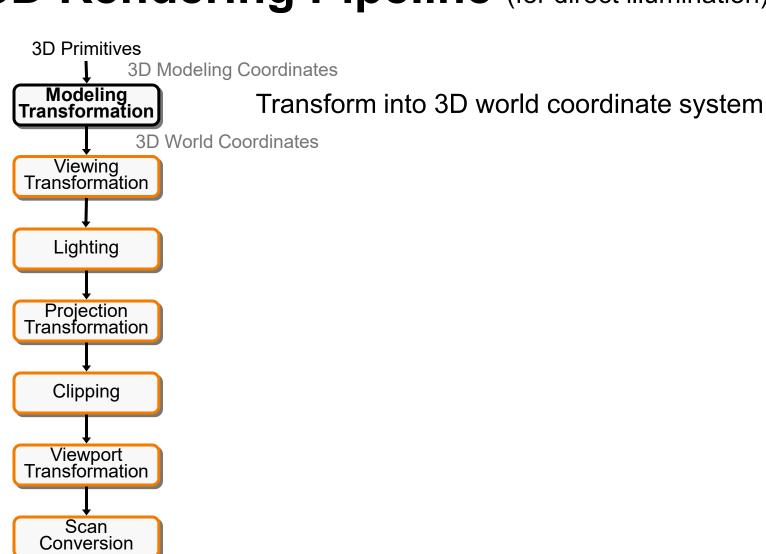




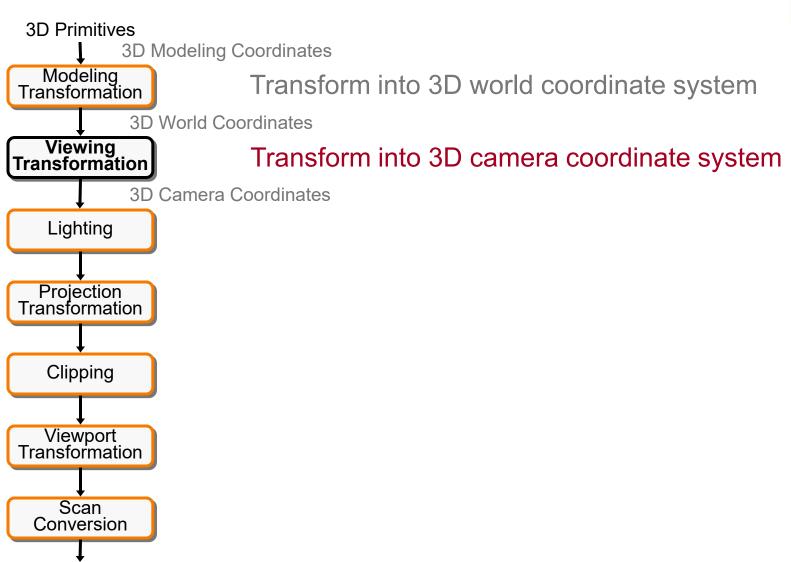


2D Viewport

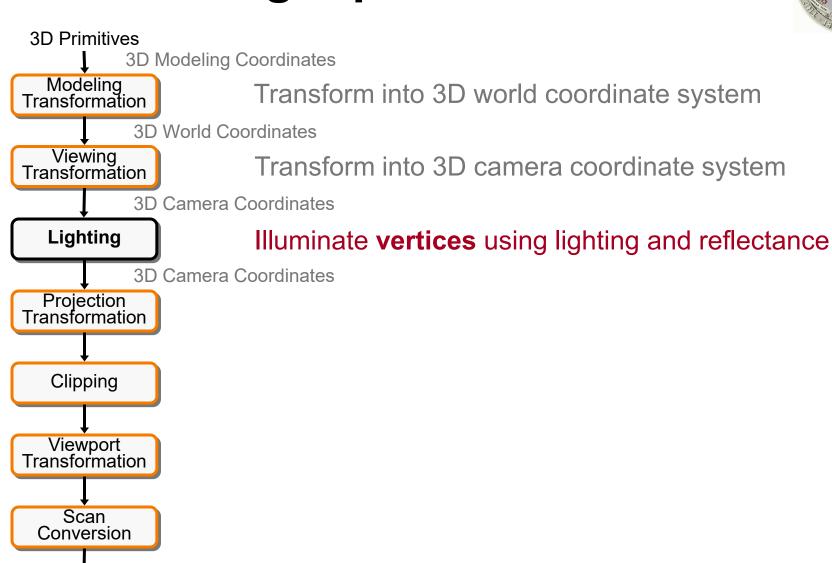




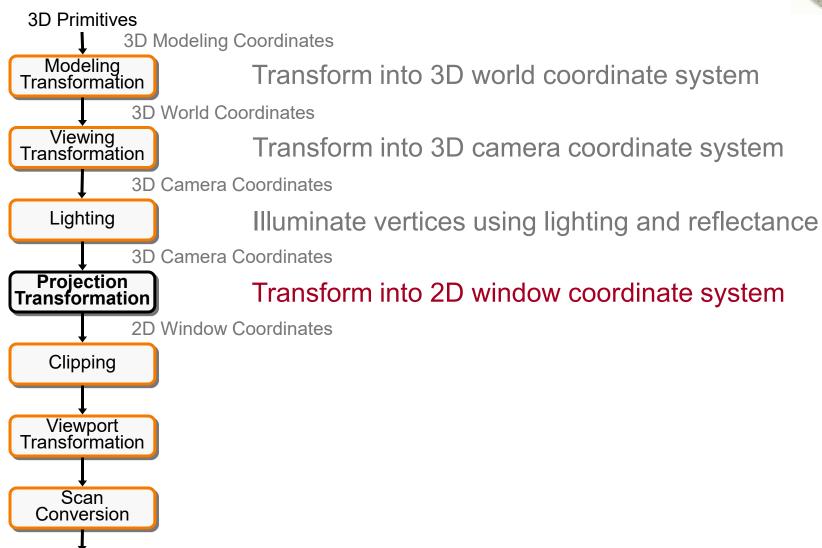




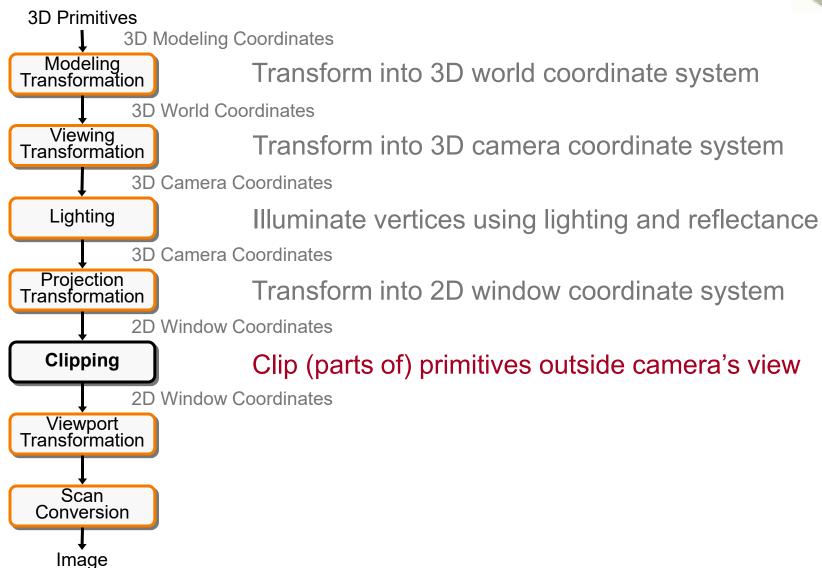




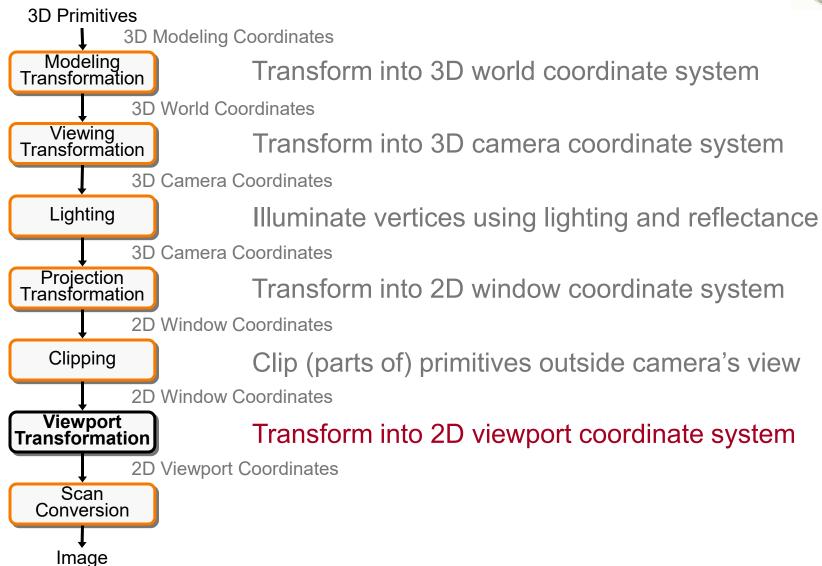




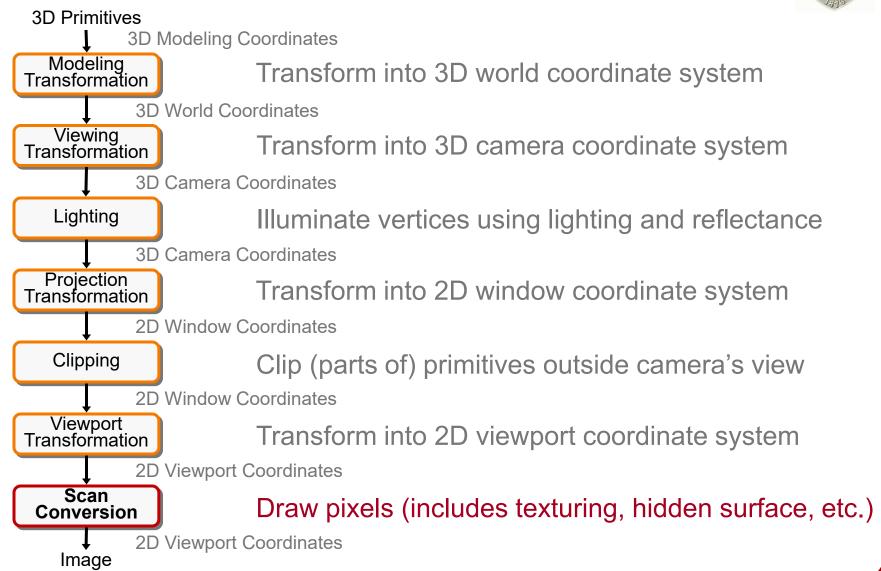




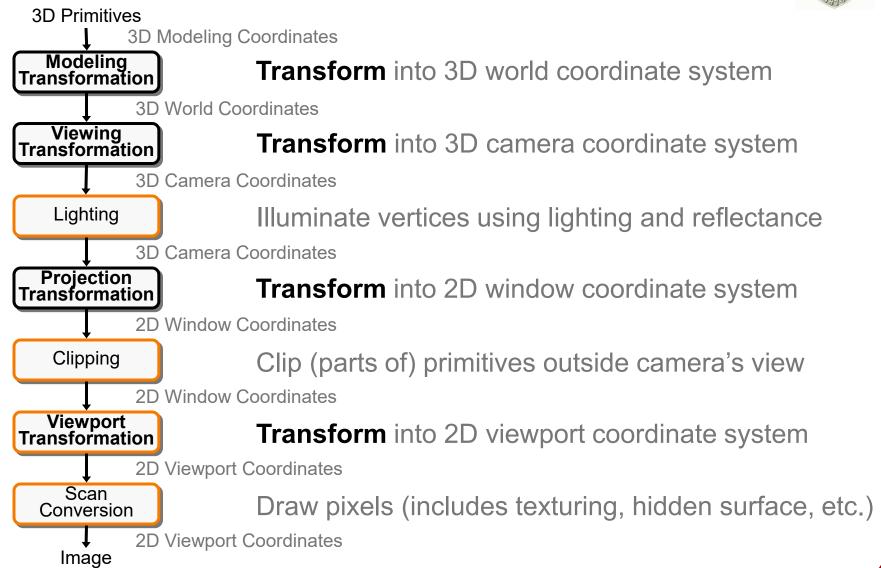












Recall: Homogeneous Coordinates



Add a 4th coordinate to every 3D point

- (x, y, z, w) represents a point at location $(\frac{x}{w}, \frac{y}{w}, \frac{z}{w})$
- (x, y, z, 0) represents the (unsigned) direction $\frac{\pm (x, y, z)}{\sqrt{x^2 + y^2 + z^2}}$
- (0,0,0,0) is not allowed

Recall: 3D Transformations



Using homogenous coordinates, we have two types of transformations:

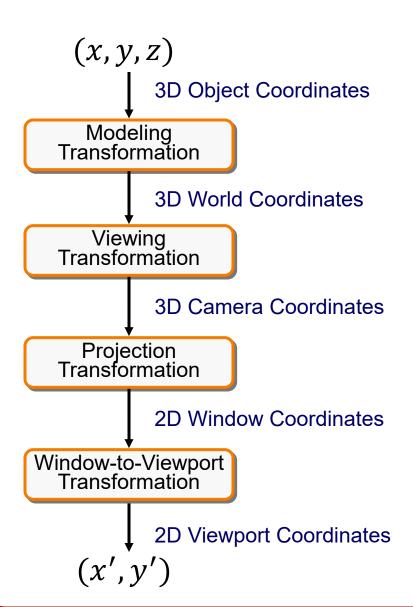
Affine

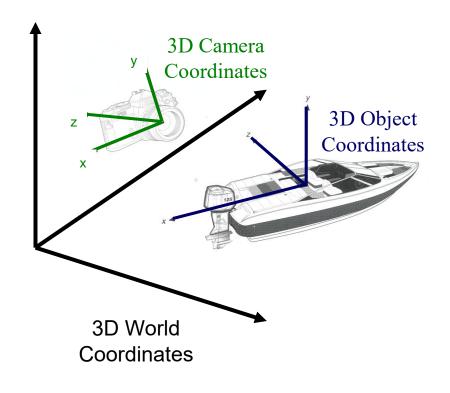
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Projective

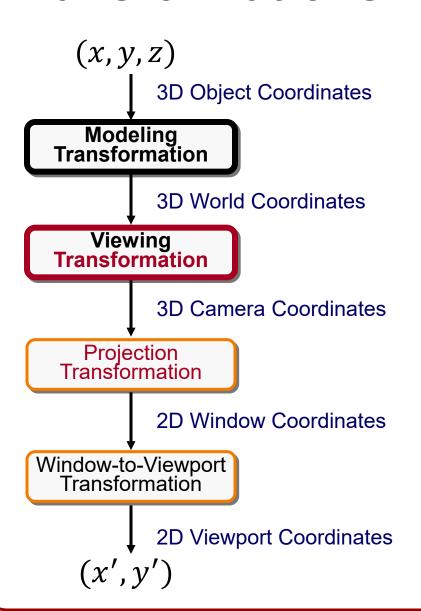
$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$











Modelview Transformations

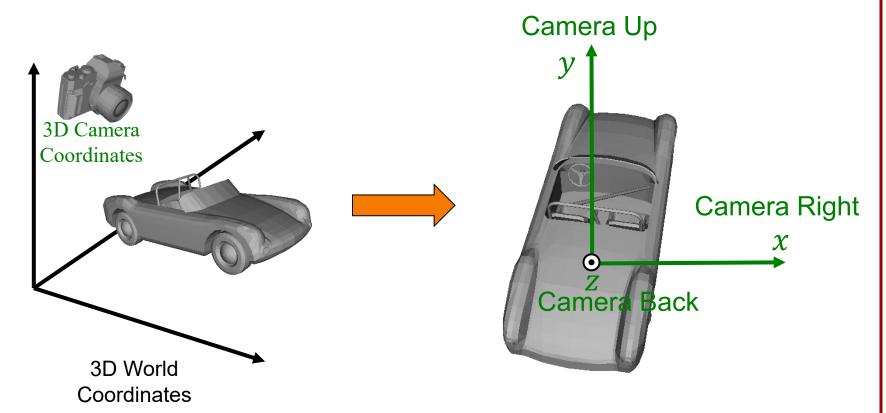


```
(x, y, z)
              3D Object Coordinates
  Modeling
Transformation
              3D World Coordinates
  Viewing
Transformation
              3D Camera Coordinates
  Projection
Transformation
              2D Window Coordinates
Window-to-Viewport
Transformation
              2D Viewport Coordinates
    (x', y')
```



Canonical viewing coordinate system

- \circ Convention is right-handed (looking down -z axis)
- Convenient for projection, clipping, etc.





The transformation, $T_{W\to C}$, taking us from world coordinates to camera coordinates should map:

The right vector to the *x*-axis:

$$(R_x, R_y, R_z, 0) \rightarrow (1,0,0,0)$$

• The up vector to the *y*-axis:

$$(U_x, U_y, U_z, 0) \rightarrow (0,1,0,0)$$

• The back vector to the z-axis:

$$(B_x, B_y, B_z, 0) \rightarrow (0,0,1,0)$$

The eye position to the origin:

$$(E_x, E_y, E_z, 1) \rightarrow (0,0,0,1)$$

How should we define this transformation/matrix?



Consider the inverse transformation, $T_{C\to W}$, taking us from camera coordinates to world coordinates:

$$(R_{\chi}, R_{y}, R_{z}, 0) \leftarrow (1,0,0,0)$$
$$(U_{\chi}, U_{y}, U_{z}, 0) \leftarrow (0,1,0,0)$$
$$(B_{\chi}, B_{y}, B_{z}, 0) \leftarrow (0,0,1,0)$$
$$(E_{\chi}, E_{y}, E_{z}, 1) \leftarrow (0,0,0,1)$$

This is described by the camera-to-world matrix:

$$\begin{pmatrix} x^{w} \\ y^{w} \\ z^{w} \end{pmatrix} = \begin{pmatrix} R_{x} & U_{x} & B_{x} & E_{x} \\ R_{y} & U_{y} & B_{y} & E_{y} \\ R_{z} & U_{z} & B_{z} & E_{z} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^{c} \\ y^{c} \\ z^{c} \\ 1 \end{pmatrix}$$



The world-to-camera matrix is its inverse:

$$\begin{pmatrix} x^{c} \\ y^{c} \\ z^{c} \\ 1 \end{pmatrix} = \begin{pmatrix} R_{x} & U_{x} & B_{x} & E_{x} \\ R_{y} & U_{y} & B_{y} & E_{y} \\ R_{z} & U_{z} & B_{z} & E_{z} \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} x^{w} \\ y^{w} \\ z^{w} \\ 1 \end{pmatrix}$$

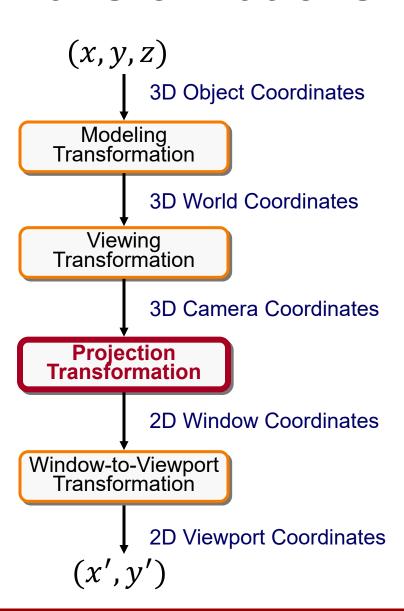
$$\mathbf{T}_{W \to c} \stackrel{!}{=} \mathbf{T}_{c \to W}^{-1}$$

This is described by the camera-to-world matrix:

$$\begin{pmatrix} x^{w} \\ y^{w} \\ z^{w} \end{pmatrix} = \begin{pmatrix} R_{x} & U_{x} & B_{x} & E_{x} \\ R_{y} & U_{y} & B_{y} & E_{y} \\ R_{z} & U_{z} & B_{z} & E_{z} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^{c} \\ y^{c} \\ z^{c} \\ 1 \end{pmatrix}$$

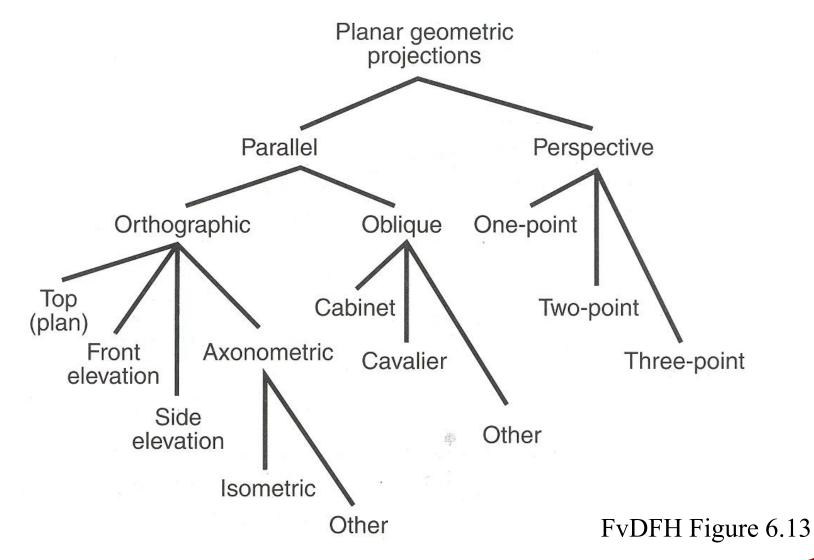
$$\mathbf{T}_{C \to W}$$





Taxonomy of Projections



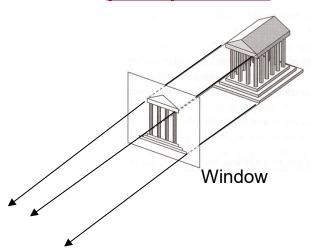


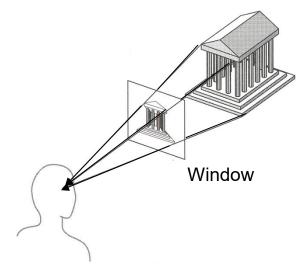
Projection



Two general classes. Both shoot rays *from* the 3D scene, *through* the 2D window:

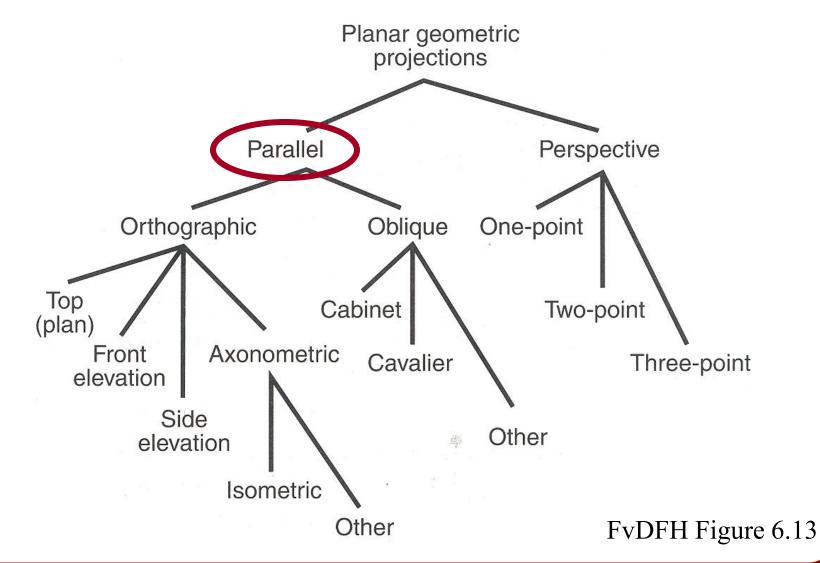
- Parallel Projection:
 - » Rays converge at a point at infinity and are parallel
- Perspective "Projection":
 - » Rays converge at a finite point, giving rise to perspective distortion





Taxonomy of Projections



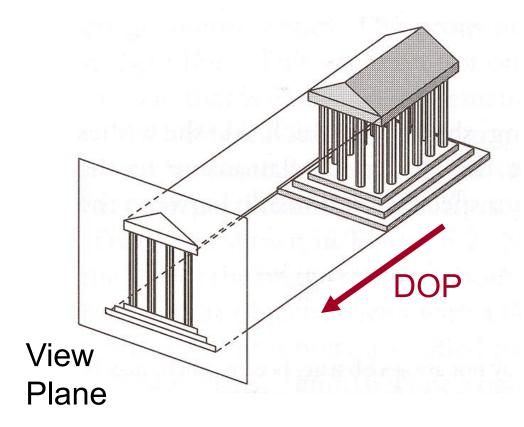


Parallel Projection



Center of projection is at infinity

Direction of projection (DoP) same for all points

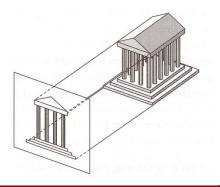


Angel Figure 5.4

Parallel Projection

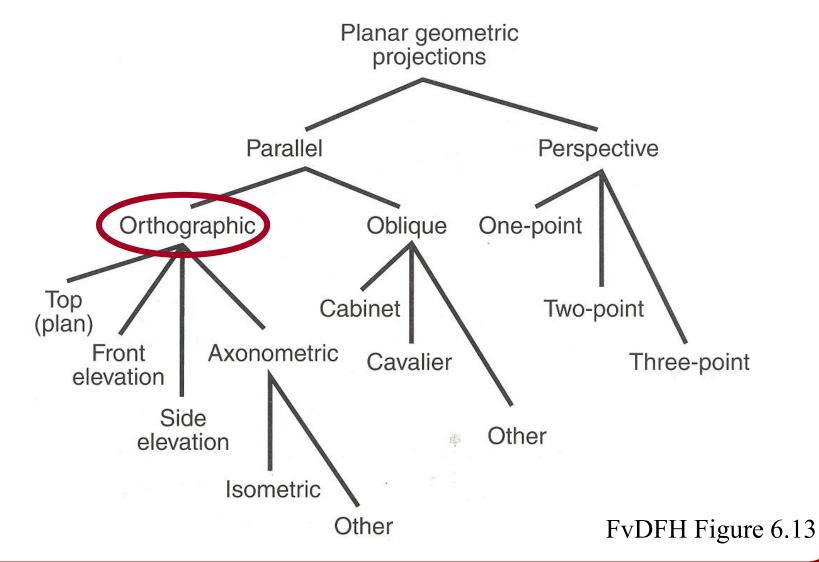


- ✓ Parallel lines remain parallel
- ✓ Proportions are preserved (no foreshortening)
- × (Some) angles are not preserved
- Less realistic looking



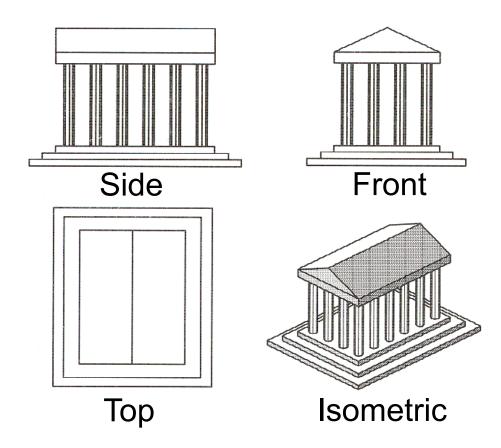
Taxonomy of Projections







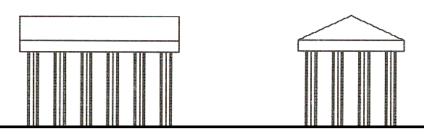
DoP perpendicular to view plane



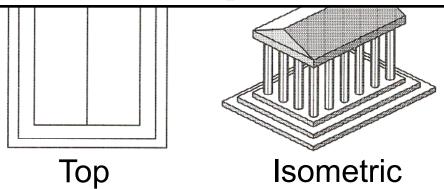
Angel Figure 5.5



DoP perpendicular to view plane



- Lines perpendicular to the view plane vanish
- Faces parallel to the view plane are un-distorted.

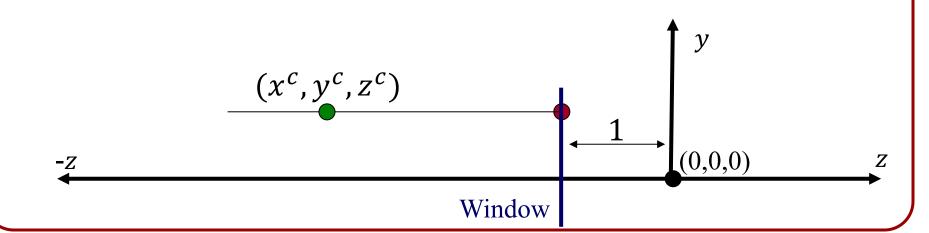




DoP perpendicular to view plane

• Maps a point in 3D space to the (x, y, -1)-plane, by projecting out the z-component:

$$(x^c, y^c, z^c) \rightarrow (x^c, y^c, -1)$$





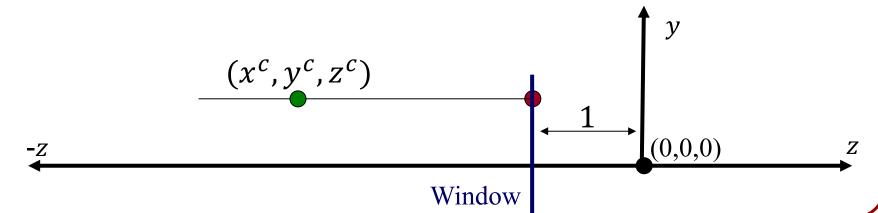
DoP perpendicular to view plane

• Maps a point in 3D space to the (x, y, -1)-plane, by projecting out the z-component:

$$(x^c, y^c, z^c, 1) \rightarrow (x^c, y^c, -1, 1)$$

In terms of homogenous coordinates:

$$\begin{bmatrix} x^c \\ y^c \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^c \\ y^c \\ z^c \\ 1 \end{bmatrix}$$





DoP perpendicular to view plane

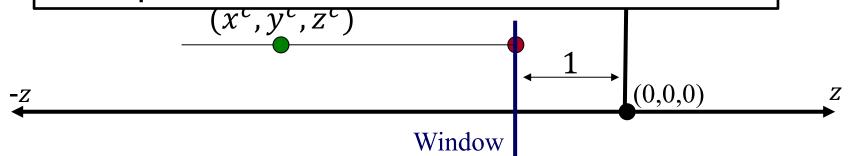
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$$\begin{bmatrix} x^c \\ y^c \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^c \\ y^c \\ z^c \\ 1 \end{bmatrix}$$

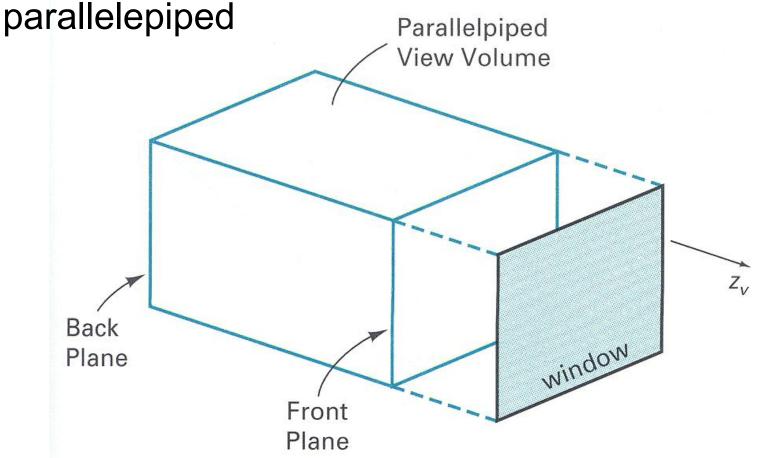
As expected, this is an affine transformation



Parallel Projection View Volume



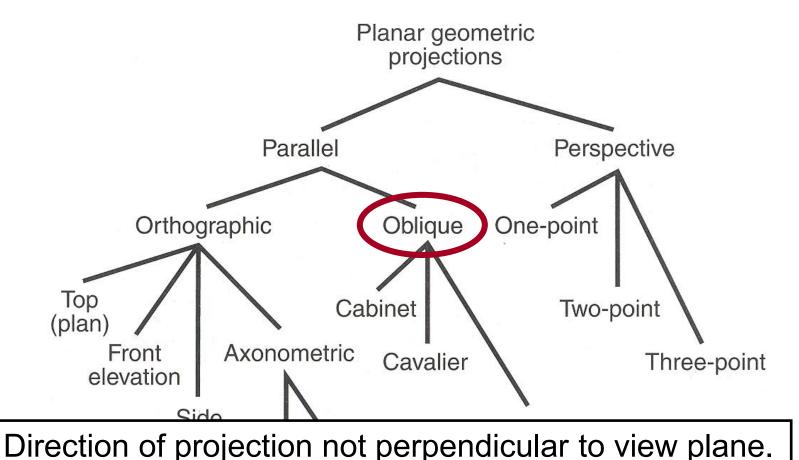
The volume projecting onto the window is a



Taxonomy of Projections

Isometric



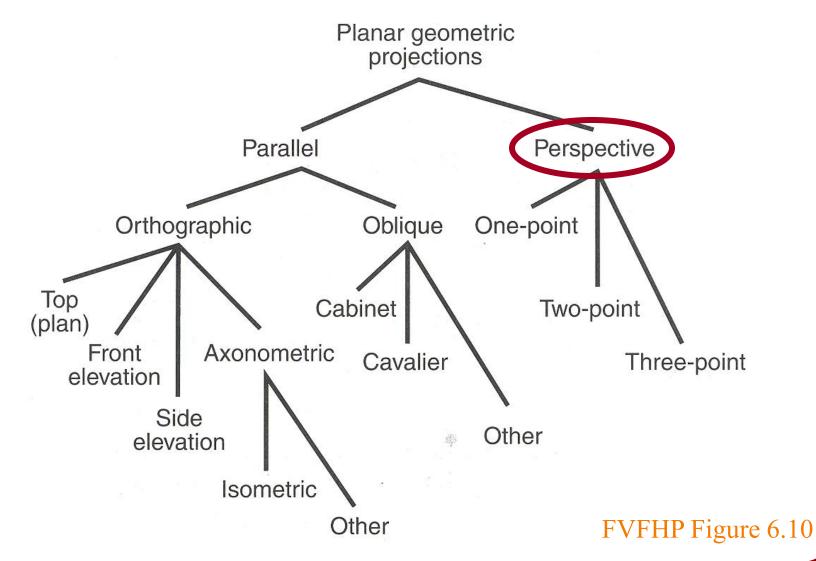


Other

FvDFH Figure 6.13

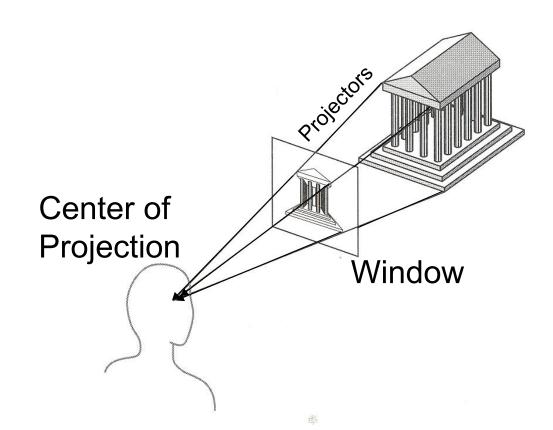
Taxonomy of Projections





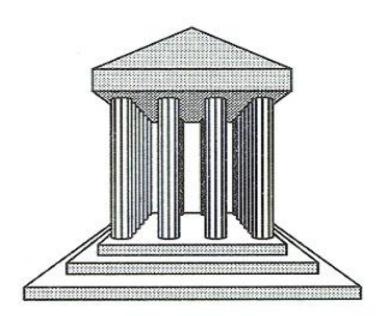


Map points onto view plane along "projectors" emanating from the **center of projection** (CoP)



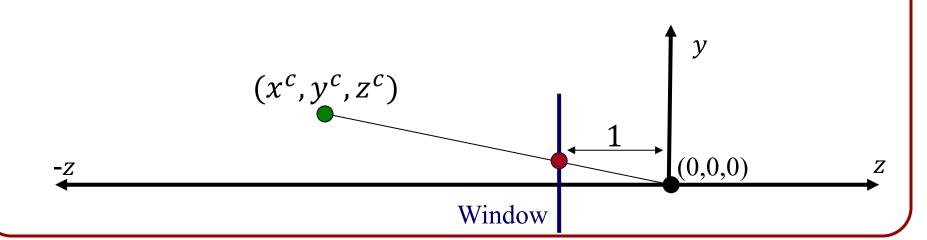


Not all parallel lines remain parallel!



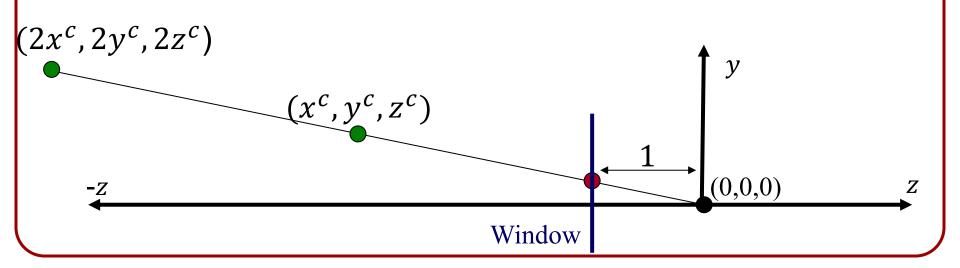


Q: What are the coordinates of the point resulting from projection of (x^c, y^c, z^c) onto the camera screen a <u>unit</u> distance back along the *z*-axis?





A: For any point (x^c, y^c, z^c) and any scalar α , the points (x^c, y^c, z^c) and $(\alpha x^c, \alpha y^c, \alpha z^c)$ map to the same location.

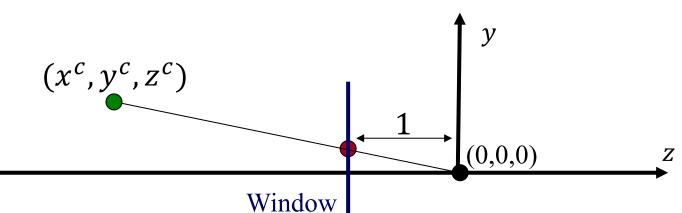




A: For any point (x^c, y^c, z^c) and any scalar α , the points (x^c, y^c, z^c) and $(\alpha x^c, \alpha y^c, \alpha z^c)$ map to the same location.

Since we want the position on the window that intersects the line from (x^c, y^c, z^c) to the origin:

$$(x^c, y^c, z^c) \rightarrow \left(\frac{x^c}{-z^c}, \frac{y^c}{-z^c}, -1\right)$$

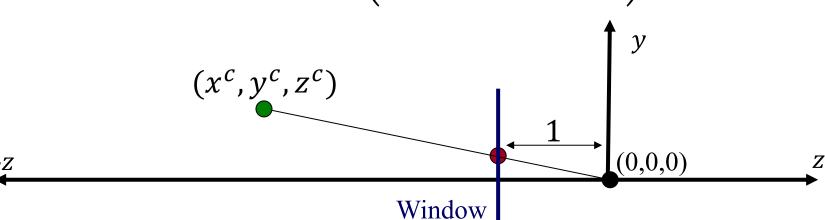




A: For any point (x^c, y^c, z^c) and any scalar α , the points (x^c, y^c, z^c) and $(\alpha x^c, \alpha y^c, \alpha z^c)$ map to the same location.

Since we want the position on the window that intersects the line from (x^c, y^c, z^c) to the origin:

$$(x^c, y^c, z^c, 1) \rightarrow \left(\frac{x^c}{-z^c}, \frac{y^c}{-z^c}, -1, 1\right)$$



Perspective Projection Matrix



$$(x^c, y^c, z^c, 1) \rightarrow \left(\frac{x^c}{-z^c}, \frac{y^c}{-z^c}, -1, 1\right)$$

Division by z^c can't represented with a 3×3 matrix!

In homogenous coordinates, we can write this as:

$$(x^c, y^c, z^c, 1) \rightarrow (x^c, y^c, z^c, -z^c)$$

In matrix form, this gives:

$$\begin{bmatrix} -x^c/z^c \\ -y^c/z^c \\ -1 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} x^c \\ y^c \\ z^c \\ -z^c \end{bmatrix}$$

Perspective Projection Matrix



$$(x^c, y^c, z^c, 1) \rightarrow \left(\frac{x^c}{-z^c}, \frac{y^c}{-z^c}, -1, 1\right)$$

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Perspective Projection Matrix



$$(x^c, y^c, z^c, 1) \rightarrow \left(\frac{x^c}{-z^c}, \frac{y^c}{-z^c}, -1, 1\right)$$

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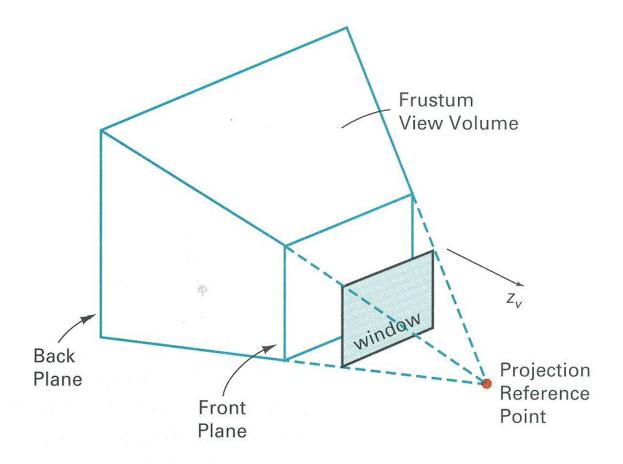
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As expected, this describes a projective transformation

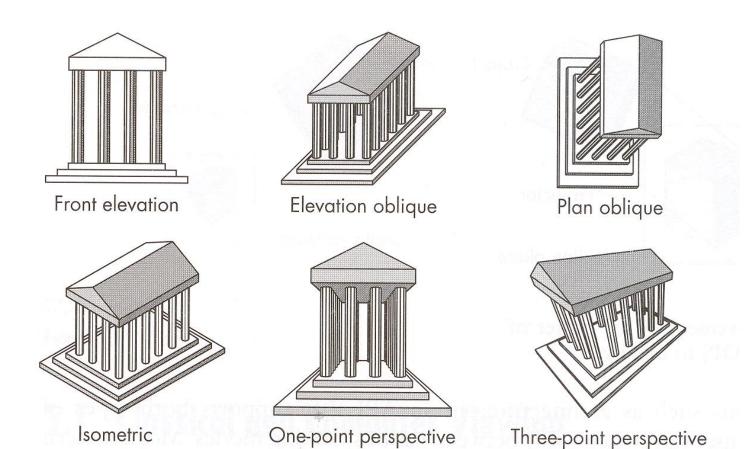
Perspective Projection View Volume

The volume projecting onto the window is a pyramid



Classical Projections

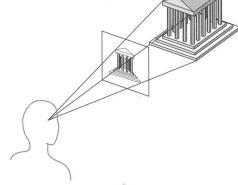




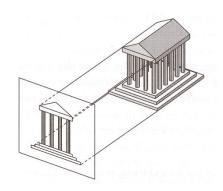
Perspective vs. Parallel



- Perspective projection
 - ✓ Size varies inversely with distance looks realistic
 - ✓ Angles are preserved on faces parallel to the view plane
 - Distance are not preserved

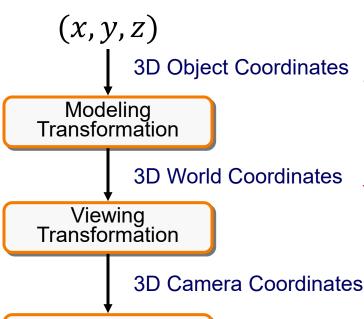


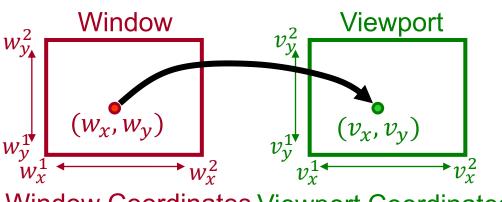
- Parallel (orthographic) projection
 - ✓ Parallel lines remain parallel
 - ✓ Angles and distances are preserved on faces parallel to the view plane
 - Less realistic looking
 - ✓ Good for exact measurements



Transformations







Window Coordinates Viewport Coordinates

Projection Transformation

2D Window Coordinates

Window-to-Viewport Transformation

2D Viewport Coordinates

(x',y')

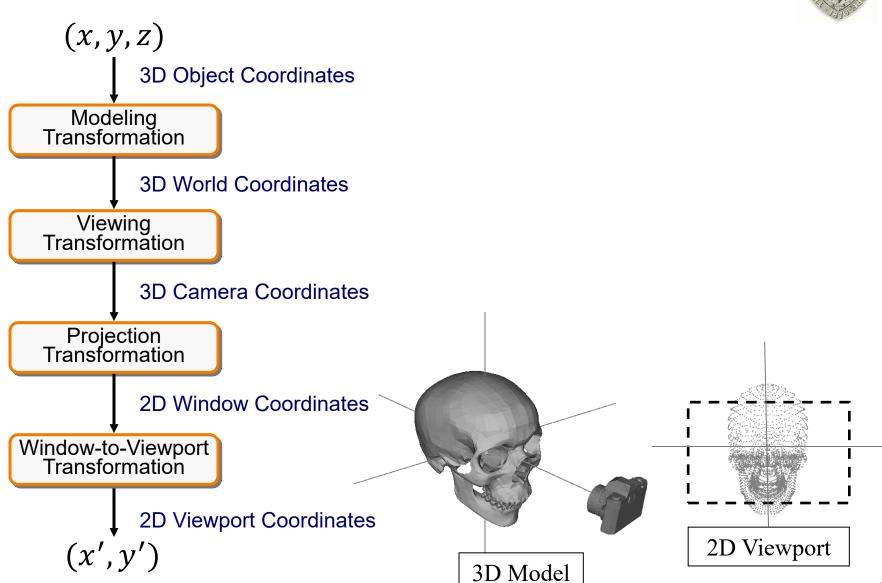
V = viewport transform

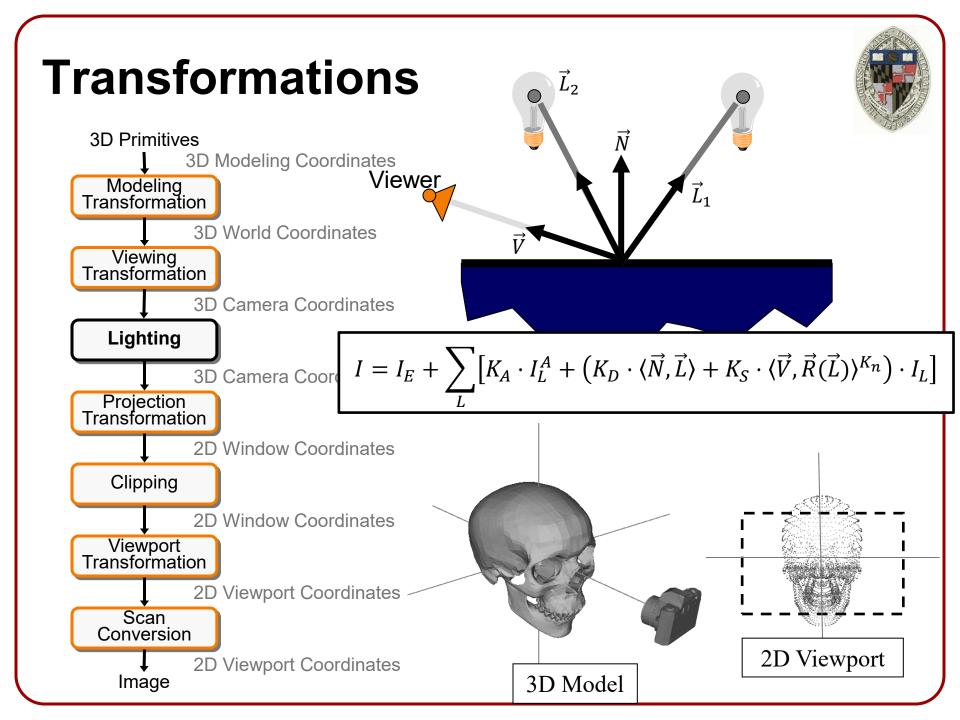
es
$$\mathbf{V} = \begin{bmatrix} 1 & 0 & v_x^1 \\ 0 & 1 & v_x^2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{v_x^2 - v_x^1}{w_x^2 - w_x^1} & 0 & 0 \\ \frac{v_y^2 - v_y^1}{w_y^2 - w_y^1} & 0 \\ 0 & \frac{v_y^2 - v_y^1}{w_y^2 - w_y^1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -w_x^1 \\ 0 & 1 & -w_y^1 \\ 0 & 0 & 1 \end{bmatrix}$$
tes

Note that this may scale non-uniformly.

3D Rendering Pipeline (for direct illumination)

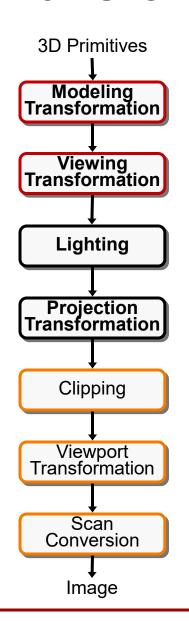






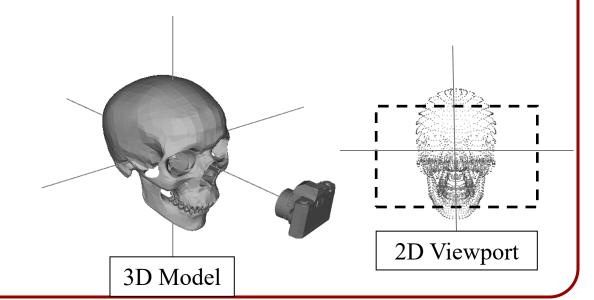
Transformations





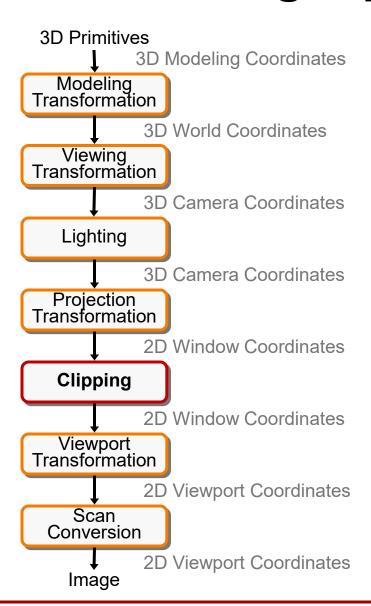
Vertex processing

- Originally, vertex processing was <u>fixed</u>
- Now this is programmable in the <u>vertex shader</u>



3D Rendering Pipeline (for direct illumination)





Clipping



Avoid drawing parts of primitives outside window

- Window defines the subset of the scene being viewed
- Must draw geometric primitives only inside window



Clipping



Avoid drawing parts of primitives outside window

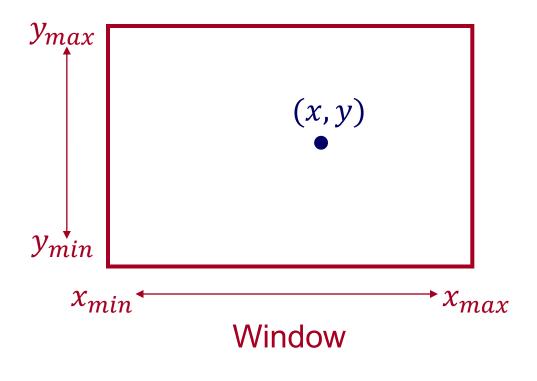
- Points
- Line Segments
- Polygons



Point Clipping



Is point (x, y) inside the clip window?



```
inside =
  (x >= x_min) &&
  (x < x_max) &&
  (y >= y_min) &&
  (y < y_max);
```

Clipping



Avoid drawing parts of primitives outside window

- Points
- Line Segments
- Polygons

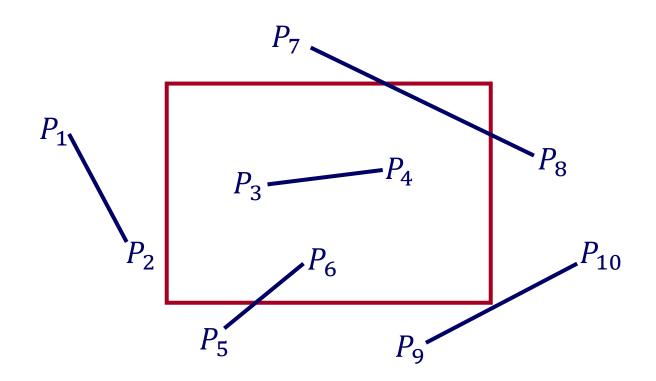


Line Segment Clipping



Find the part of a line inside the clip window

 Do this as efficiently as possible by identifying the easiest cases first

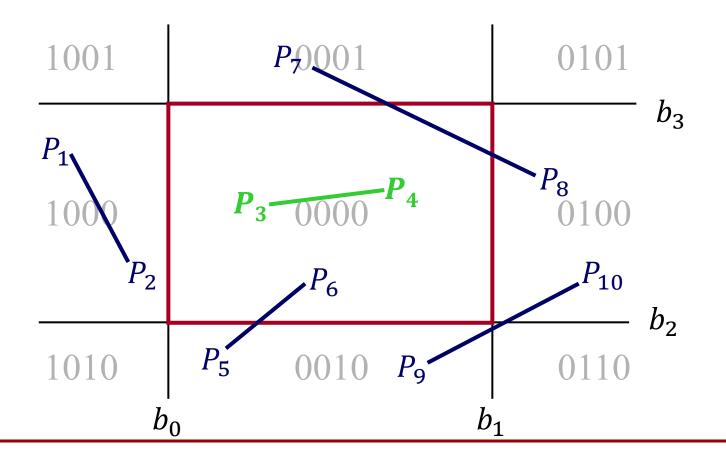




- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
 - $b_0 = 1$ if the vertex is <u>left of</u> the window
 - $b_1 = 1$ if the vertex is <u>right of</u> the window
 - $b_2 = 1$ if the vertex is <u>below</u> the window
 - $b_3 = 1$ if the vertex is <u>above</u> the window 0101 1001 0001 b_3 1000 0100 0000 1010 0010 0110

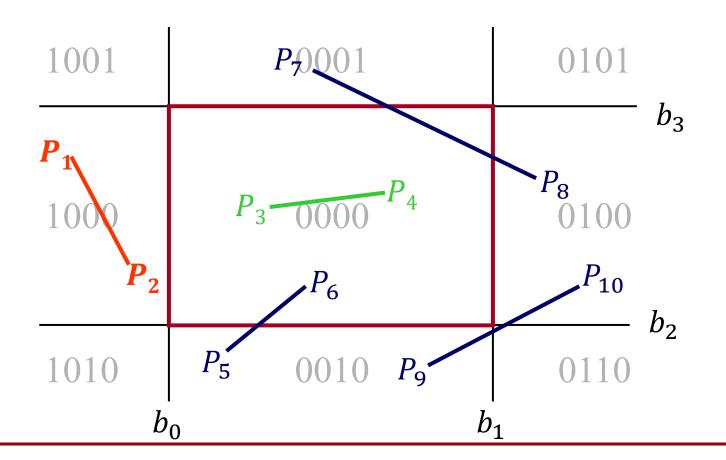


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside



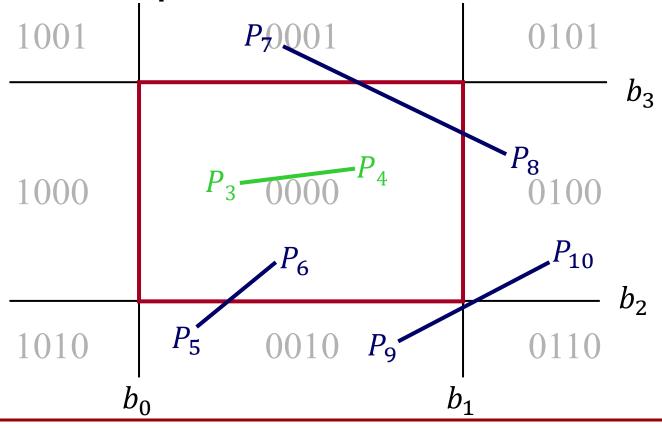


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside



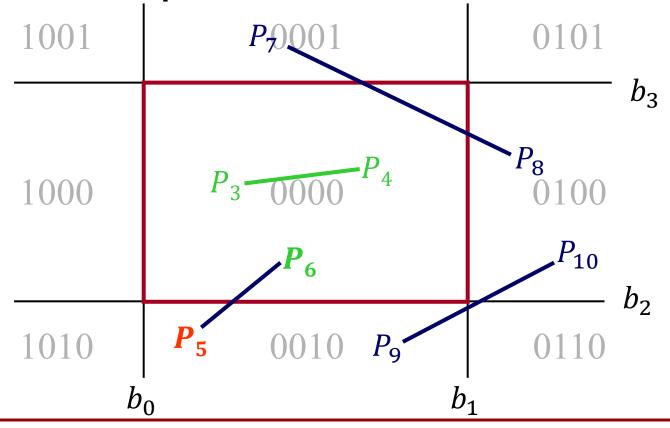


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



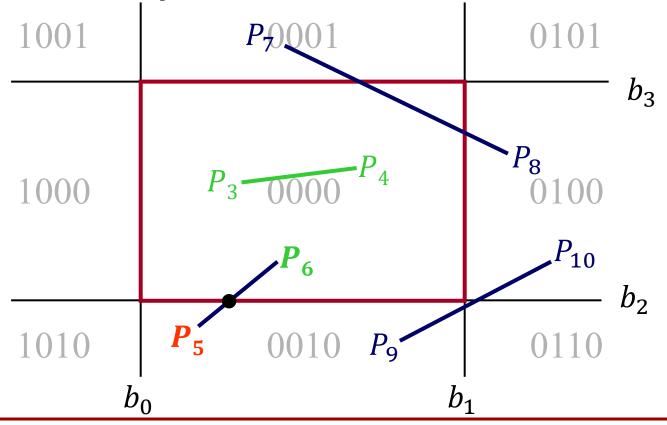


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



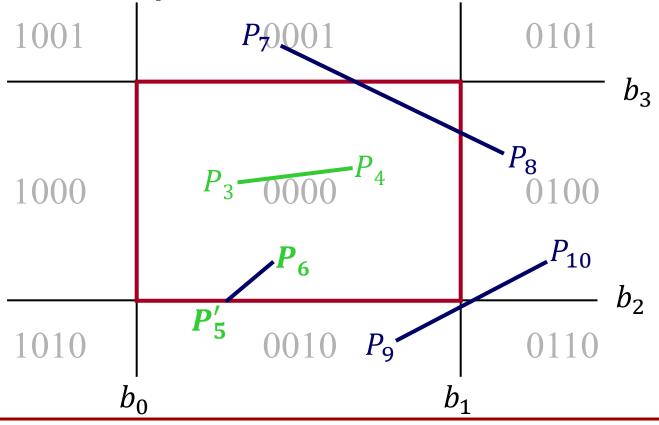


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



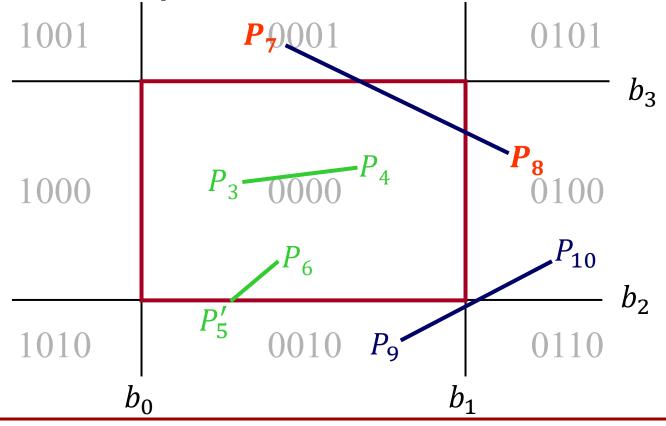


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



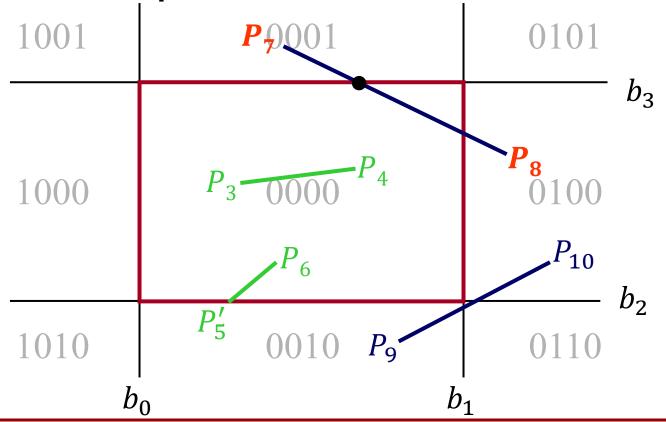


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



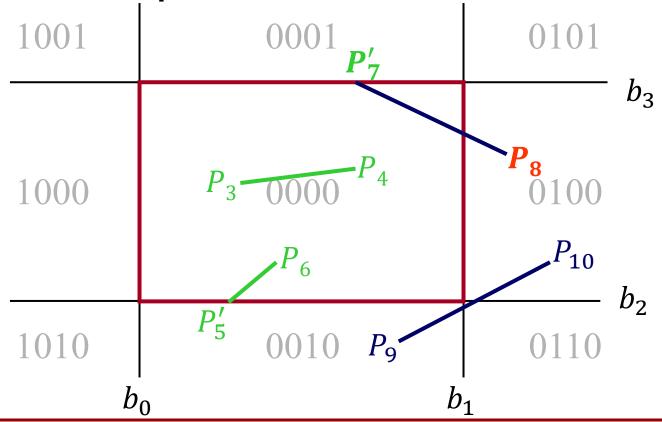


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



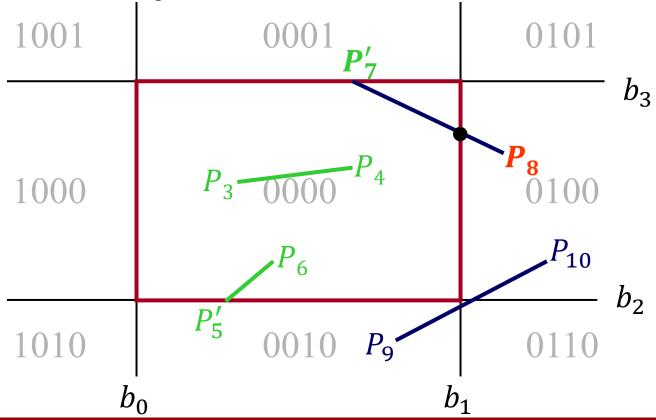


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



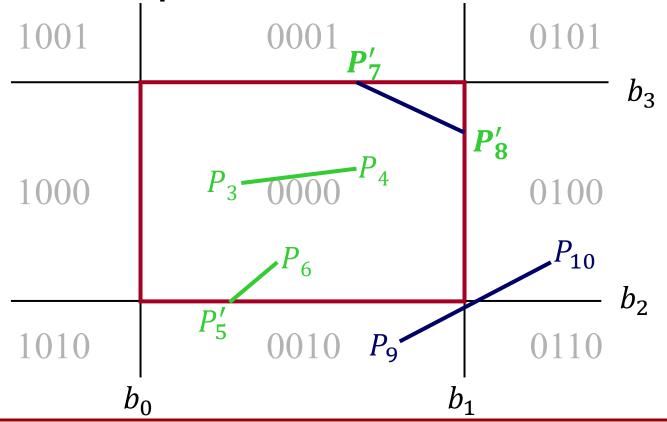


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



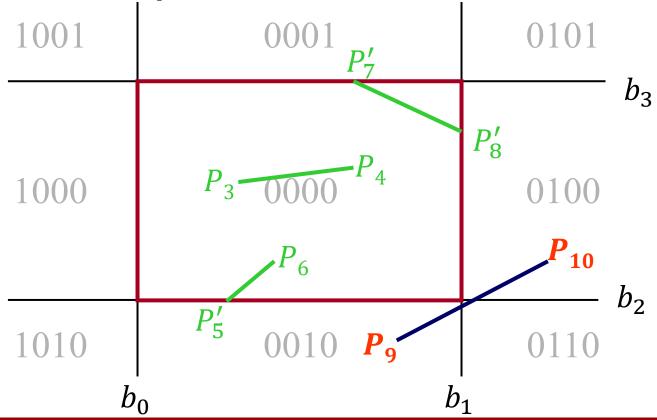


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



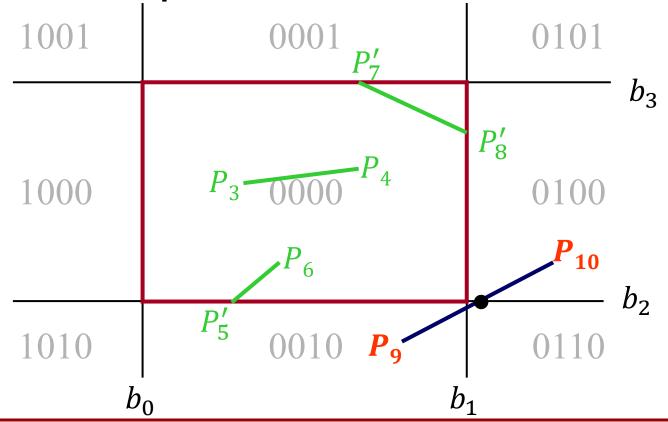


- Associate a 4-bit <u>outcode</u> b₀b₁b₂b₃ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



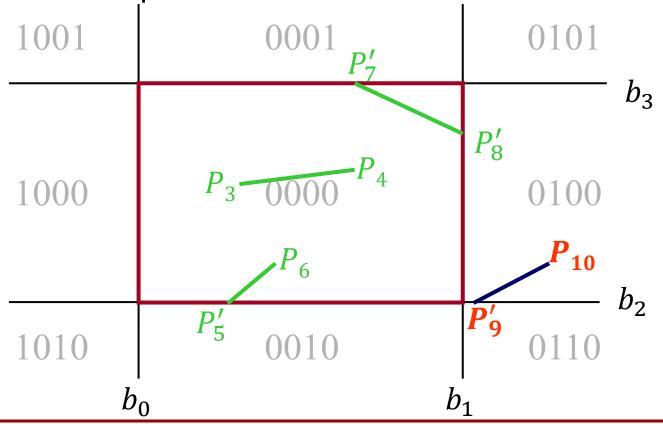


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



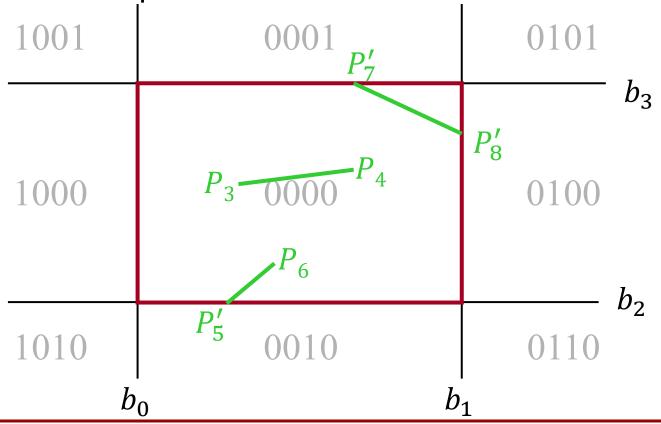


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



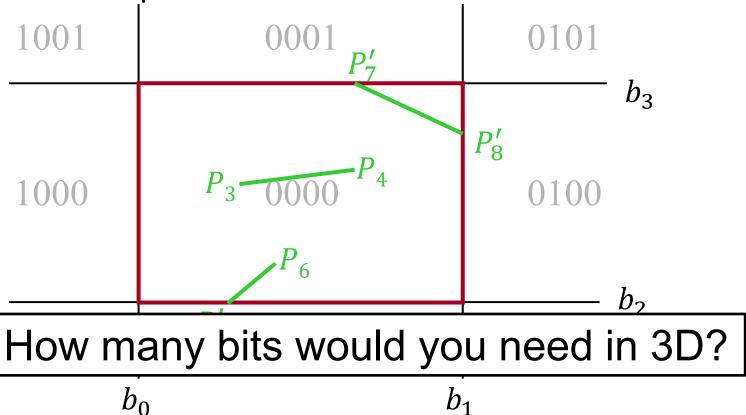


- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test





- Associate a 4-bit outcode $b_0b_1b_2b_3$ to each vertex
- If both points' outcodes are 0, line segment is inside
- If AND of outcodes is not 0, line segment is outside
- Otherwise clip and test



Clipping



Avoid drawing parts of primitives outside window

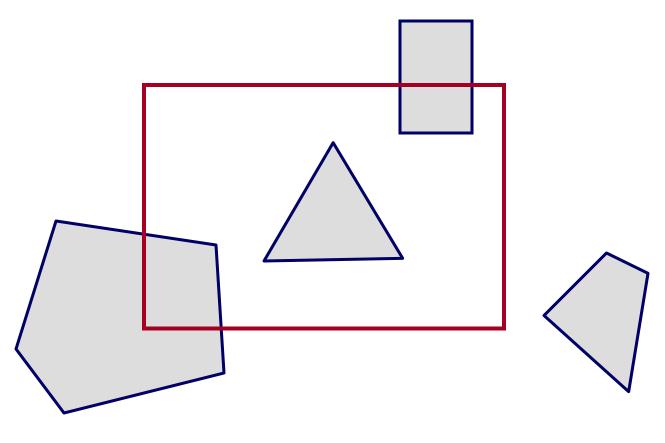
- Points
- Line Segments
- Polygons



Polygon Clipping

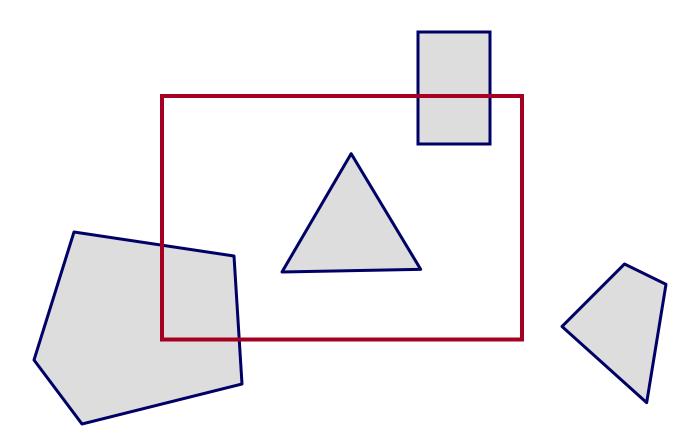


Find the part of a polygon inside the clip window

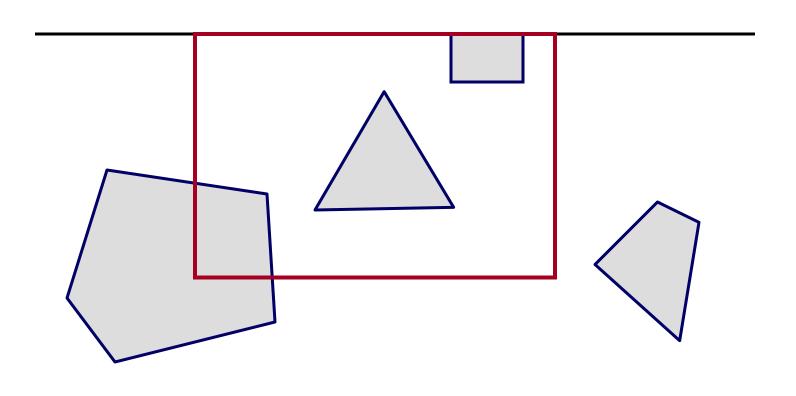


Before Clipping

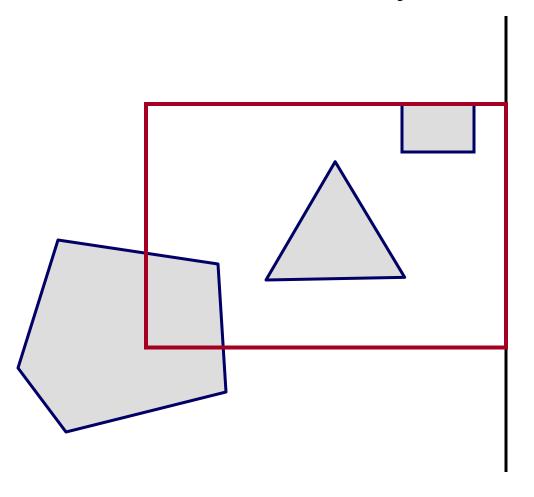




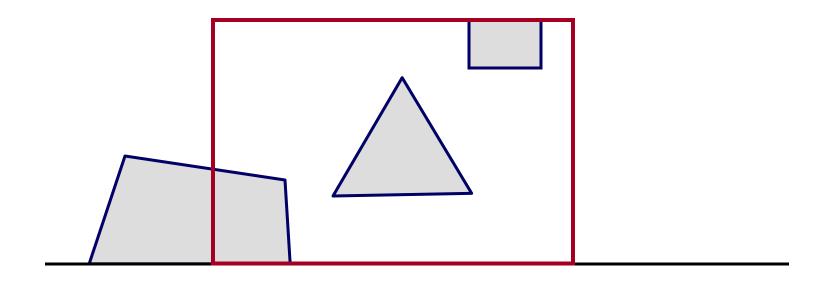




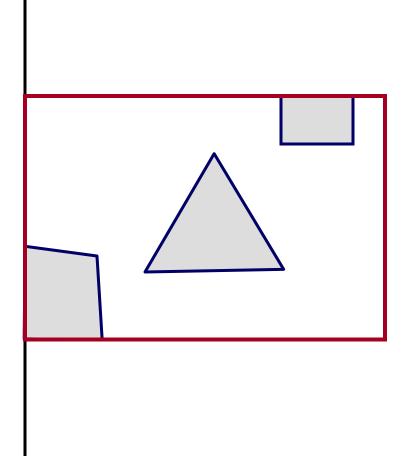






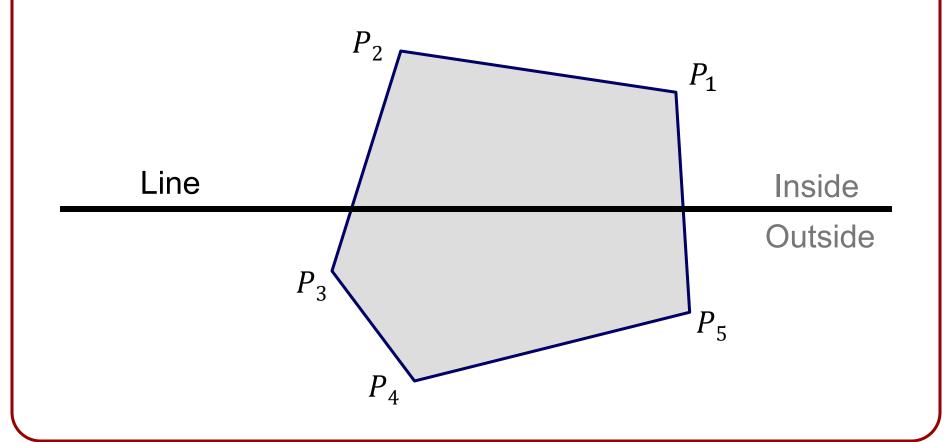






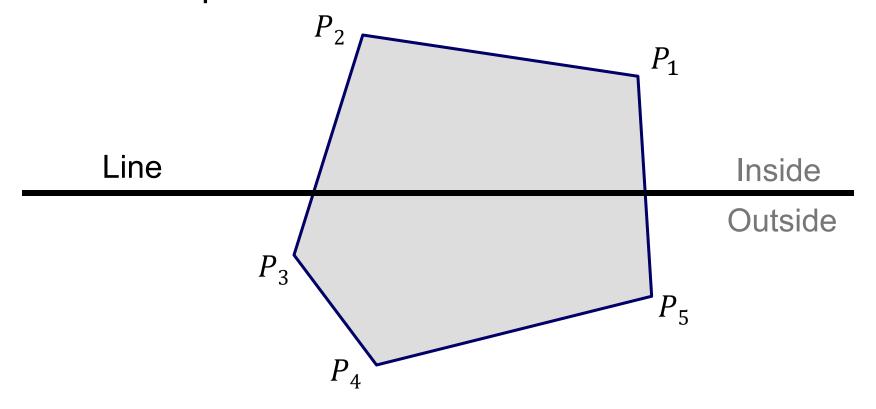


How do we clip a <u>convex</u> polygon with respect to a (window boundary) line?



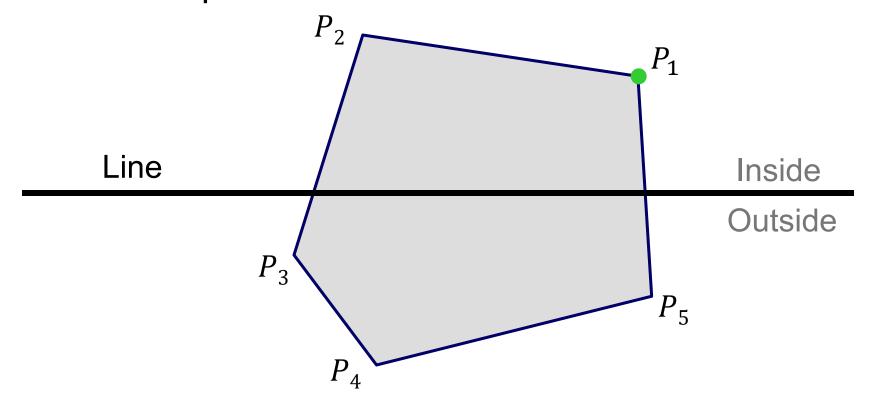


- In sequence, do interior test for each point
- Insert a new point when crossing the line.
- Remove points outside the line.



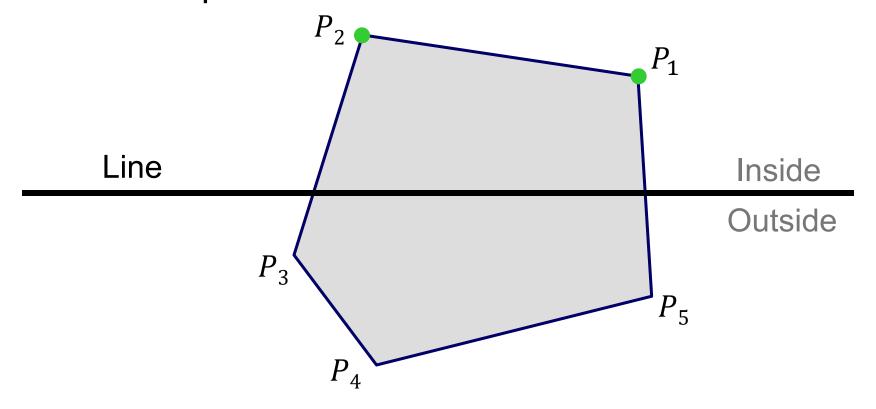


- In sequence, do interior test for each point
- Insert a new point when crossing the line.
- Remove points outside the line.



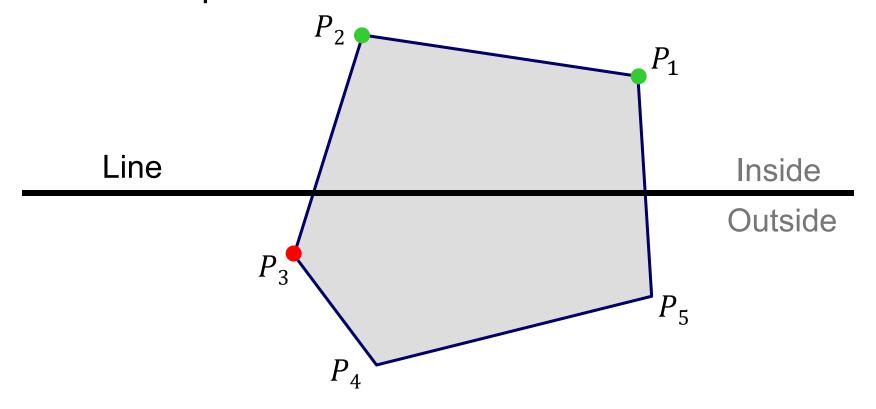


- In sequence, do interior test for each point
- Insert a new point when crossing the line.
- Remove points outside the line.



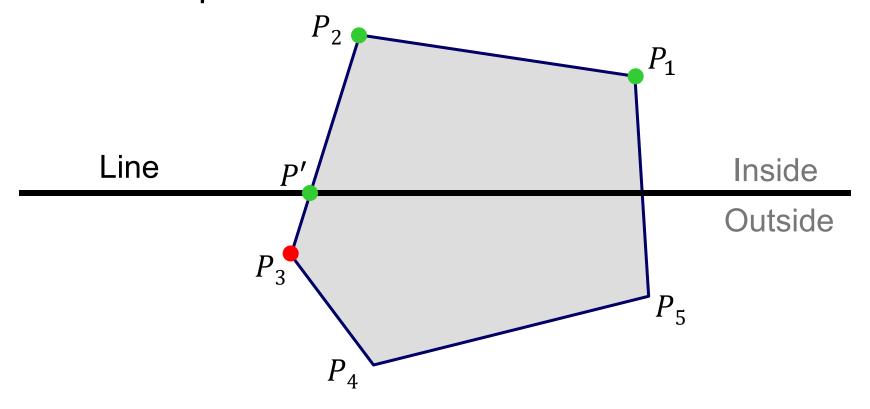


- In sequence, do interior test for each point
- Insert a new point when crossing the line.
- Remove points outside the line.



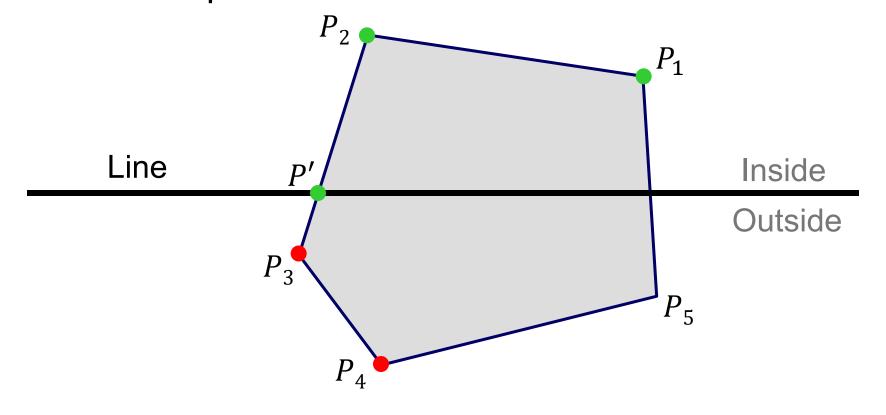


- In sequence, do interior test for each point
- Insert a new point when crossing the line.
- Remove points outside the line.



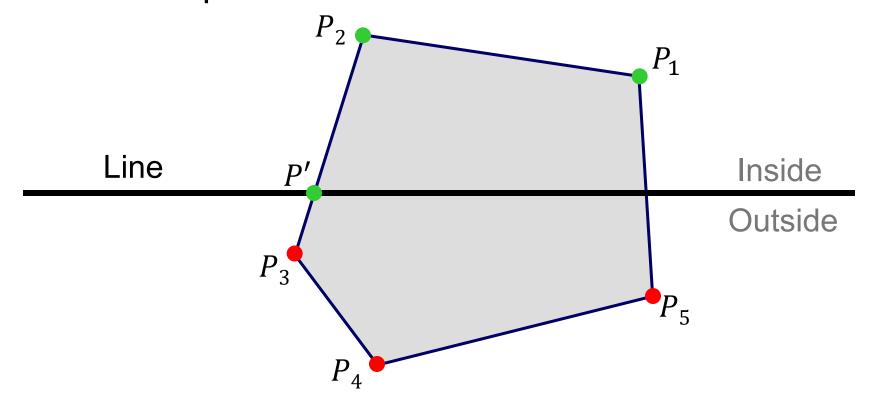


- In sequence, do interior test for each point
- Insert a new point when crossing the line.
- Remove points outside the line.



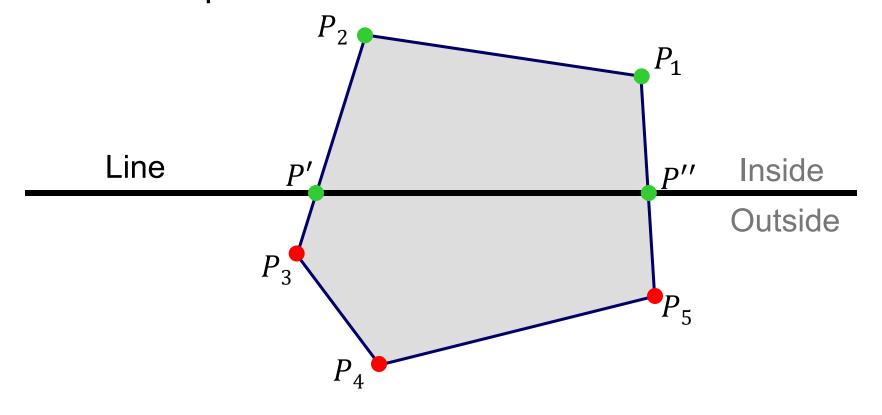


- In sequence, do interior test for each point
- Insert a new point when crossing the line.
- Remove points outside the line.



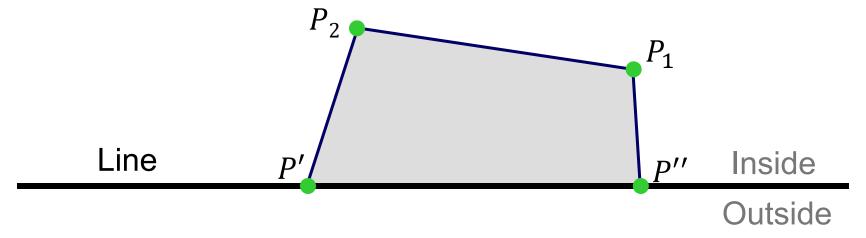


- In sequence, do interior test for each point
- Insert a new point when crossing the line.
- Remove points outside the line.





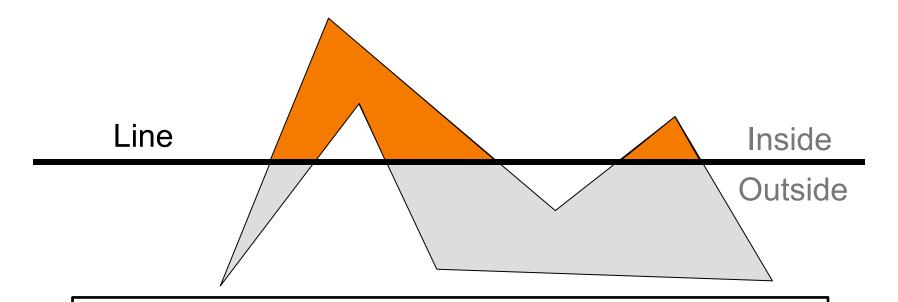
- In sequence, do interior test for each point
- Insert a new point when crossing the line.
- Remove points outside the line.



When polygons are clipped, per-vertex properties (e.g. lighting) is interpolated to the new vertices.



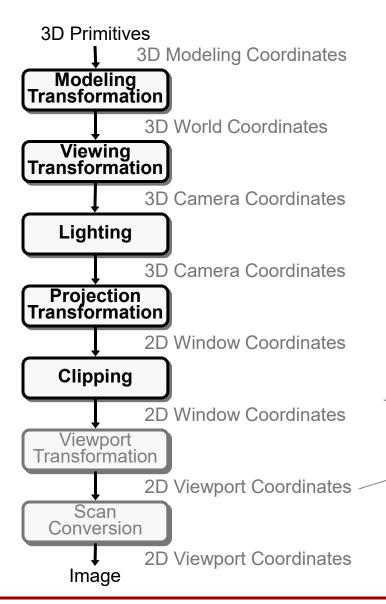
- In sequence, do interior test for each point
- Insert a new point when crossing the line.
- Remove points outside the line.



[WARNING] If the polygon is not convex, we may end up with more than one polygon!

3D Rendering Pipeline (for direct illumination)





At this point we have the:

- Positions of the mesh vertices (including new vertices obtained through clipping)
- Color at each vertex.
- A list of (possibly clipped) polygons describing the intersection of the projected 3D polygons with the window.

