Direct Illumination

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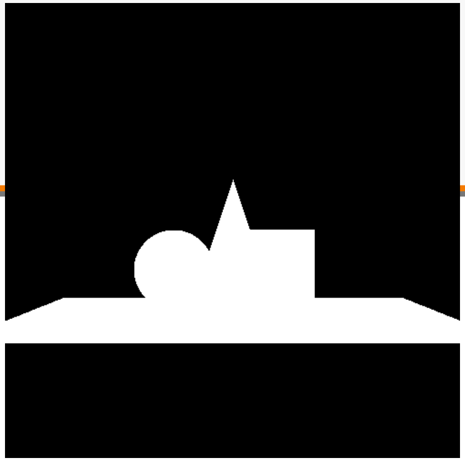
(601.457/657)

Ray Casting

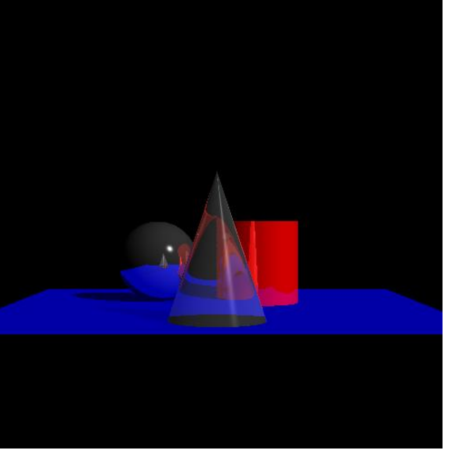
```
Image RayCast( Camera camera , Scene scene , int width , int height )
{
    Image image( width , height );
    for( int j=0 ; j<height ; j++ ) for( int i=0 ; i<width ; i++ )
    {
        Ray< 3 > ray = ConstructRayThroughPixel( camera , i , j );
        Intersection hit = FindIntersection( ray , scene );
        image(i,j) = GetColor( scene , ray , hit );
    }
    return image;
}
```



Without Illumination

Ray Casting

```
Image RayCast( Camera camera , Scene scene , int width , int height )
{
    Image image( width , height );
    for( int j=0 ; j<height ; j++ ) for( int i=0 ; i<width ; i++ )
    {
            Ray< 3 > ray = ConstructRayThroughPixel( camera , i , j );
            Intersection hit = FindIntersection( ray , scene );
            image(i,j) = GetColor( scene , ray , hit );
    }
    return image;
}
```

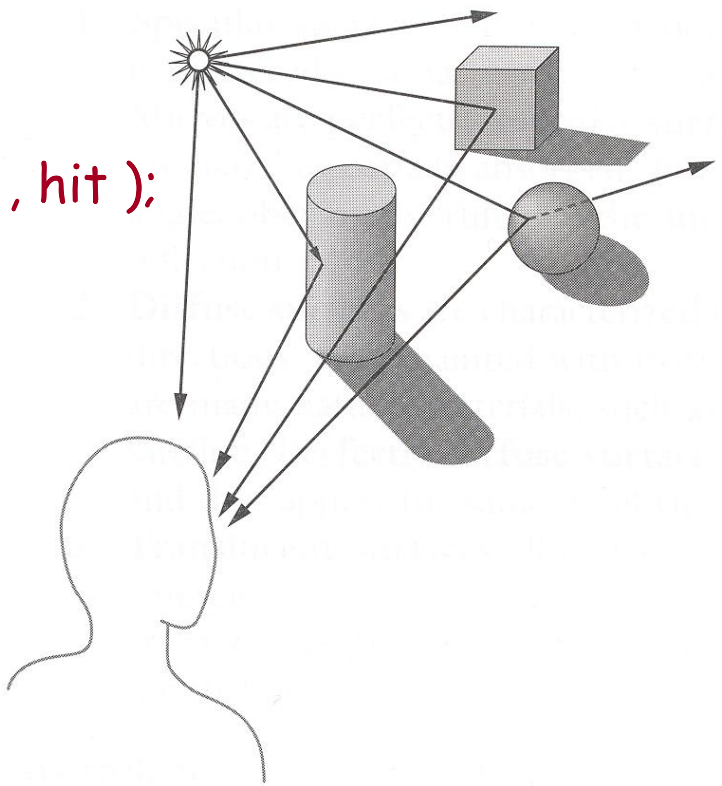


With Illumination

Illumination

How do we compute color for a sample ray?

```
image(i,j) = GetColor( scene , ray , hit );
```

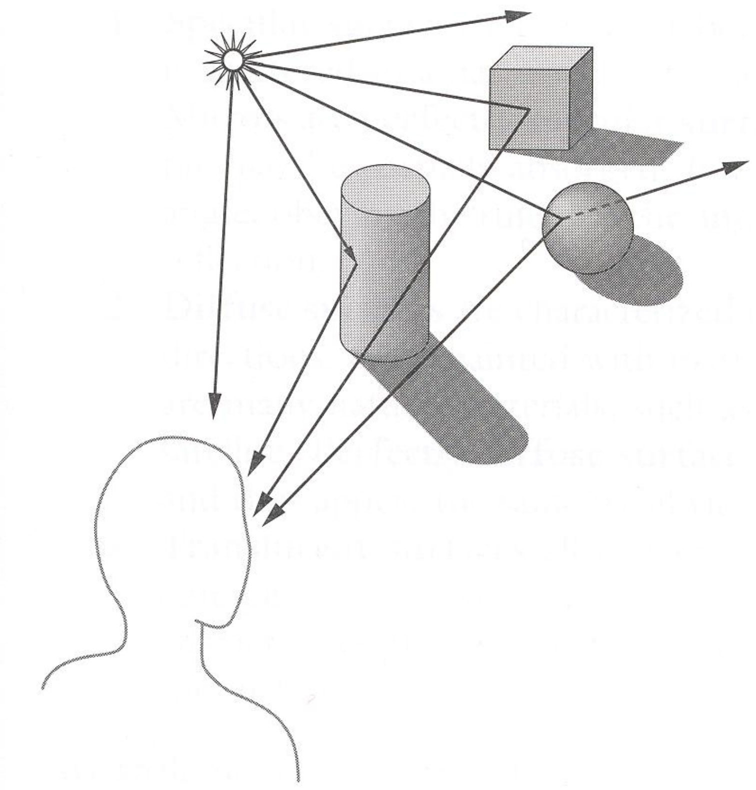


Angel Figure 6.2

Goal

Must derive models for ...

- Emission at light sources
- Direct reflection
- Indirect scattering



Desirable features …

- Concise
- Efficient to compute
- Convincing

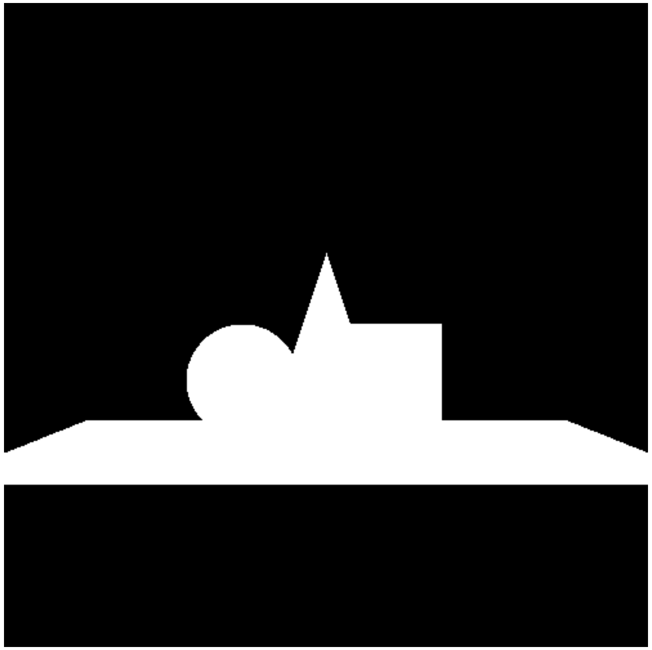
Overview

Direct Illumination

- Emission at a light source
- Direct reflection

Global illumination

- Shadows
- Inter-object reflections
- Transmission



Intersection Testing

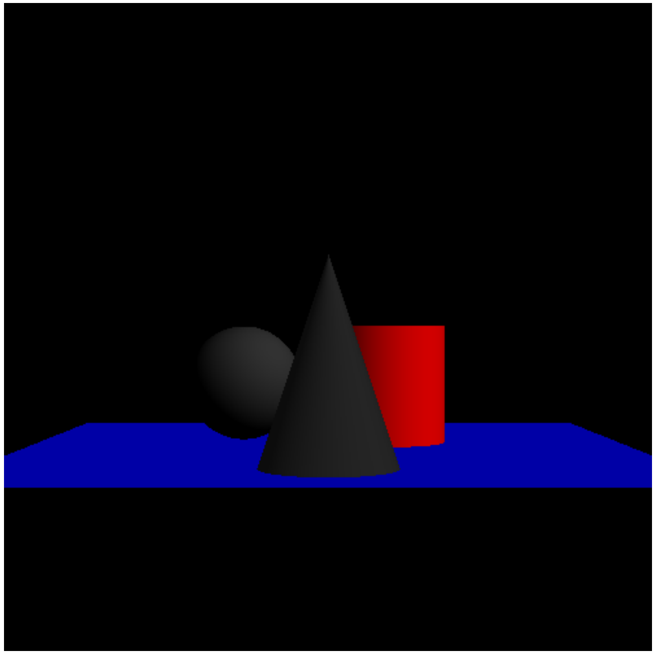
Overview

Direct Illumination

- Emission at light sources
- Direct reflection

Global illumination

- Shadows
- Inter-object reflections
- Transmission



Lambertian Shading

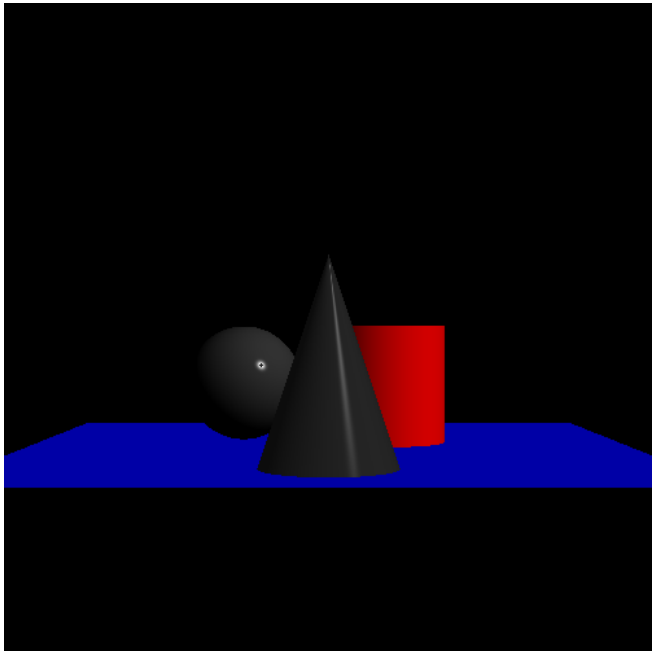
Overview

Direct Illumination

- Emission at light sources
- Direct reflection

Global illumination

- Shadows
- Inter-object reflections
- Transmission



Phong Shading

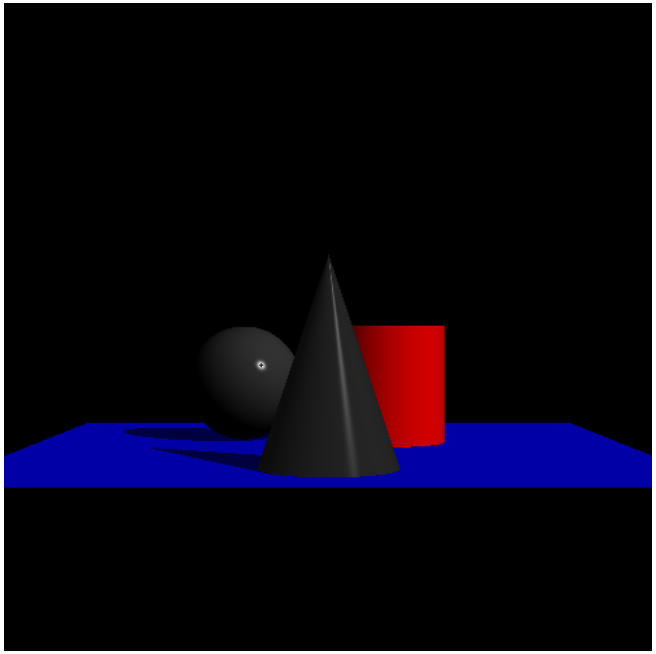
Overview

Direct Illumination

- Emission at light sources
- Direct reflection

Global illumination

- Shadows
- Inter-object reflections
- Transmission



Shadow Computation

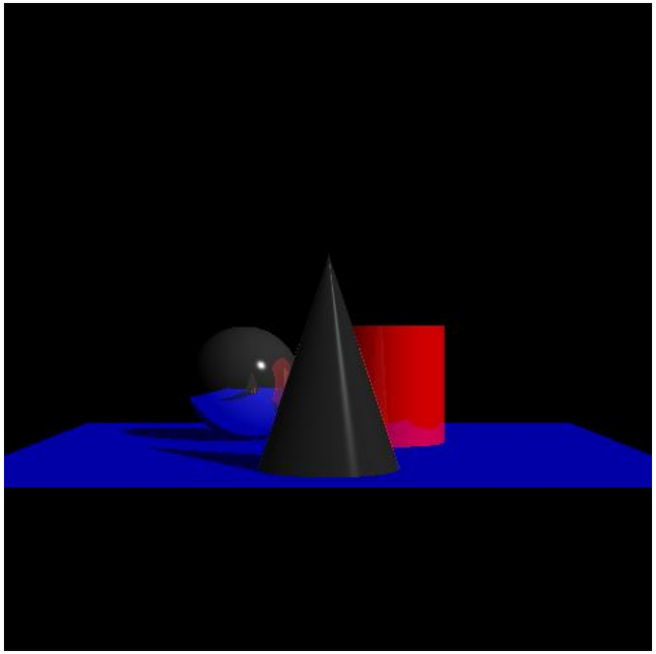
Overview

Direct Illumination

- Emission at light sources
- Direct reflection

Global illumination

- Shadows
- Inter-object reflections
- Transmission



Reflections

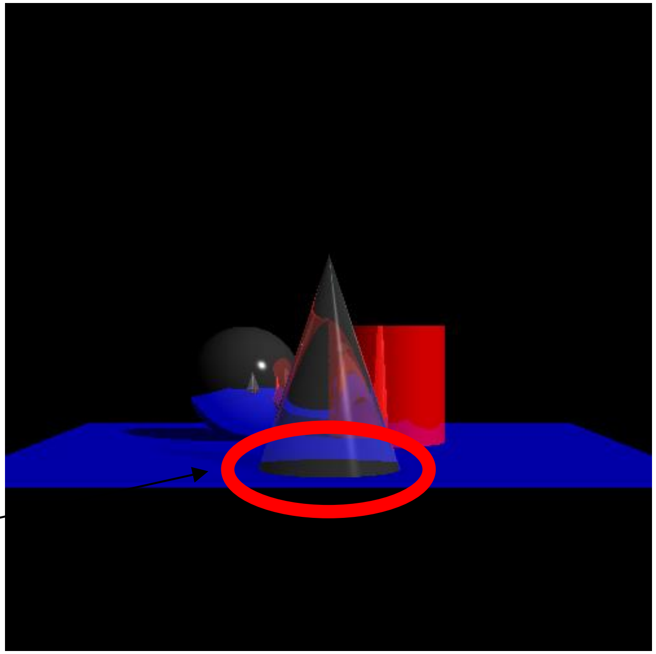
Overview

Direct Illumination

- Emission at light sources
- Direct reflection

Global illumination

- Shadows
- Inter-object reflections
- Transmission



Ignore this

Refractions

Aside

Recall:

Given two **unit** vectors $\vec{v}$ and $\vec{w}$, the angle θ between them satisfies:

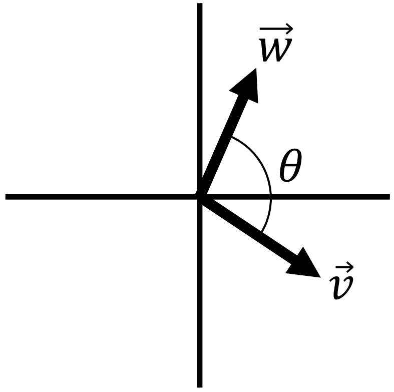
$$\langle \vec{v}, \vec{w} \rangle = \cos(\theta) \in [-1,1]$$

- Equal to 1 when $\vec{v}$ and $\vec{w}$ are aligned
- Equal to 0 when $\vec{v}$ and $\vec{w}$ are perpendicular
- Negative when $\vec{v}$ and $\vec{w}$ point in opposite directions

$\Rightarrow$ For fixed vector $\vec{v}$, a simple parametric function measuring the alignment of a vector $\vec{w}$ to $\vec{v}$ can have the form:

$$C_{\vec{v},\alpha}(\vec{w}) = \left(\max(0, \langle \vec{v}, \vec{w} \rangle)\right)^{\alpha} \in [0,1]$$

for $\alpha \geq 0$.



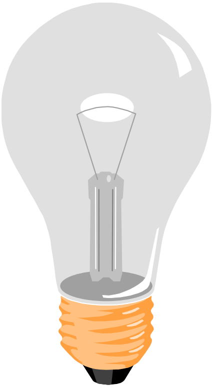
Overview

Direct Illumination

- Emission at light sources
- Direct reflection

Global illumination

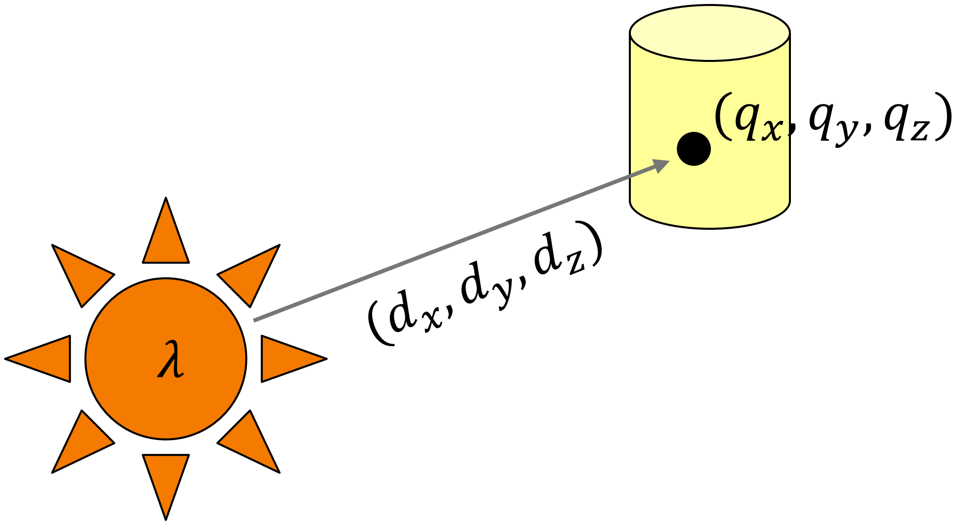
- Shadows
- Inter-object reflections
- Transmissions



Modeling Light Sources

$I_L(q, d, \lambda)$ describes the intensity of energy:

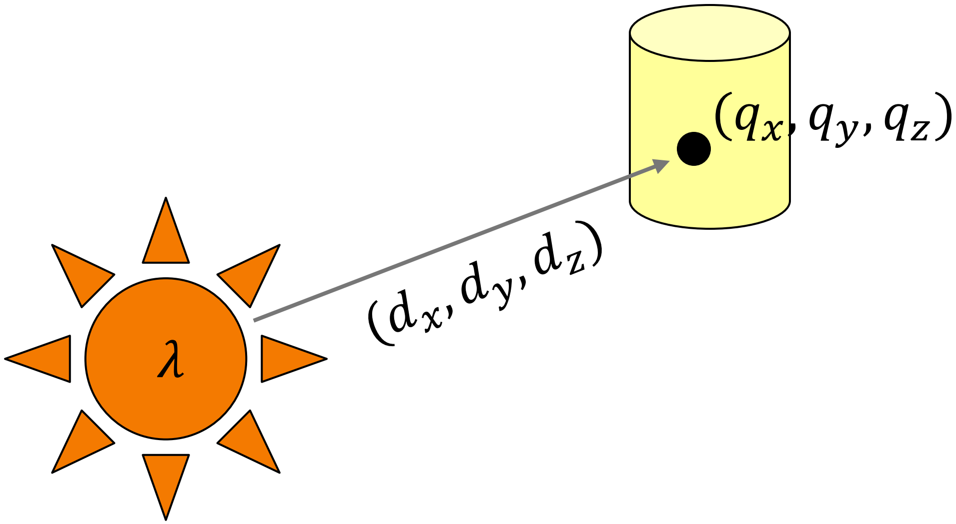
- arriving at (a patch of surface area at) q,
- from direction d,
- with wavelength λ



Empirical Models

Ideally, have a fully general model that can describe all light sources

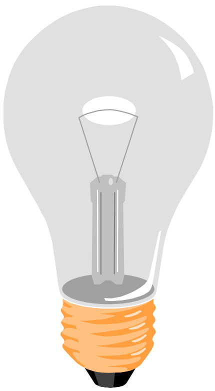
- ✗ Difficult in practice



Simplified Light Source Models

Simple mathematical models:

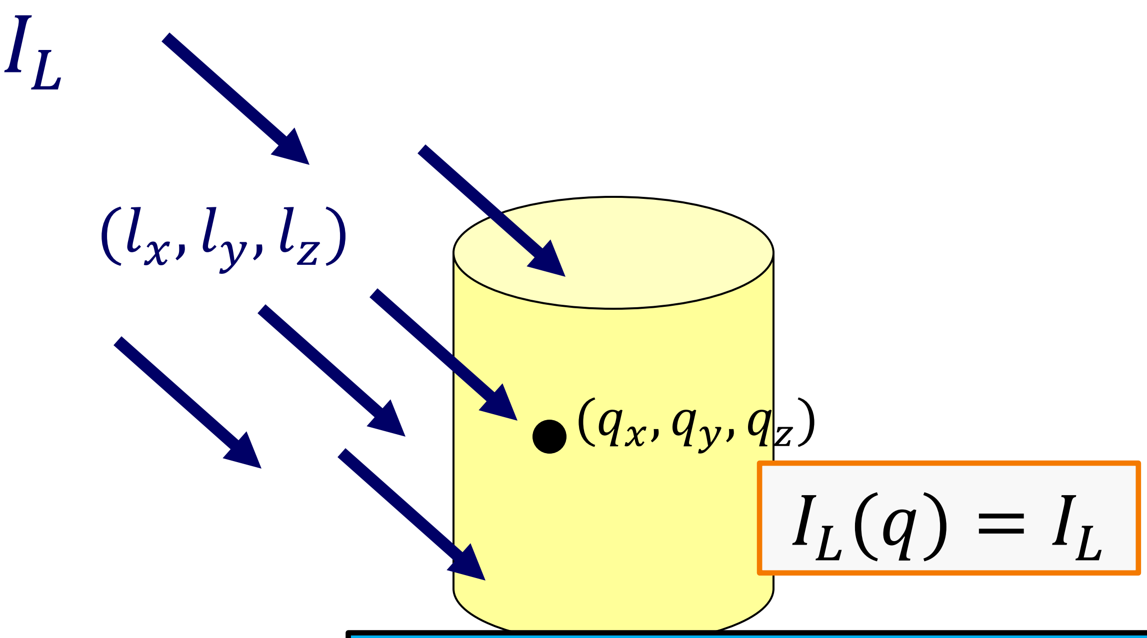
- Directional light
- Point light
- Spot light



Directional Light Source

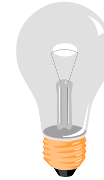
Models point light source at infinity

- intensity I_L, (typically a three-channel value)
- direction $\vec{l} = (l_x, l_y, l_z)$



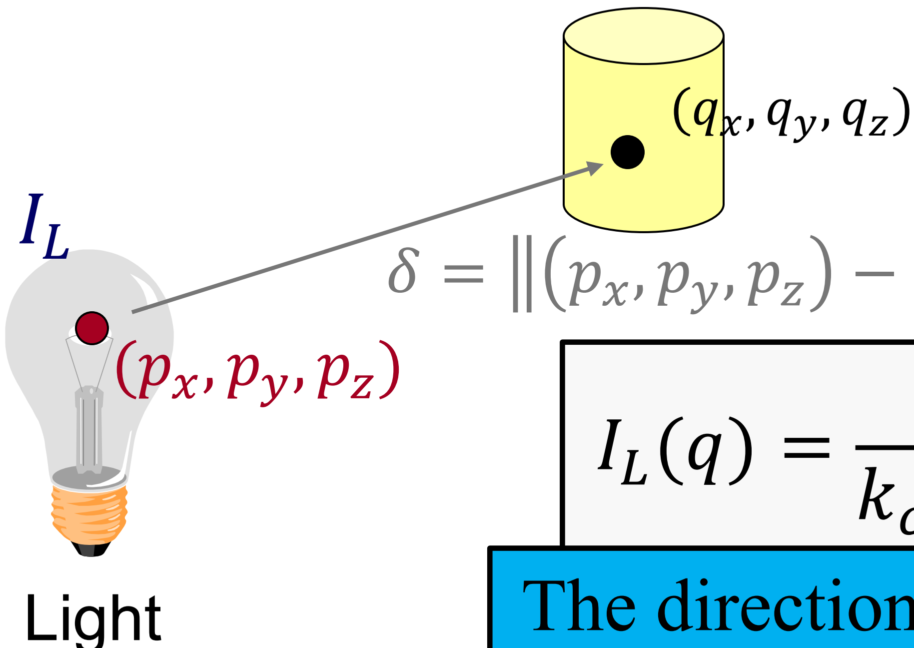
The direction **from** the light to the surface point q is $\vec{l}$.

Point Light Source



Models omni-directional point source

- Intensity I_L, (typically a three-channel value)
- position $p = (p_x, p_y, p_z)$,
- factors (k_c, k_l, k_q) for attenuation with distance (δ)



$$\delta = \|(p_x, p_y, p_z) - (q_x, q_y, q_z)\|$$

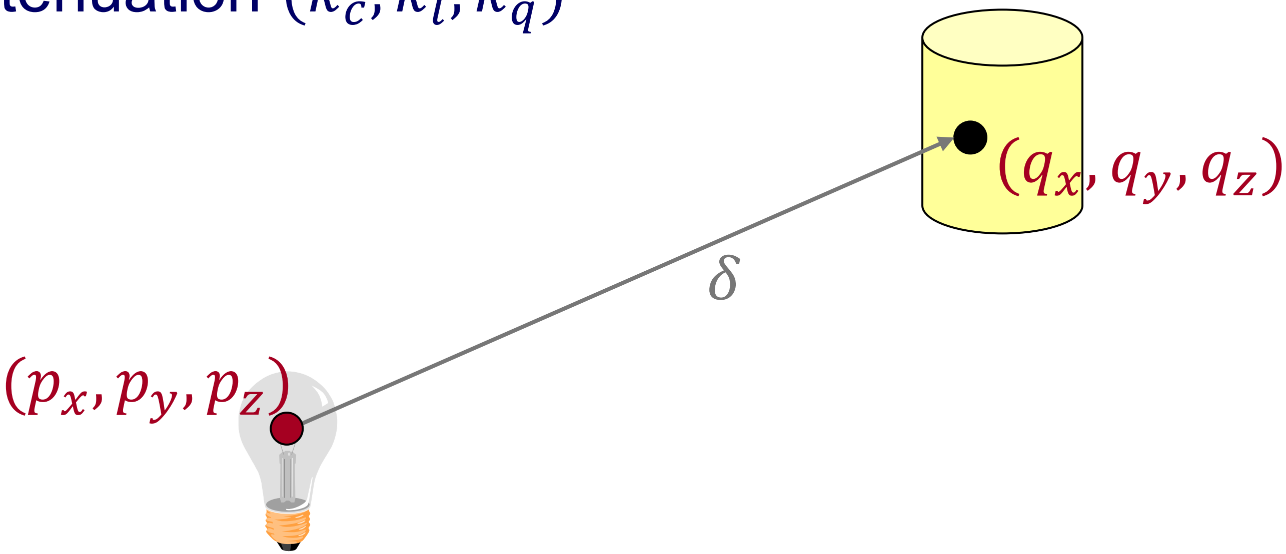
$$I_L(q) = \frac{I_L}{k_c + k_l \cdot \delta + k_q \cdot \delta^2}$$

The direction **from** the light to the surface point is $\vec{l} = \frac{q-p}{|q-p|}$.

Spot Light Source

Models point light source

- Intensity I_L, (typically a three-channel value)
- position $p = (p_x, p_y, p_z)$,
- attenuation (k_c, k_l, k_q)



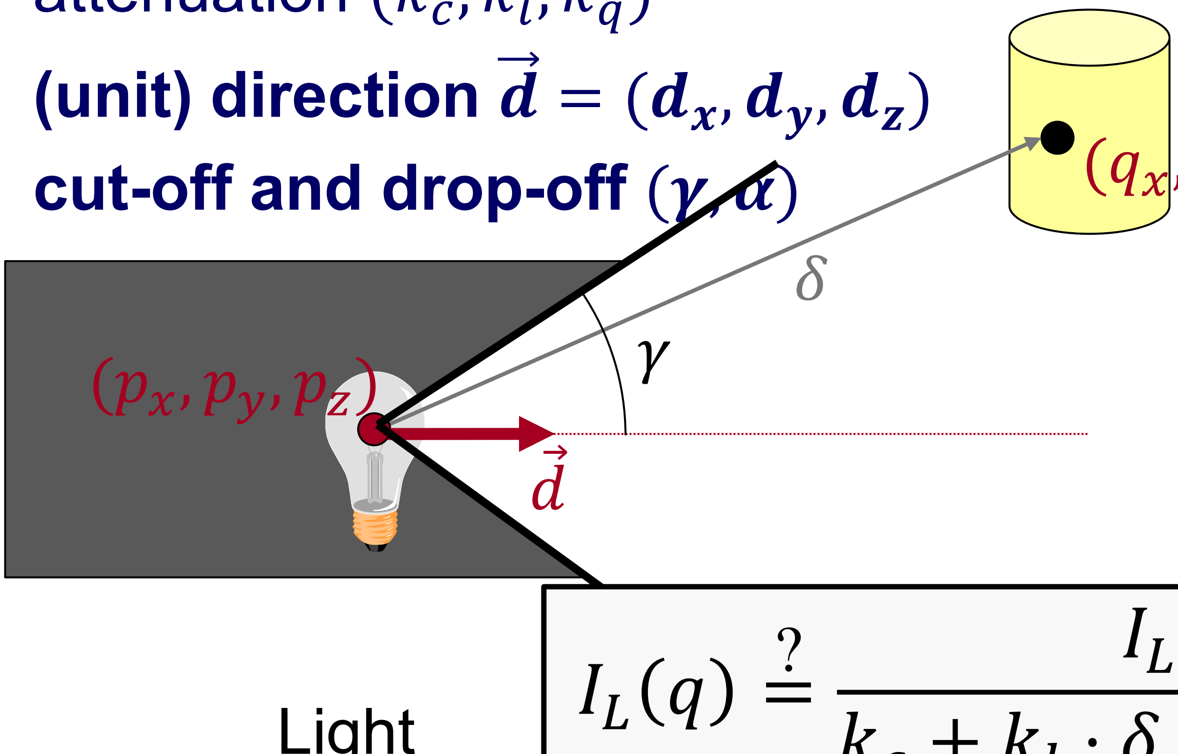
Light

$$I_L(q) = \frac{I_L}{k_c + k_l \cdot \delta + k_q \cdot \delta^2}$$

Spot Light Source

Models point light source <u>with direction and fall-off</u>

- Intensity I_L, (typically a three-channel value)
- position $p = (p_x, p_y, p_z)$,
- attenuation (k_c, k_l, k_q)
- **(unit) direction $\vec{d} = (d_x, d_y, d_z)$**
- **cut-off and drop-off (γ, α)**



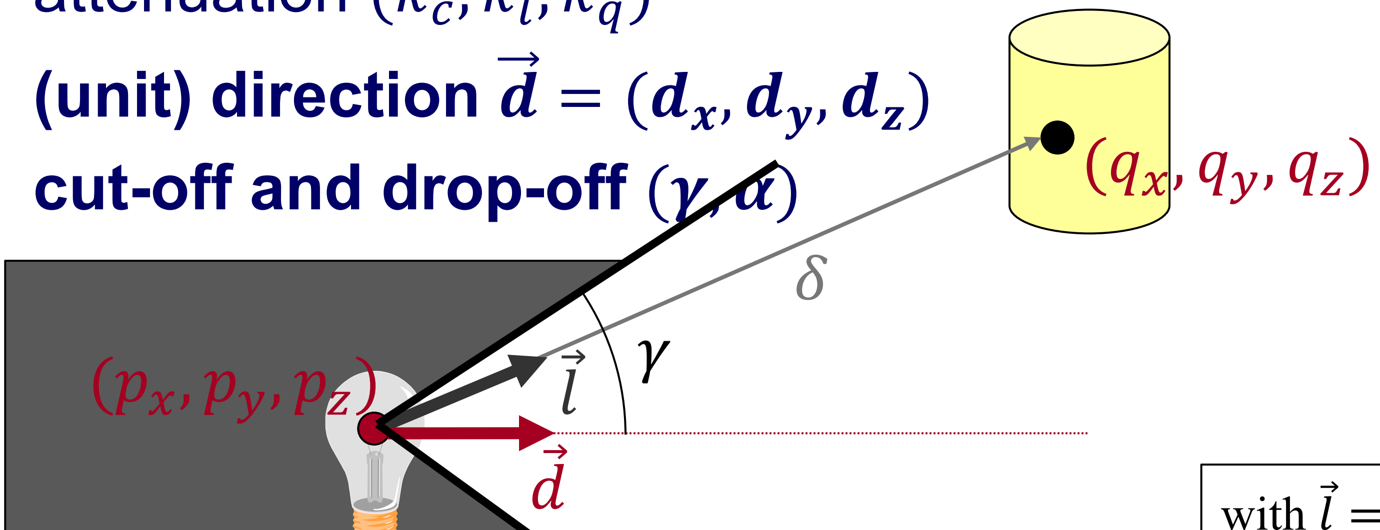
How can we modify the intensity of a point light to decrease as γ increases?

$$I_L(q) \stackrel{?}{=} \frac{I_L}{k_c + k_l \cdot \delta + k_q \cdot \delta^2}$$

Spot Light Source

Models point light source with direction and fall-off

- Intensity I_L, (typically a three-channel value)
- position $p = (p_x, p_y, p_z)$,
- attenuation (k_c, k_l, k_q)
- **(unit) direction $\vec{d} = (d_x, d_y, d_z)$**
- **cut-off and drop-off (γ, α)**



with $\vec{l} = \frac{q-p}{|q-p|}$

$$I_L(q) = \begin{cases} \dfrac{I_L \cdot \langle \vec{d}, \vec{l} \rangle^\alpha}{k_c + k_l \cdot \delta + k_q \cdot \delta^2} & \text{if } \langle \vec{d}, \vec{l} \rangle > \cos\gamma \\ 0 & \text{otherwise} \end{cases}$$

Spot Light Source

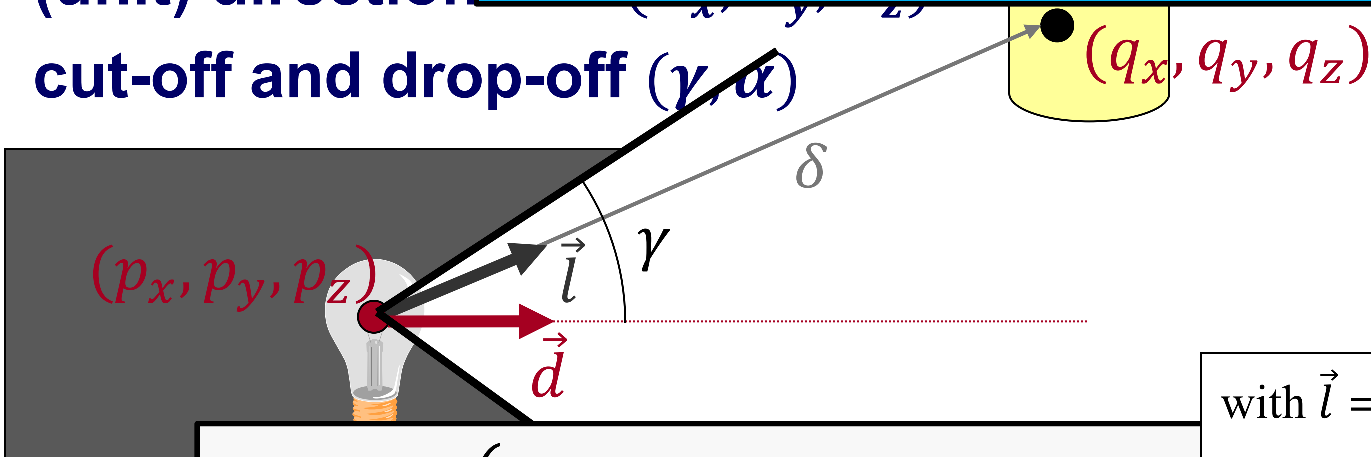
Models point light

- Intensity I_L, (typ
- position $p = (p_x$
- attenuation $(k_c,$
- **(unit) direction**
- **cut-off and drop-off** (γ, α)

$$\gamma \in \left[0, \frac{\pi}{2}\right) \text{ so that } \left\langle \vec{d}, \vec{l} \right\rangle \in [0,1].$$

$$\alpha \in [0, \infty)$$

The direction **from** the light to the surface point is $\vec{l} = \frac{q-p}{|q-p|}$ (not $\vec{d}$).



with $\vec{l} = \frac{q-p}{|q-p|}$

$$I_L(q) = \begin{cases} \dfrac{I_L \cdot \langle \vec{d}, \vec{l} \rangle^\alpha}{k_c + k_l \cdot \delta + k_q \cdot \delta^2} & \text{if } \langle \vec{d}, \vec{l} \rangle > \cos \gamma \\ 0 & \text{otherwise} \end{cases}$$

Directional/Point/Spot Light Source

In addition to the light directly reaching a point, light can arrive at the point <u>indirectly</u> by bouncing around the scene.

Approximate with a constant intensity represented by a (directionless) **ambient** term:

$$I_L^A(q) = I_L^A$$

<u>Note</u>:
In some applications, there is a single ambient light, independent of the number of lights in the scene.
⇒ Makes it hard to turn on/off individual lights.

Overview

Direct Illumination

- Emission at light sources
- Direct reflection

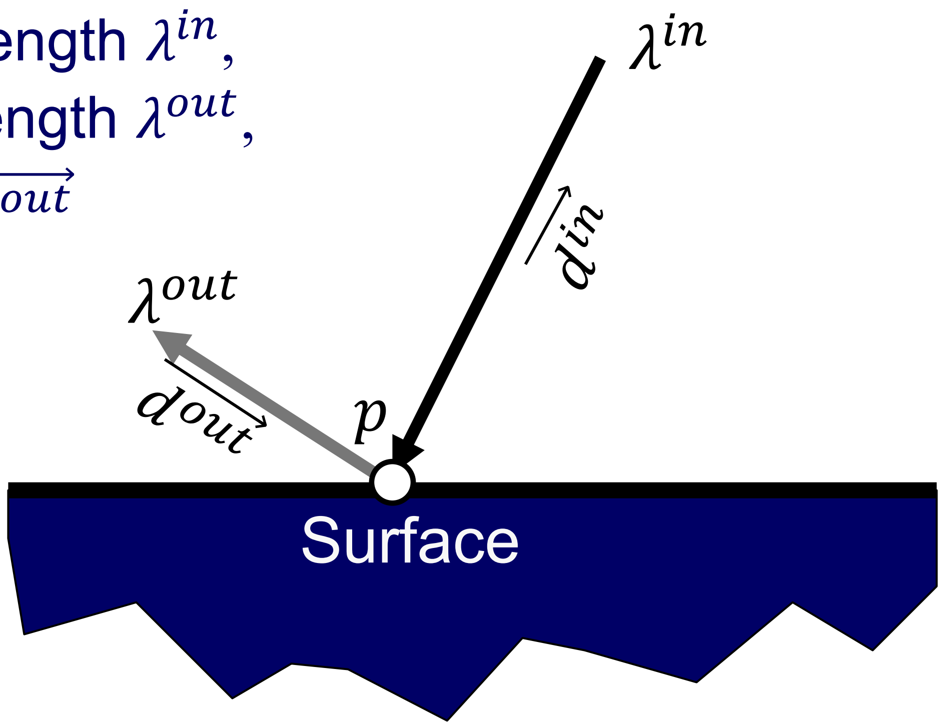
Global illumination

- Shadows
- Transmissions
- Inter-object reflections

Modeling Surface Reflectance

$SVBRDF(p, \overrightarrow{d^{in}}, \lambda^{in}, \overrightarrow{d^{out}}, \lambda^{out})$ describes the fraction of incident energy (R)

- arriving at point p
- from direction $\overrightarrow{d^{in}}$,
- with incoming wavelength λ^{in},
- with outgoing wavelength λ^{out},
- leaving in direction $\overrightarrow{d^{out}}$

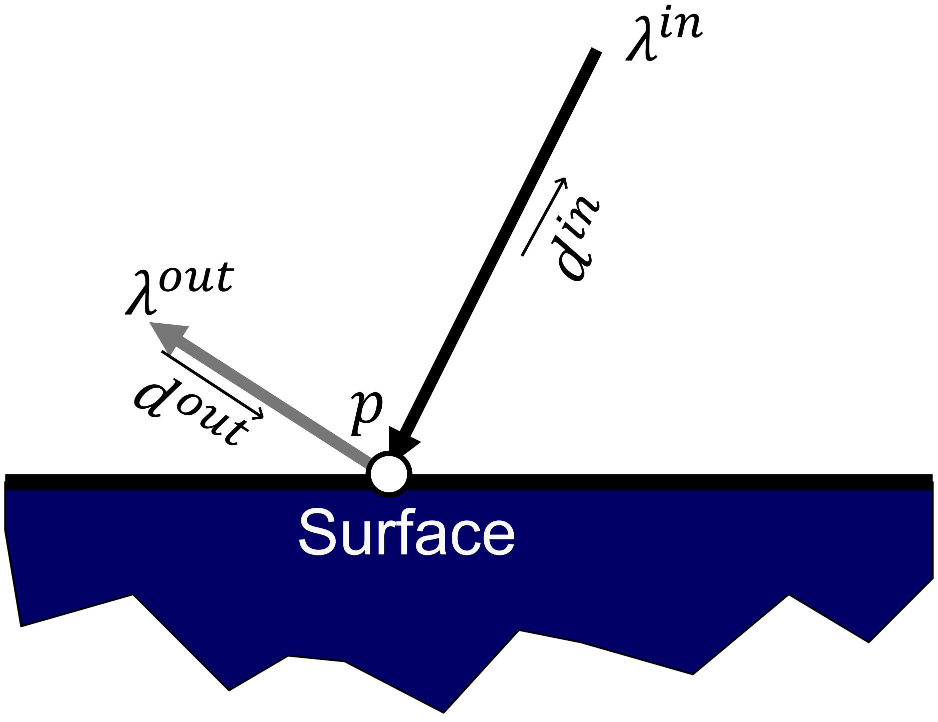


SVBRDF = Spatially Varying Bi-Directional Reflectance Distribution Function

Empirical Models

Ideally, model reflectance for all combinations of incoming/outgoing angles/wavelengths, and all surface positions

- ✘ Difficult in practice

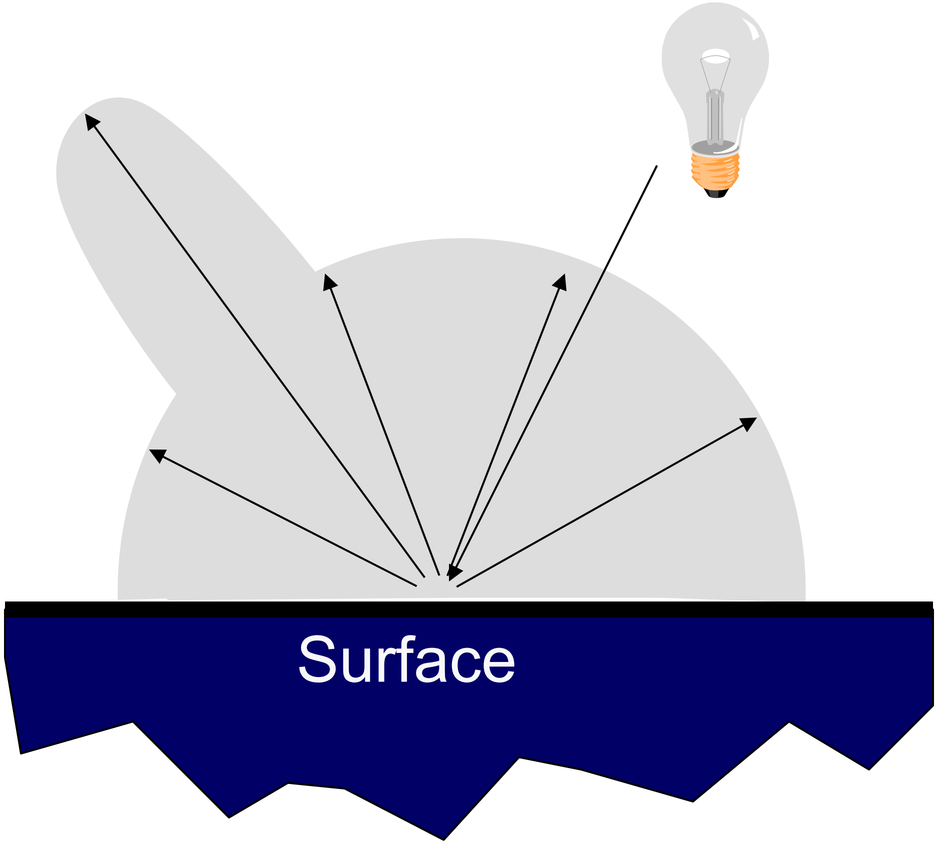


Simple Reflectance Model

Simple analytic model:

- diffuse reflection +
- specular reflection +
- emission +
- "ambient"

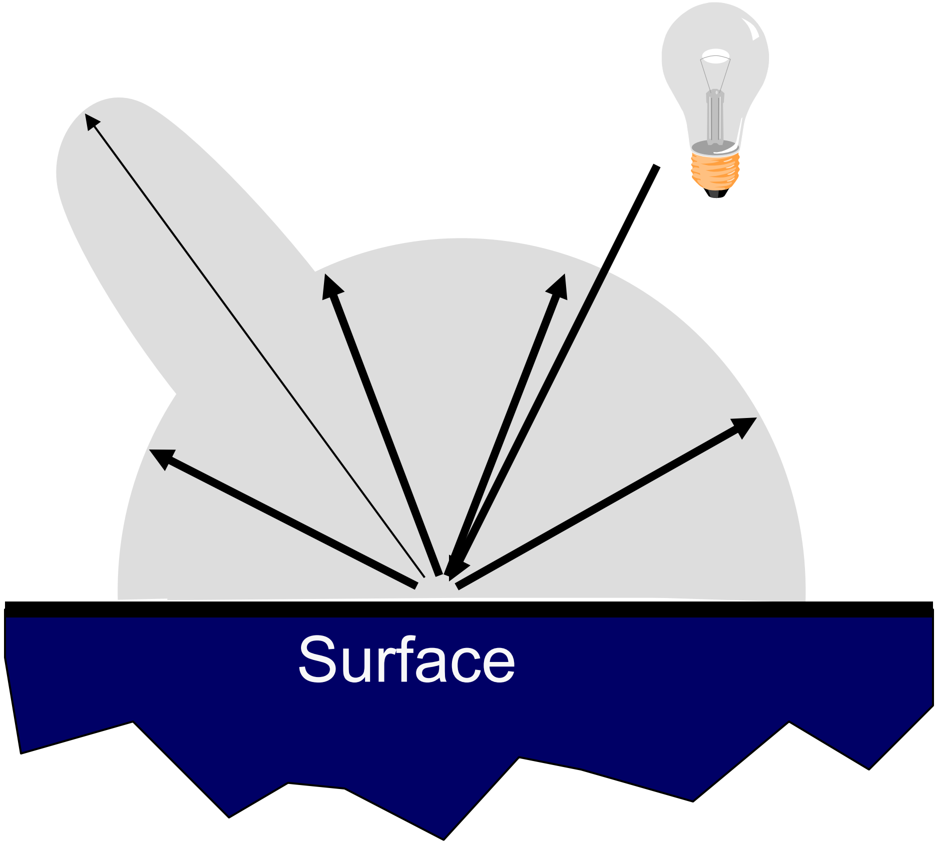
Based on model proposed by Phong



Simple Reflectance Model

Simple analytic model:

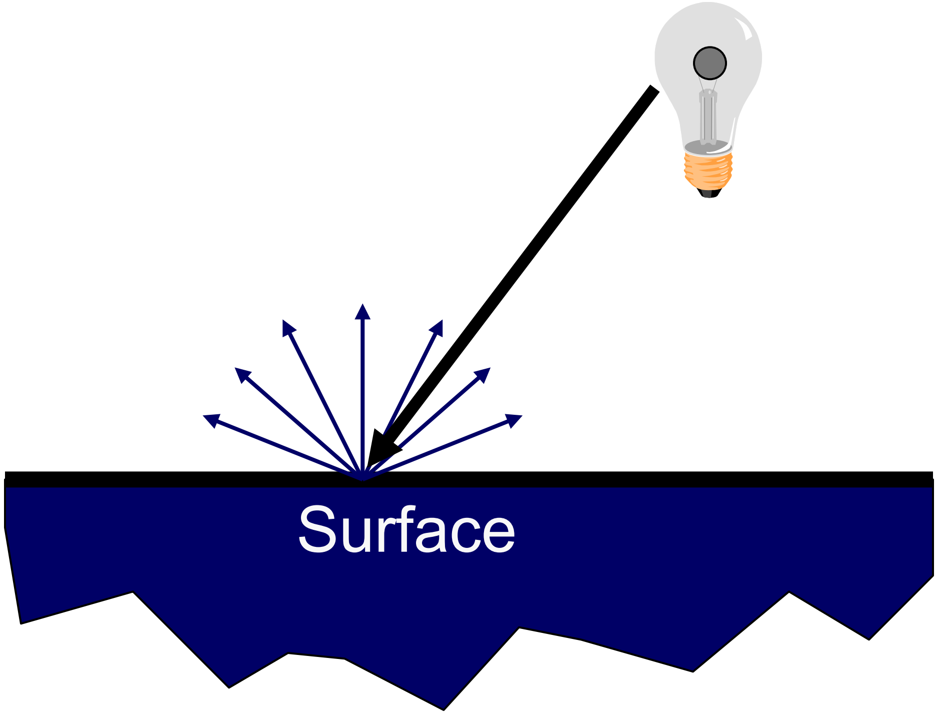
- **diffuse reflection +**
- specular reflection +
- emission +
- “ambient”



Diffuse Reflection

Assume surface intensity is viewer independent

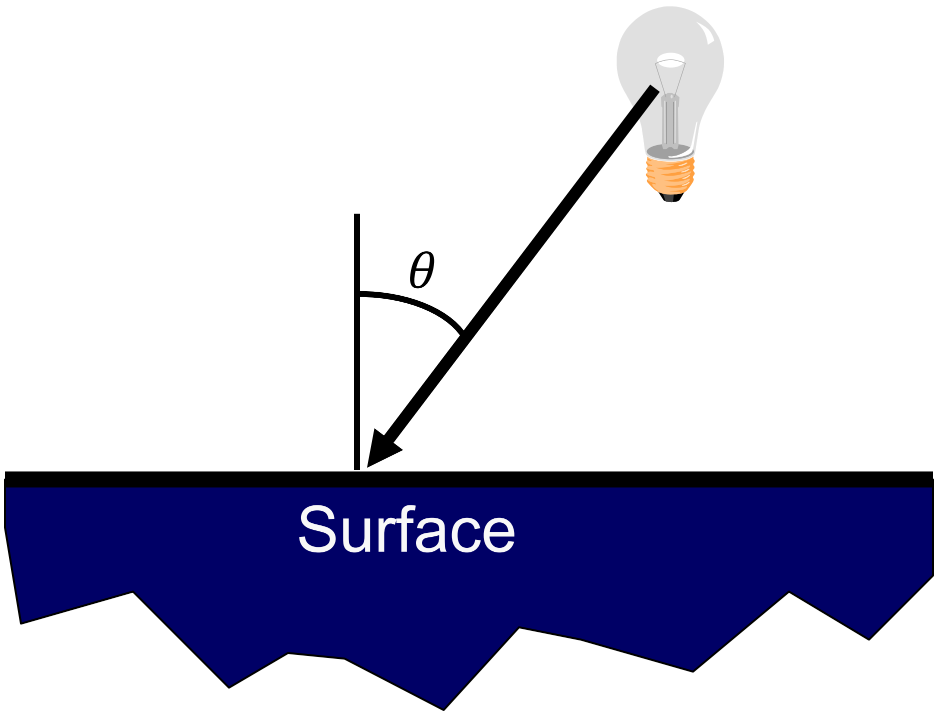
- Examples: chalk, clay



Diffuse Reflection

How much light is reflected?

- Lambertian: Only depends on angle of incident light

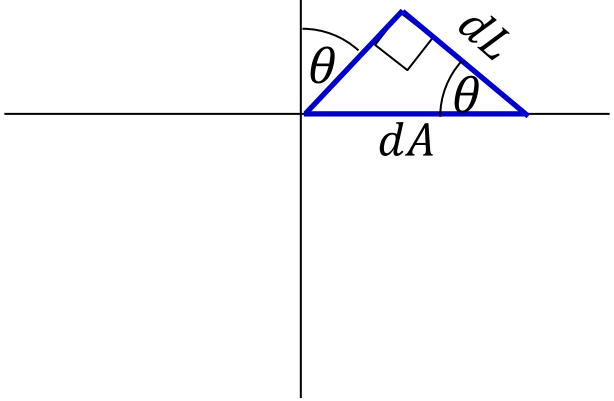


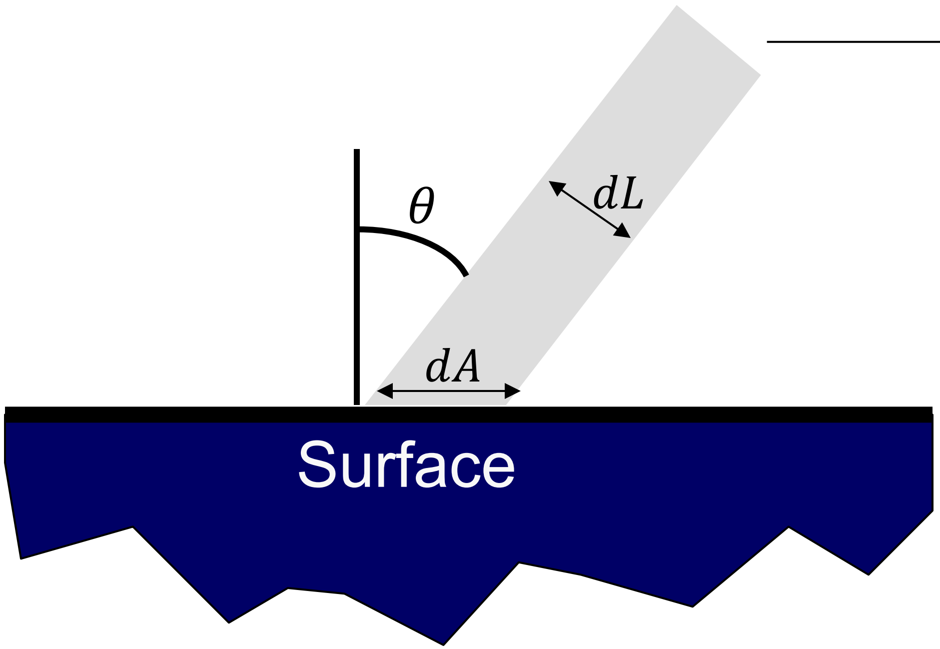
Diffuse Reflection

How much light is reflected?

- Lambertian: Only depends on angle of incident light

$$dL = dA \cdot \cos\theta$$





Diffuse Reflection

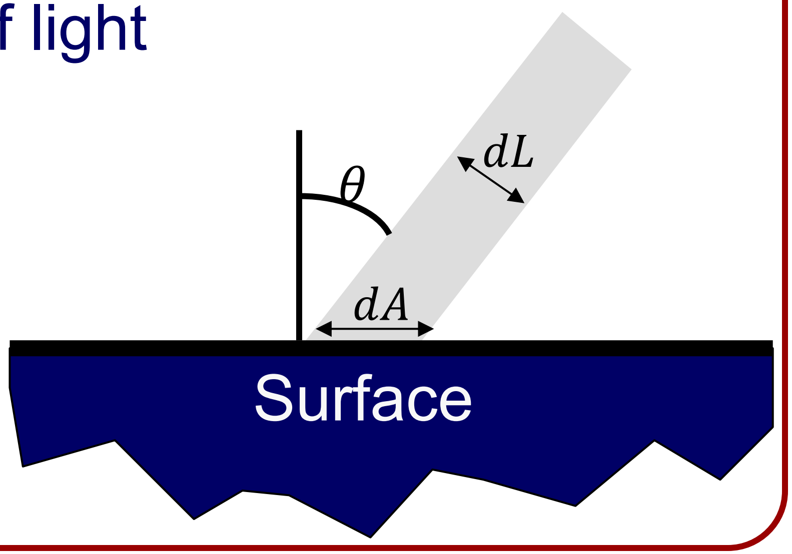
How much light is reflected?

- Lambertian: Only depends on angle of incident light

$$dL = dA \cdot \cos\theta$$

A unit area of light along the beam is spread across a patch of surface with area $1/\cos\theta$.

$\Rightarrow$ A unit surface area receives only $\cos\theta$ of the light coming through a unit area of light along the beam.



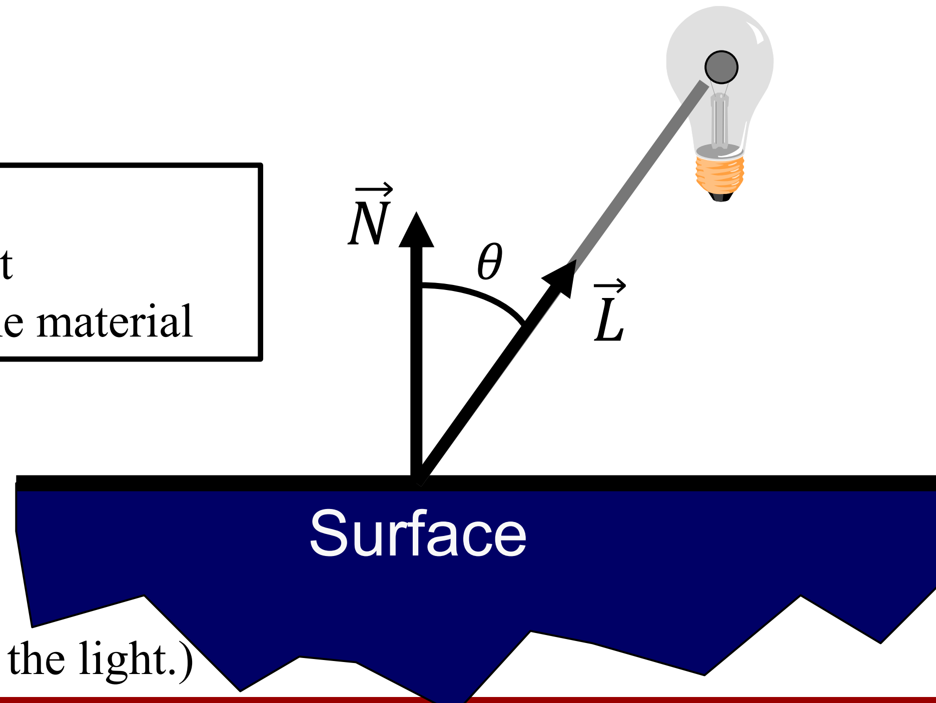
Diffuse Reflection

Lambertian model:

$$I_D = K_D \cdot \langle \vec{N}, \vec{L} \rangle \cdot I_L$$

- cosine law: $\cos\theta = \langle \vec{N}, \vec{L} \rangle$, with $\vec{N}$ and $\vec{L}$ **unit** vectors
- K_D is surface/material property (how much is reflected)
- I_L is incoming light

<u>Throughout</u>:
- Variable I_L^* denotes intensity of light
- Variable K_* denotes a property of the material



<u>Note</u>:
Here, $\vec{L}$ is the direction **to** the light,
(i.e. The negative of $\vec{l}$, the direction **from** the light.)

Diffuse Reflection

Light/surface properties have RGB components!

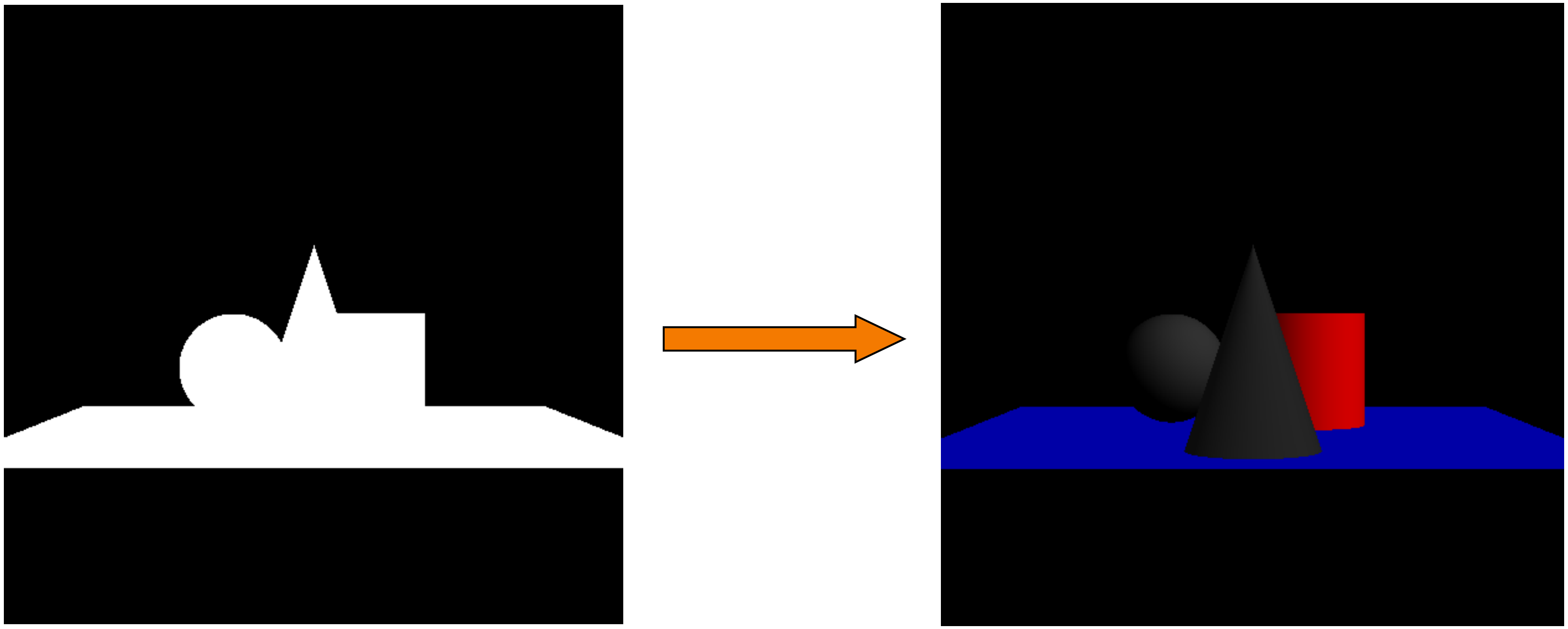
- Run calculation on EACH color channel
- This holds true for all lighting calculations

$$I_D^C = K_D^C \cdot \langle \vec{N}, \vec{L} \rangle \cdot I_L^C, \qquad C \in \{R, G, B\}$$

Diffuse Reflection

Assume surface reflects equally in all directions

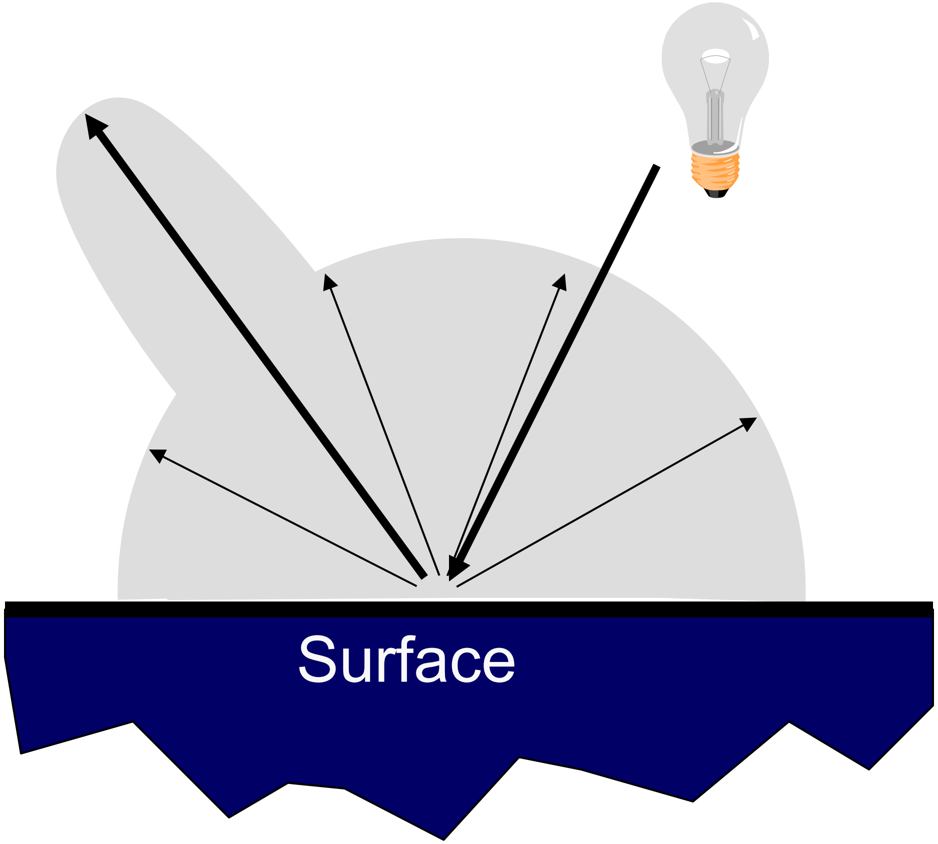
- Examples: chalk, clay



Simple Reflectance Model

Simple analytic model:

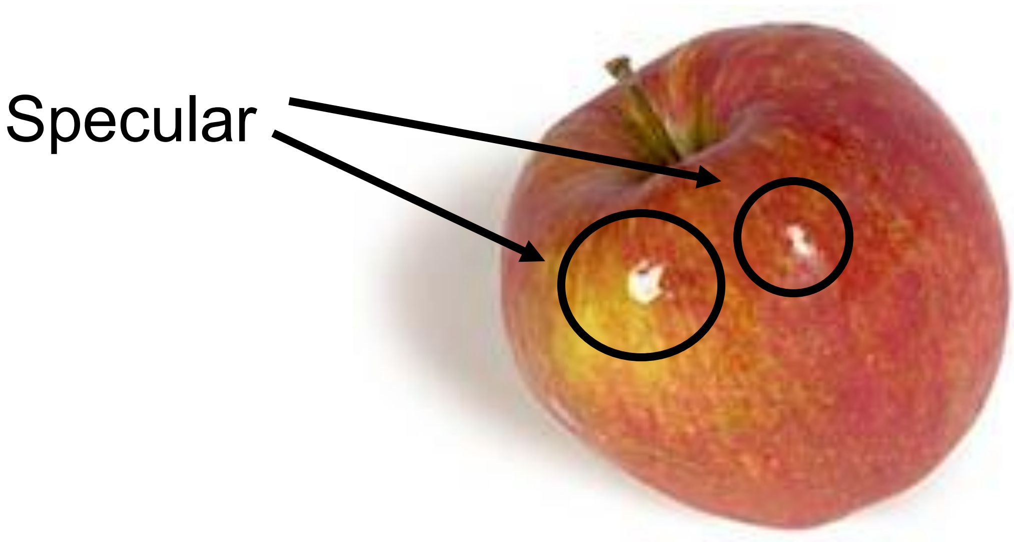
- diffuse reflection +
- **specular reflection +**
- emission +
- "ambient"



Specular Reflection

Reflection is strongest near mirror angle

- Examples: metals, shiny apples

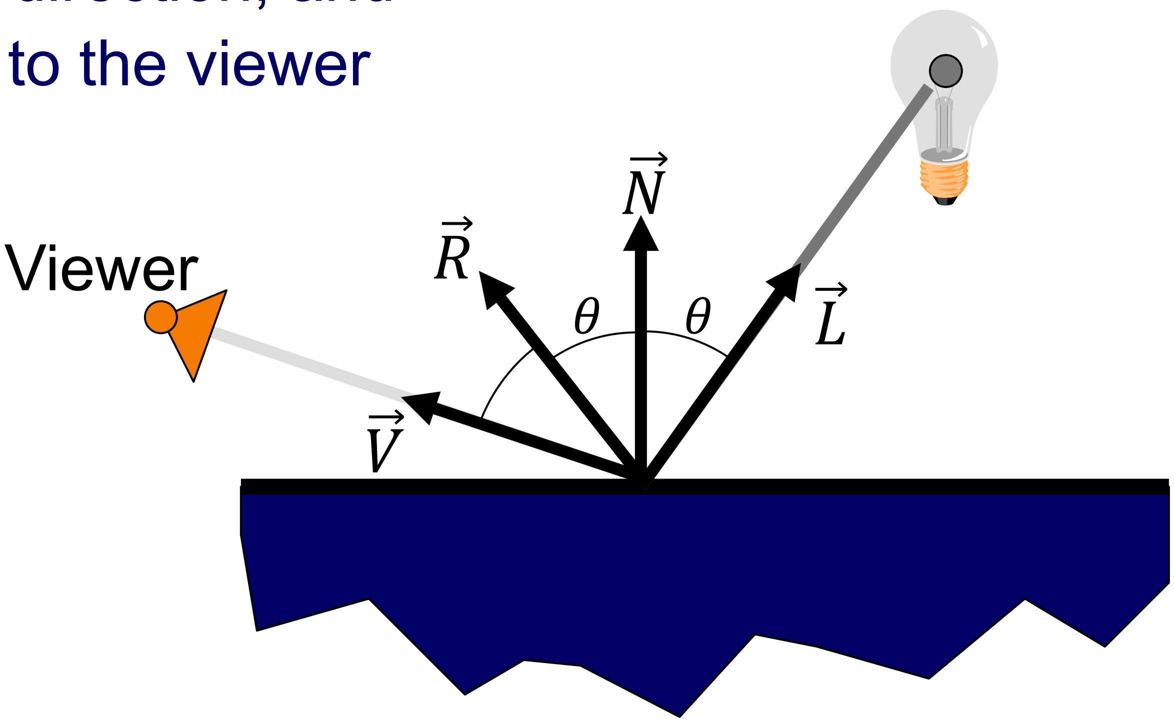


Specular Reflection

How much light is seen?

Depends on the alignment of the:

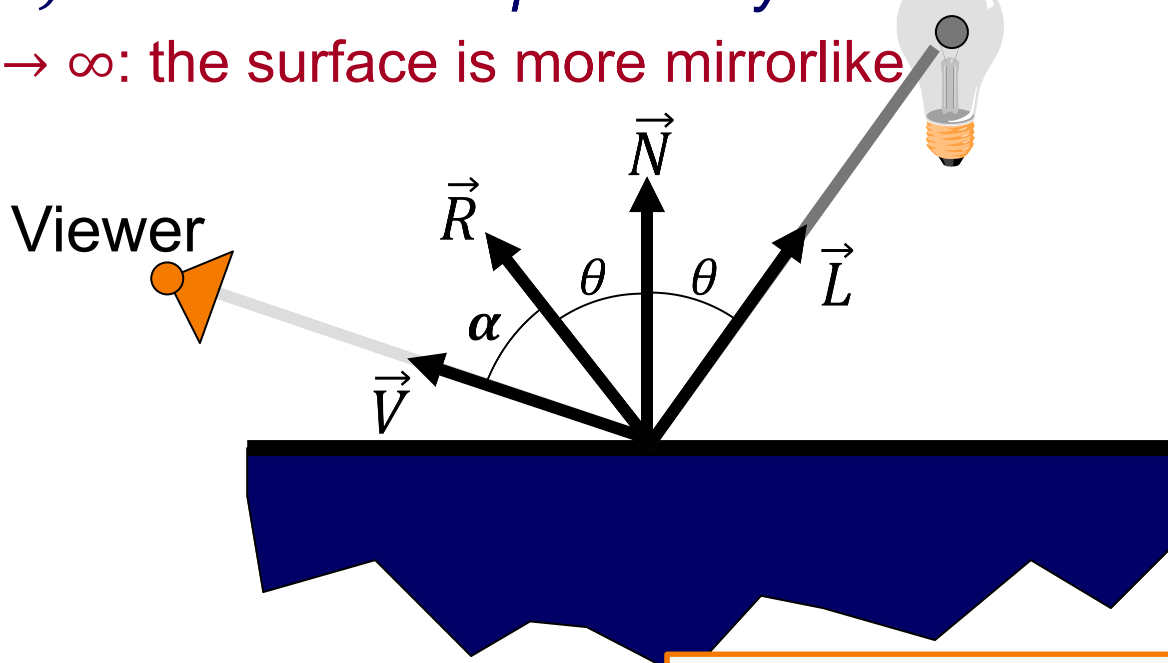
- reflected direction, and
- direction to the viewer



Specular Reflection

Phong Model:

- $\cos(\alpha) = \langle \vec{V}, \vec{R} \rangle \in [-1,1]$ describes how aligned the reflected and view directions are
- $K_n \in [0, \infty)$ describes the *specularity* of the surface
  - » As $K_n \to \infty$: the surface is more mirrorlike

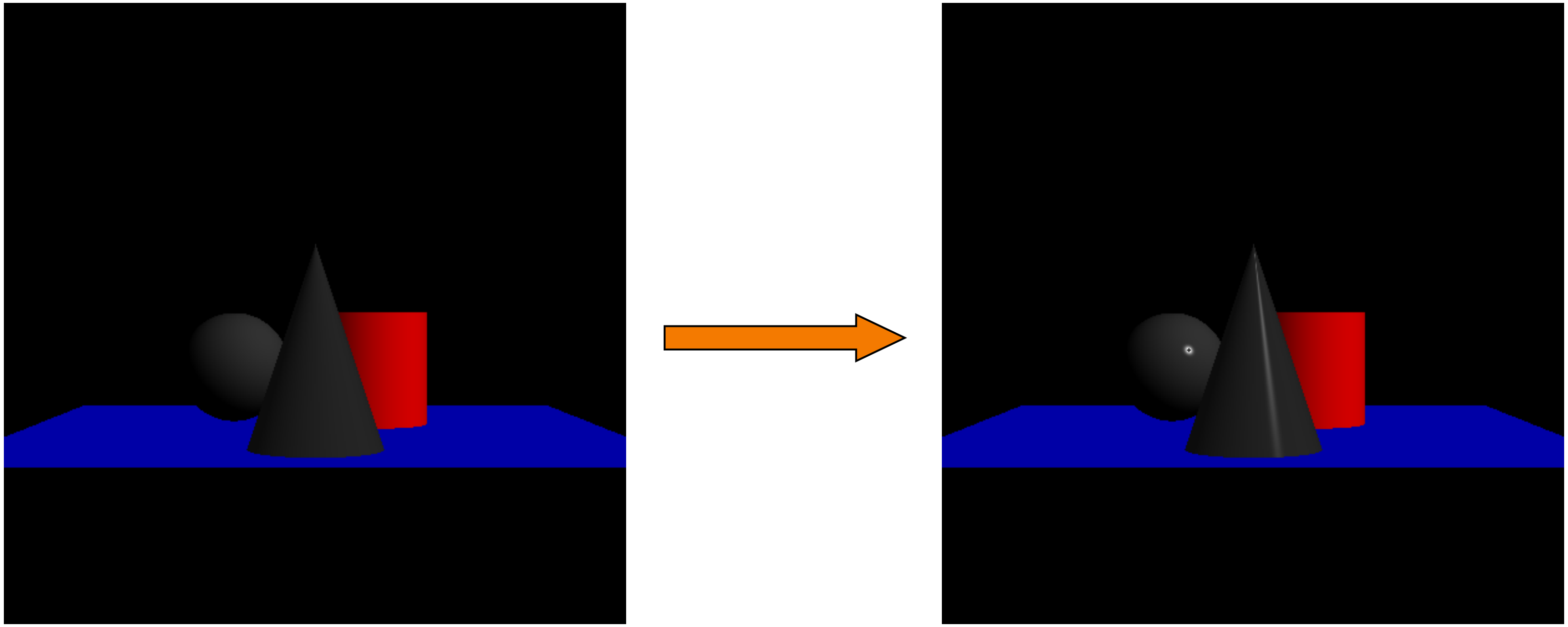


$$I_S = K_S \cdot \langle \vec{V}, \vec{R} \rangle^{K_n} \cdot I_L$$

Specular Reflection

Reflection is strongest near mirror angle

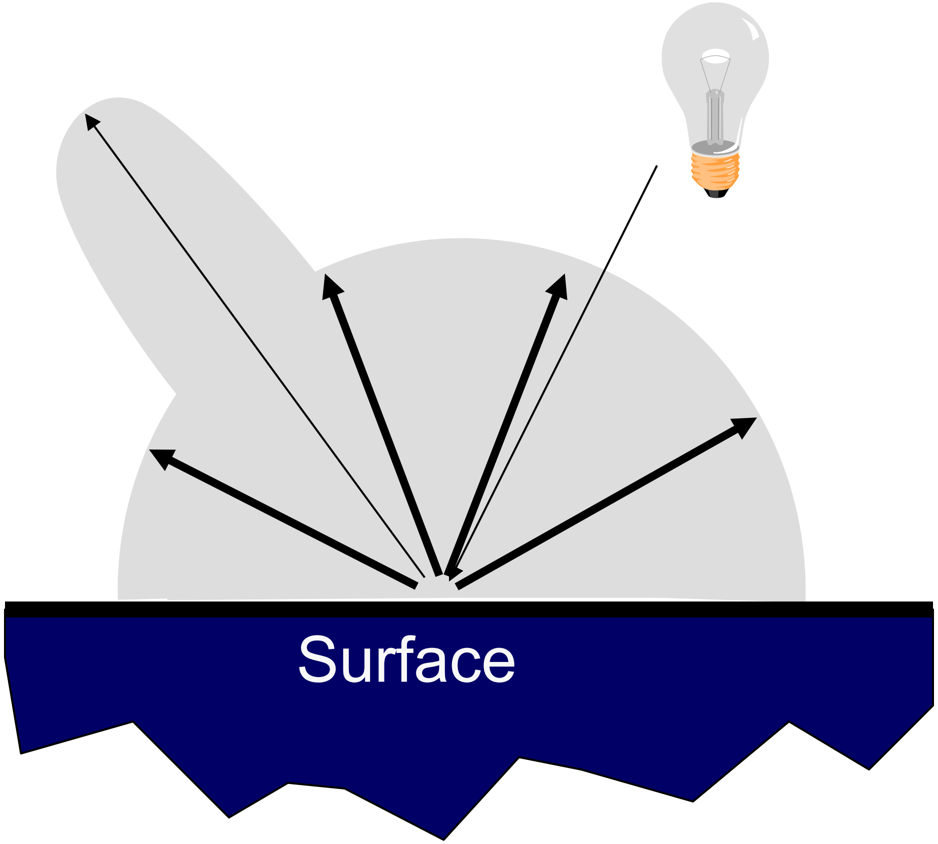
- Examples: metals, shiny apples



Simple Reflectance Model

Simple analytic model:

- diffuse reflection +
- specular reflection +
- **emission +**
- “ambient”



Emission

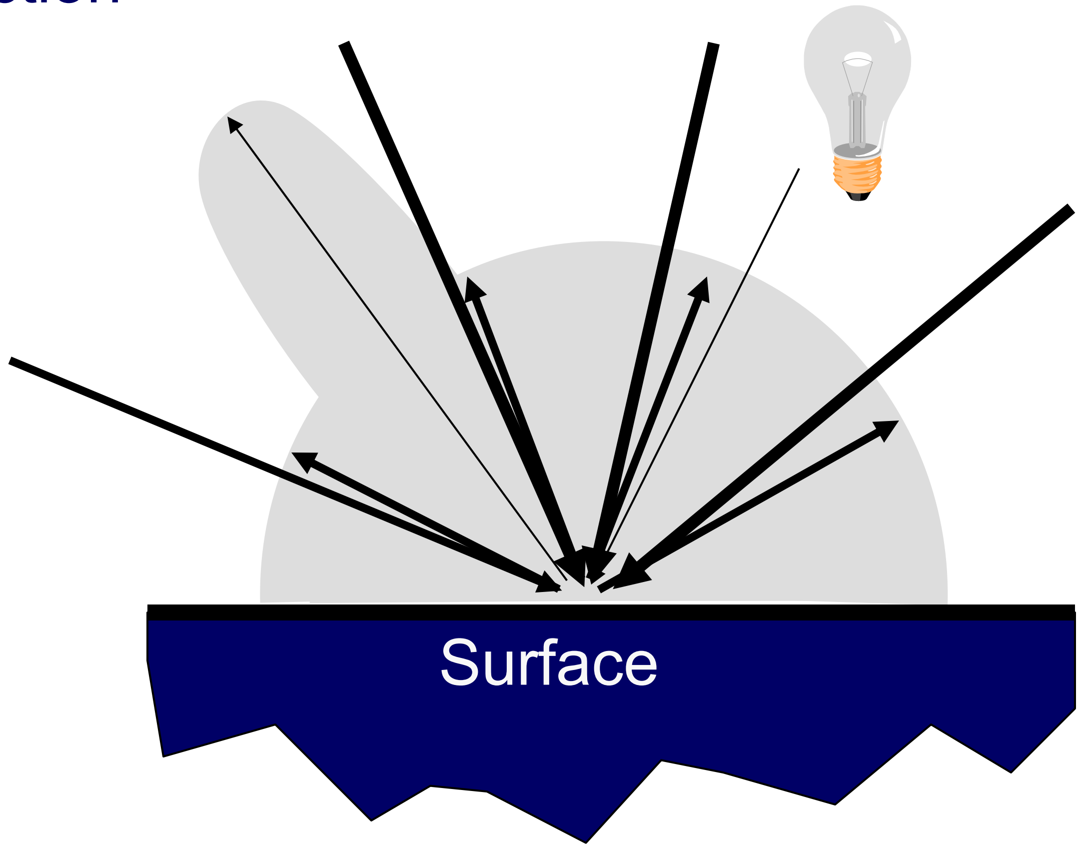
Represents light emanating **uniformly** from a surface that cannot be described by the three light sources (e.g. area lights)

$$K_E = K_E$$

Simple Reflectance Model

Simple analytic model:

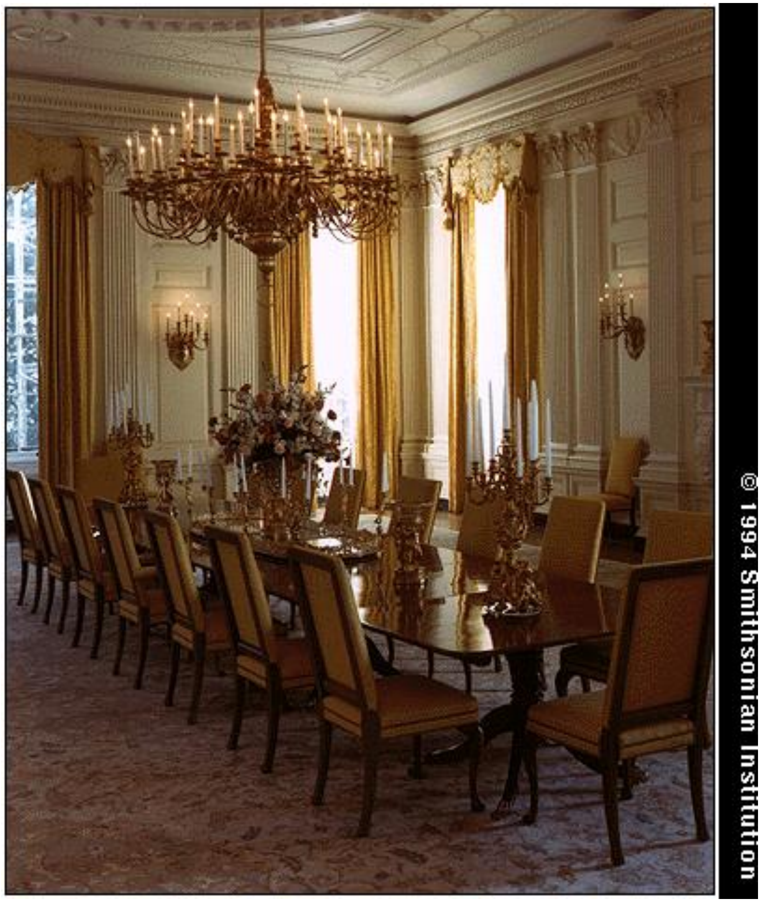
- diffuse reflection +
- specular reflection +
- emission +
- **“ambient”**



Ambient Term

Represents reflection from all indirect illumination

$$I_A = K_A \cdot I_L^A$$



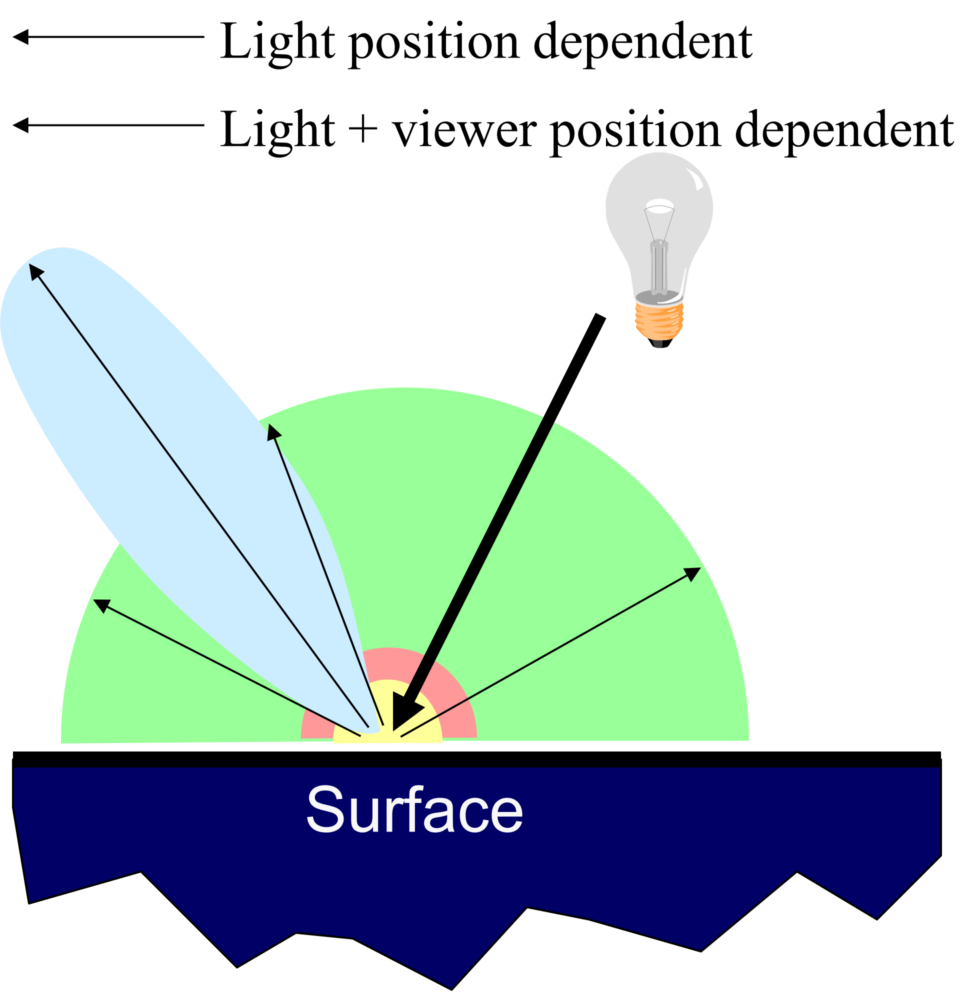
Note:
Because it is directionless, ambient light cannot be incorporated in the specular/diffuse calculations.

Typically, $K_A = K_D$ describe the color of the material.

Simple Reflectance Model

Simple analytic model:

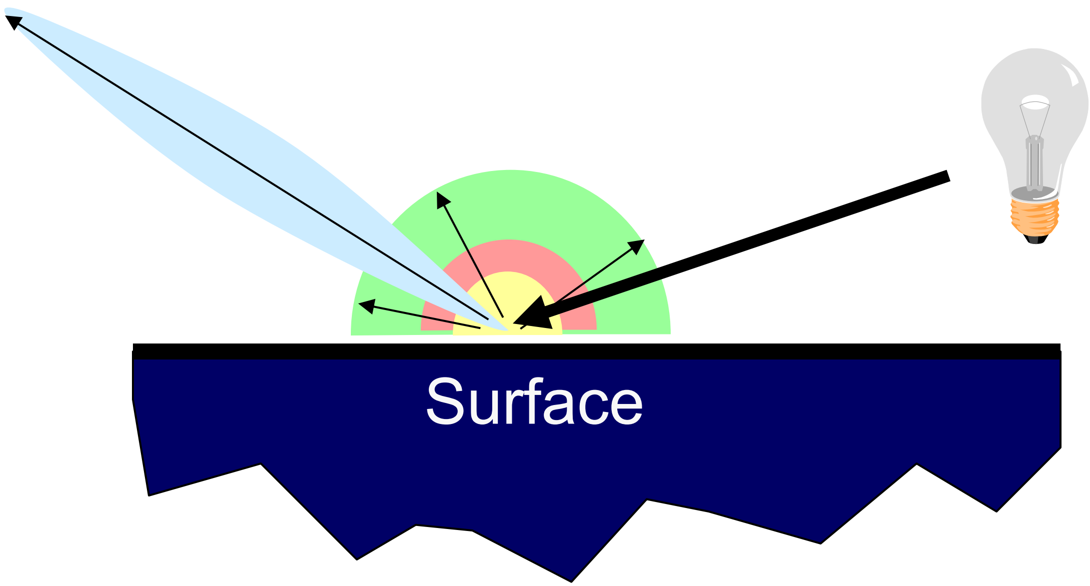
- diffuse reflection +
- specular reflection +
- emission +
- “ambient”



Simple Reflectance Model

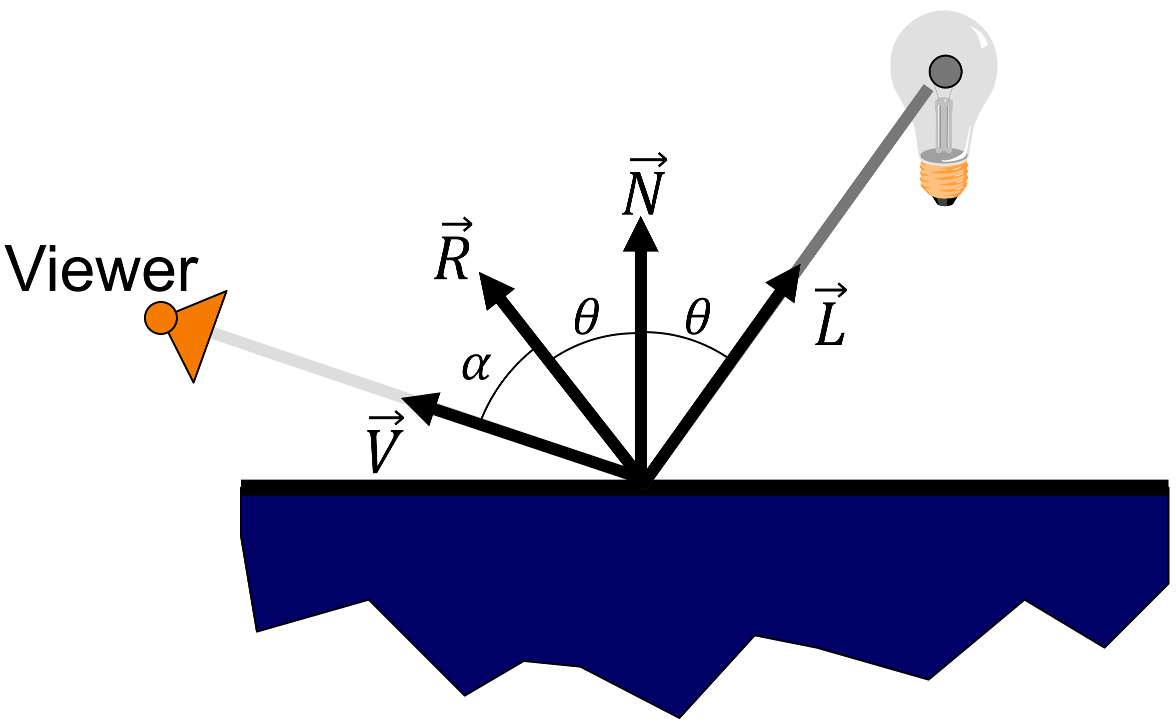
Simple analytic model:

- diffuse reflection + ← Light position dependent
- specular reflection + ← Light + viewer position dependent
- emission +
- "ambient"



Surface Illumination Calculation

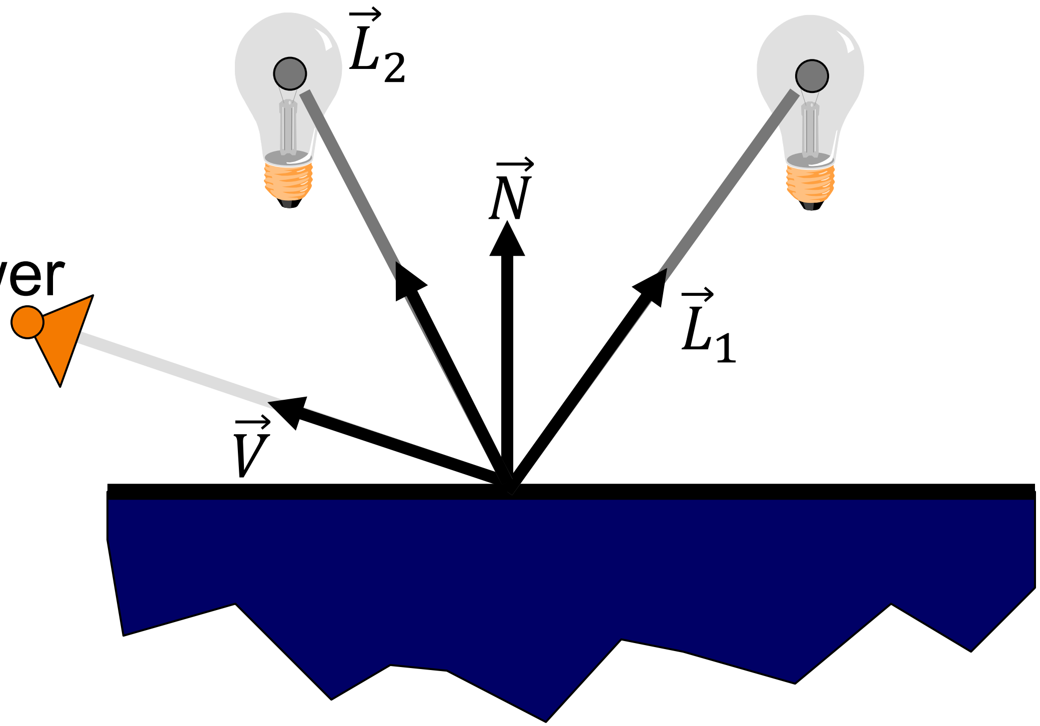
Single light source:



$$I = K_E + K_A \cdot I_L^A + \left(K_D \cdot \langle \vec{N}, \vec{L} \rangle + K_S \cdot \langle \vec{V}, \vec{R}(\vec{L}) \rangle^{K_n}\right) \cdot I_L$$

Surface Illumination Calculation

Multiple light source:



$$I = K_E + \sum_L \left[K_A \cdot I_L^A + \left(K_D \cdot \langle \vec{N}, \vec{L} \rangle + K_S \cdot \langle \vec{V}, \vec{R}(\vec{L}) \rangle^{K_n} \right) \cdot I_L \right]$$