Accelerated Ray Casting

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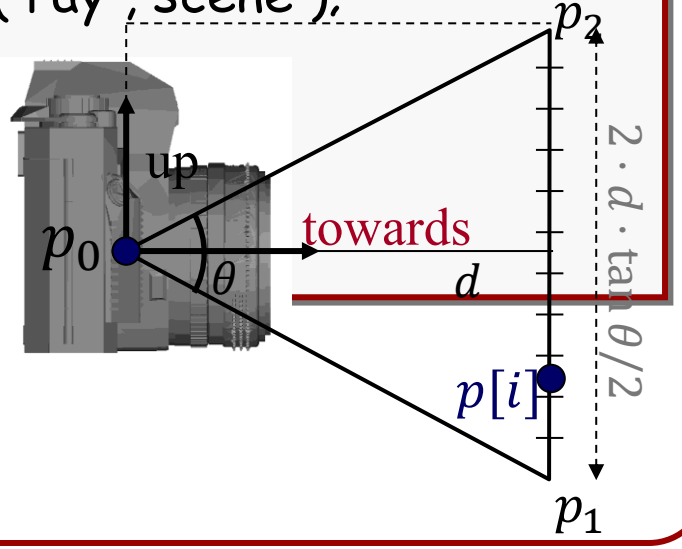
(601.457/657)

Ray Casting

- Simple implementation:

```
Image RayCast( Camera camera , Scene scene , int width , int height)
{
    Image image( width , height );
    for( int j=0 ; j<height ; j++ ) for( int i=0 ; i<width ; i++ )
    {
        Ray< 3 > ray = ConstructRayThroughPixel( camera , i , j );
        Intersection hit = FindIntersection( ray , scene );
        image[i][j] = GetColor( hit );
    }
    return image;
}
```



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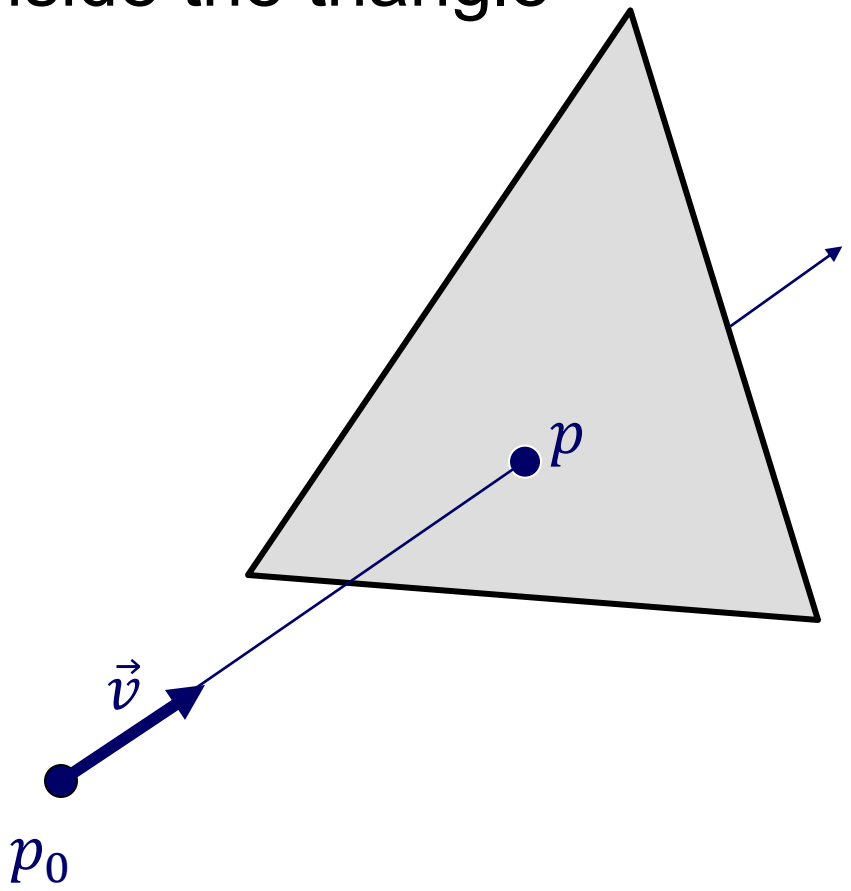
Ray-Triangle Intersection

1. Intersect ray with plane
2. Check if the point is inside the triangle



Ray-Plane Intersection

Ray: $p(t) = p_0 + t \cdot \vec{v}, \quad (0 \le t < \infty)$

Plane: $\Phi(p) = \langle p, \vec{n} \rangle - d = 0$

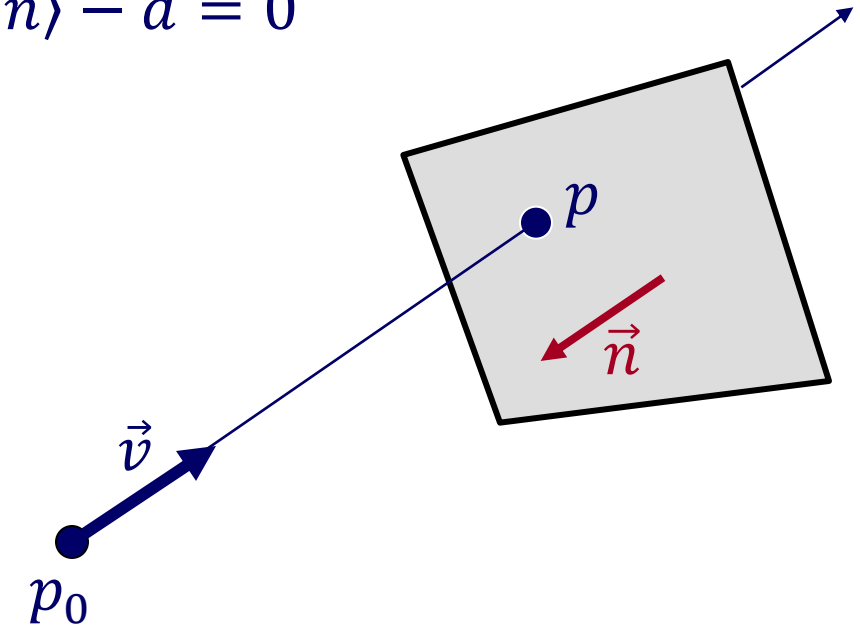
Algebraic Method

Substituting for p we get:

$$\Phi(t) \equiv \Phi\big(p(t)\big) = \langle e + t \cdot \vec{v}, \vec{n} \rangle - d = 0$$

Solution:

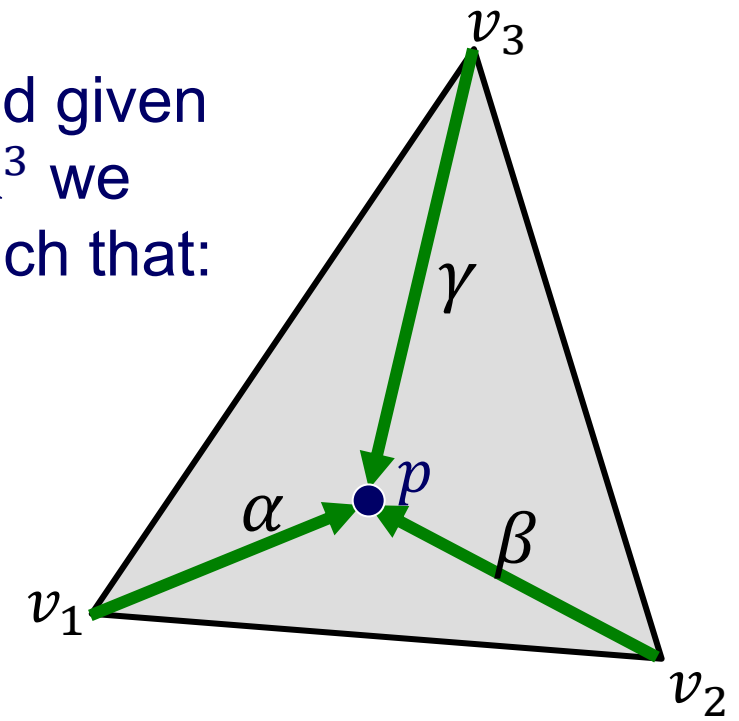
$$t = -\frac{\langle p_0, \vec{n} \rangle - d}{\langle \vec{v}, \vec{n} \rangle}$$



Ray-Triangle Intersection

- Check for point-triangle intersection parametrically
  - In general, given $p \in \mathbb{R}^3$ and given three points $\{v_1, v_2, v_3\} \subset \mathbb{R}^3$ we can solve for $\alpha, \beta, \gamma \in \mathbb{R}$ such that:

$$p = \alpha v_1 + \beta v_2 + \gamma v_3$$



p is in the plane containing $\{v_1, v_2, v_3\}$ if and only if (iff.):

$$\alpha + \beta + \gamma = 1$$

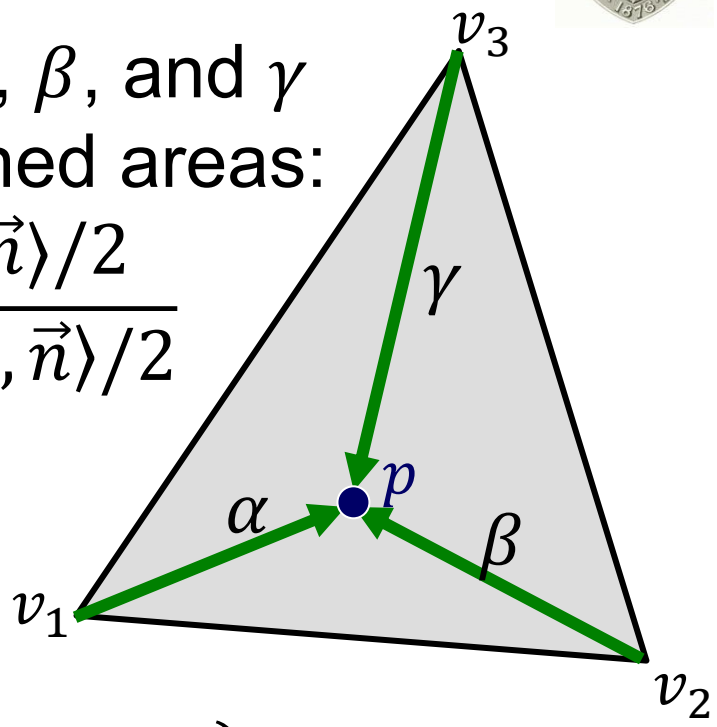
p is inside the triangle with vertices $\{v_1, v_2, v_3\}$ iff.:

$$\alpha, \beta, \gamma \geq 0$$

Ray-Triangle Intersection

We can compute the weights α, β, and γ by considering the ratios of signed areas:

$$\alpha = \frac{\langle (v_2 - p) \times (v_3 - p), \vec{n} \rangle / 2}{\langle (v_2 - v_1) \times (v_3 - v_1), \vec{n} \rangle / 2}$$
$$\vdots$$



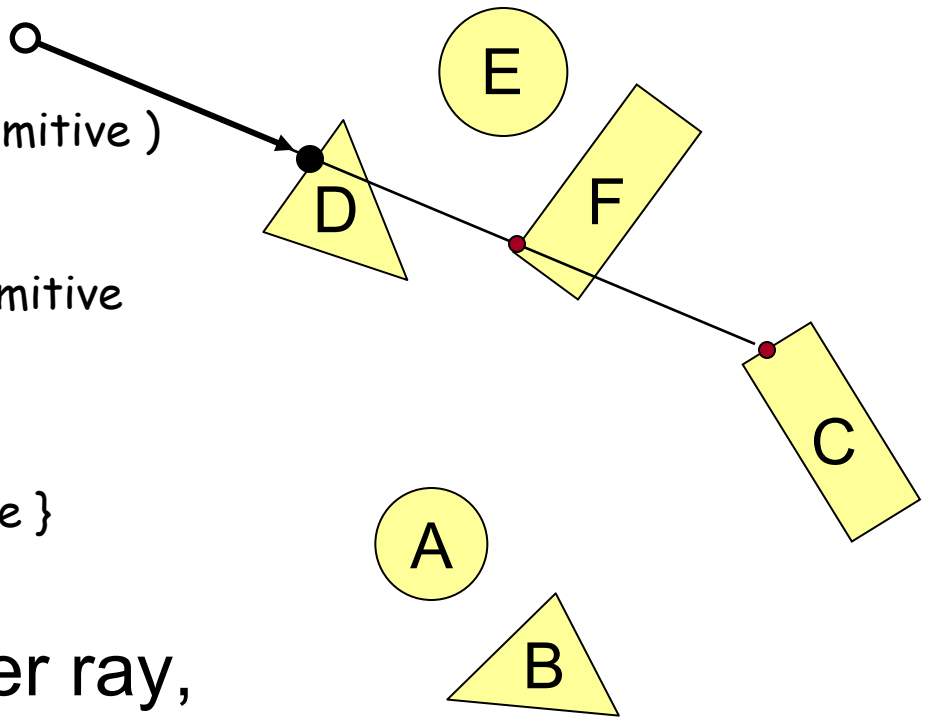
where $\vec{n}$ is a unit vector that is perpendicular to the triangle:

$$\vec{n} = \frac{(v_2 - v_1) \times (v_3 - v_1)}{|(v_2 - v_1) \times (v_3 - v_1)|}$$

Ray-Scene Intersection

A direct (naïve) approach:

```
Intersection FindIntersection( Ray< 3 > ray, Scene scene )
{
    { min_t , first_shape } = { ∞ , NULL }
    for each primitive in scene
    {
        t = Intersect( ray , primitive )
        if( t<min_t )
        {
            first_shape = primitive
            min_t = t
        }
    }
    return { min_t , first_shape }
}
```



Complexity is $O(N)$ per ray,
with N the number of primitives.

Overview

Acceleration techniques

- Data Partitions
  - » Bounding volume hierarchy (BVH)
- Space Partitions
  - » Uniform (voxel) grid
  - » Octree
  - » Binary space partition (BSP) tree

Acceleration techniques

Both data and space partitions accelerate intersections testing by leveraging:

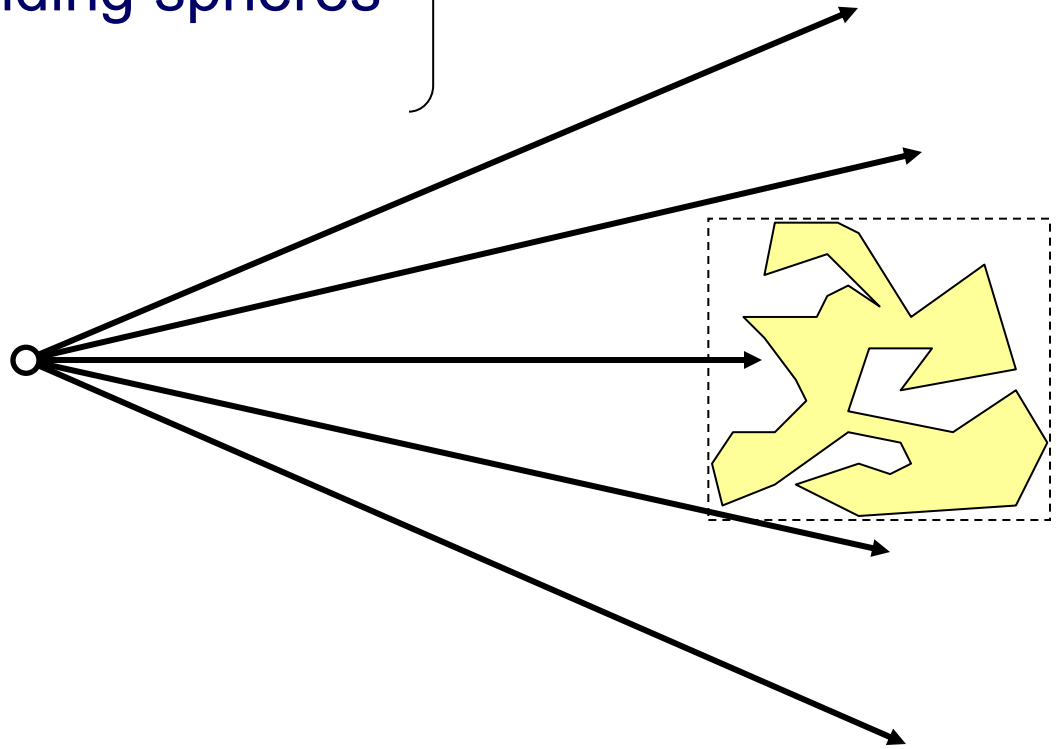
- Trivial Rejection:
  Discard groups of primitives / regions of space that can be (easily) guaranteed to be missed by the ray.
- Ordering:
  Test (likely) nearer primitives / regions of space first, allowing for early termination if there is a hit.

Data Partition: Bounding Volume

Check for intersection with the bounding volume:

- Bounding cubes
- Bounding boxes
- Bounding spheres
- Etc.

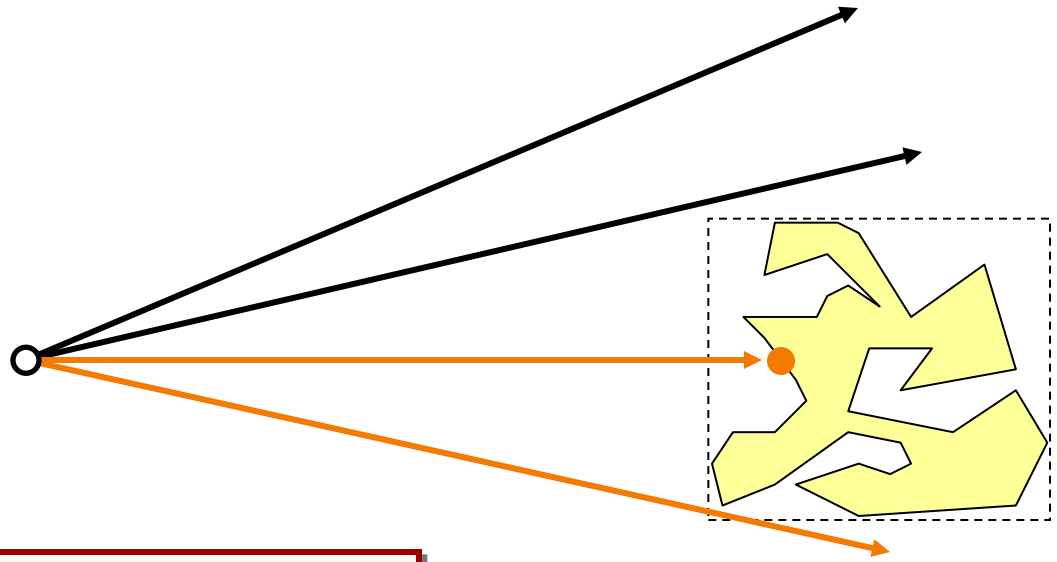
Stuff that's easy to intersect



Data Partition: Bounding Volume

Check for intersection with the bounding volume

If the ray misses the bounding volume,
it can't intersect its contents



Still need to check for intersections with shape.

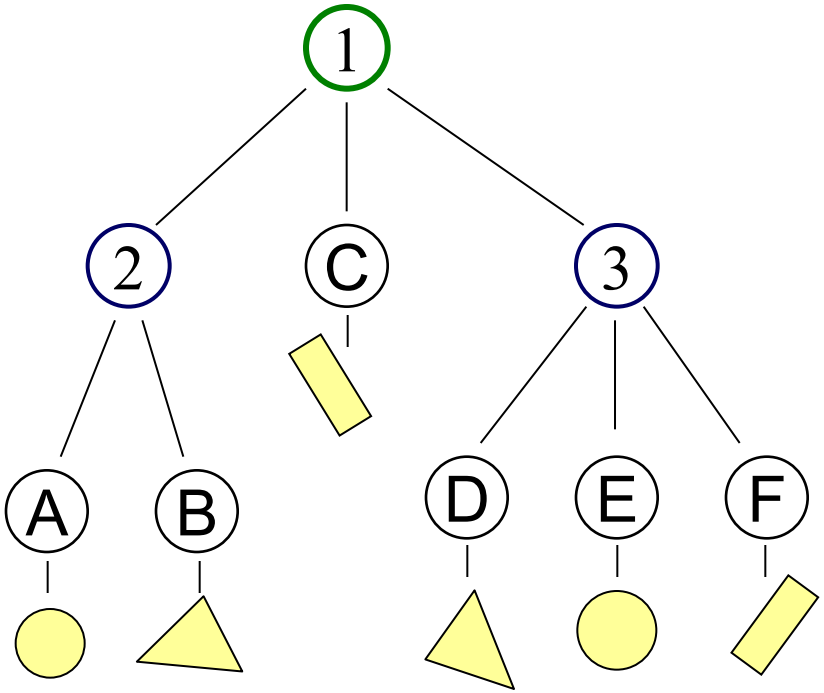
Data Partition: BVH

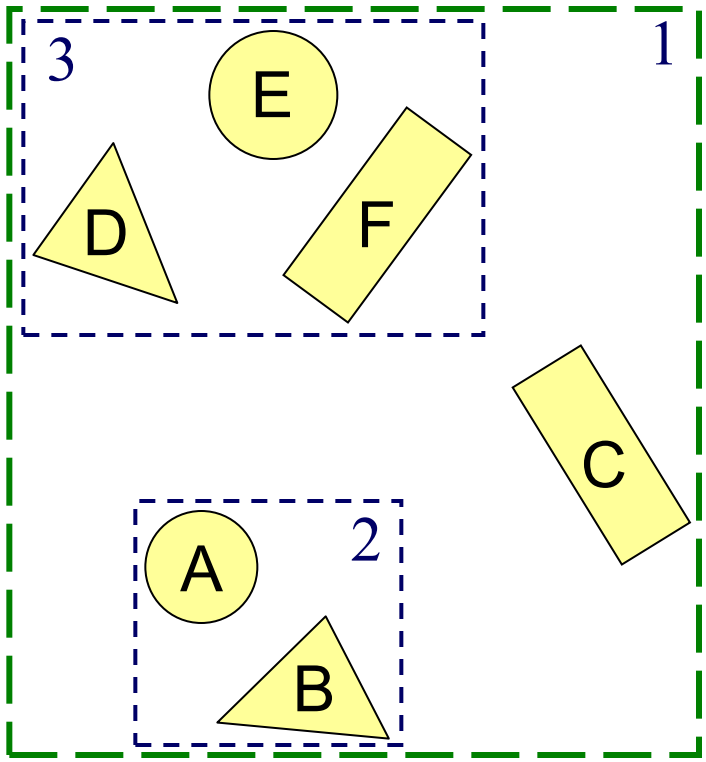
Build a bounding volume hierarchy (BVH) tree

A node in the tree consists of a bounding box as well as:

» A subset of (enclosed) shapes

» Child nodes





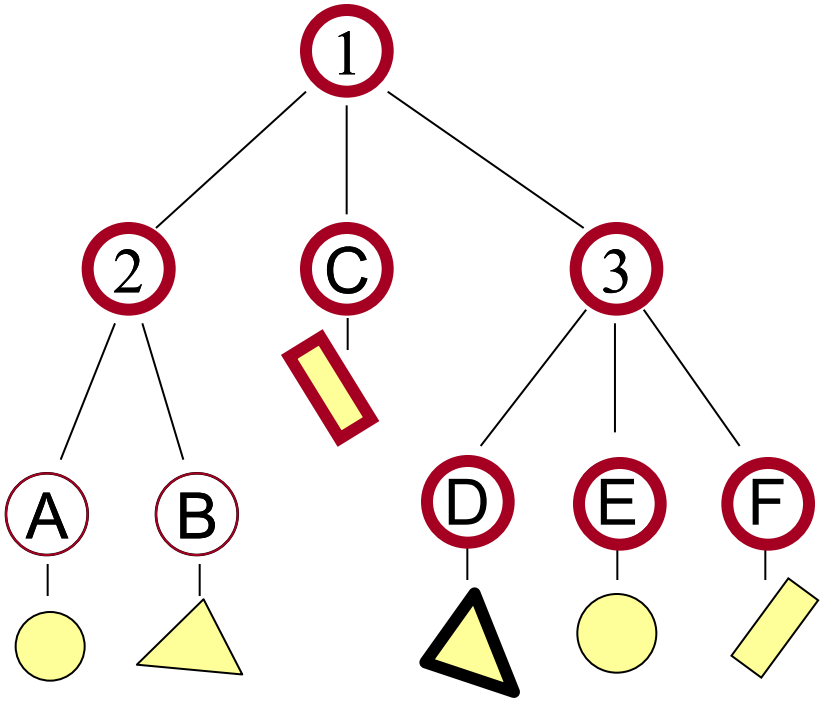
Data Partition: BVH

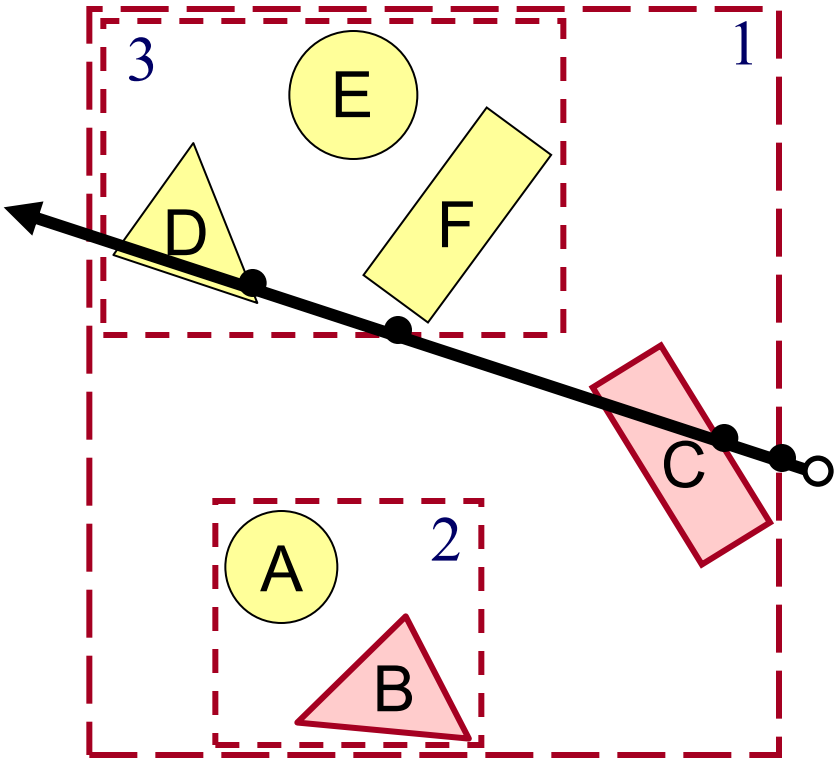
Trivial Rejection:

```
Intersection FindIntersection( Ray< 3 > ray , BVHNode< 3 > node )
{
      { min_t , first_shape } = { ∞ , NULL }

      if( !intersect( ray , node.bBox ) )                    // Test Bounding box
            return { ∞ , NULL }

      foreach shape in node                                  // Test node's shape
      {
            t = Intersect( ray , shape )
            if( t<min_t ) { min_t , first_shape } = { t , shape }
      }

      for each child_node of node                            // Test node's children
      {
            { t , shape } = FindIntersection( ray , child_node )
            if( t<min_t ) { min_t , first_shape } = { t , shape }
      }
      return { min_t , first_shape }
}
```

Data Partition: BVH

Trivial Rejection:
Discard groups of primitives that can be (easily) guaranteed to be missed by the ray.





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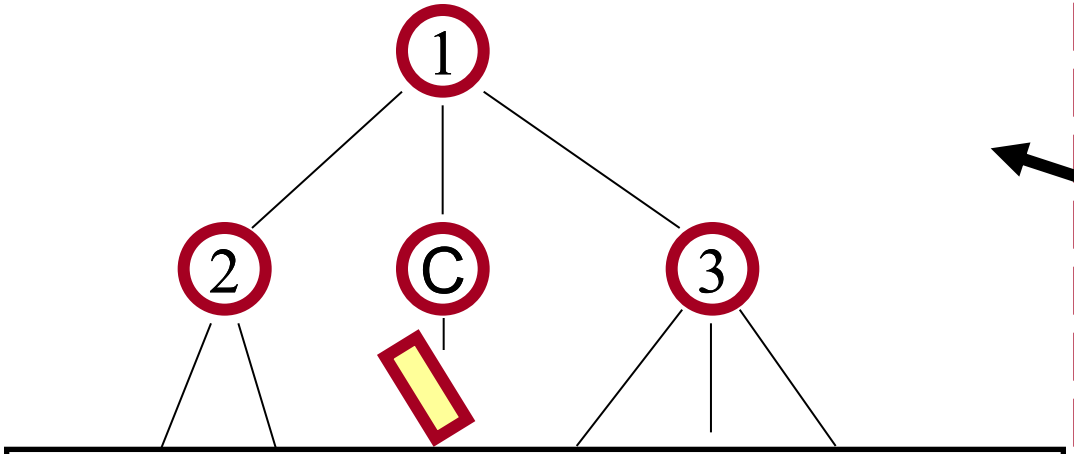

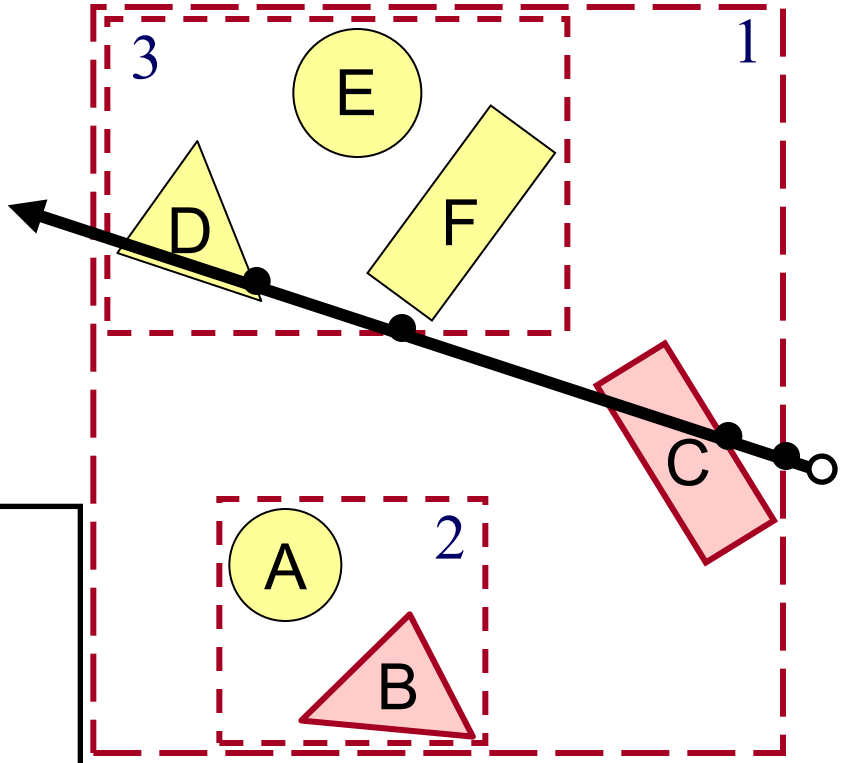

- Don't need to test shapes A or B
- Need to test volumes 1, 2, and 3
- Need to test shapes C, D, E, and F

Data Partition: BVH

Ordering:

```
Intersection FindIntersection( Ray< 3 > ray , BVHNode< 3 > node )
{
      // Find intersections with the nearest shape stored in node
      …
      // Find intersections with all child bounding volumes
      …
      // Sort child BVH node intersections front to back
      // and store distances to child bounding boxes in bv_t[]
      …

      // Process intersections
      for each intersected child bounding volume
      {

            { t , shape } = FindIntersection( ray , child_bBox )
            if( t<min_t ) { min_t , min_shape } = { t , shape }
      }
      return { min_t , min_shape }
}
```

Data Partition: BVH

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```
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      …
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      …
      // Sort child BVH node intersections front to back
      // and store distances to child bounding boxes in bv_t[]
      …

      // Process intersections
      for each intersected child bounding volume
      {
            if( min_t<bv_t[child_bVolume] ) break
            { t , shape } = FindIntersection( ray , child_bBox )
            if( t<min_t ) { min_t , min_shape } = { t , shape }
      }
      return { min_t , min_shape }
}
```
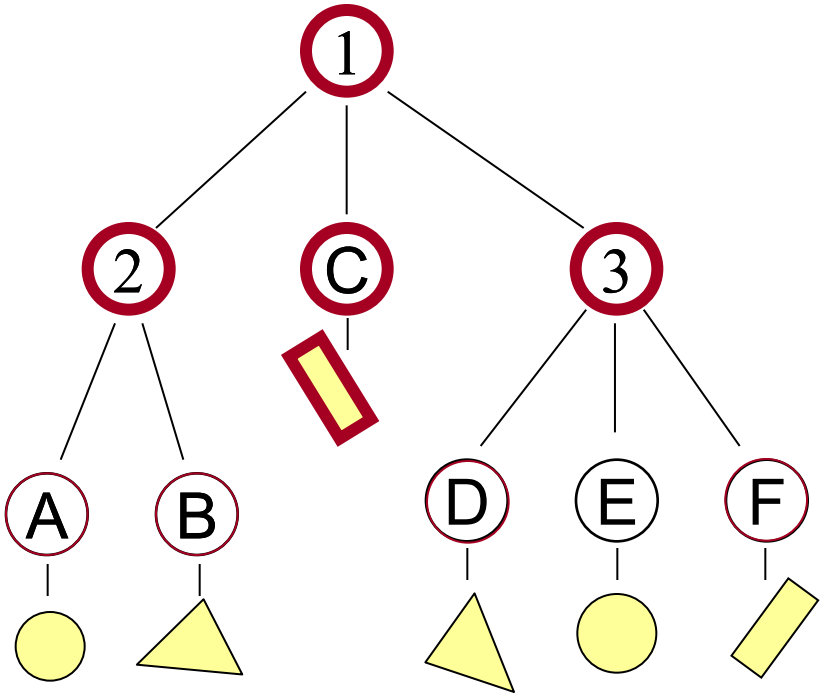
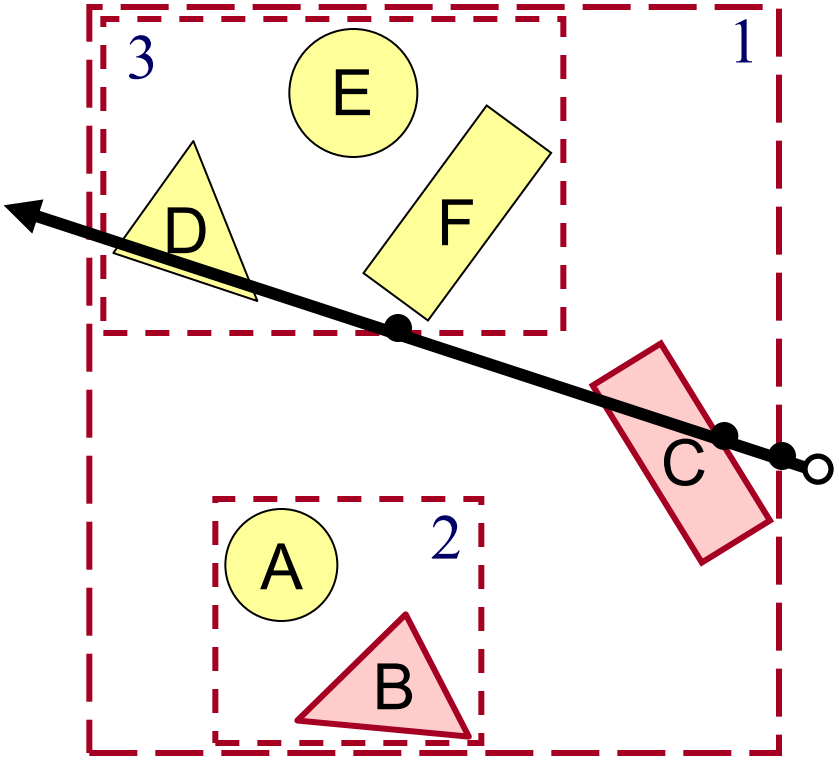

Data Partition: BVH

Ordering:
Test (likely) nearer intersections first, allowing for early termination if there is a hit.





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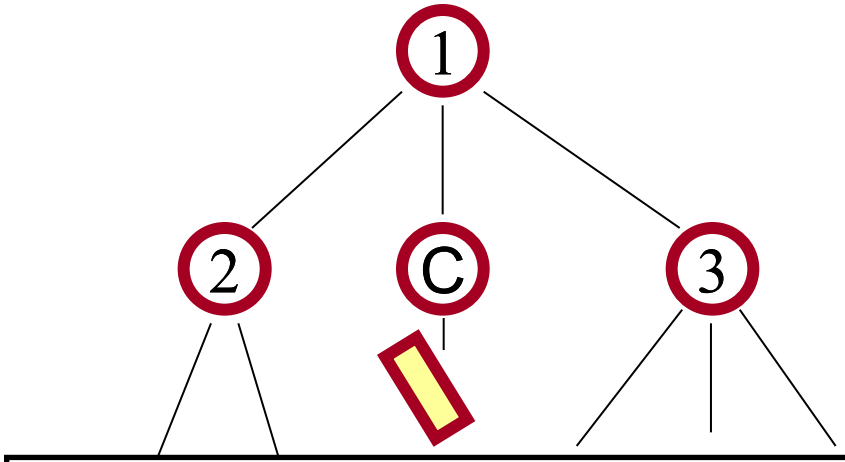
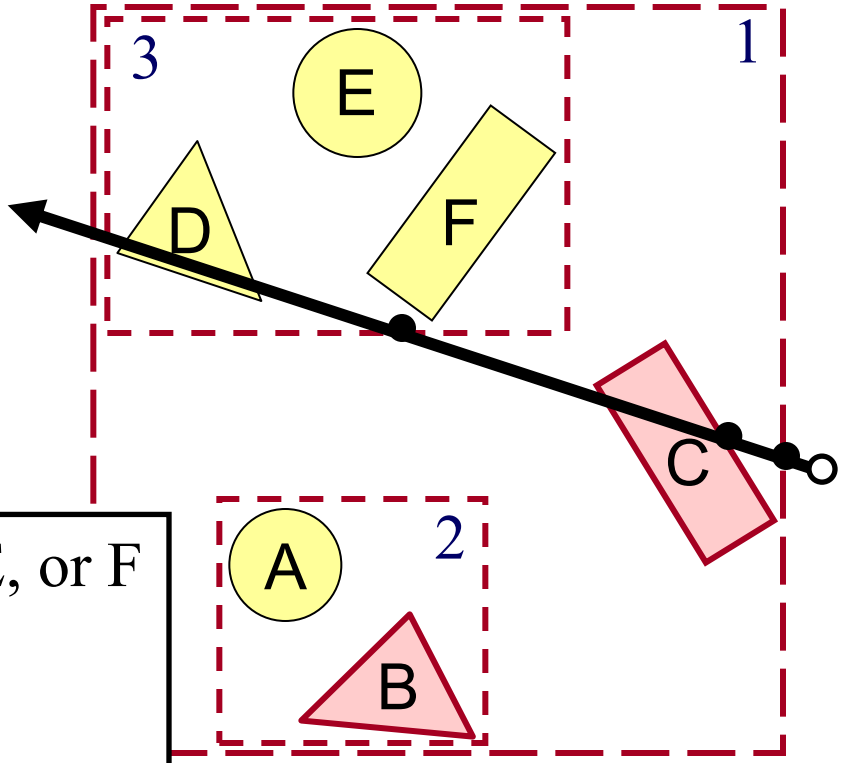




- Don't need to test shapes A, B, D, E, or F
- Need to test groups 1, 2, and 3
- Need to test shape C

Data Partition: BVH

[WARNING]:
A bounding box may be singular – e.g. if it incapsulates planar, axis-aligned geometry.

To avoid potential numerical-precision issues, you can thicken the bounding box by a small amount in each dimension.

Overview

Acceleration techniques

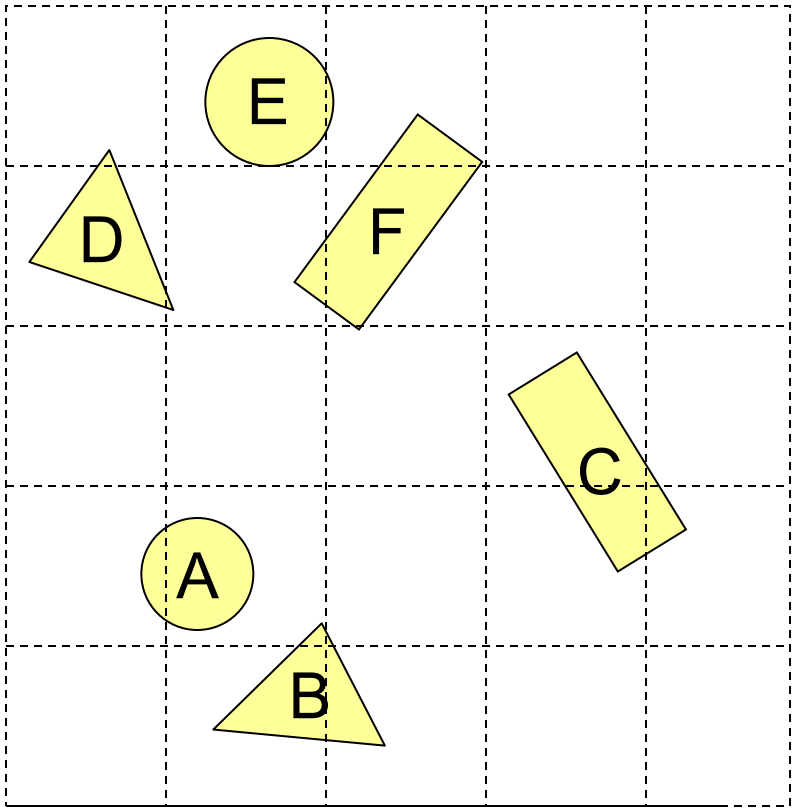
- Data partitions
  - » Bounding volume hierarchy (BVH)
- Space Partitions
  - » Uniform (Voxel) grid
  - » Octree
  - » Binary space partition (BSP) tree

Space Partitions: Uniform Grid

Construct uniform grid over the scene

Store a list of intersected primitive with each grid cell

- A primitive may belong to multiple cells
- A cell may have multiple primitives

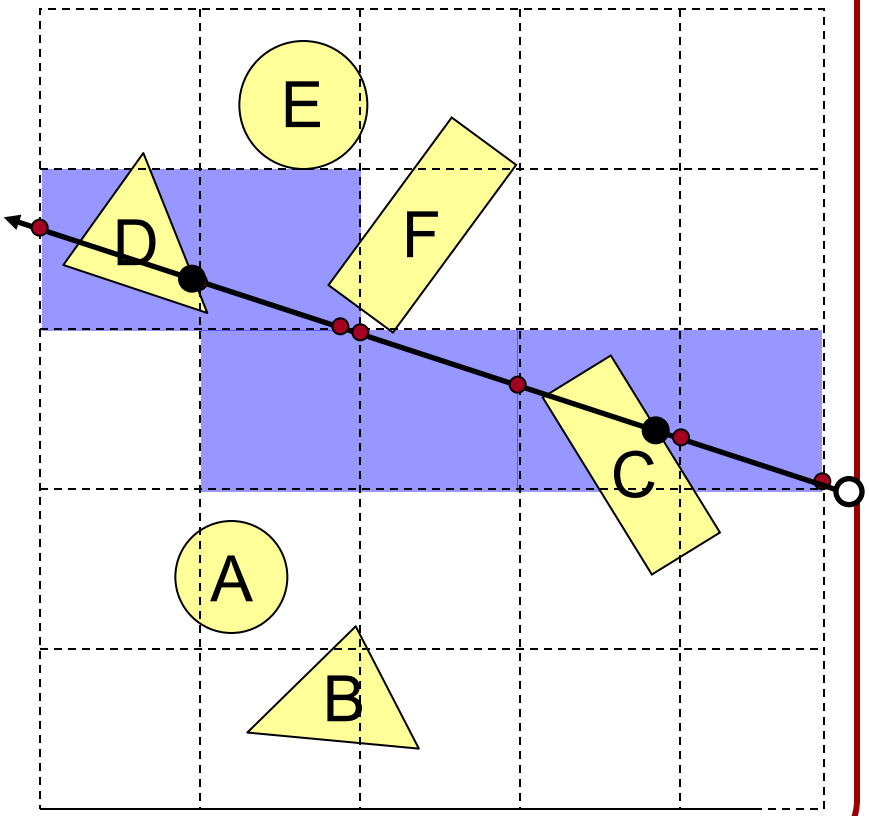


Space Partitions: Uniform Grid

Trace rays through grid cells

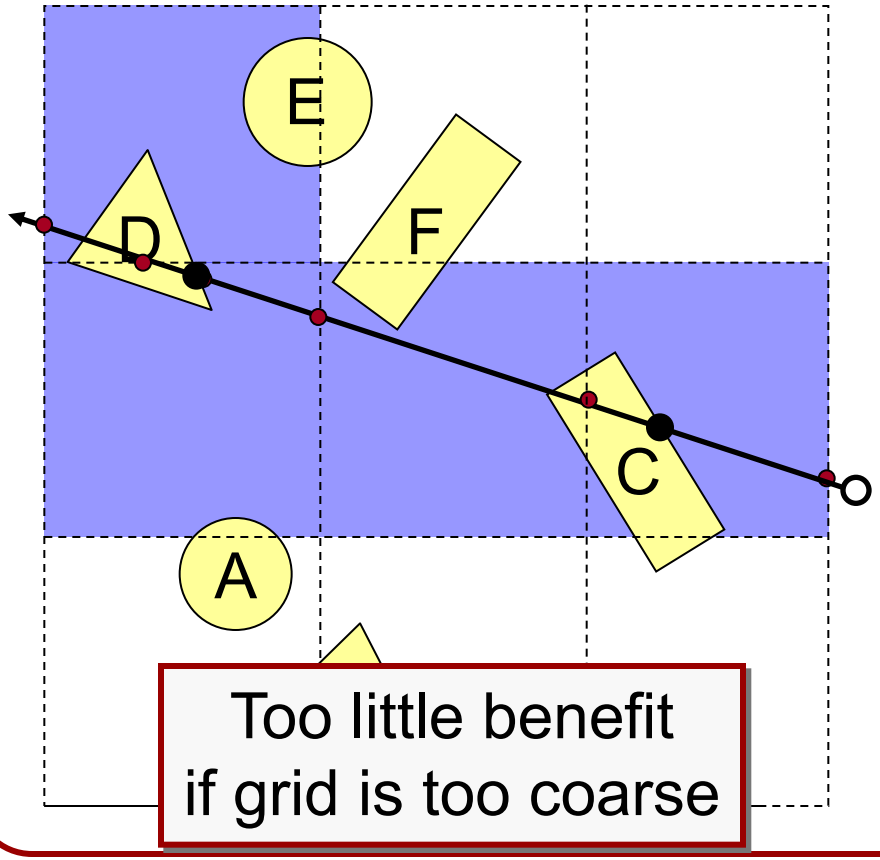
- Trivial Rejection:
  Avoid testing parts of space the ray doesn't go through
- Ordering:
  Traverse the cells in closest-to-furthest order

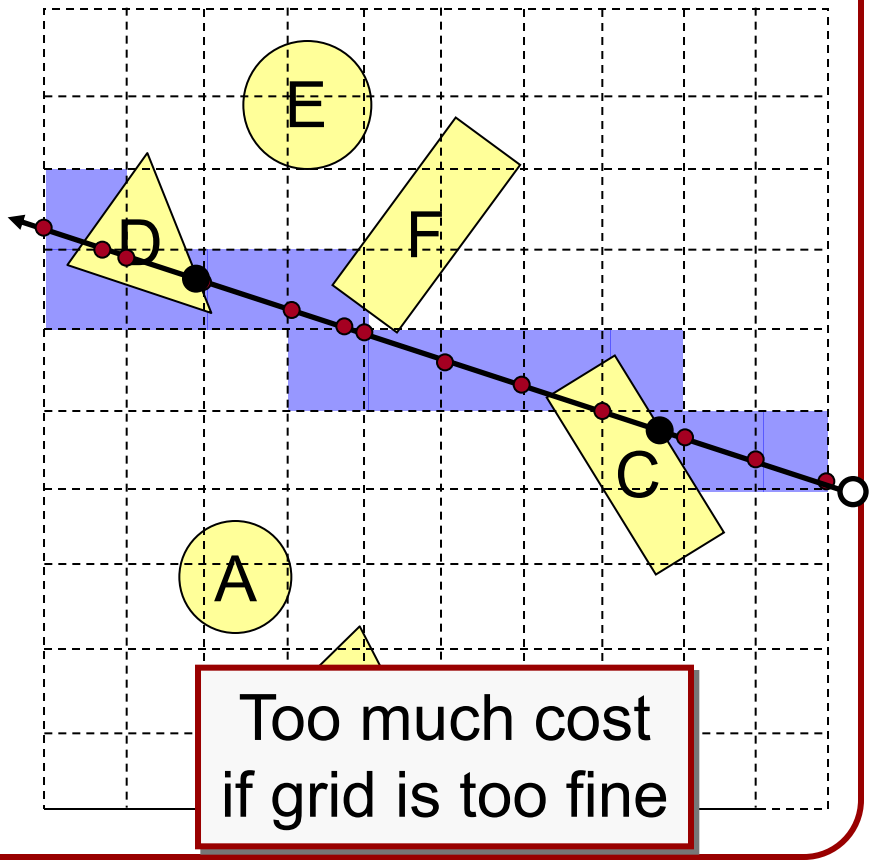
Only check primitives in intersected grid cells



Space Partitions: Uniform Grid

Challenge: How to choose suitable grid resolution?





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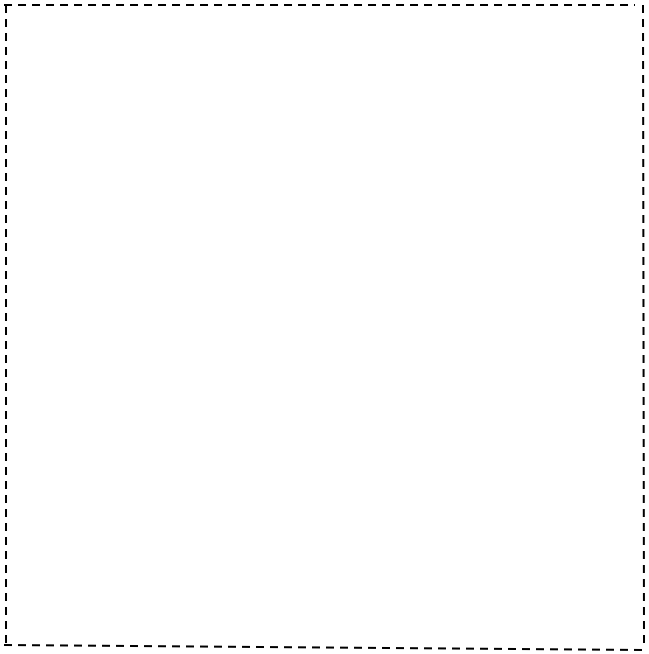
Space Partition: Octree

Think of a (power-of-two) voxel grid as a tree.

- The root node is the entire region
- Each node has eight children obtained by subdividing the parent into eight equal regions



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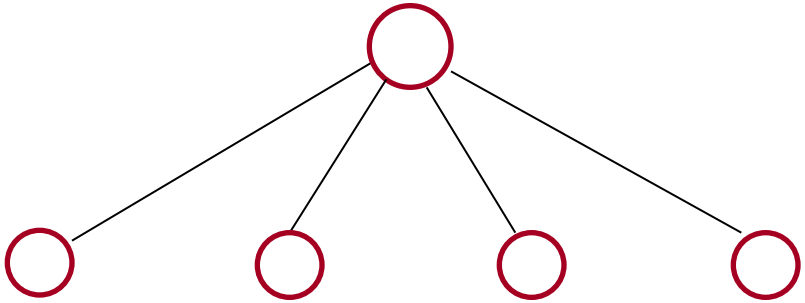

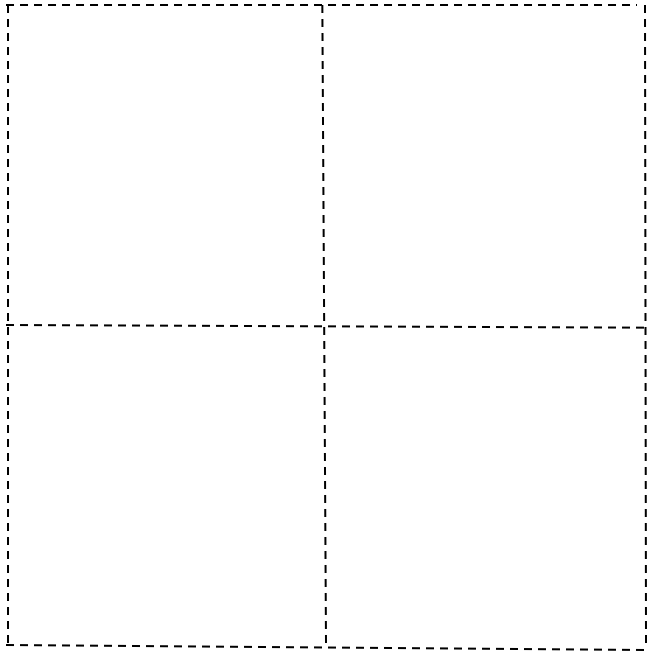

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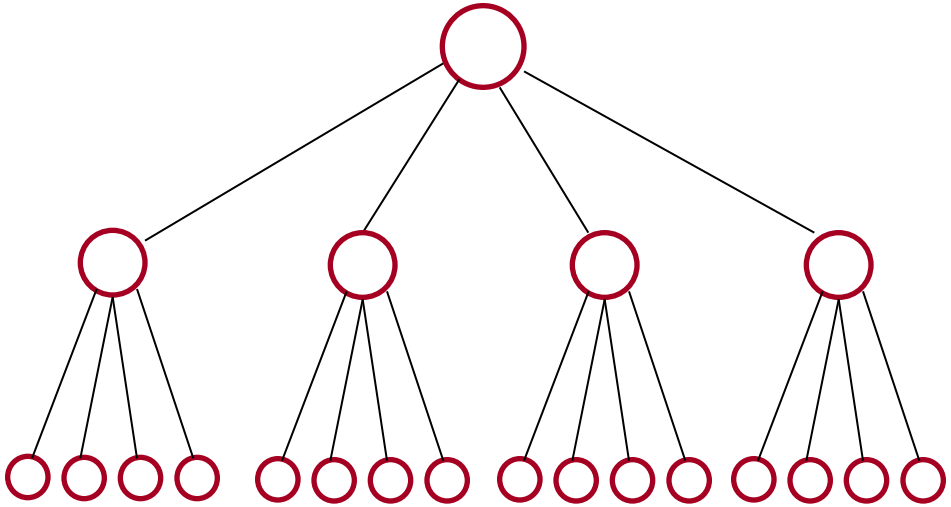

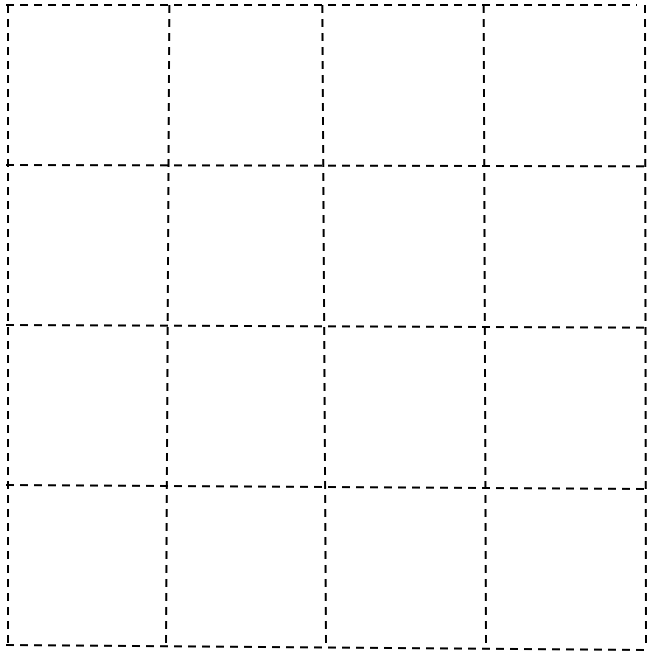

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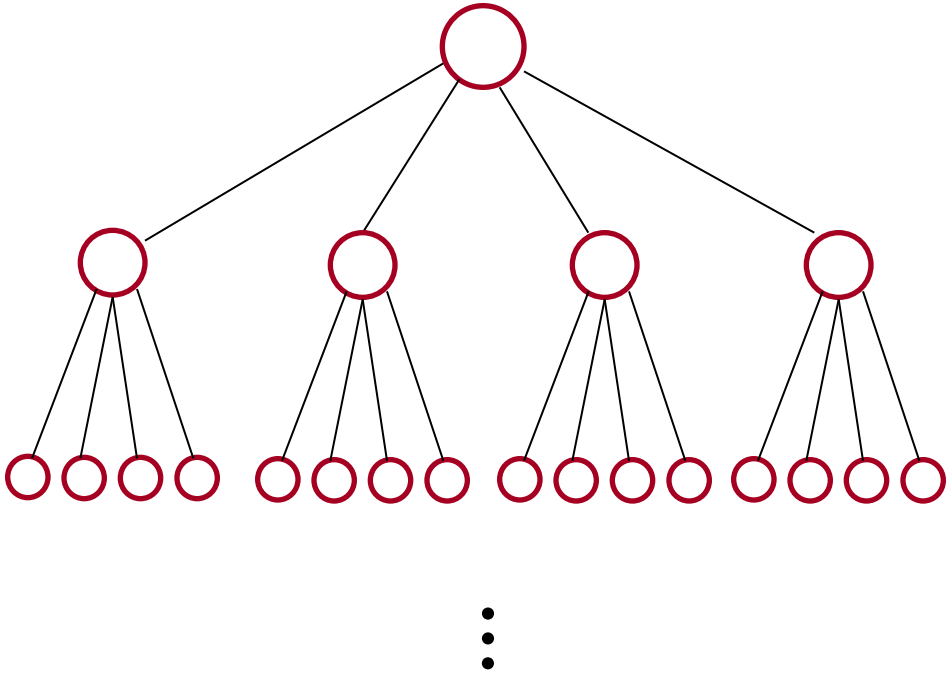
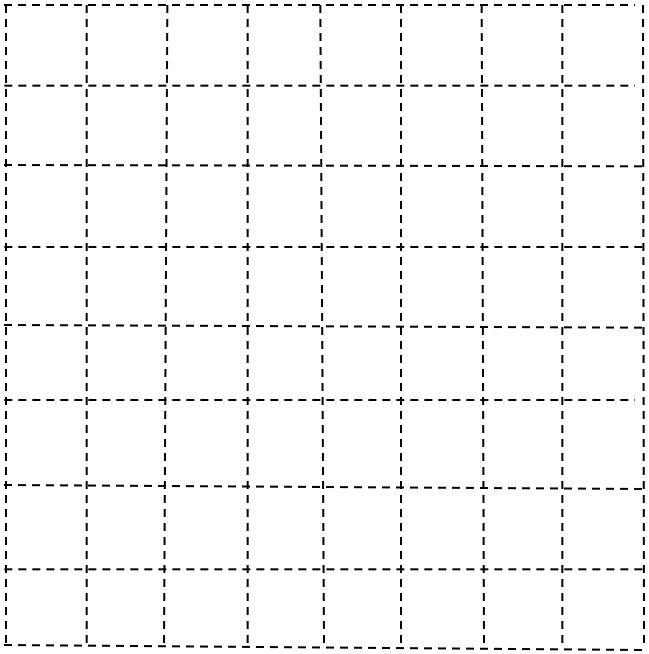
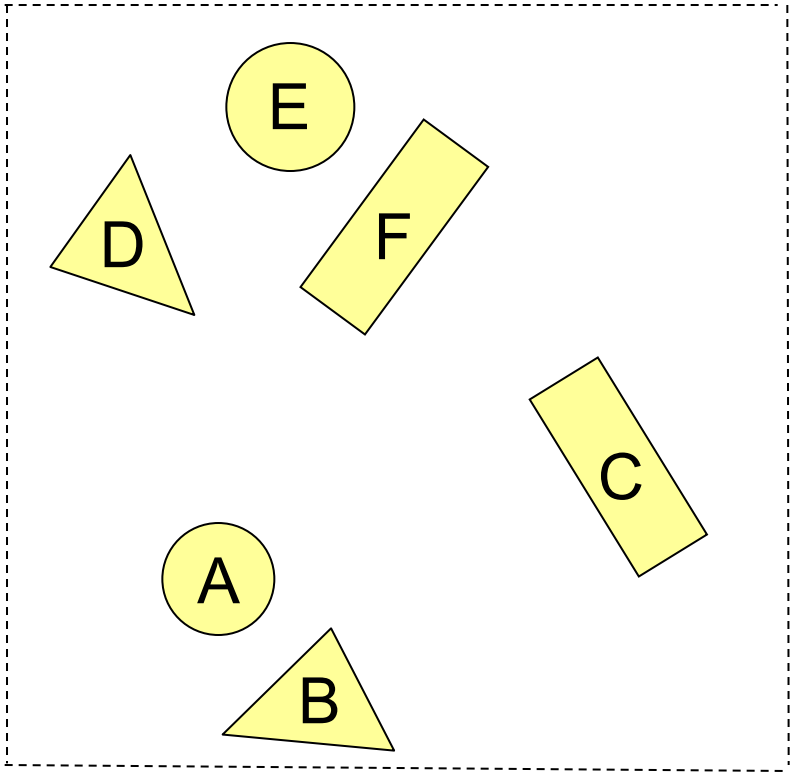




Space Partition: Octree

In an octree, we subdivide adaptively (e.g. only refining cells containing more than one shape).



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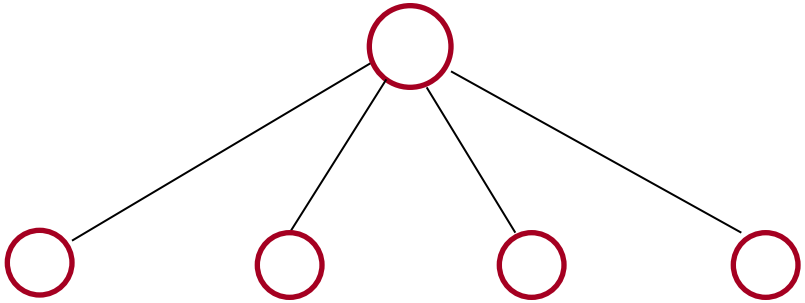

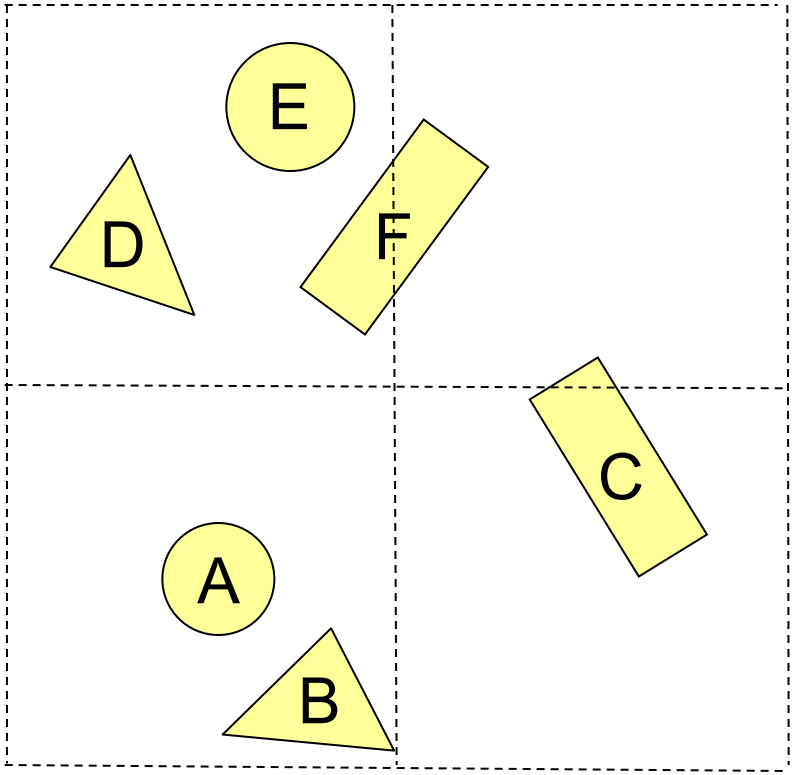

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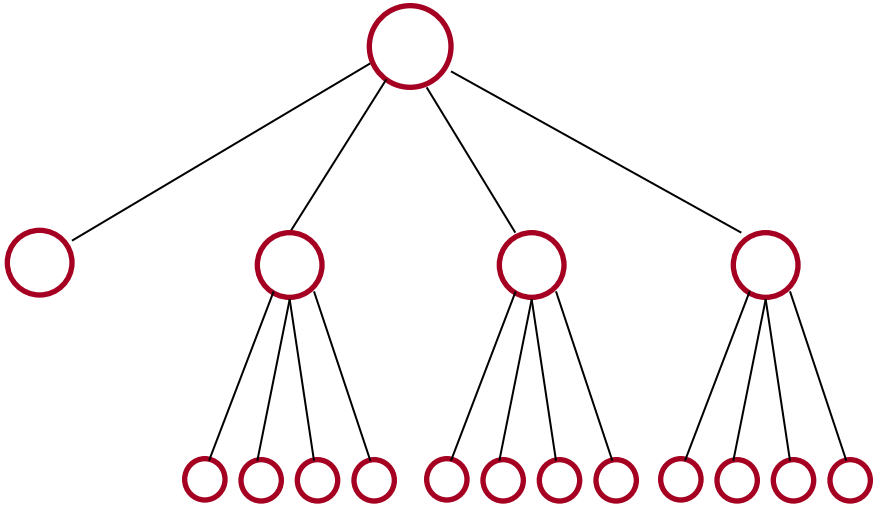

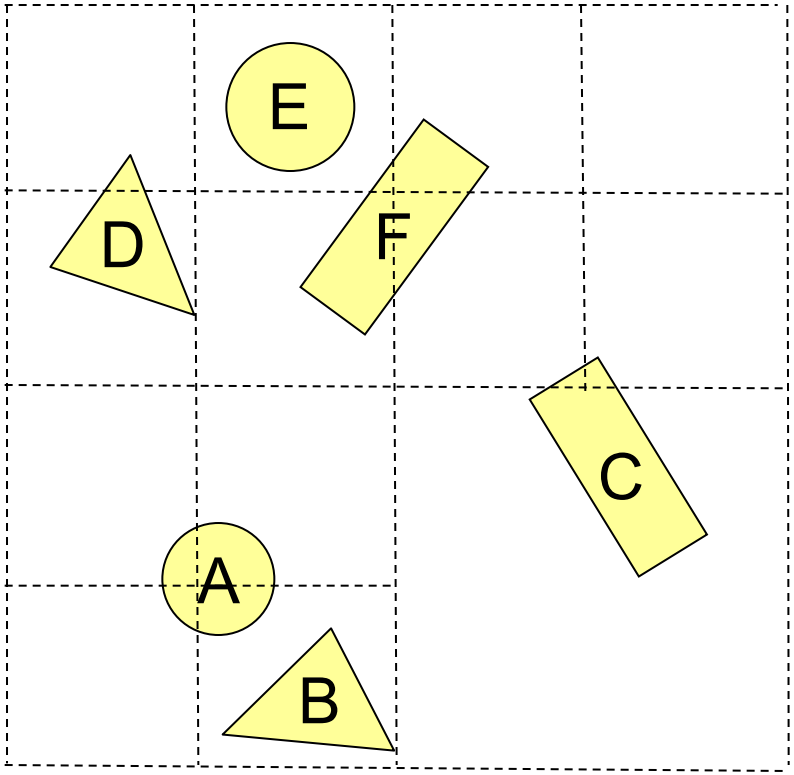

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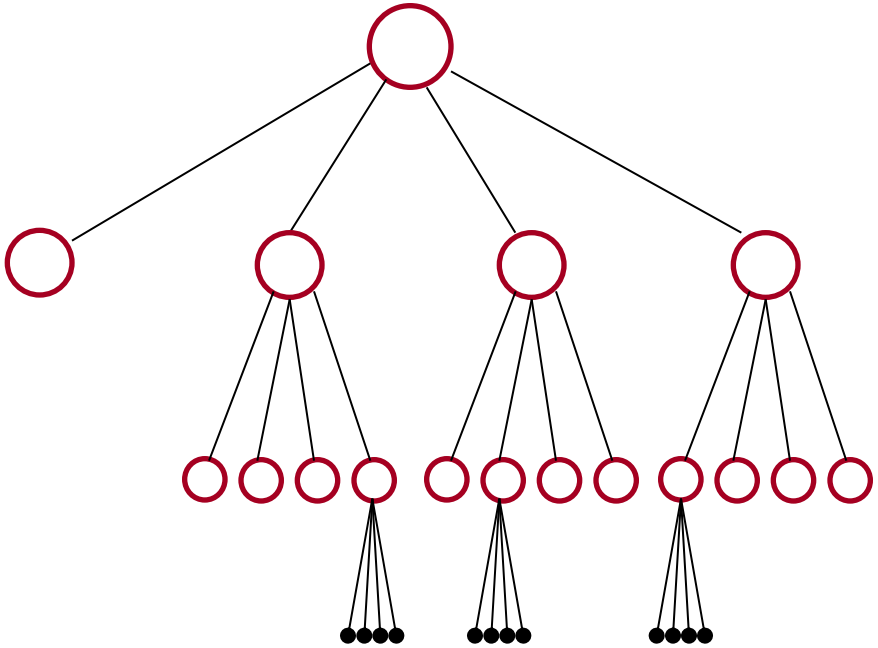

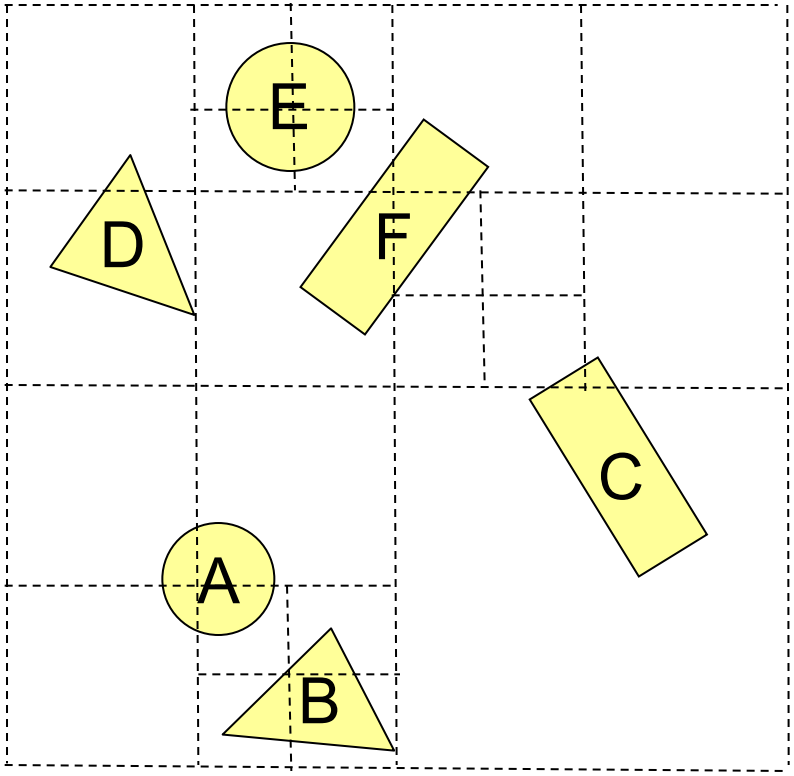

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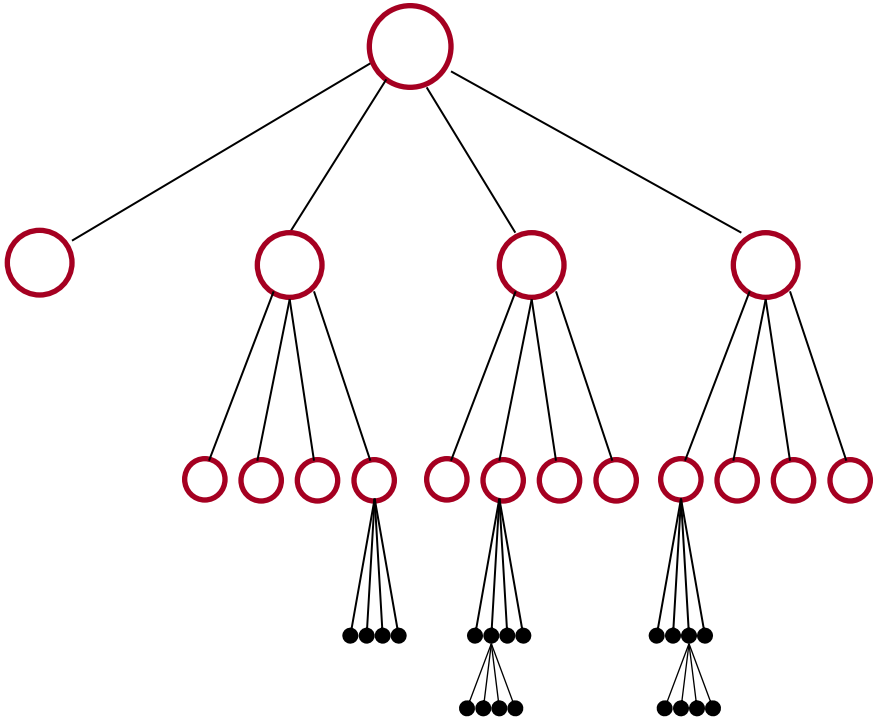

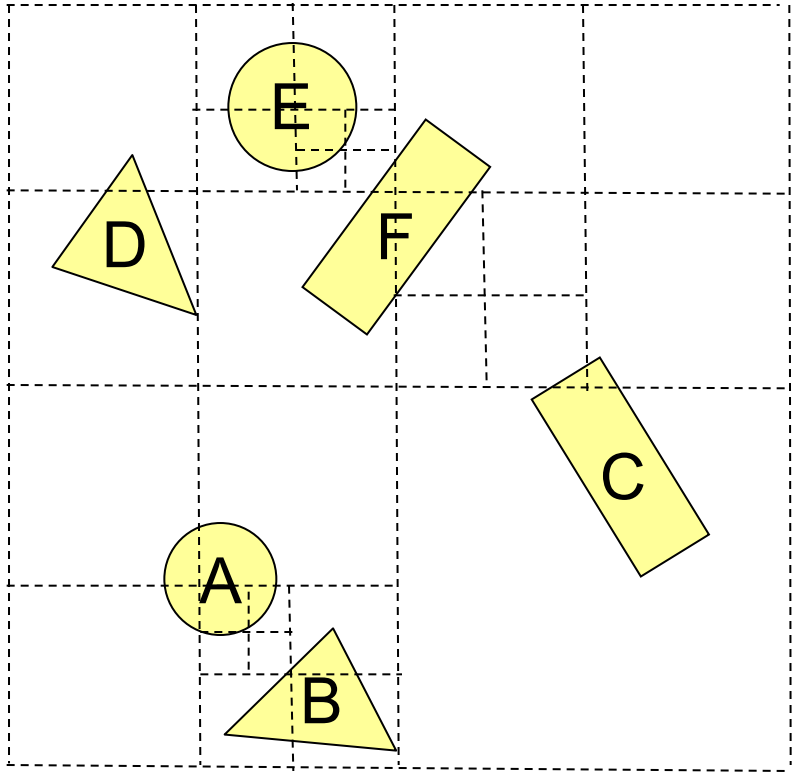

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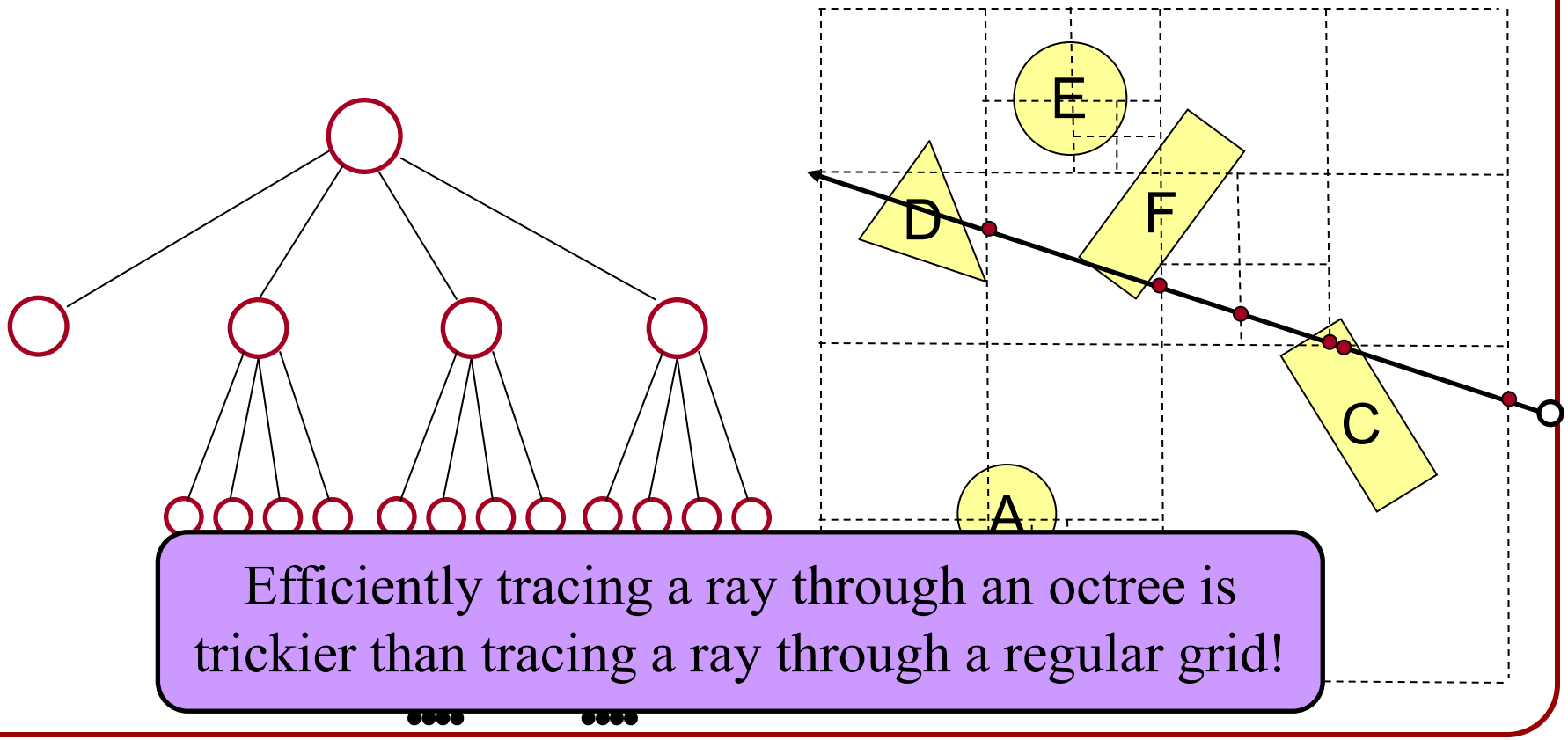

Efficiently tracing a ray through an octree is trickier than tracing a ray through a regular grid!

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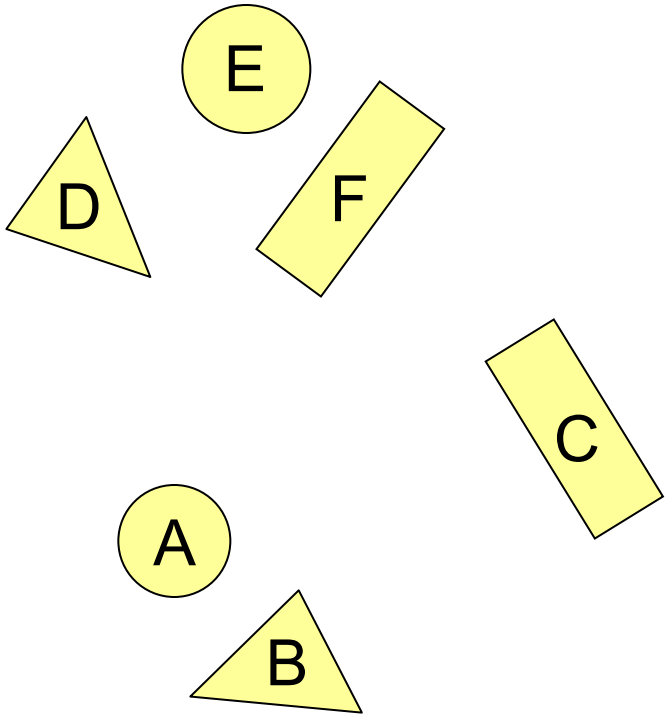
  - » Uniform (Voxel) grid
  - » Octree
  - » Binary space partition (BSP) tree
    - – k-D tree

Space Partition: k-D Trees

Alternate between splitting along the coordinate axes until (at most) one shape per region.



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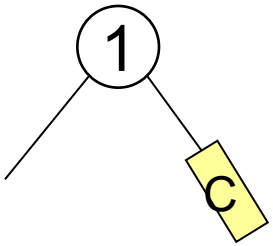

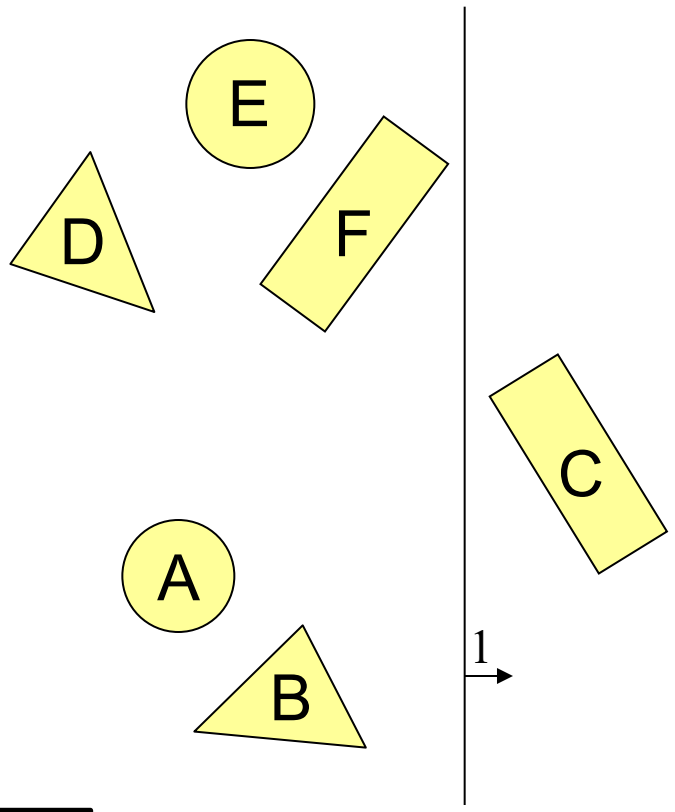

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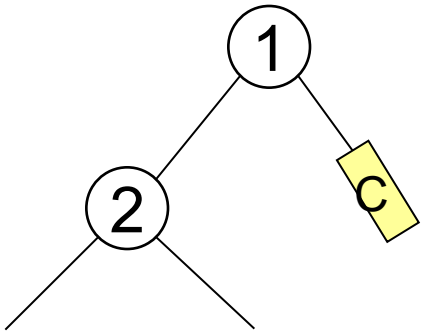

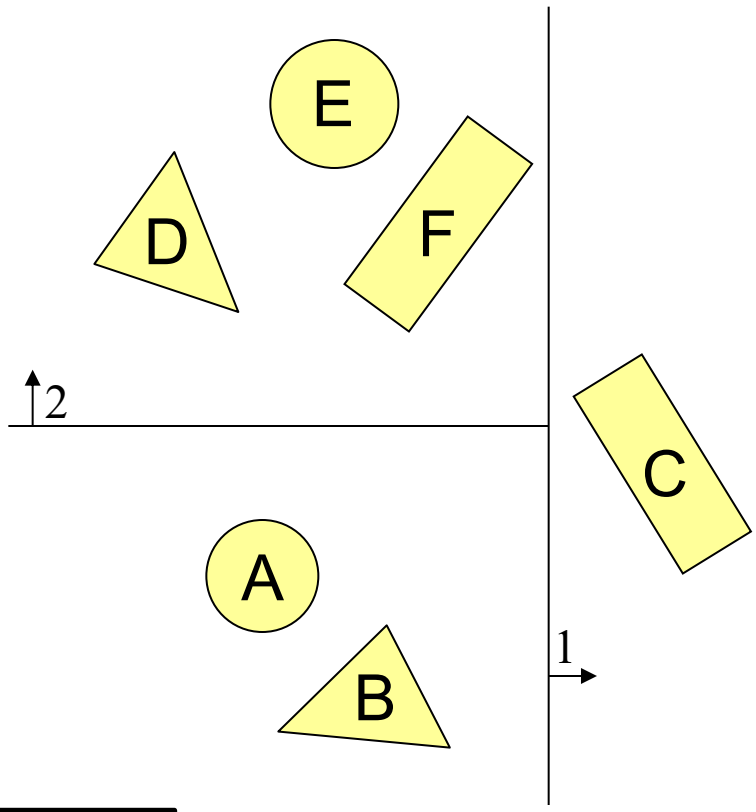

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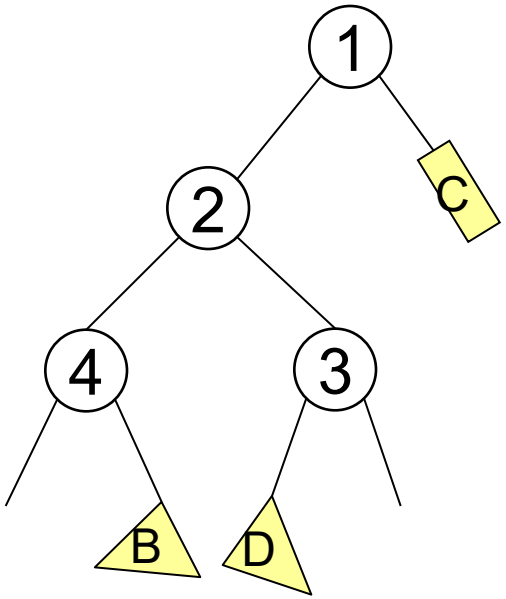

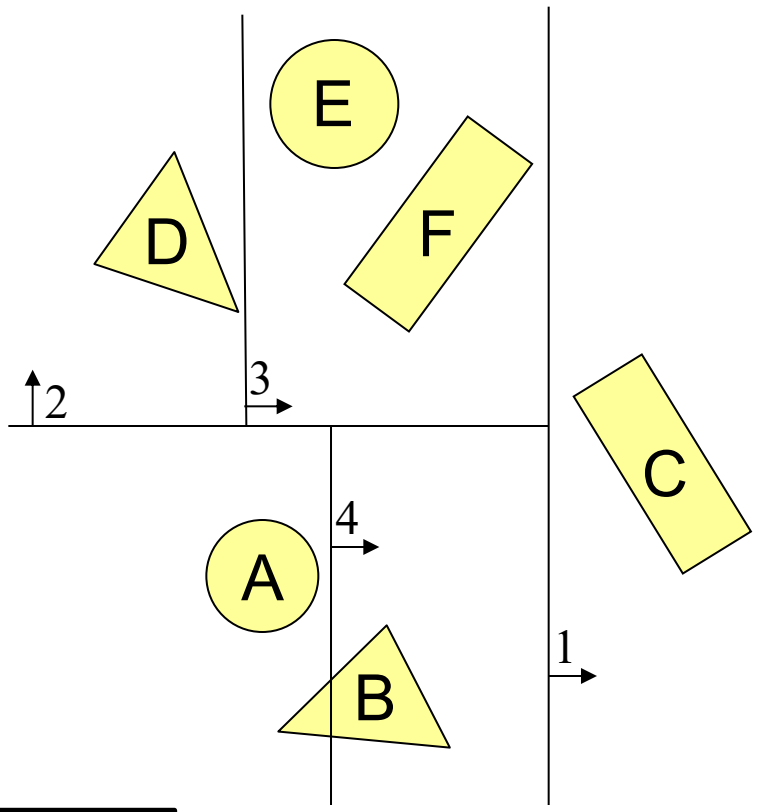

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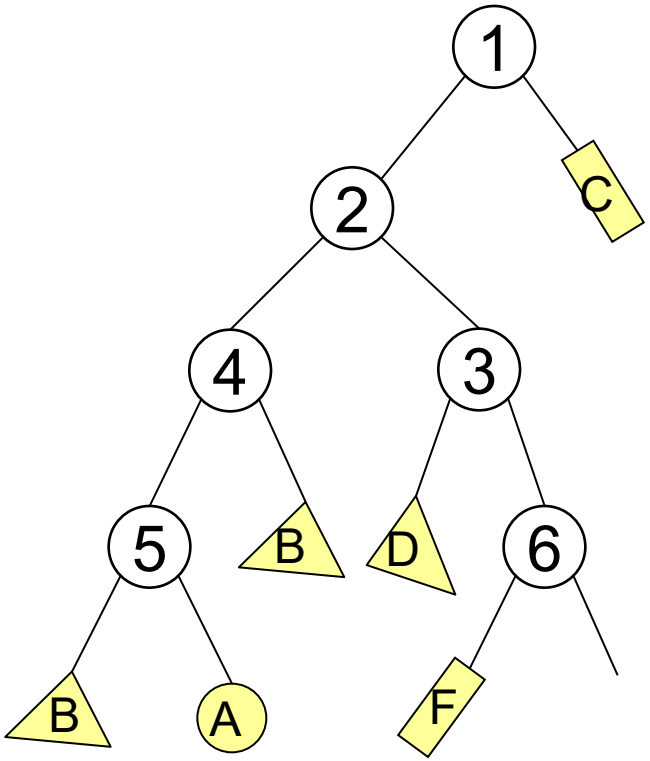

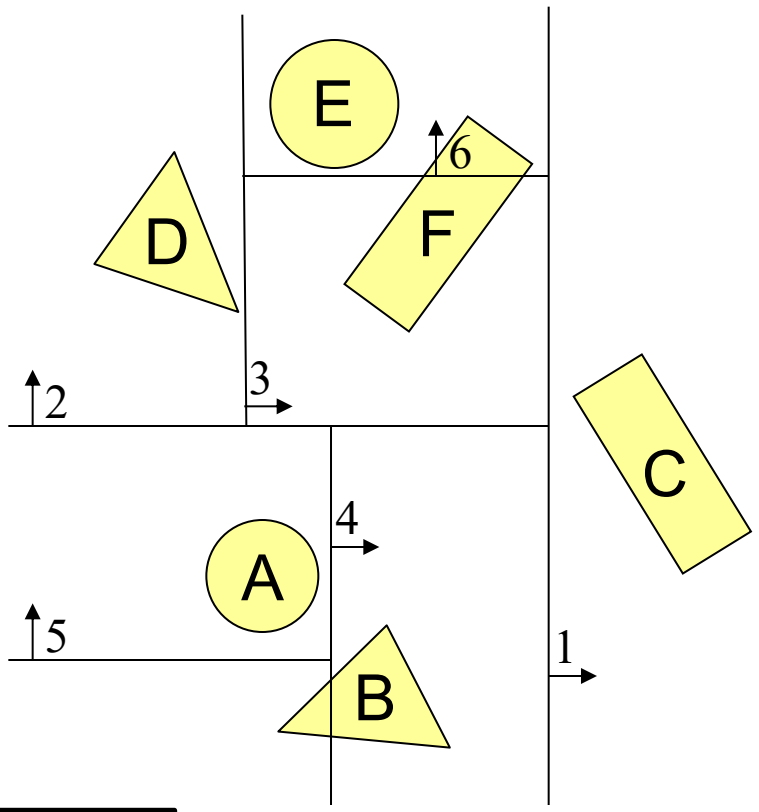

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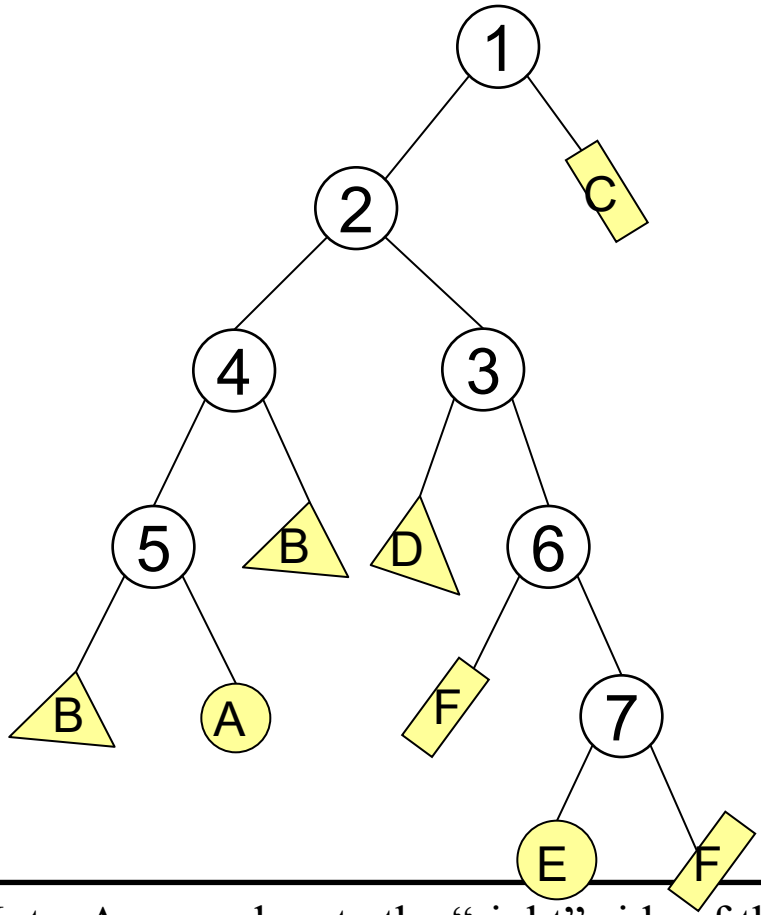

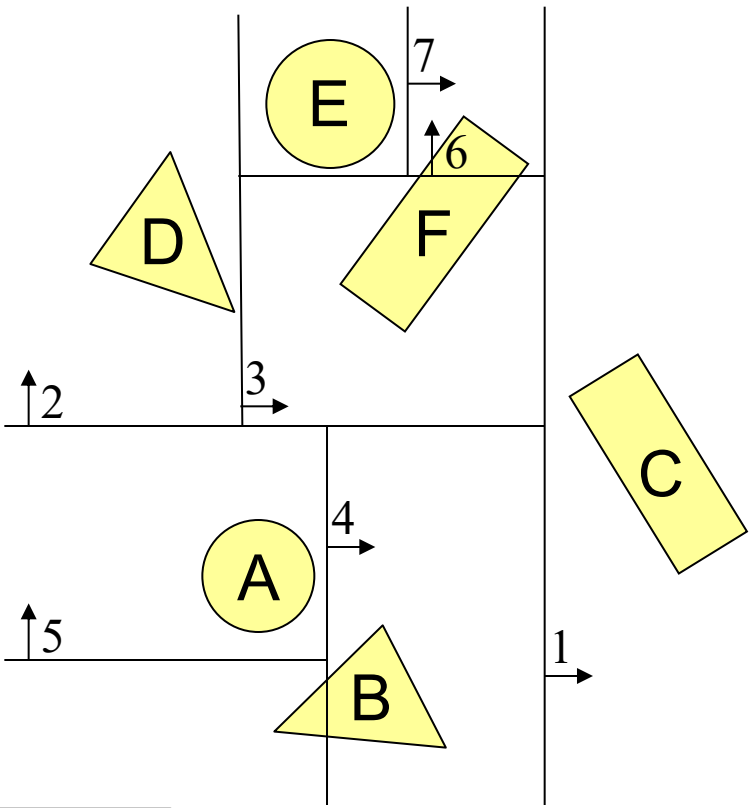

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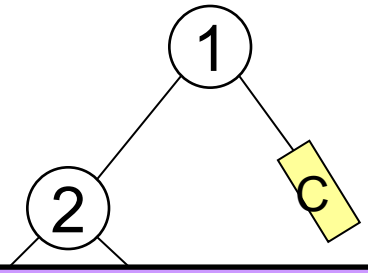
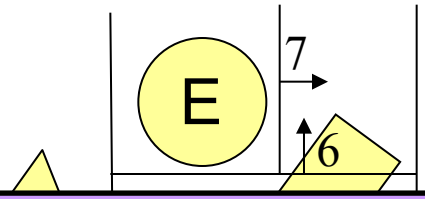
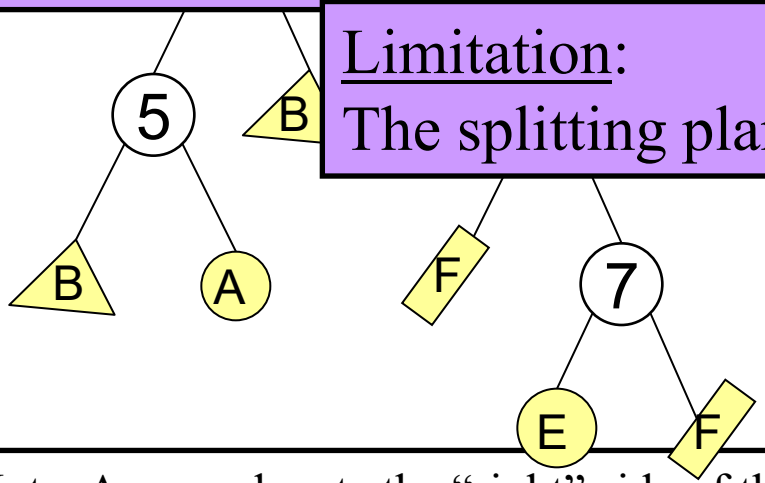
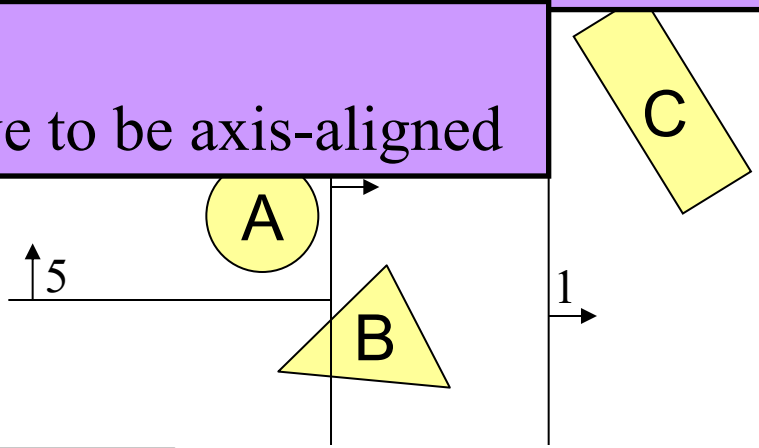




Note:
Either primitives need to be split, or they can belong to multiple nodes.

Limitation:
The splitting planes have to be axis-aligned





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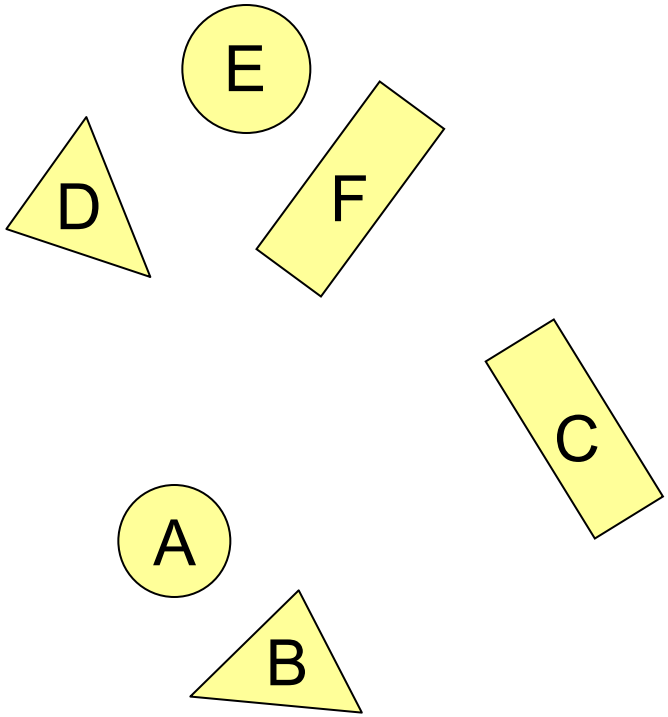
Space Partition: BSP Tree

With a Binary Space Partition (BSP) we recursively partition space by **arbitrarily** aligned planes



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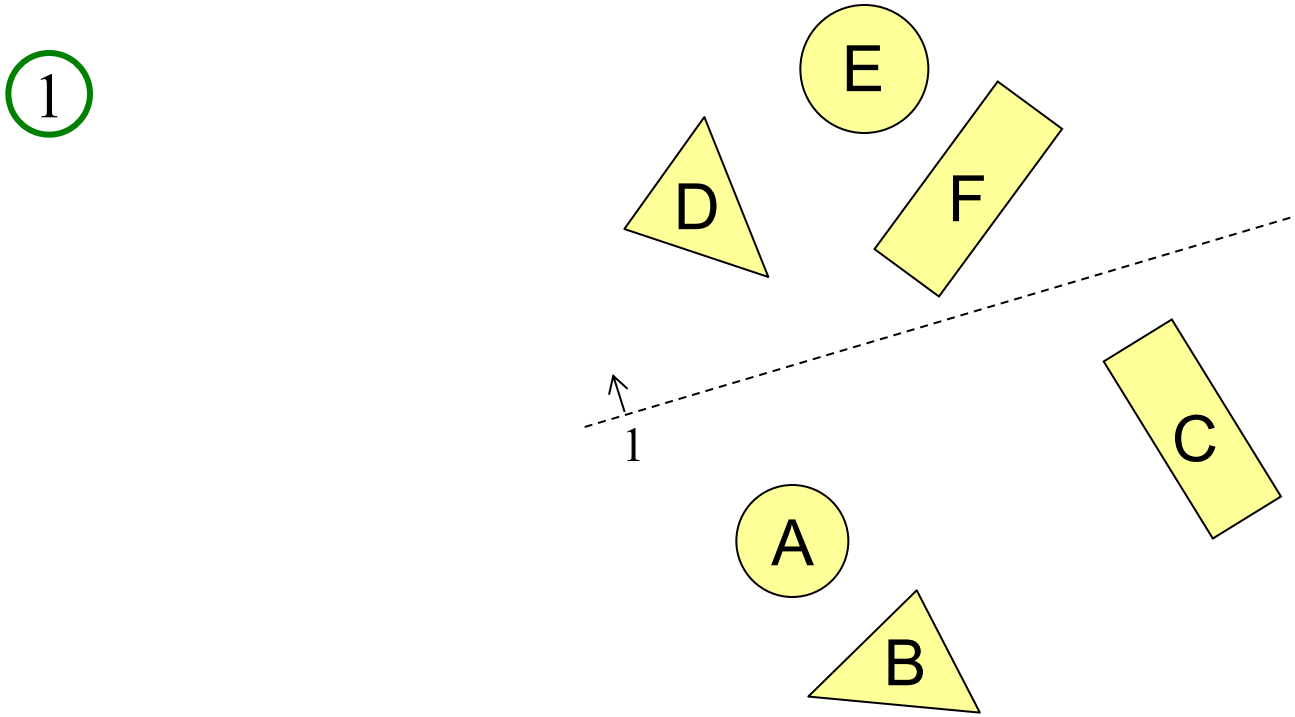
Generate a tree structure where leaves store shapes



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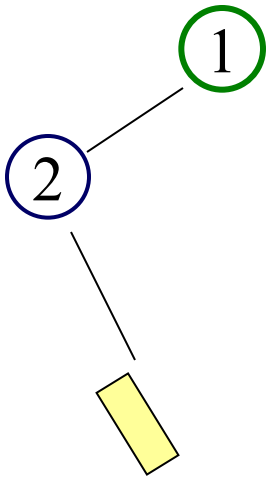

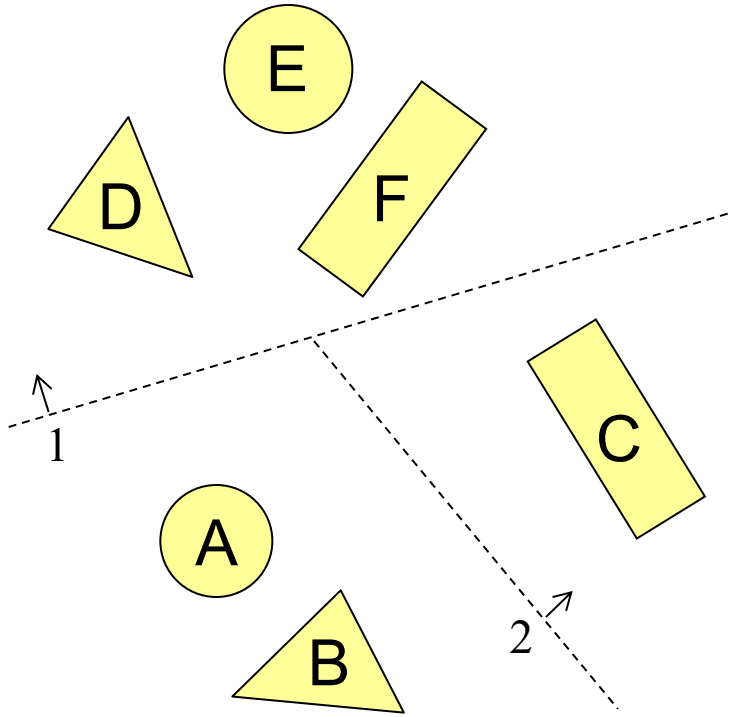

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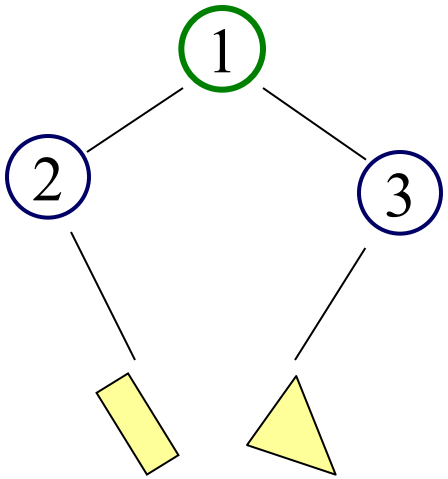

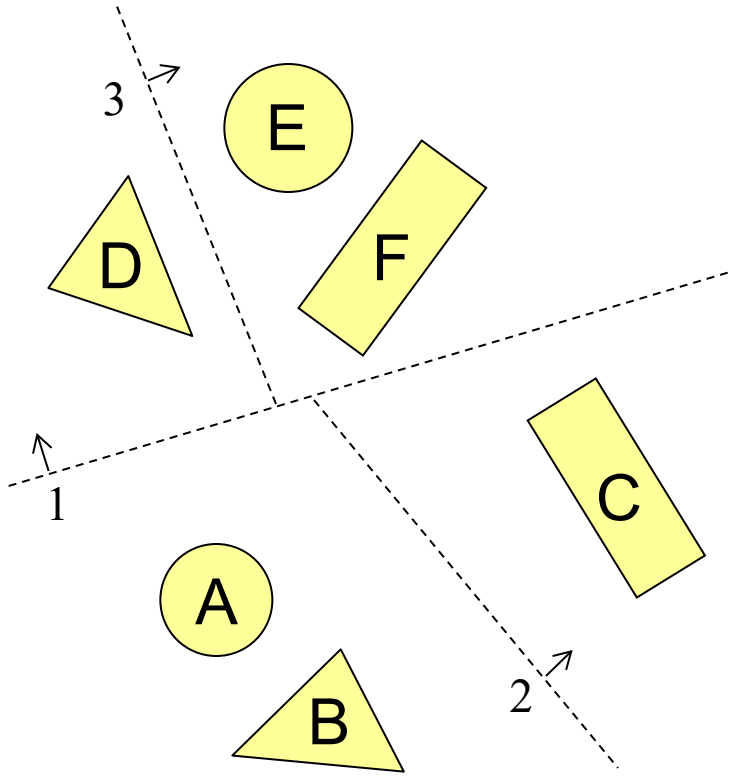

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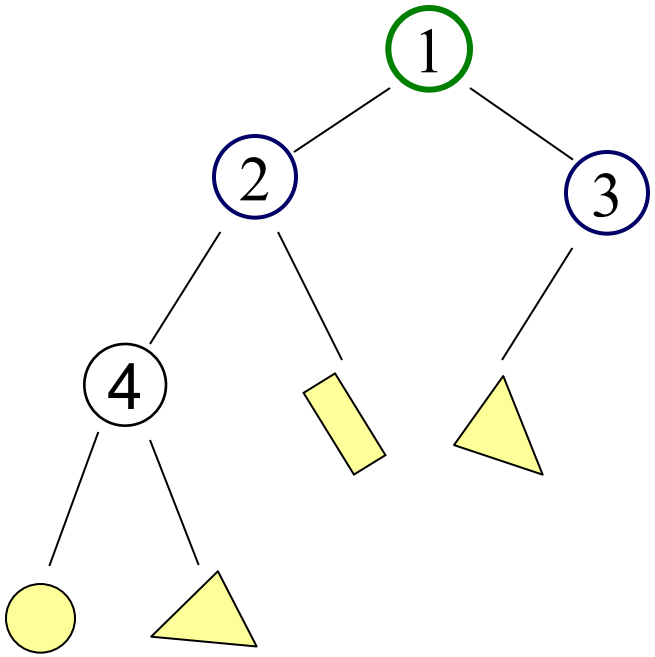

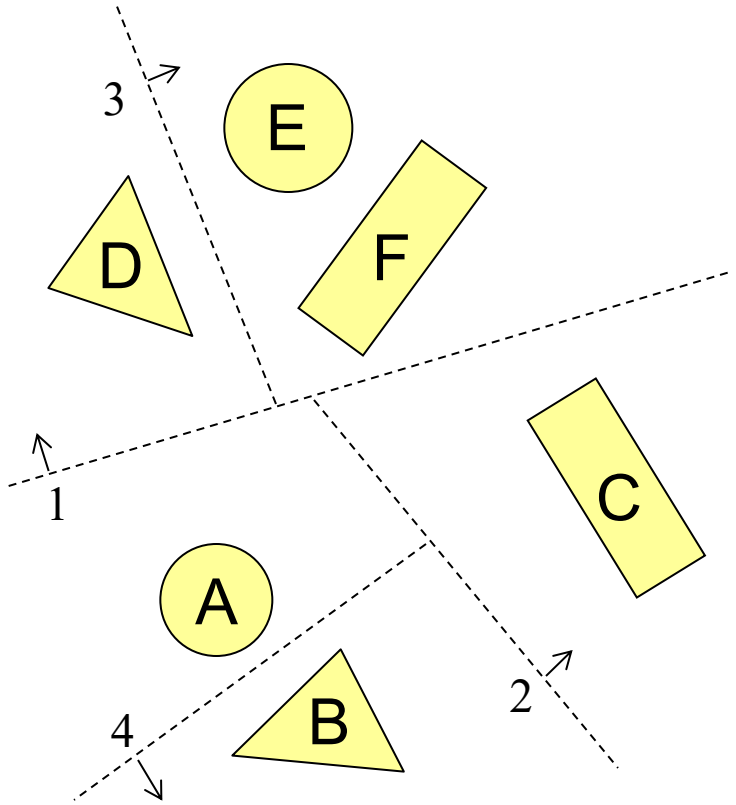

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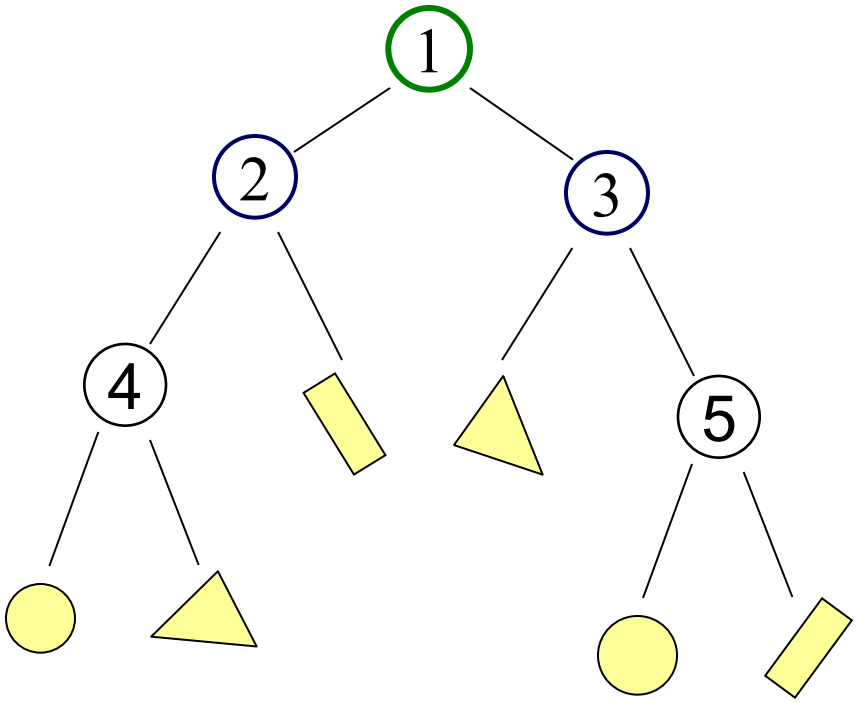

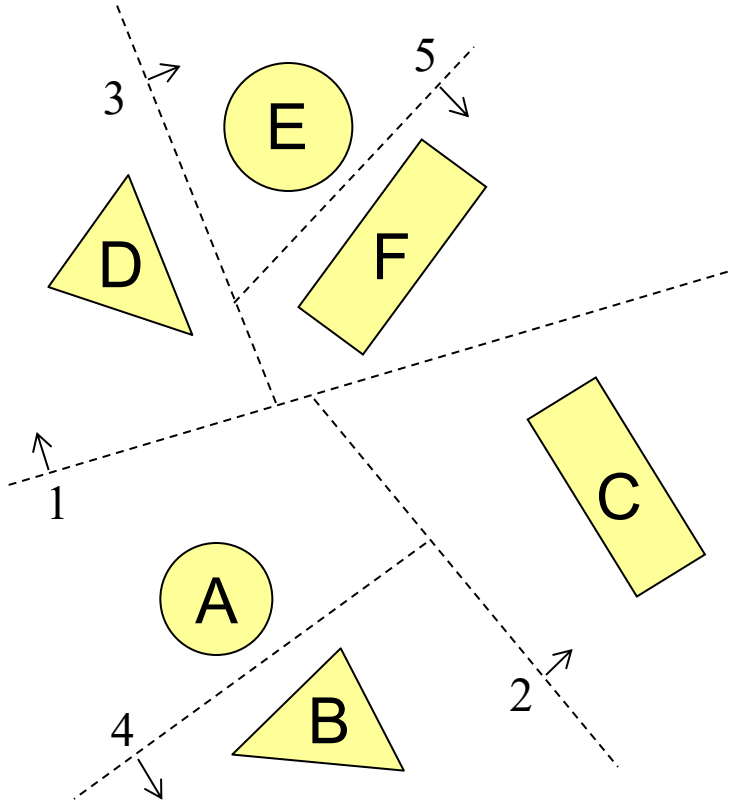

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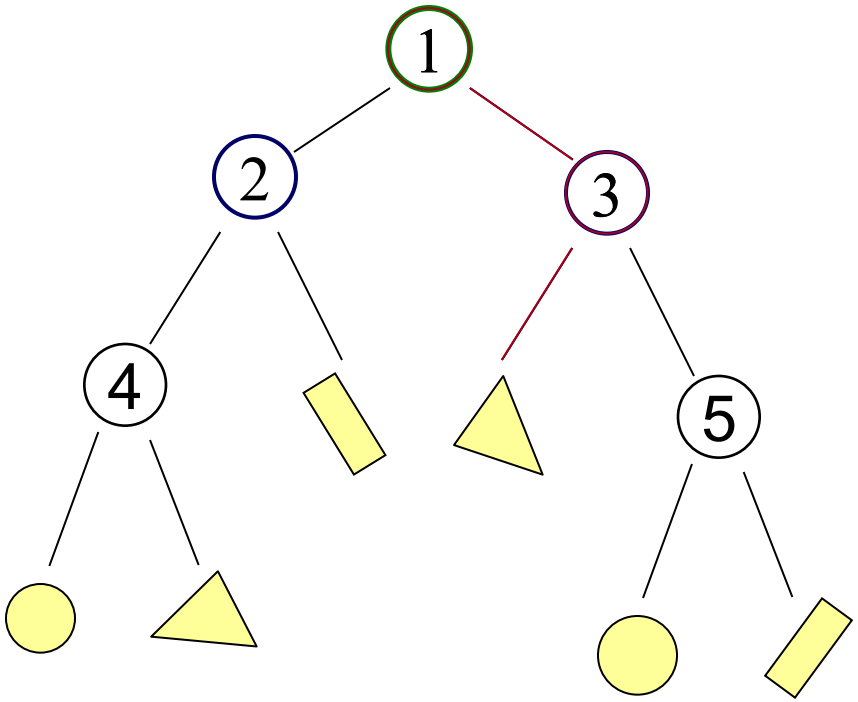
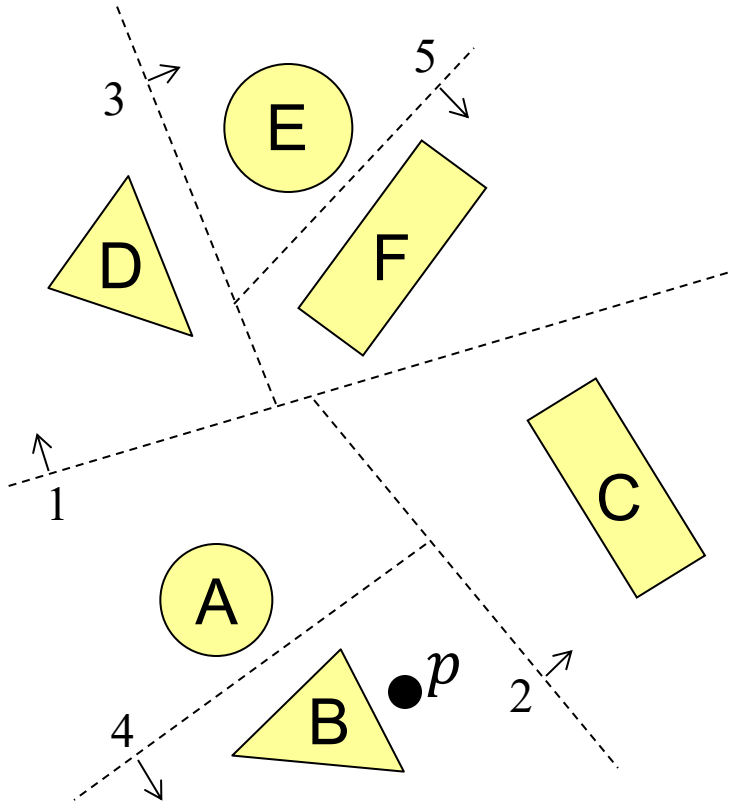
Space Partition: BSP Tree

Example 1: Point intersection query





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Space Partition: BSP Tree

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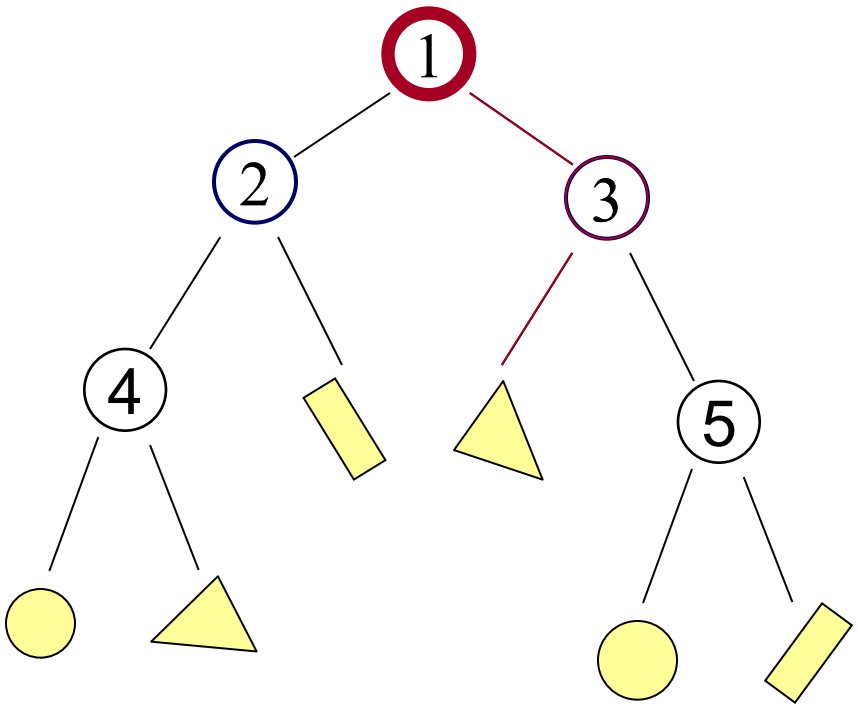

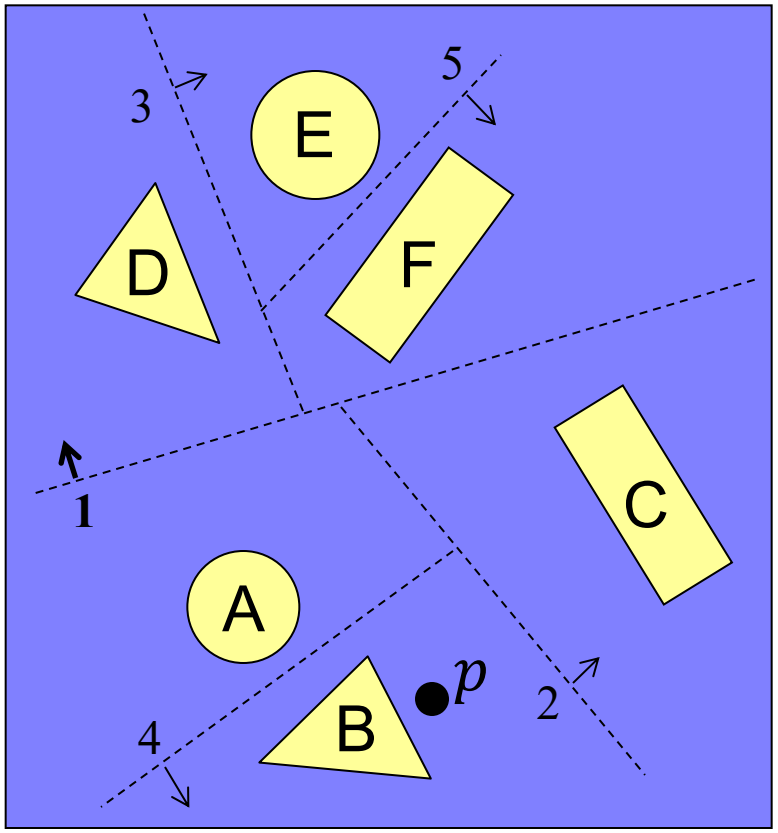

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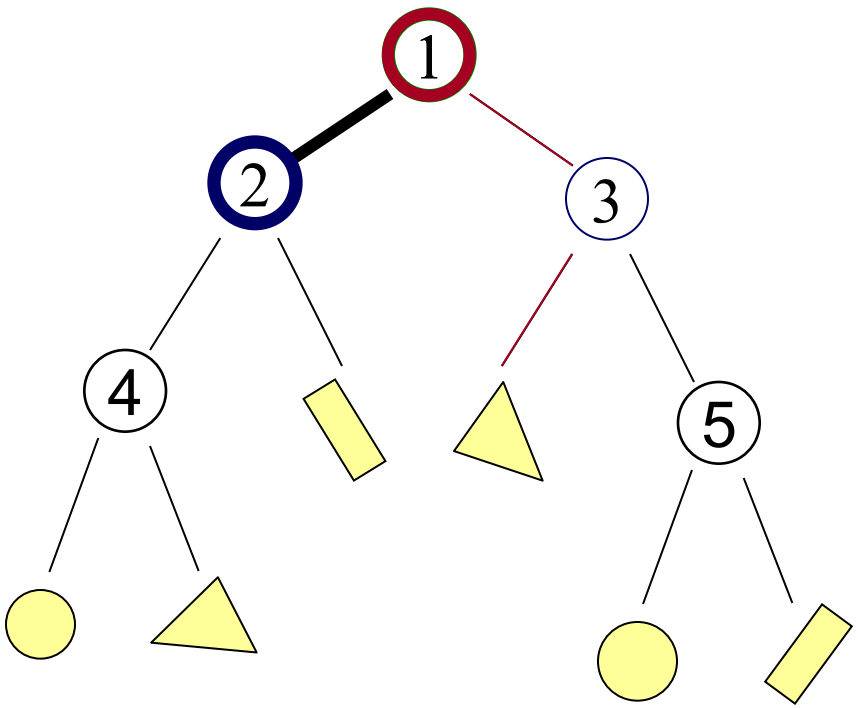
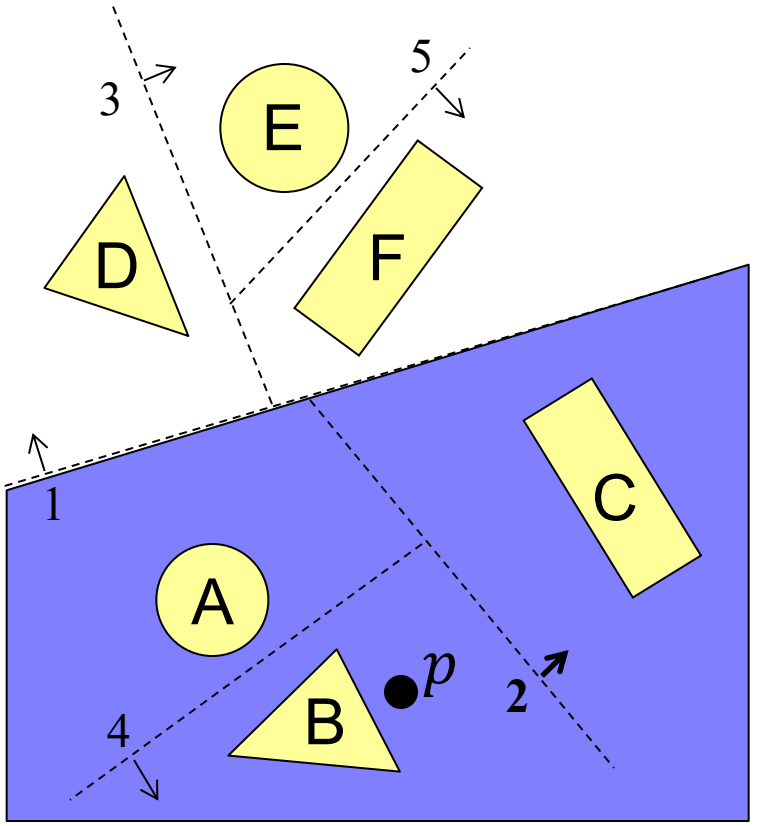
Space Partition: BSP Tree

Example 1: Point intersection query

- Recursively test what side we are on
  - » Left of 1 (root) → 2





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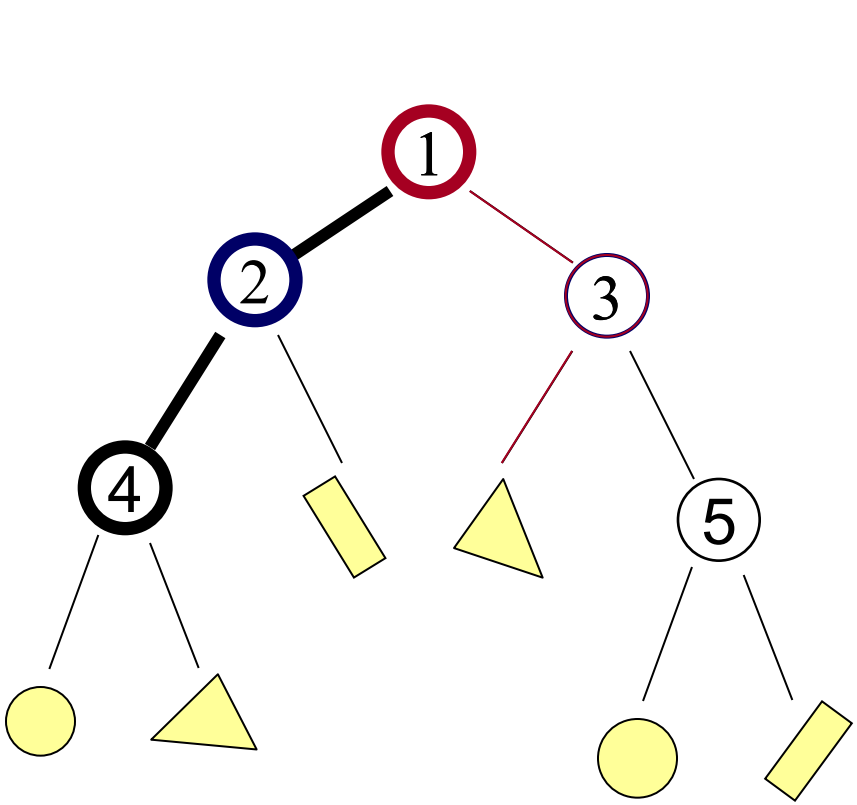
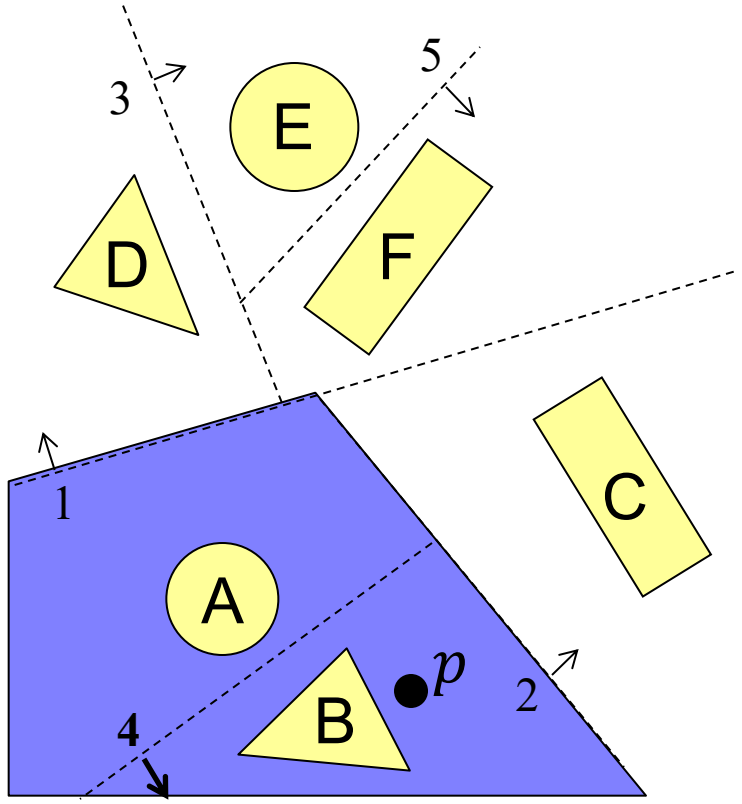
Space Partition: BSP Tree

Example 1: Point intersection query

- Recursively test what side we are on
  - » Left of 2 → 4





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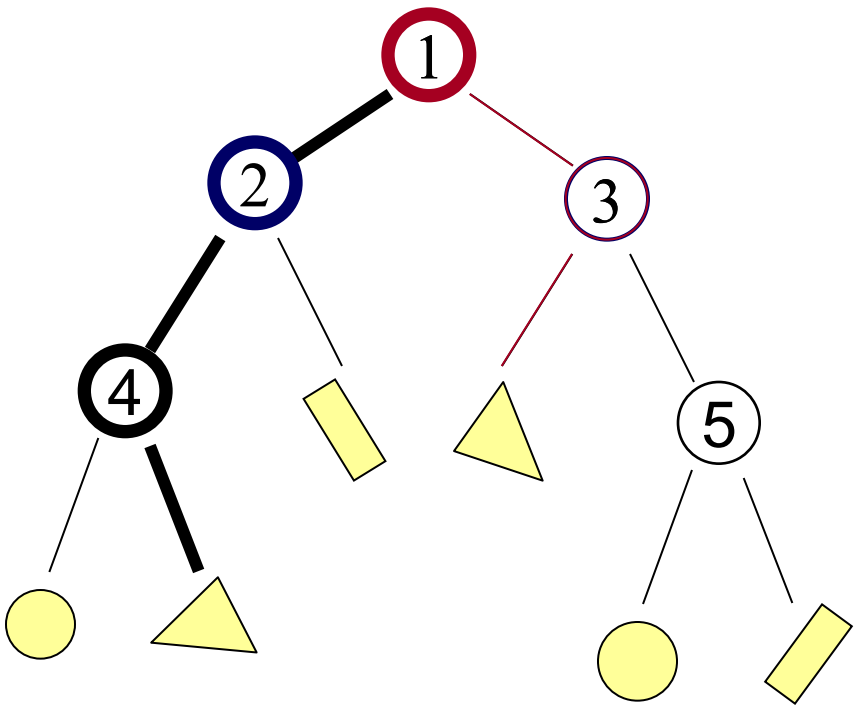
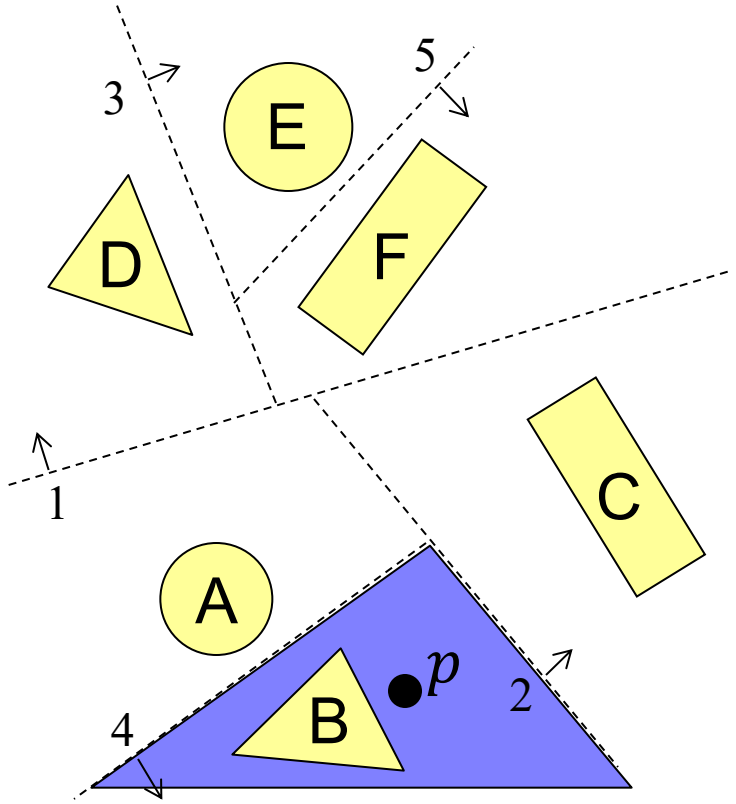
Space Partition: BSP Tree

Example 1: Point intersection query

- Recursively test what side we are on
  - » Right of 4 → Test B





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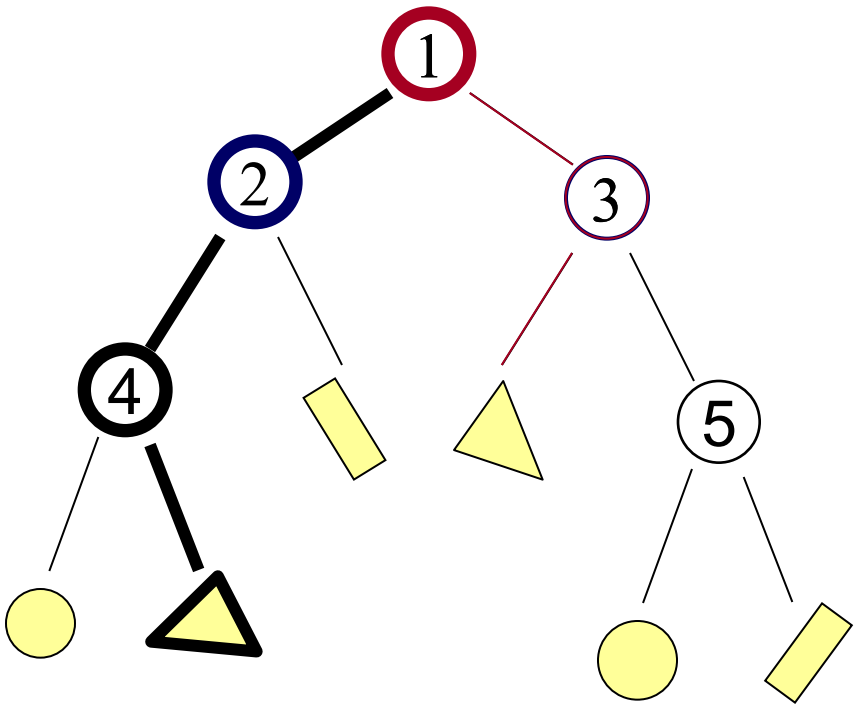
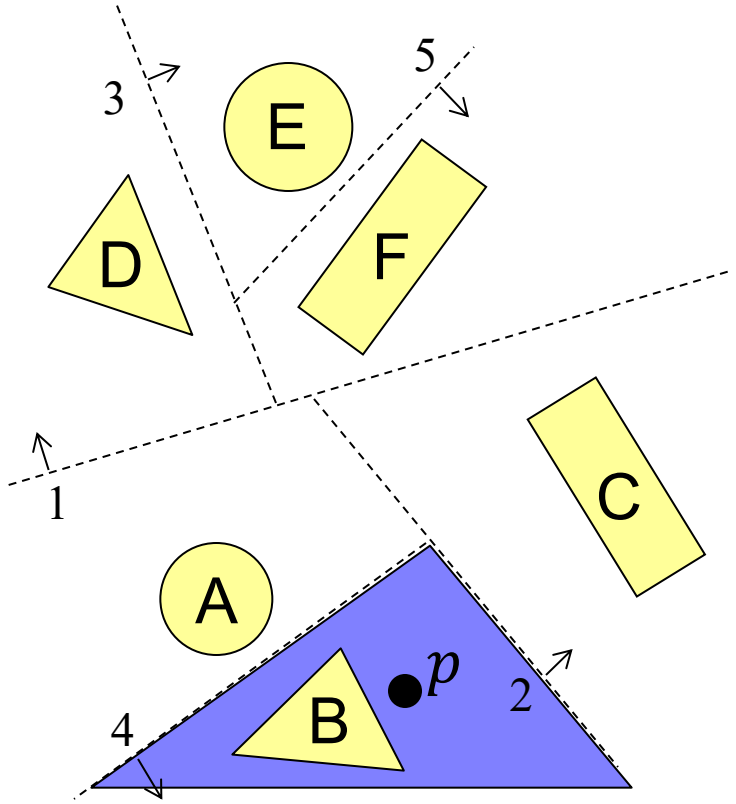
Space Partition: BSP Tree

Example 1: Point intersection query

- Recursively test what side we are on
  - » Missed B. No intersection!





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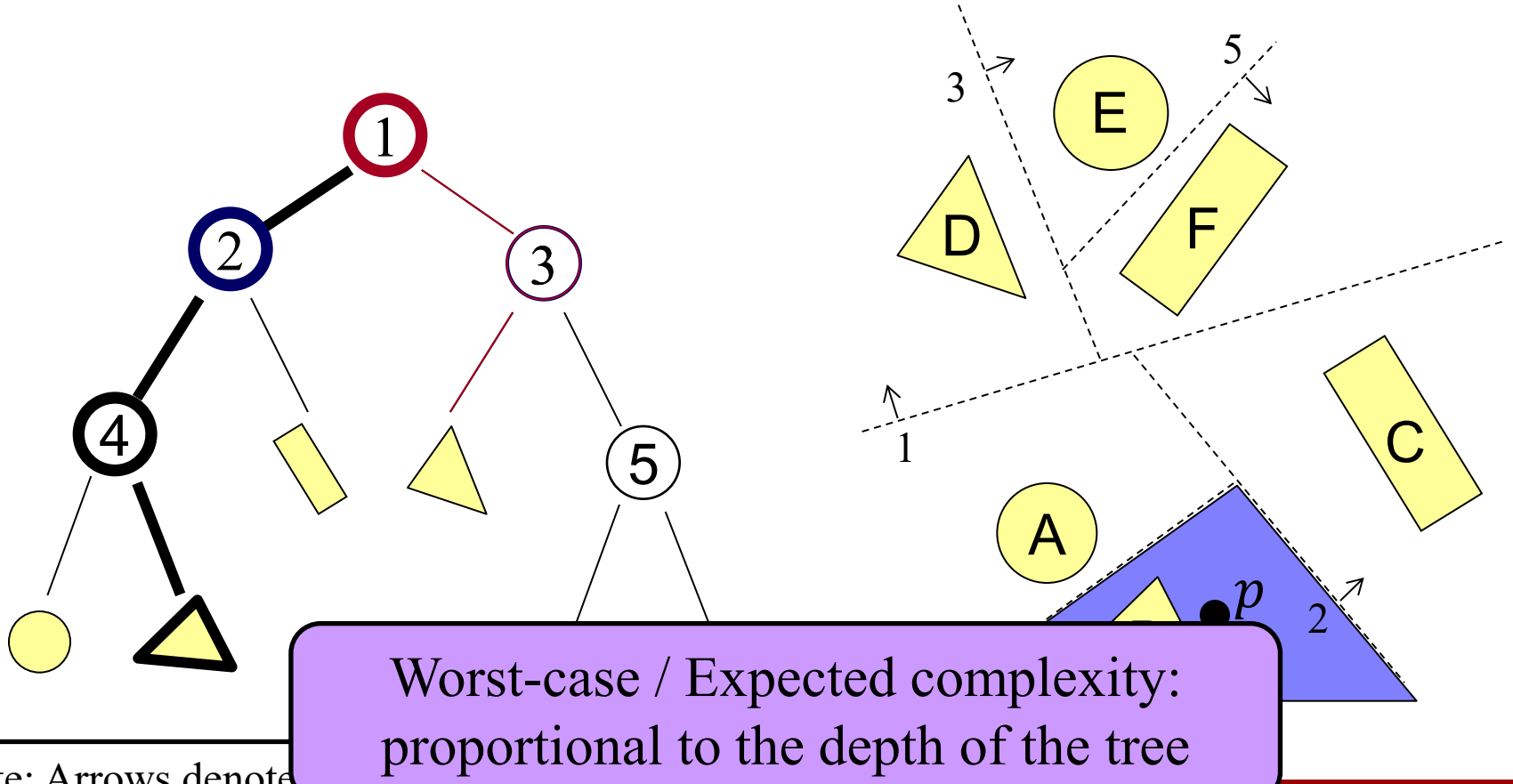


Worst-case / Expected complexity:
proportional to the depth of the tree

Note: Arrows denote

Space Partition: BSP Tree

Observation:

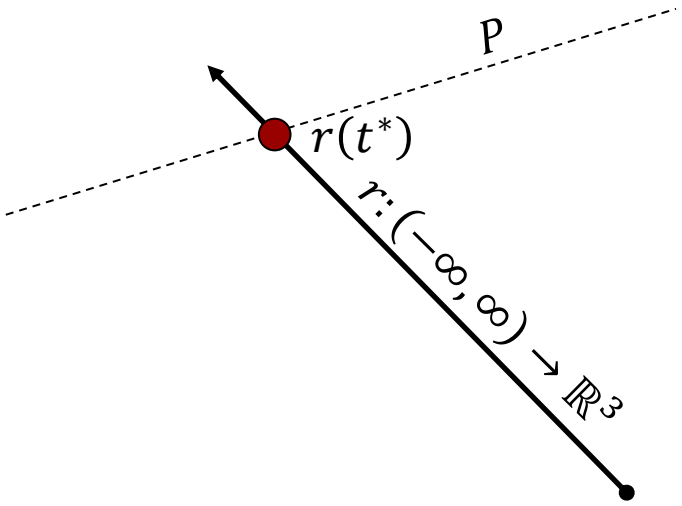
Assume we are given a ray:

$$r: (-\infty, \infty) \rightarrow \mathbb{R}^3$$

and a plane P (assuming not parallel).

- There exists a time $t^* \in (-\infty, \infty)$ at which the ray passes through the plane:

$$r(t^*) \in P.$$



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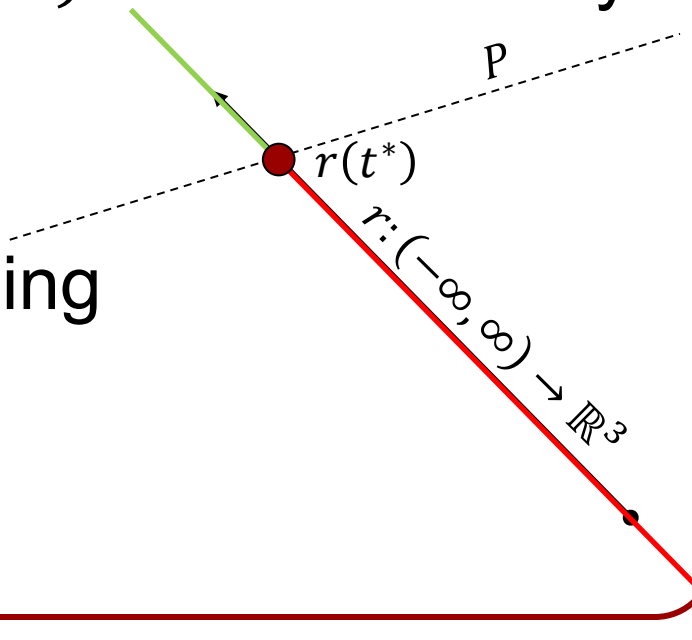
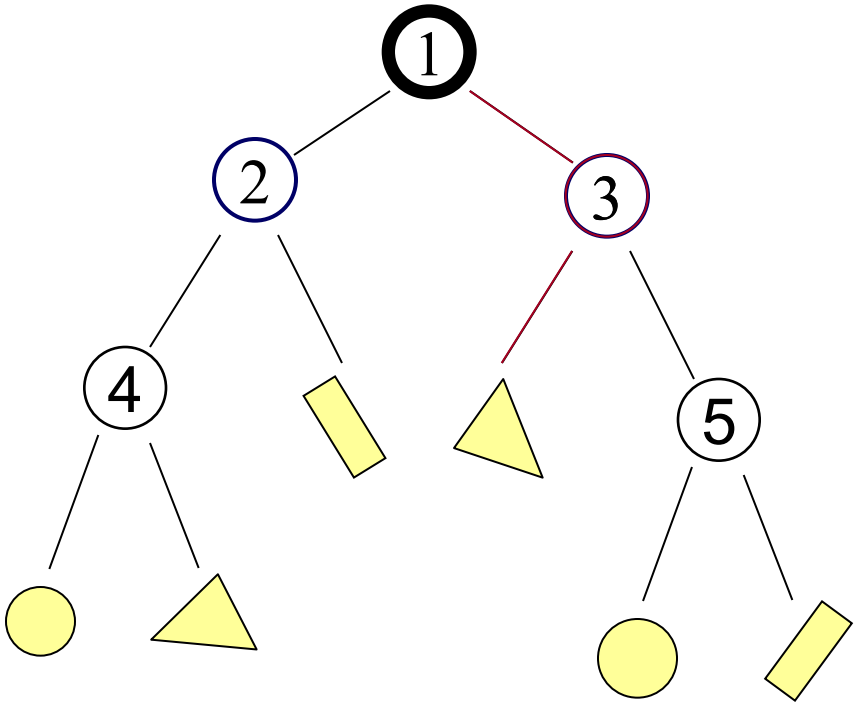


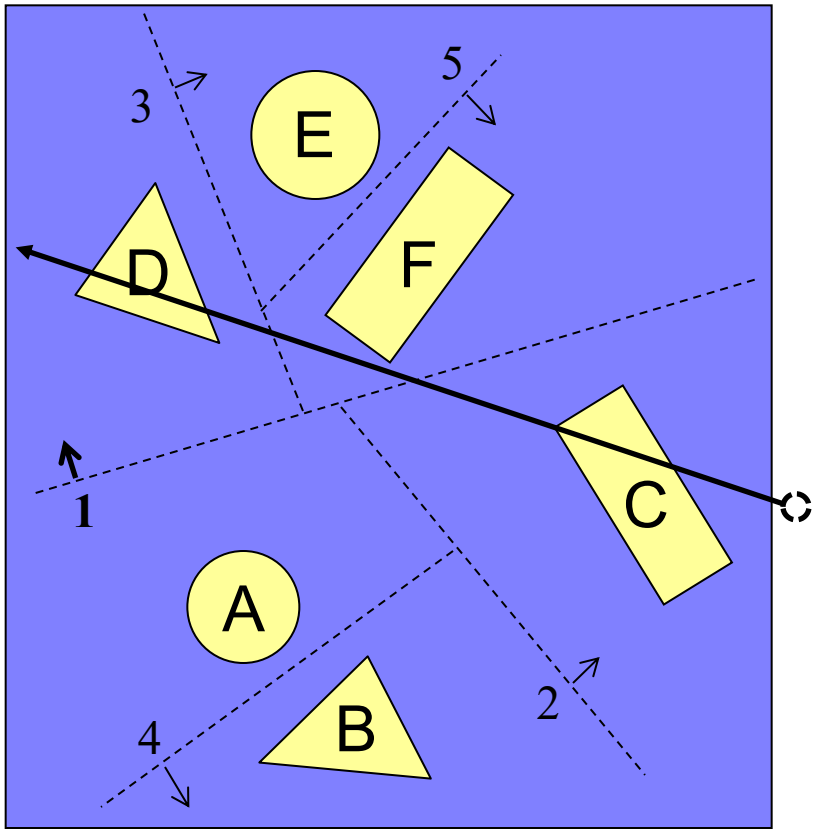
- This partitions the line containing the ray in two parts:
  - Back: $r(t)$ with $t \in (-\infty, t^*)$
  - Front: $r(t)$ with $t \in (t^*, \infty)$

Space Partition: BSP Tree

Example 2: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:





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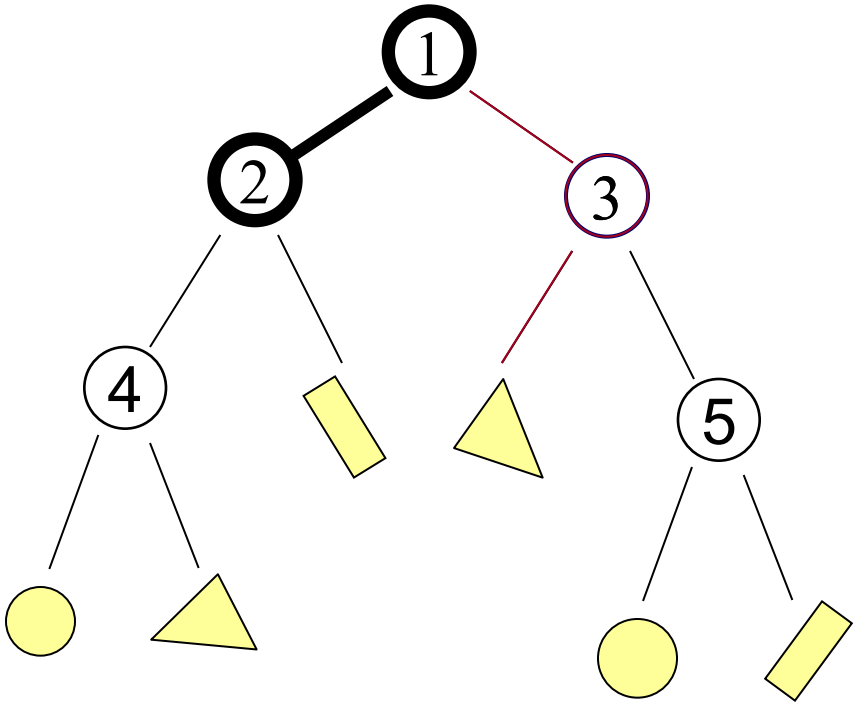
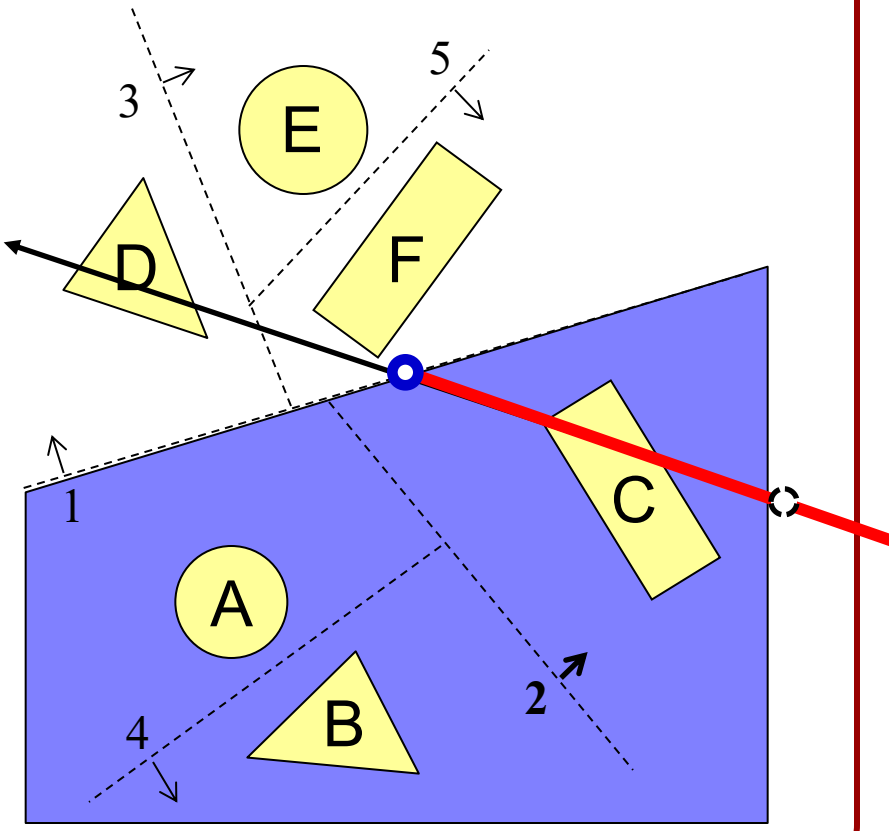
Space Partition: BSP Tree

Example 2: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Test back (left)





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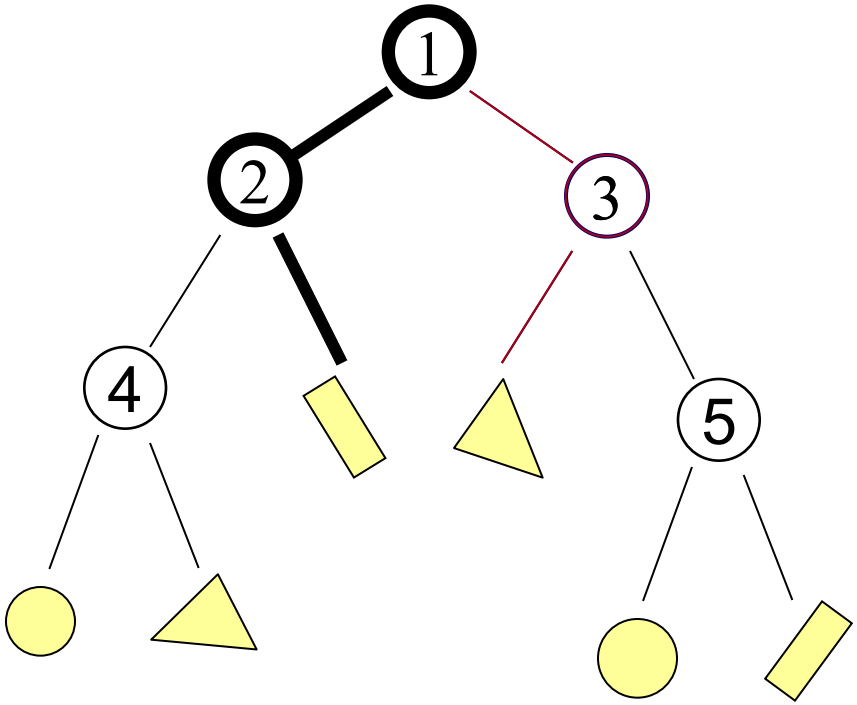
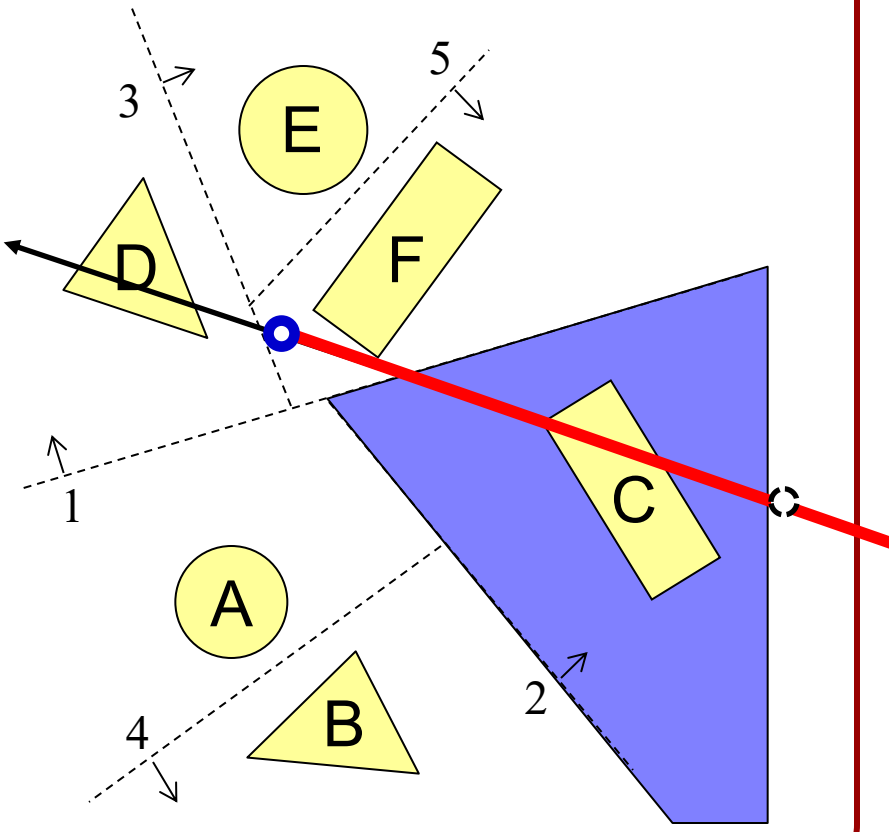
Space Partition: BSP Tree

Example 2: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Test back (right)





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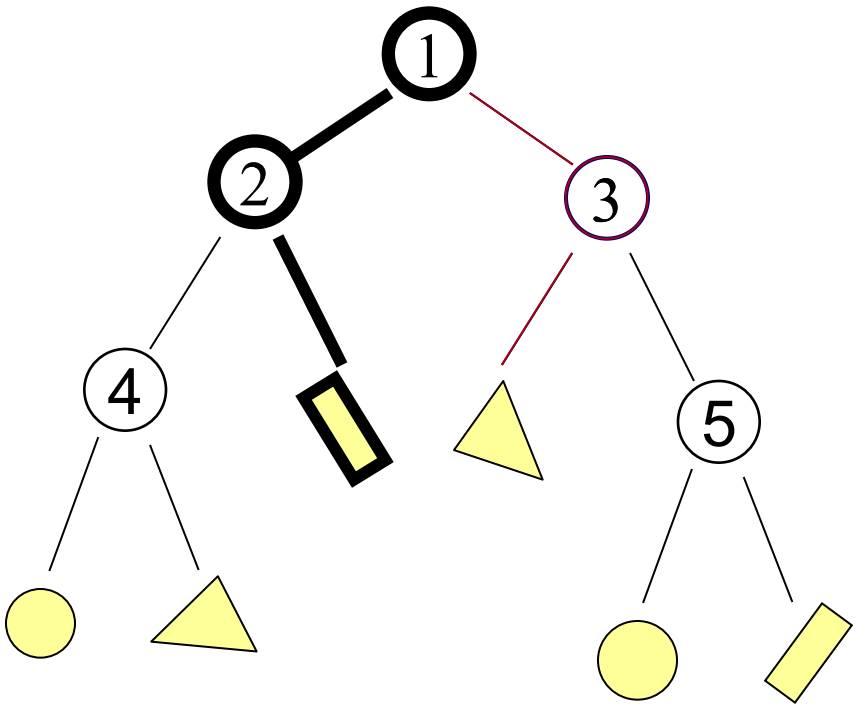
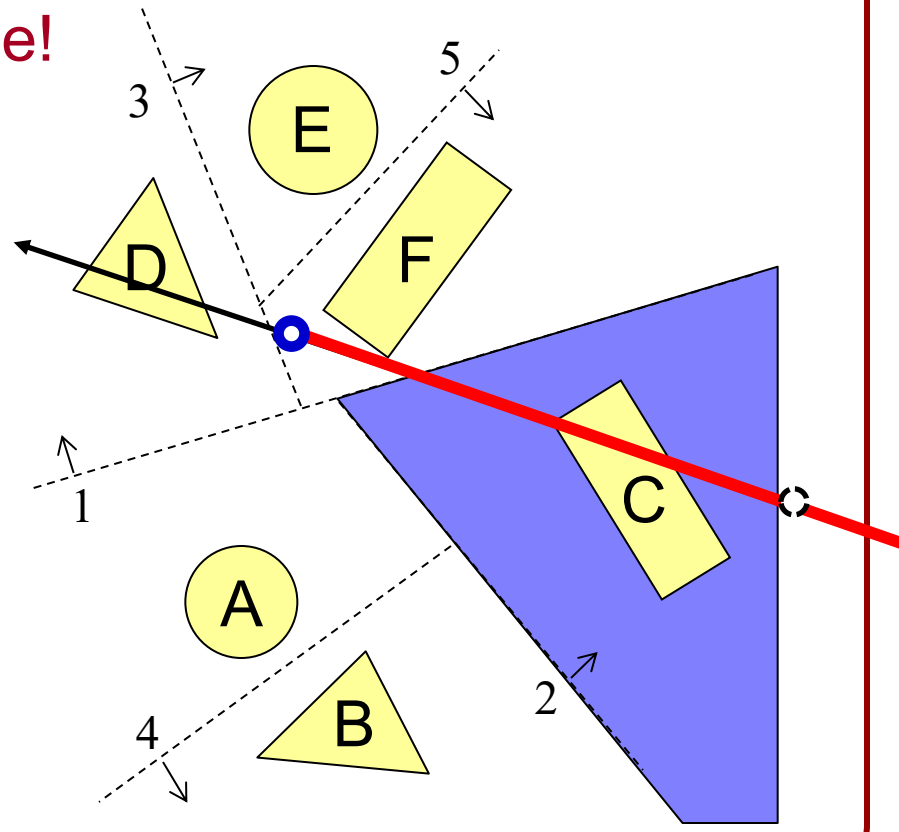
Space Partition: BSP Tree

Example 2: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Intersection with C. Done!





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Space Partition: BSP Tree

Example 3: Ray intersection query

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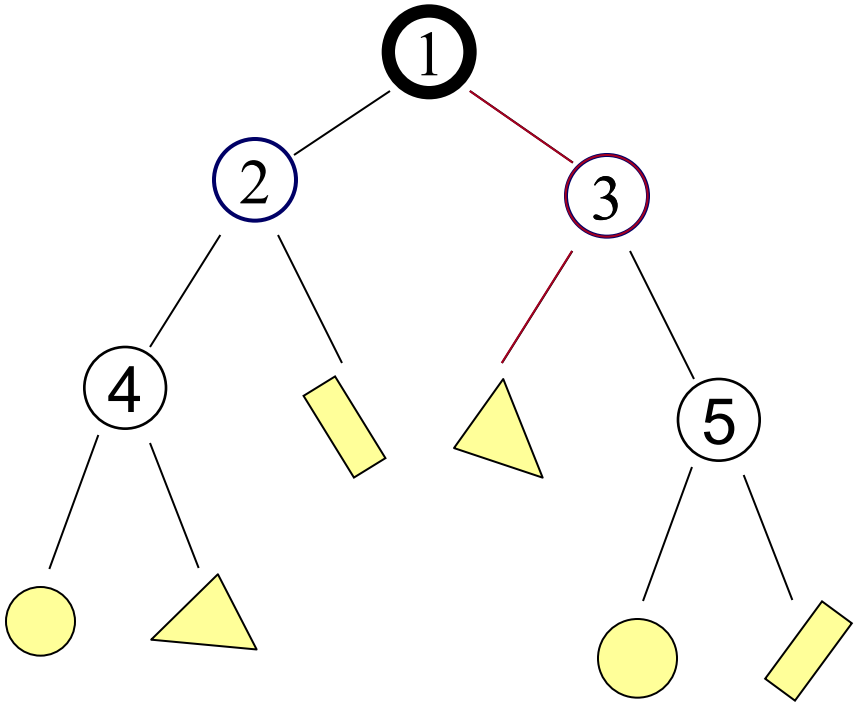

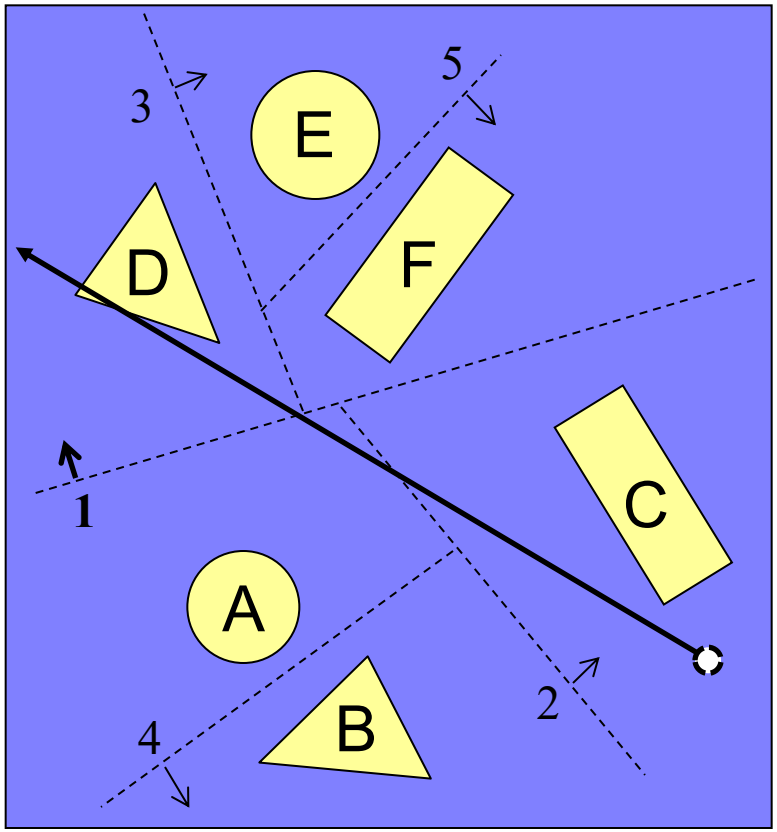

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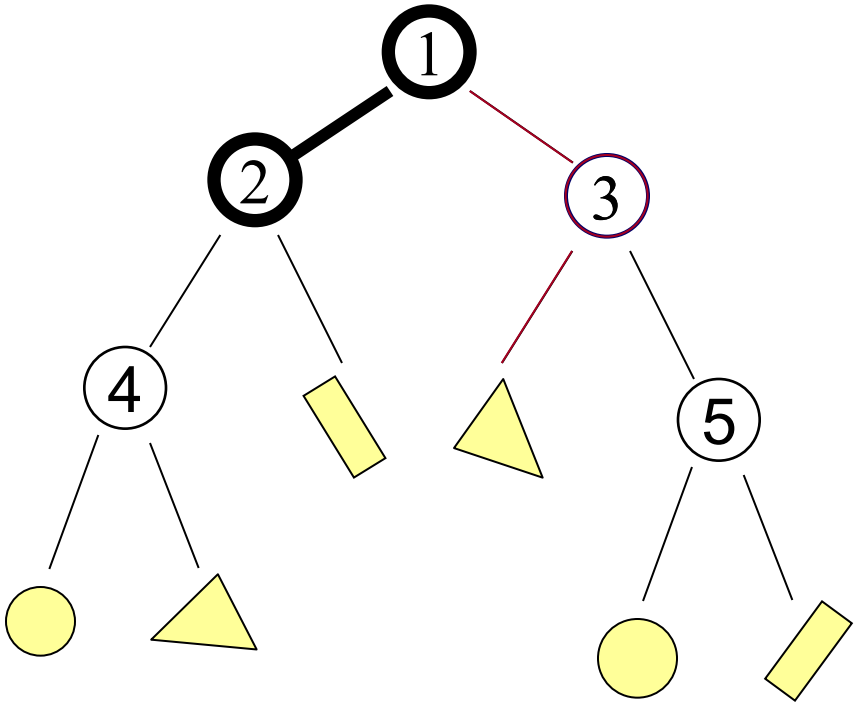
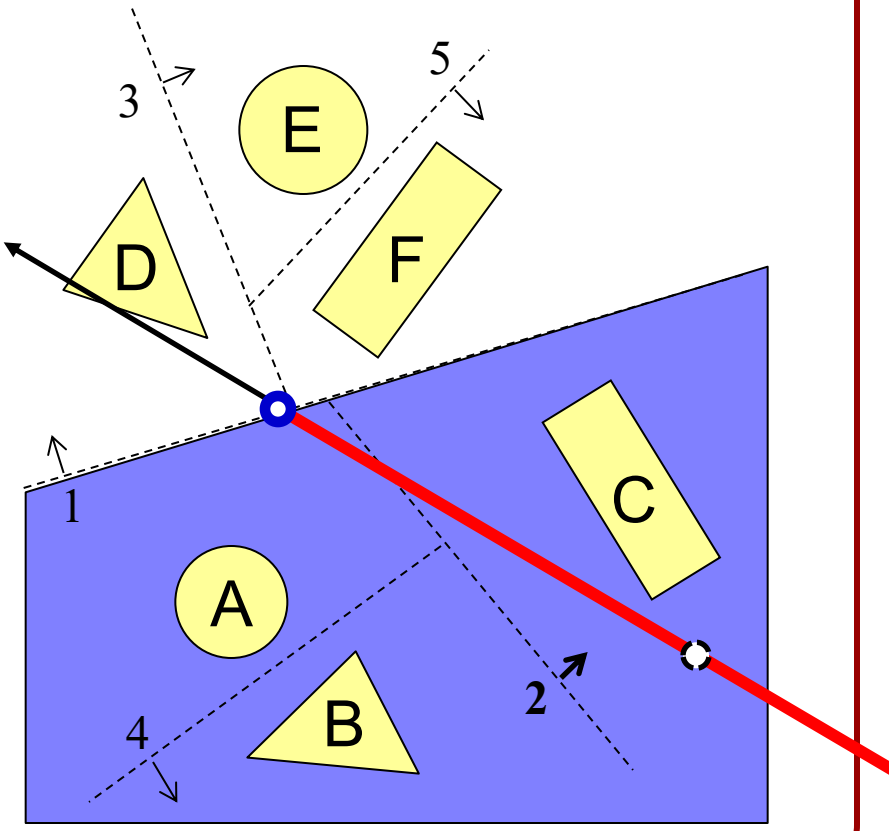
Space Partition: BSP Tree

Example 3: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Test back (left)





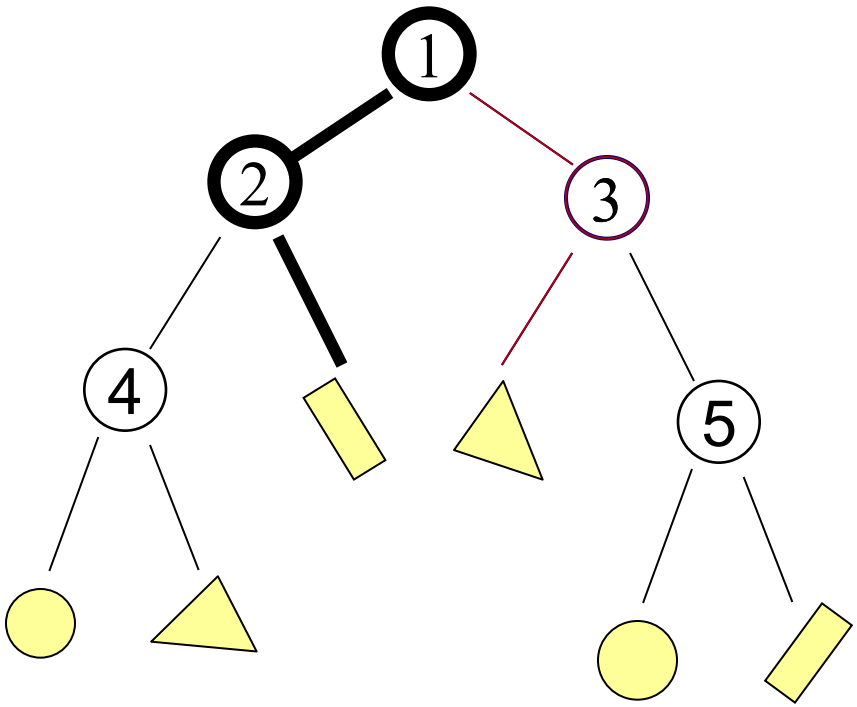
Note: Arrows denote the "right" side of the splitting plane.

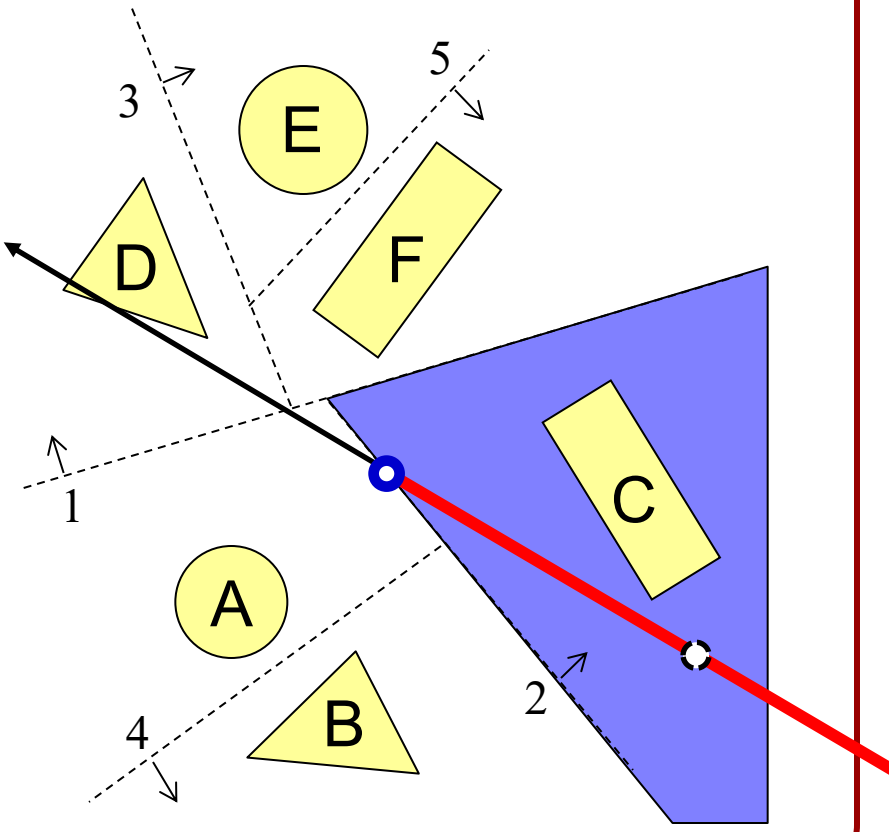
Space Partition: BSP Tree

Example 3: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Test back (right)





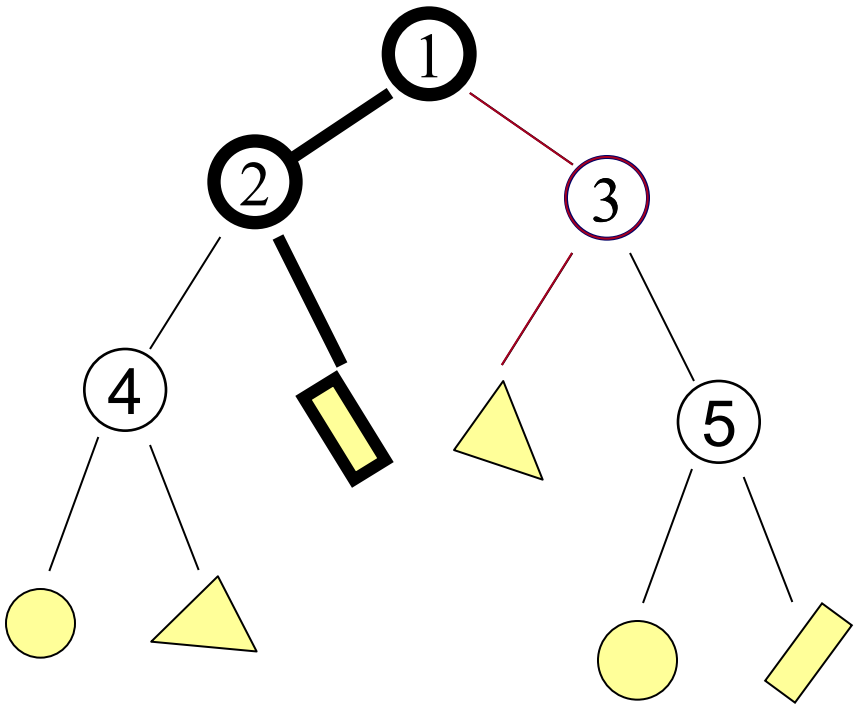
Note: Arrows denote the "right" side of the splitting plane.

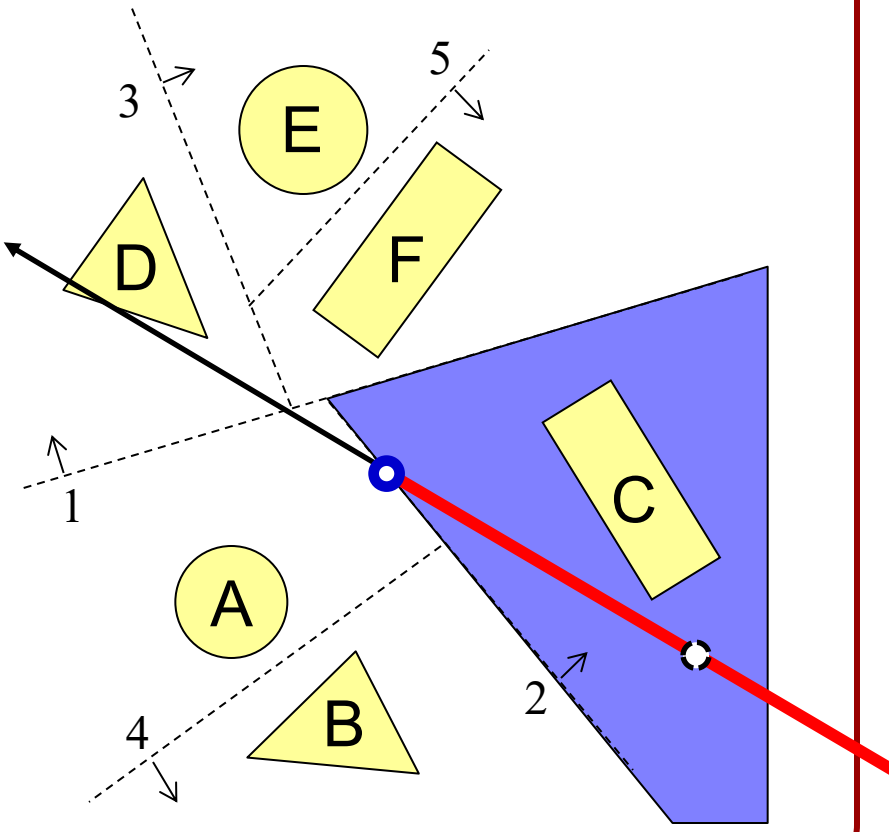
Space Partition: BSP Tree

Example 3: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Missed C. Recurse!





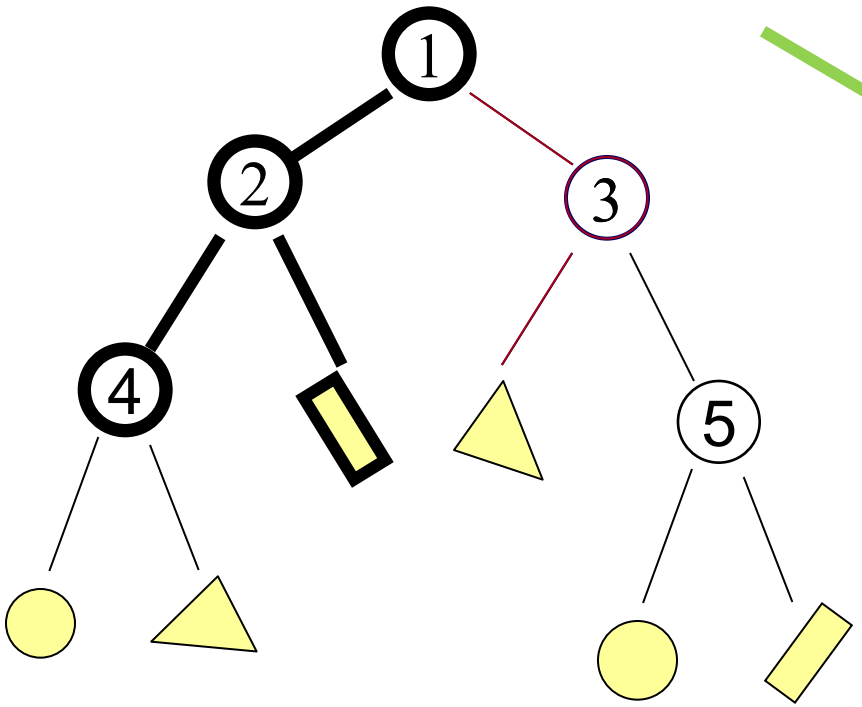
Note: Arrows denote the "right" side of the splitting plane.

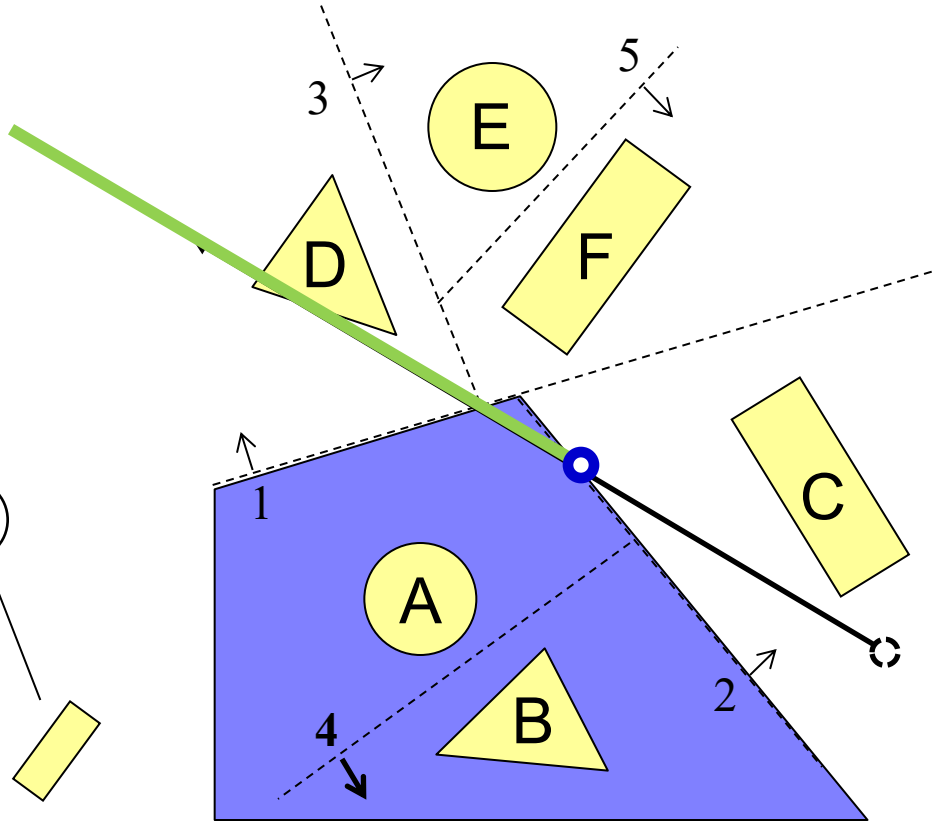
Space Partition: BSP Tree

Example 3: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Test front (left)





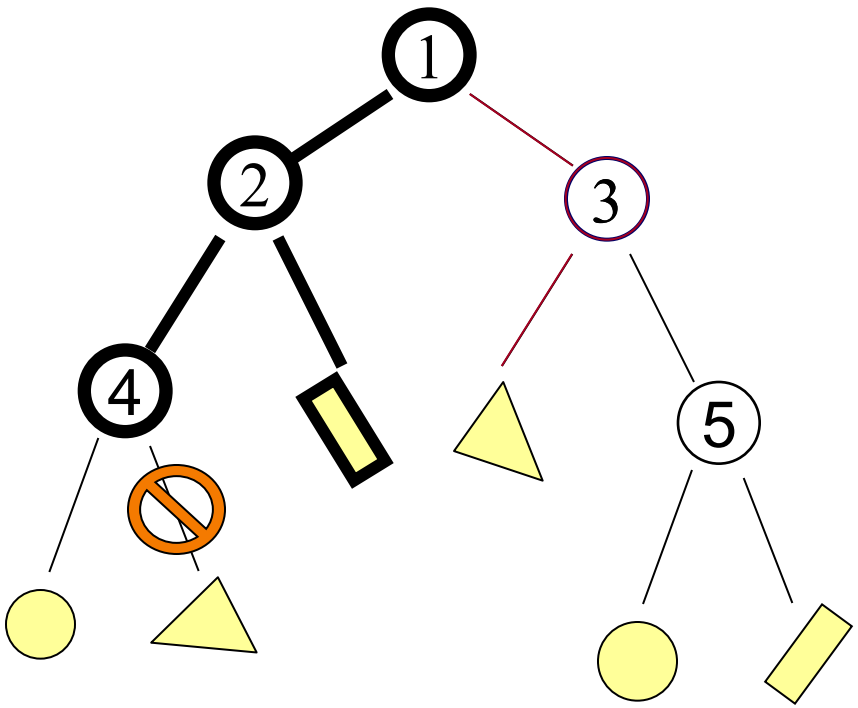
Note: Arrows denote the "right" side of the splitting plane.

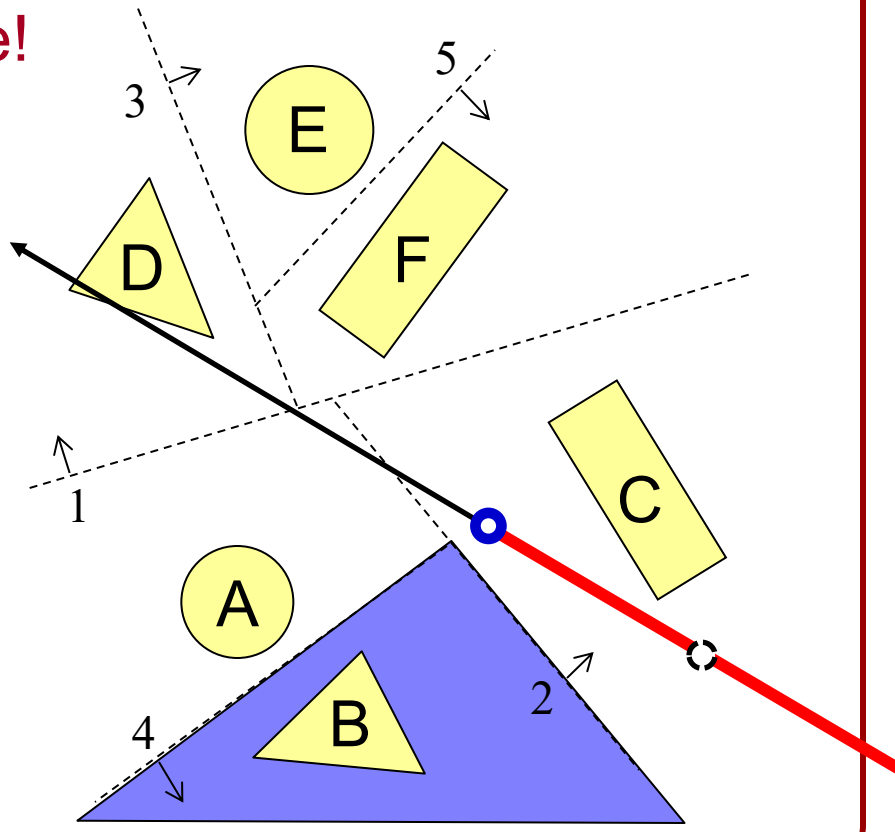
Space Partition: BSP Tree

Example 3: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» No back (right). Recurse!





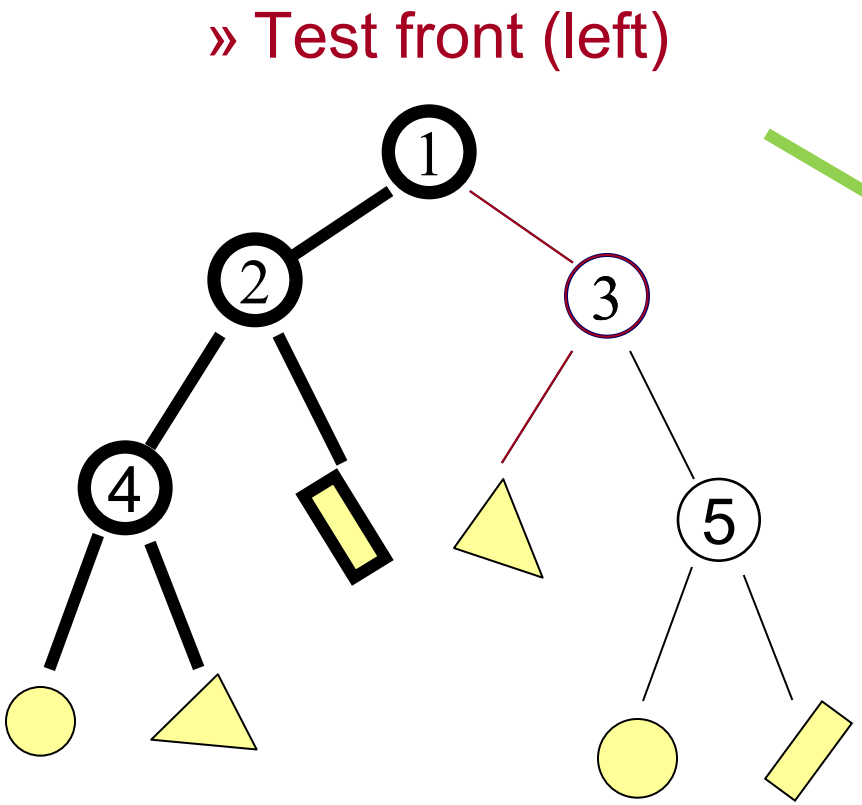
Note: Arrows denote the "right" side of the splitting plane.

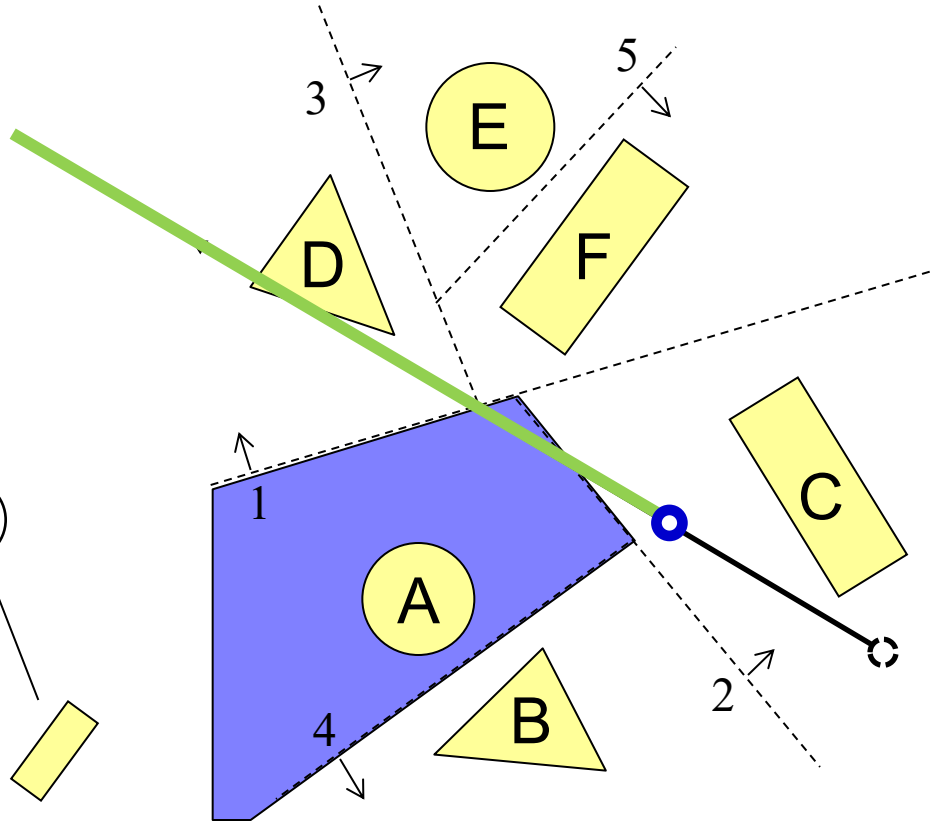
Space Partition: BSP Tree

Example 3: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Test front (left)





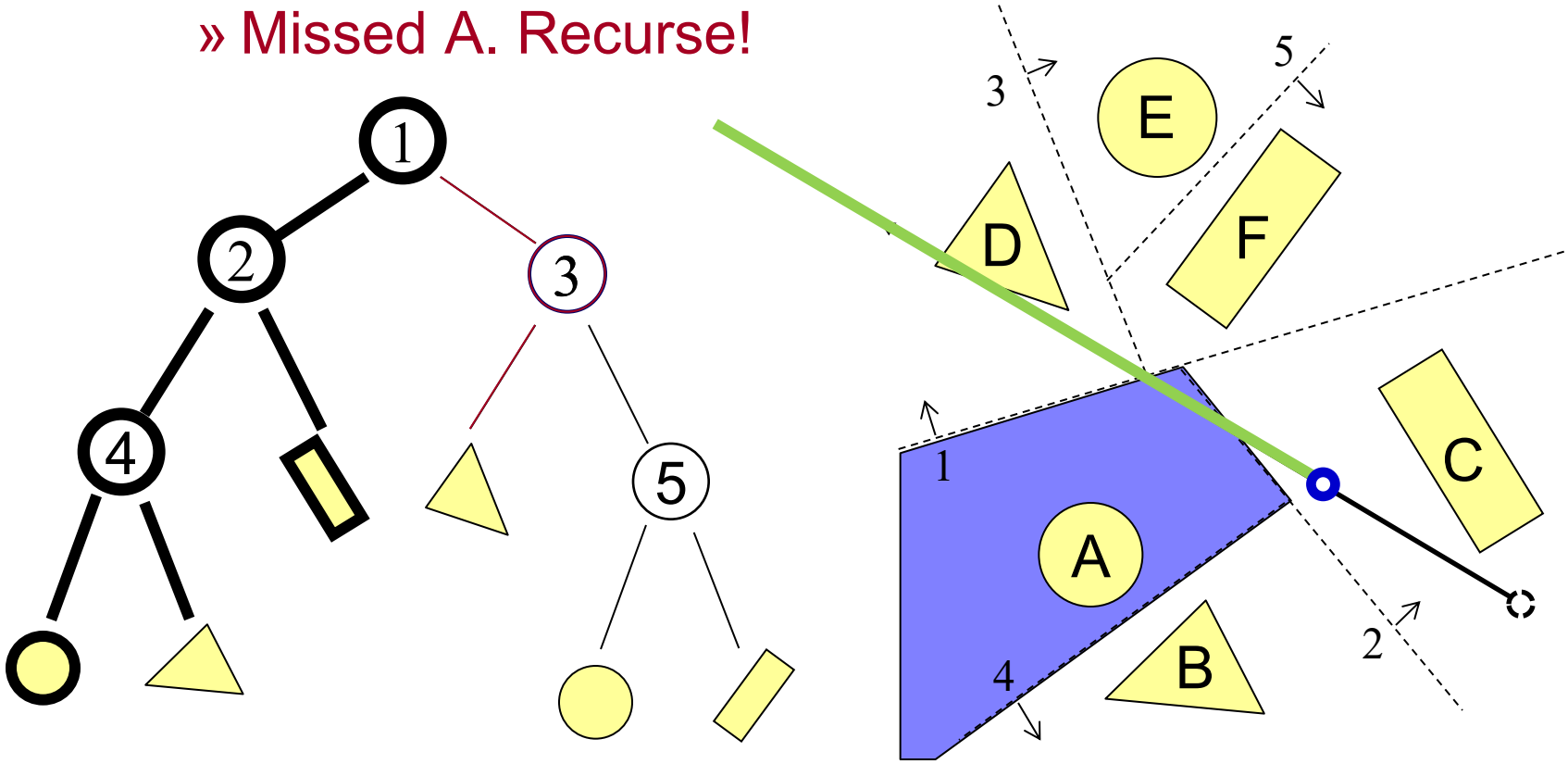
Note: Arrows denote the "right" side of the splitting plane.

Space Partition: BSP Tree

Example 3: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Missed A. Recurse!



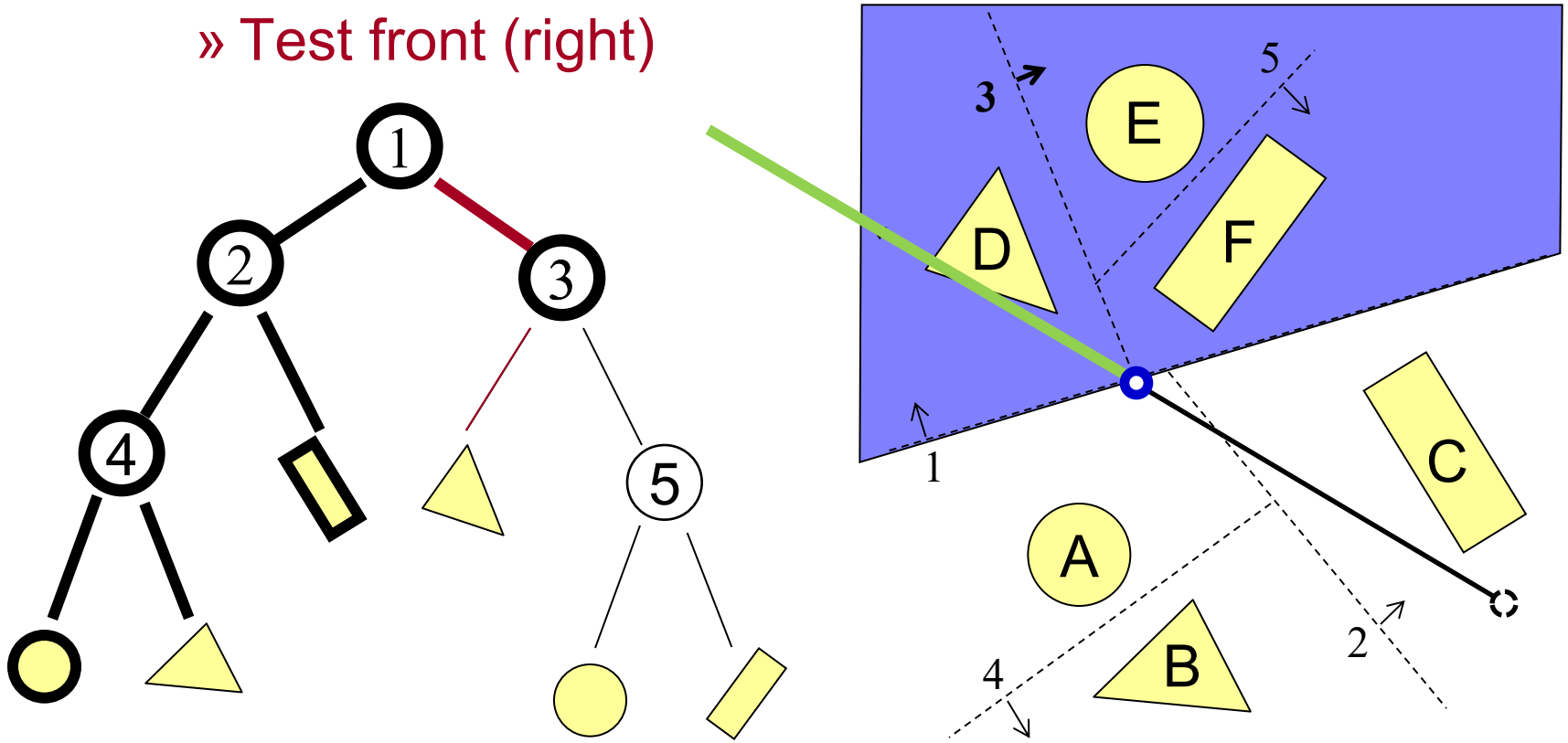
Note: Arrows denote the "right" side of the splitting plane.

Space Partition: BSP Tree

Example 3: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Test front (right)



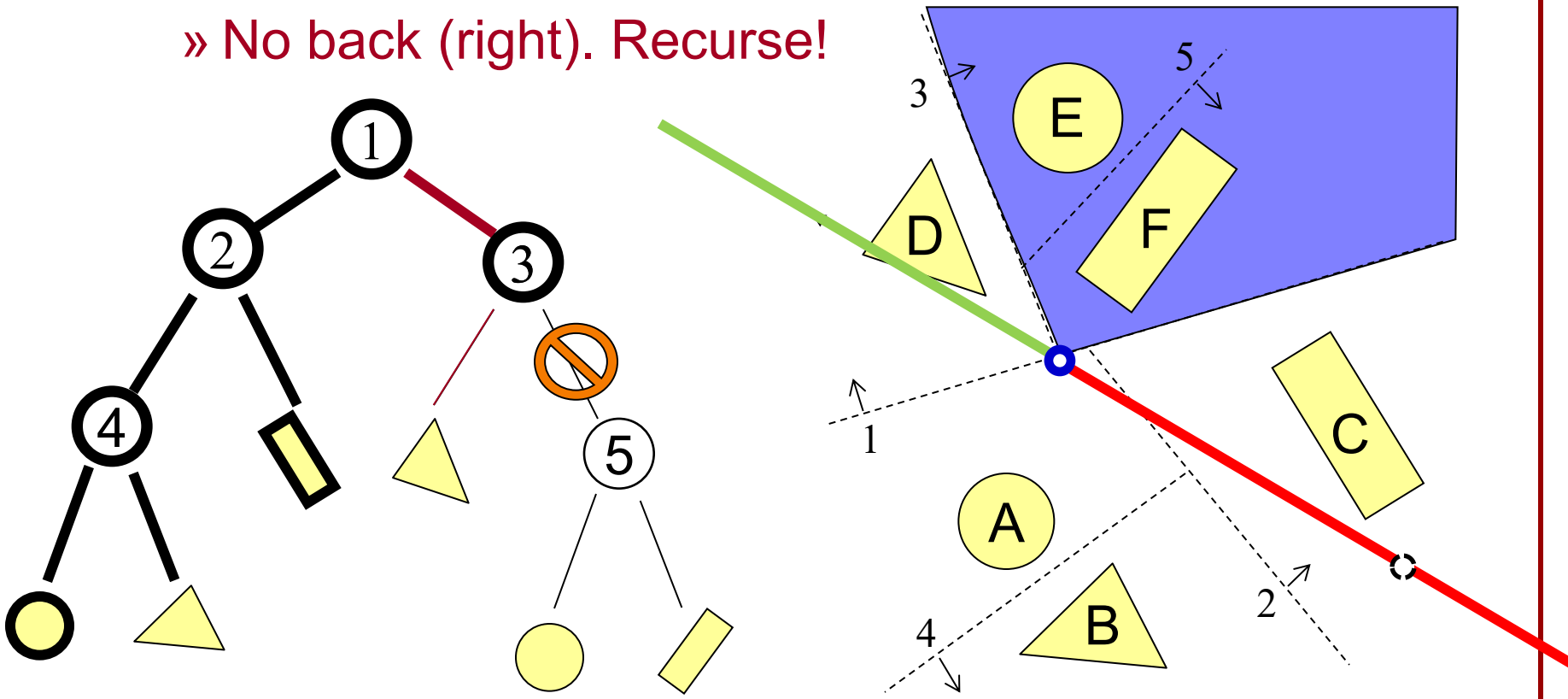
Note: Arrows denote the "right" side of the splitting plane.

Space Partition: BSP Tree

Example 3: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» No back (right). Recurse!



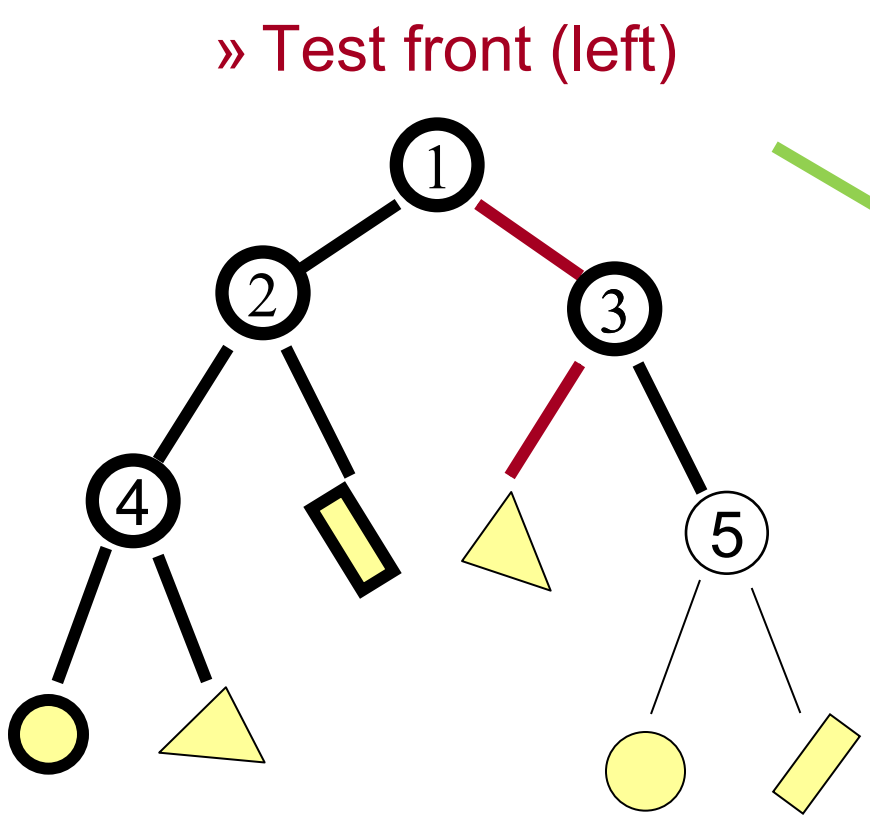
Note: Arrows denote the "right" side of the splitting plane.

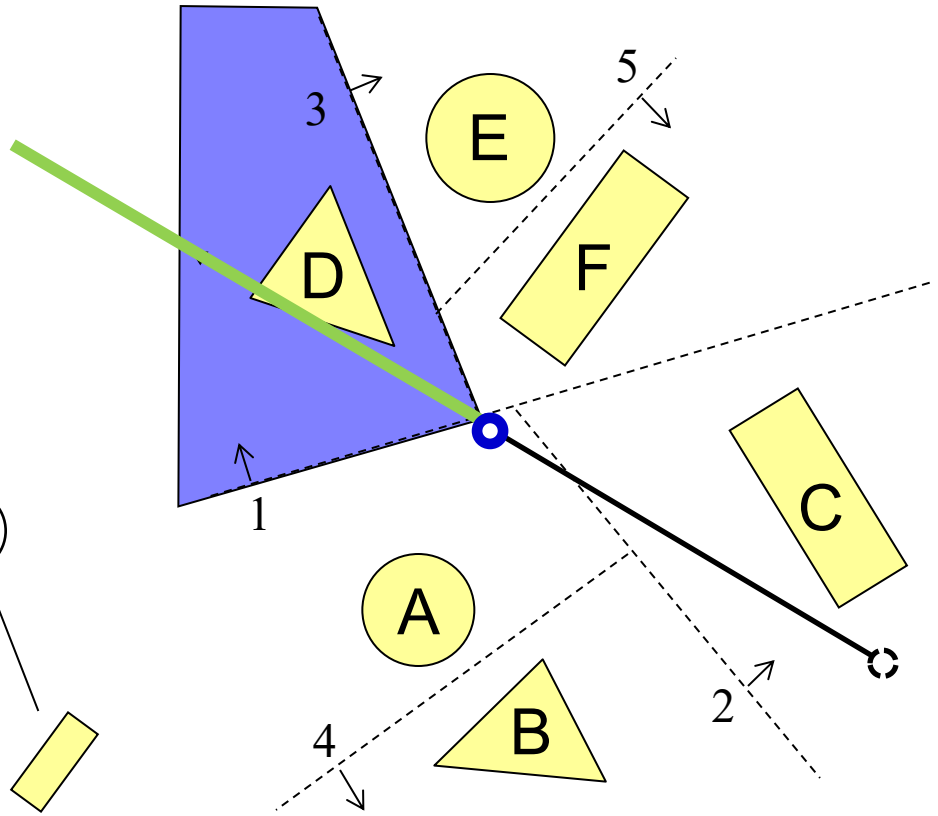
Space Partition: BSP Tree

Example 3: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Test front (left)





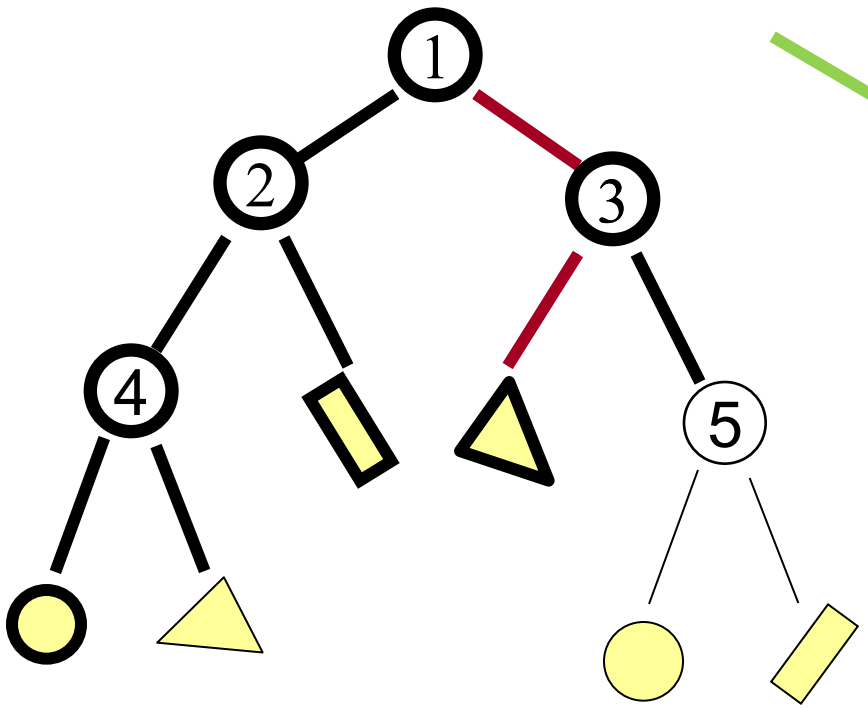
Note: Arrows denote the "right" side of the splitting plane.

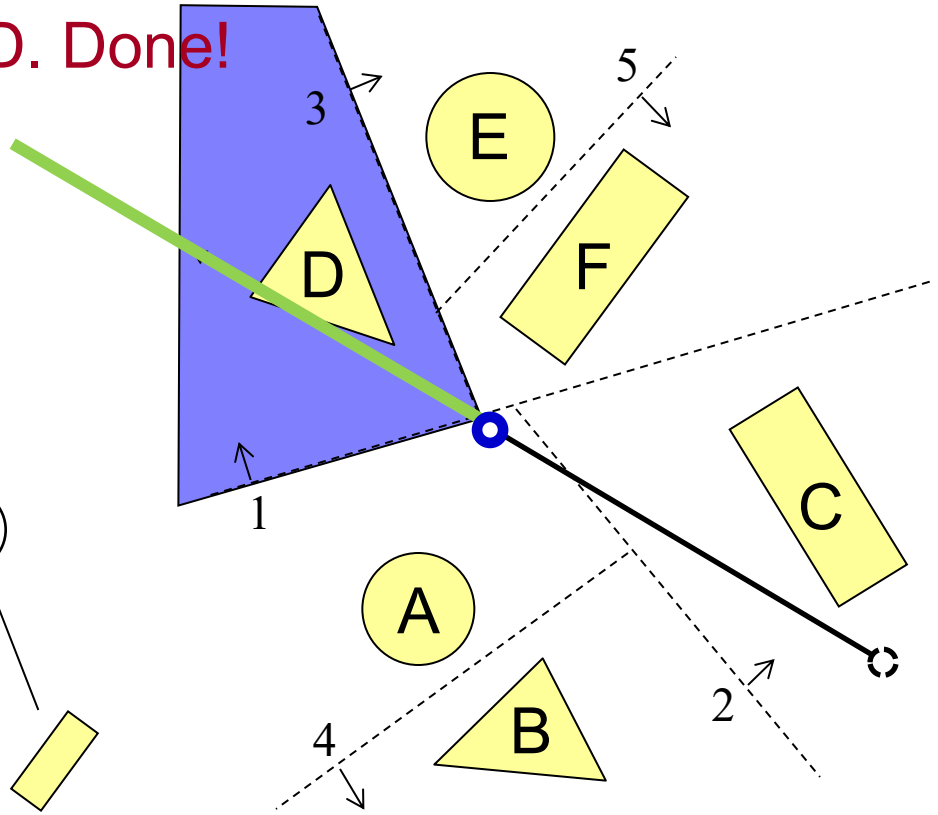
Space Partition: BSP Tree

Example 3: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Intersection with D. Done!





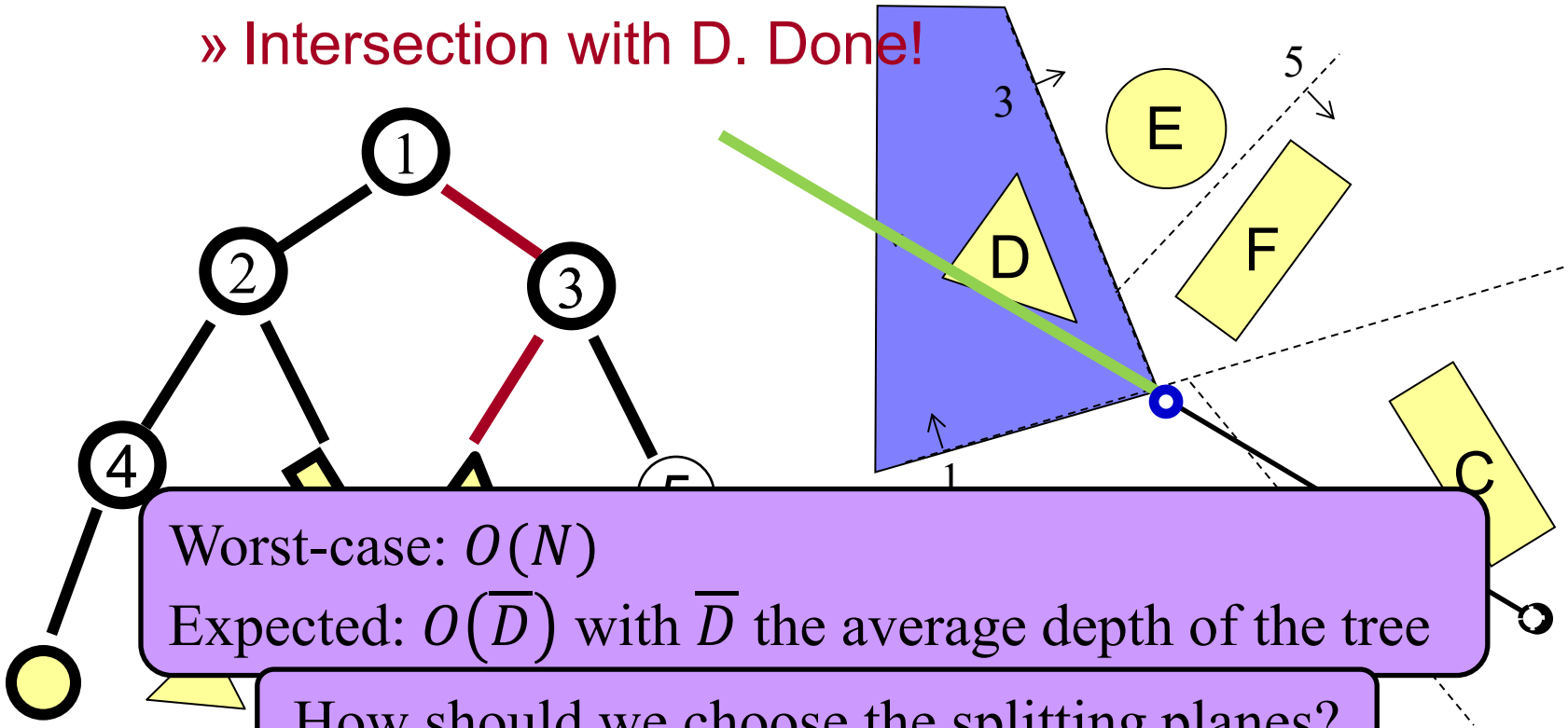
Note: Arrows denote the "right" side of the splitting plane.

Space Partition: BSP Tree

Example 3: Ray intersection query

Recursively split the ray and test both halves, testing the back part first. Stop once you hit something:

» Intersection with D. Done!



Worst-case: $O(N)$

Expected: $O(\overline{D})$ with $\overline{D}$ the average depth of the tree

How should we choose the splitting planes?

Note: Arrows denote the "right" side of the splitting plane.

Space Partition: BSP Tree

```
Intersection RayTreeIntersect( Ray ray< 3 > ,  Node node )
{
      if ( Node is a leaf ) return intersection of closest primitive in cell, or NULL if none
      else
      {
            // Find splitting plane and near and far children
            near_child = child of node that contains the start (r(−∞))
            far_child = other child of node

            // Recurse down near child first
            isect = RayTreeIntersect( ray , near_child )
            if( isect ) return isect    // If there's a hit, we are done

            // If there is no hit, test the far child
            return RayTreeIntersect( ray , far_child )
      }
}
```

Acceleration Techniques

- Data Partitions
  - » Bounding volume hierarchy (BVH)
- Space Partitions
  - » Uniform (voxel) grid
  - » Octree
  - » Binary space partition (BSP) tree

Note:

- All are independent of the viewer position
- All need to be adapted if the geometry changes/animates