Image Processing

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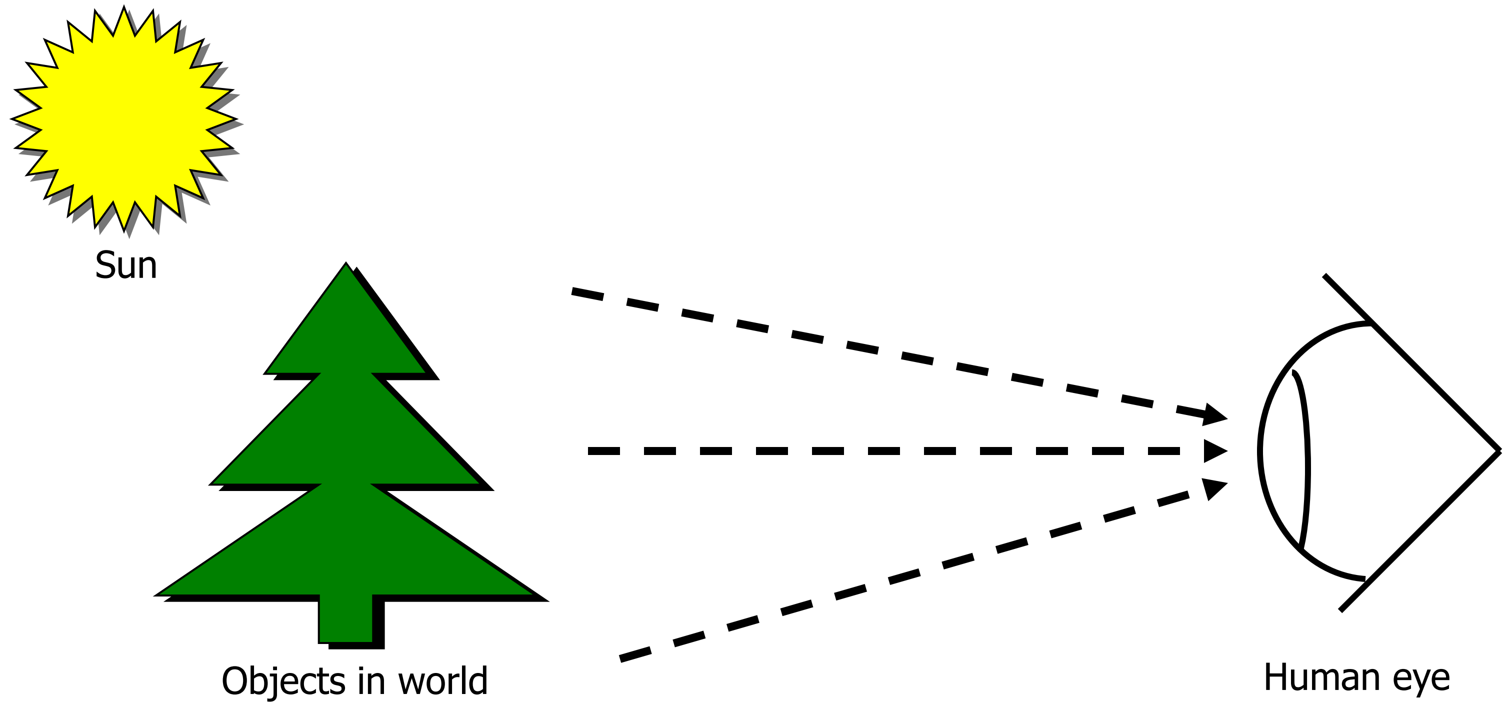
(601.457/657)

Outline

- Human Vision
- Image Representation
- Reducing Color Quantization Artifacts
- Basic Image Processing

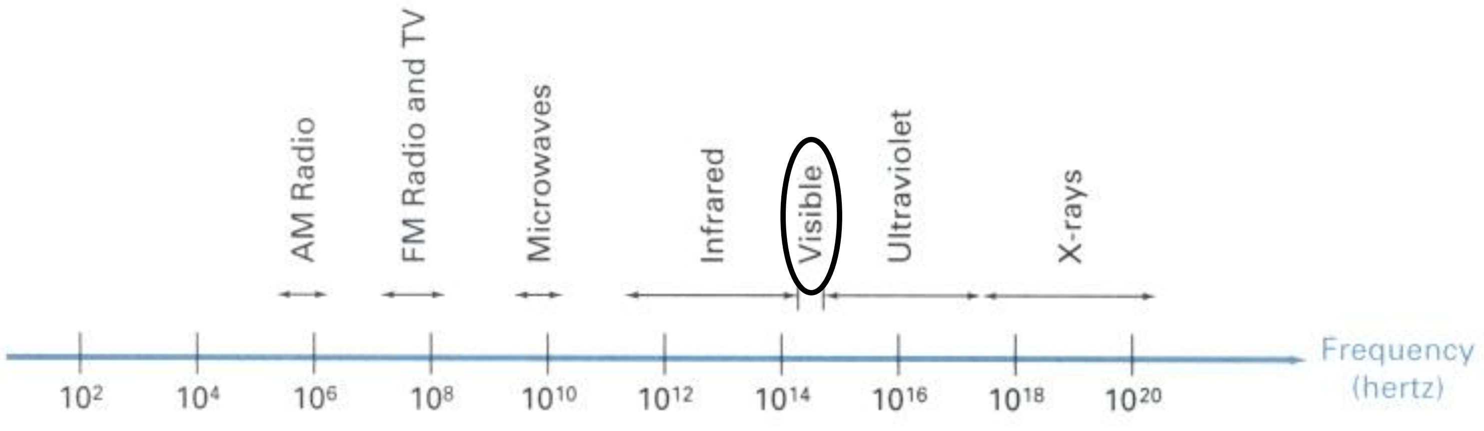
Human Vision

Model of Human Visual System



Electromagnetic Spectrum

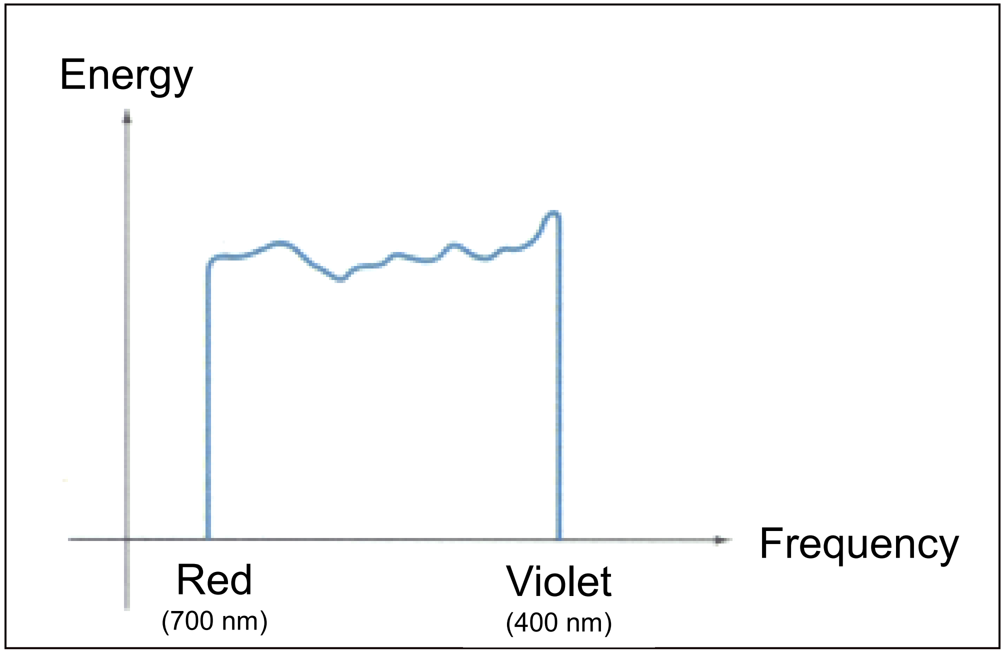
- Visible light frequencies range between ...
  - Red = 4.3×10^{14} hertz (700nm)
  - Violet = 7.5×10^{14} hertz (400nm)



Figures 15.1 from H&B

Visible Light

- What we see as “color” is described by the distribution of light across the visible range.



White Light

Figure 15.3 from H&B

Human Vision

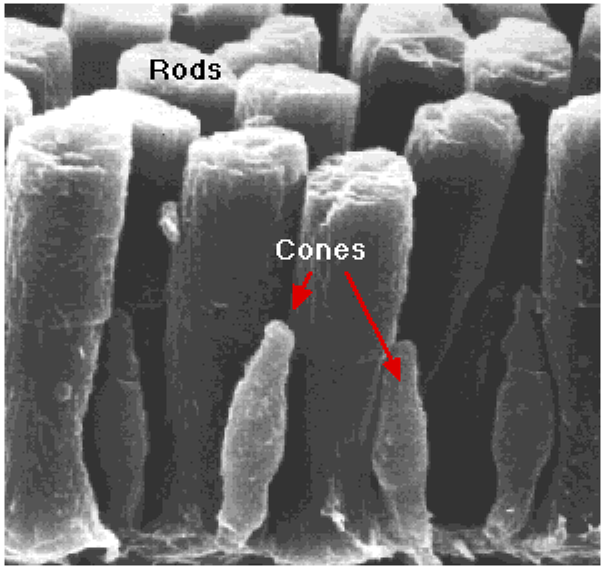
The human retina contains two types of photoreceptors, cones and rods.

Cones:

- 6-7 million cones in the retina
- Responsible for **photopic** vision
- Color sensitive:
  - 64% red, 32% green, 2% blue
- Distributed in the fovea centralis

Rods:

- 120 million rods in the retina
- 1000x more light sensitive than cones
- Responsible for **scotopic** vision
- Short-wavelength sensitive
- Responsible for peripheral vision



Tristimulus Theory of Color

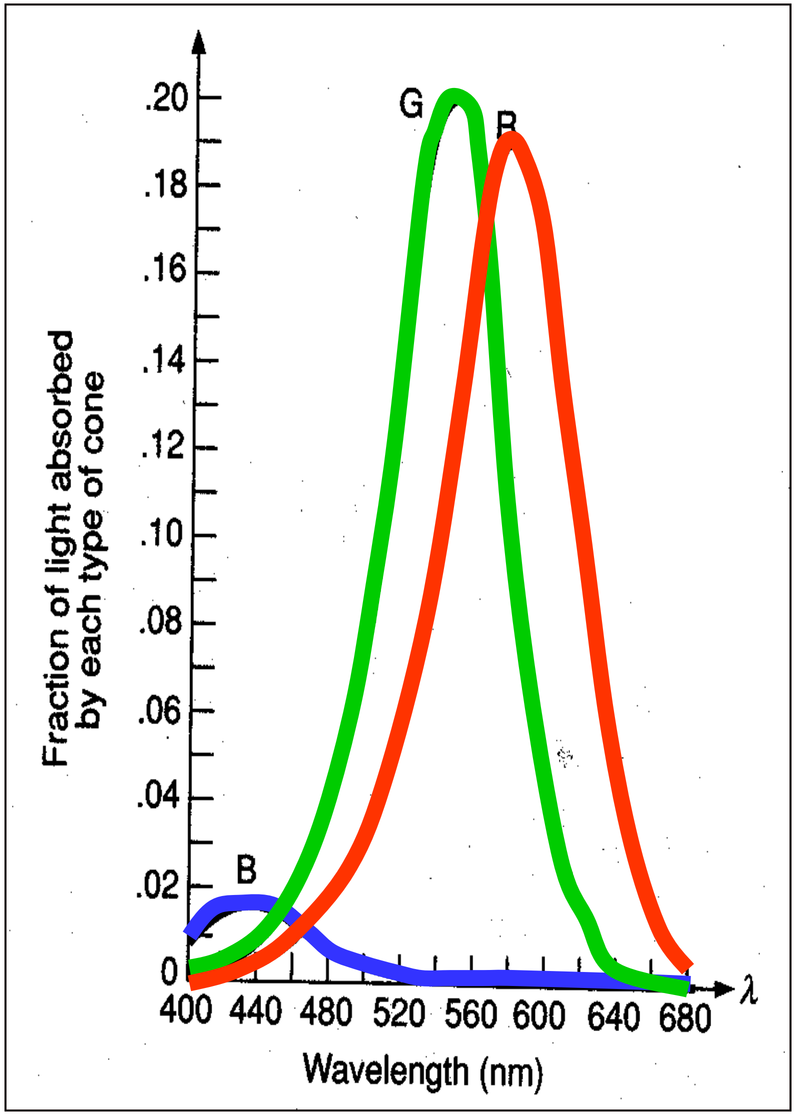


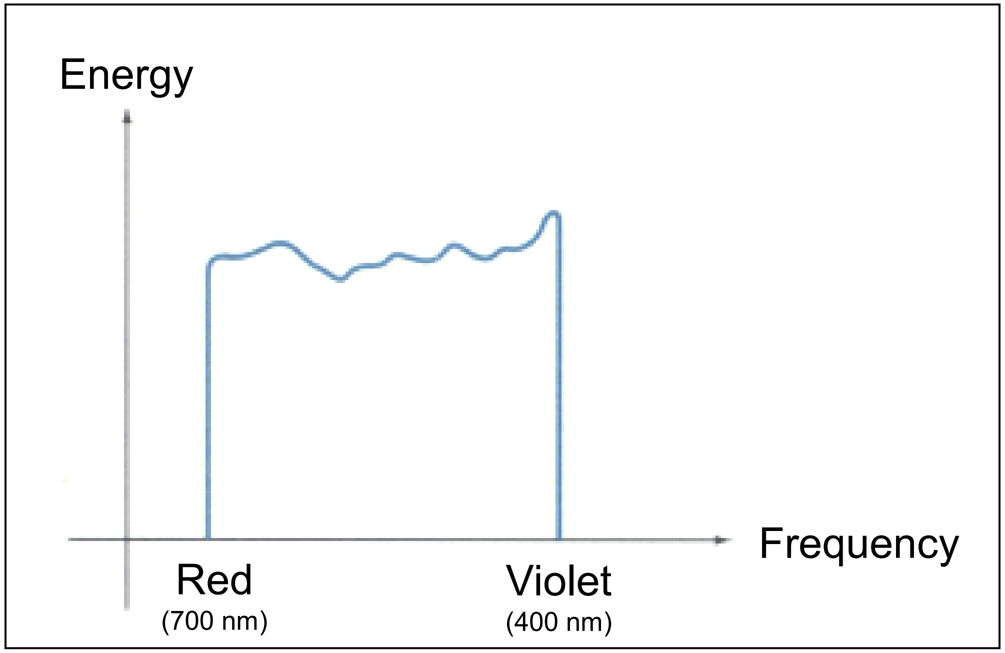
Figure 13.18 from FvDFH

Spectral-response functions of each of the three types of cones on the human retina.

This motivates encoding color as a combination of red, green, and blue (RGB).

Visible Light

- What we see as "color" is described by the distribution of light across the visible range.

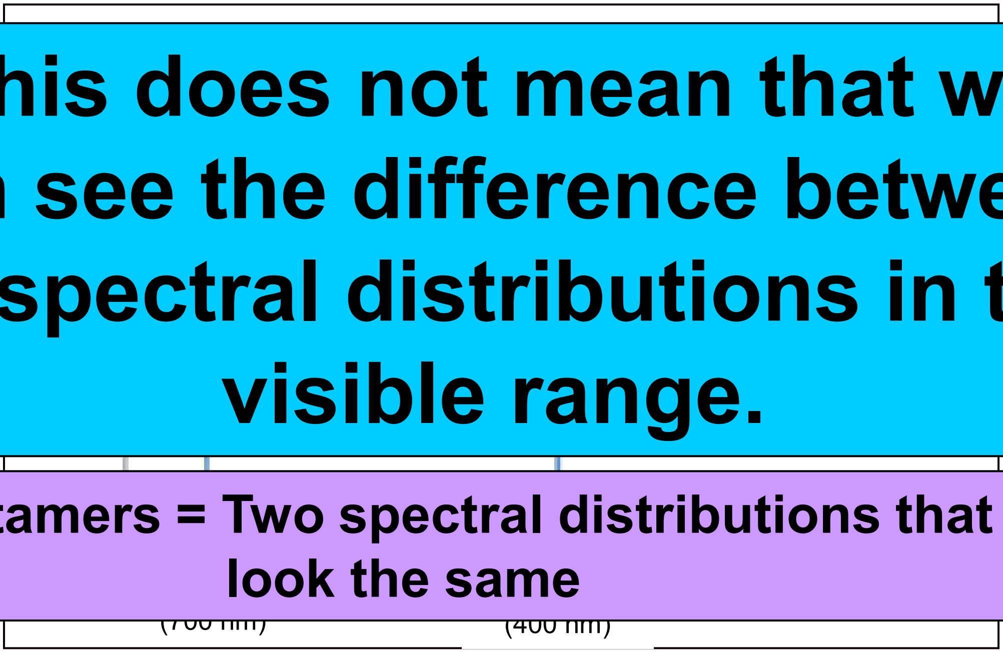


White Light

Figure 15.3 from H&B

Visible Light

- What we see as “color” is described by the distribution of light across the visible range.



This does not mean that we can see the difference between all spectral distributions in the visible range.

Metamers = Two spectral distributions that look the same

White Light

Figure 15.3 from H&B

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Image Representation

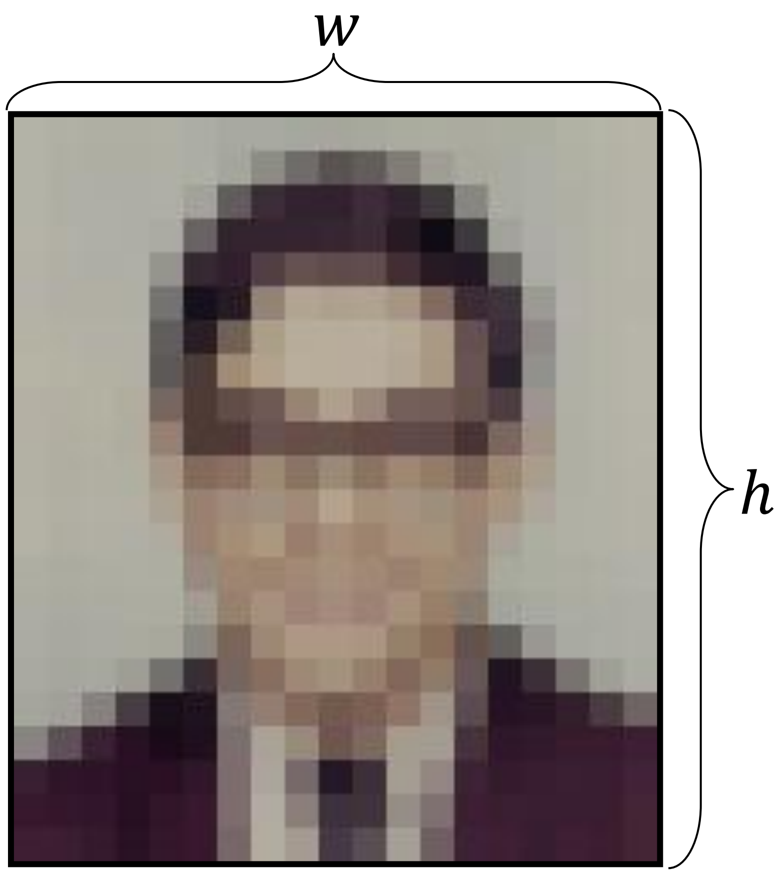
What is an image?

Image Representation

An image is a 2D rectilinear array of pixels:

- A *width* x *height* array where each entry of the array stores a single pixel.

Continuous image



Digital image

Image Representation

What is a pixel?

Continuous image

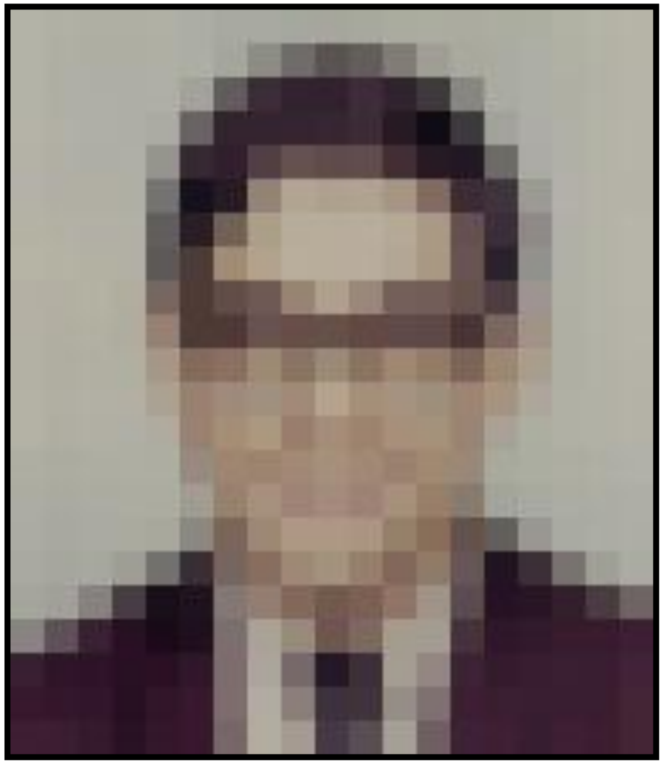

Digital image

Image Representation

A pixel is something that captures the notion of color

- Luminance pixels
  - Grey-scale images (aka “intensity images”)
- Red, Green, Blue pixels (RGB)
  - Color images

Channel value:

- Conceptually, in the continuous range [0,1)*
- In practice, in a discrete range (e.g. {0,1,…,254,255})

*[0,1) is the continuous range of numbers including 0 but not including 1.

Resolutions

- Intensity/Color: n bits per pixel
- Space: *width* x *height* pixels
- Time: n Hz (fps)

	width x height	bit depth	Hz
Handheld	2220 x 1080	24	60
Monitor (4K)	3840 x 2160	24	144
CCDs	6000 x 4000	36	50
Laser Printer	6600 x 5100	3	-

Image Quantization Artifacts

- With only a small number of bits associated to each color channel of a pixel there is a limit to intensity resolutions of an image
  - A black and white image allocates a single bit to the luminance channel of a pixel.
    - » The number of different colors that can be represented by a single pixel is 2.
  - A 24 bit image allocates 8 bits each to the red, green, and blue channels of a pixel.
    - » The number of different colors that can be represented by a single pixel is 2^{24}=~16,000,000.

Outline

- Human Vision
- Image Representation
- Reducing Color Quantization Artifacts
  - Halftoning and Dithering
- Basic Image Processing

Pixel Representation

Disclaimer:

In the next slides, we will assume that images are gray-scale (single-channel).

We assume that the original image has continuous pixel values:

$$I(x,y) \in [0,1)$$

We will represent them using a finite number of bits per pixel so that color/gray values are discrete:

$$\mathbf{I}(x,y) \in \{0,\ldots,n-1\}$$

Discretization

In particular, using b bits per pixel, we can represent $n = 2^b$ different colors.

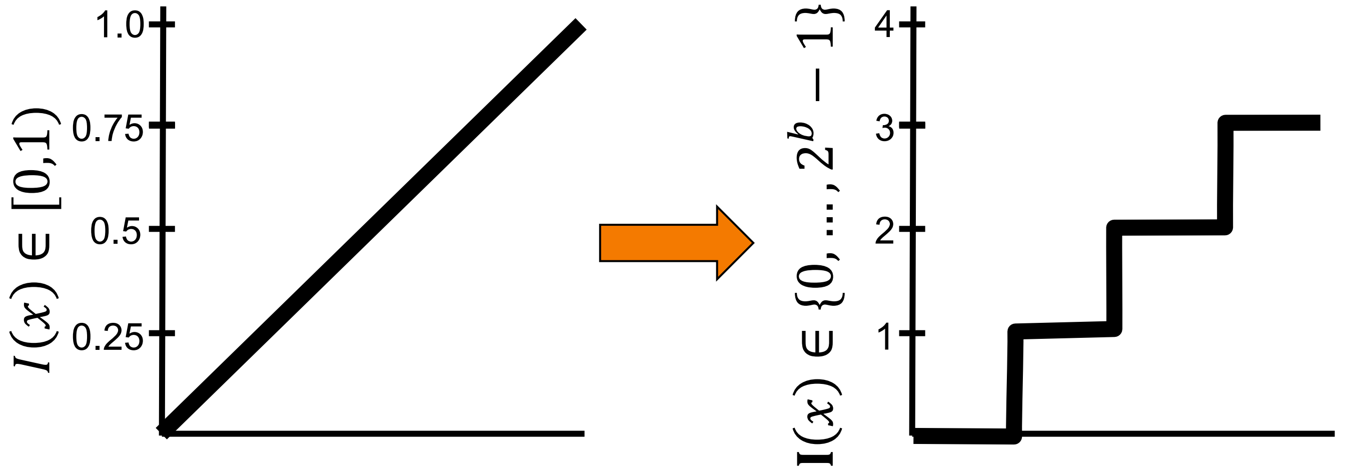
$$\Downarrow$$

$$\mathbf{I}(x, y) \in \{0, \dots, 2^b - 1\}$$

Quantization

- With b bits per pixel, you can coarsely represent an image by quantizing the color values:

$$\mathbf{I}(x,y) = Q_b\big(I(x,y)\big) = \mathrm{floor}\big(I(x,y)\cdot 2^b\big)$$



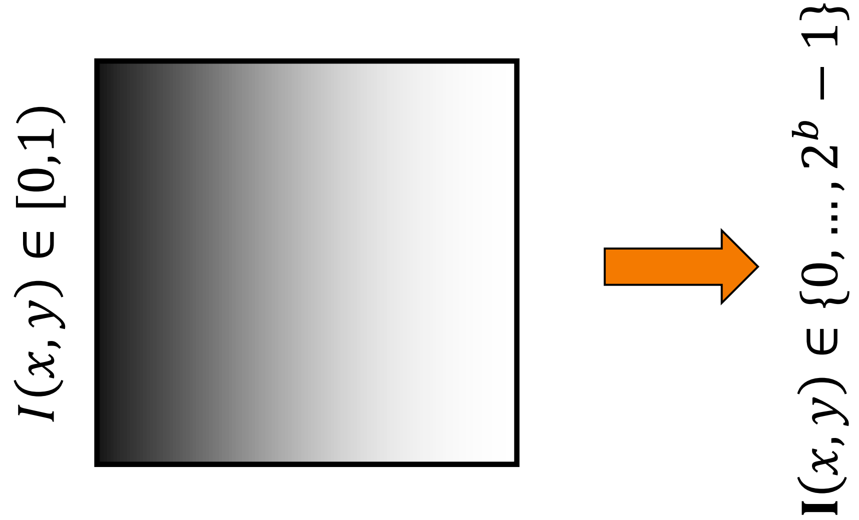
$b = 2$ bits per pixel

Note: We quantize so that each “bucket” gets the same measure of values.

Quantization

- With b bits per pixel, you can coarsely represent an image by quantizing the color values:

$$\mathbf{I}(x,y) = Q_b\big(I(x,y)\big) = \text{floor}\big(I(x,y)\cdot 2^b\big)$$

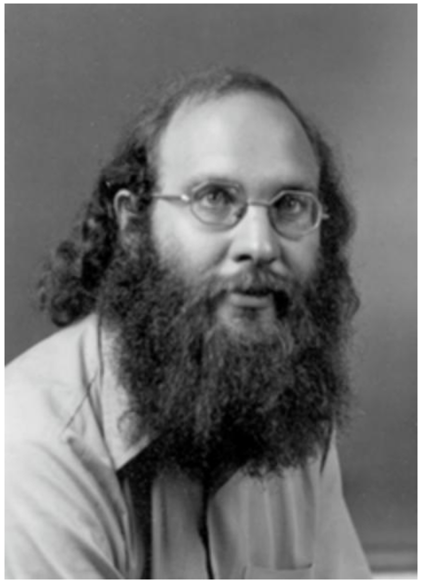


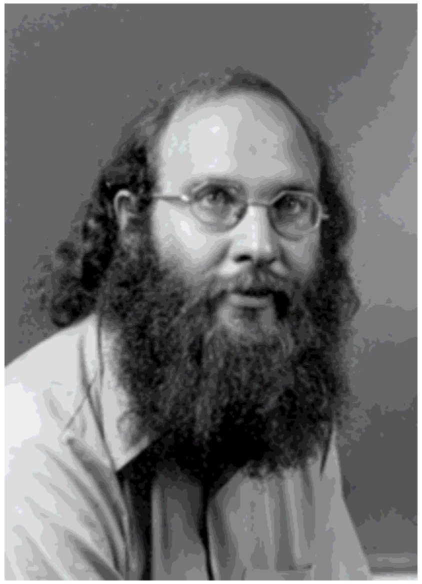
$b = 2$ bits per pixel

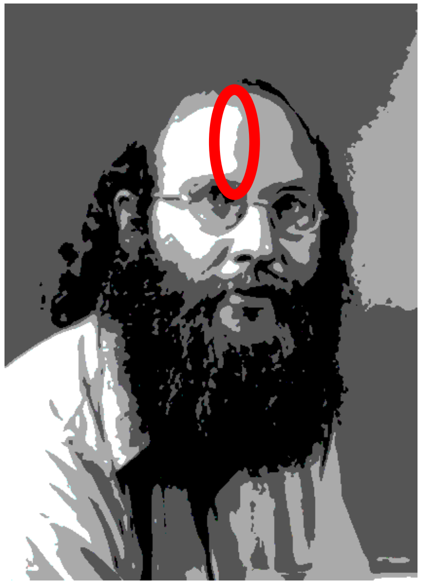
Quantization

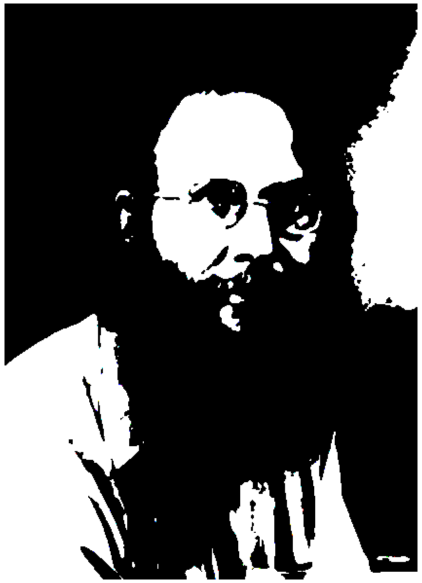
Image with decreasing bits per pixel

- Quantization can cause contours away from image edges.


$b = 8$ bits


$b = 4$ bits


$b = 2$ bits


$b = 1$ bits

Reducing Color Quantization Artifacts

For (still) images, the combination of image resolution and intensity/color resolution define the total informational content.

Key Idea:

We can trade off between these.

Reducing Effects of Quantization

Trade spatial resolution for intensity resolution:

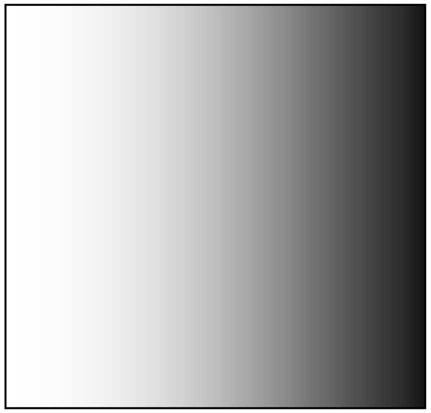
- Half-toning
- Dithering

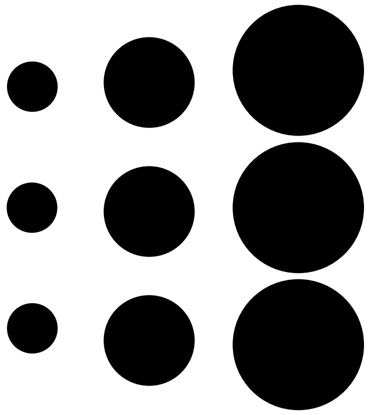
Both exploit spatial integration in our eye to display a greater range of *perceptible* intensities.

Half-Toning: Historically

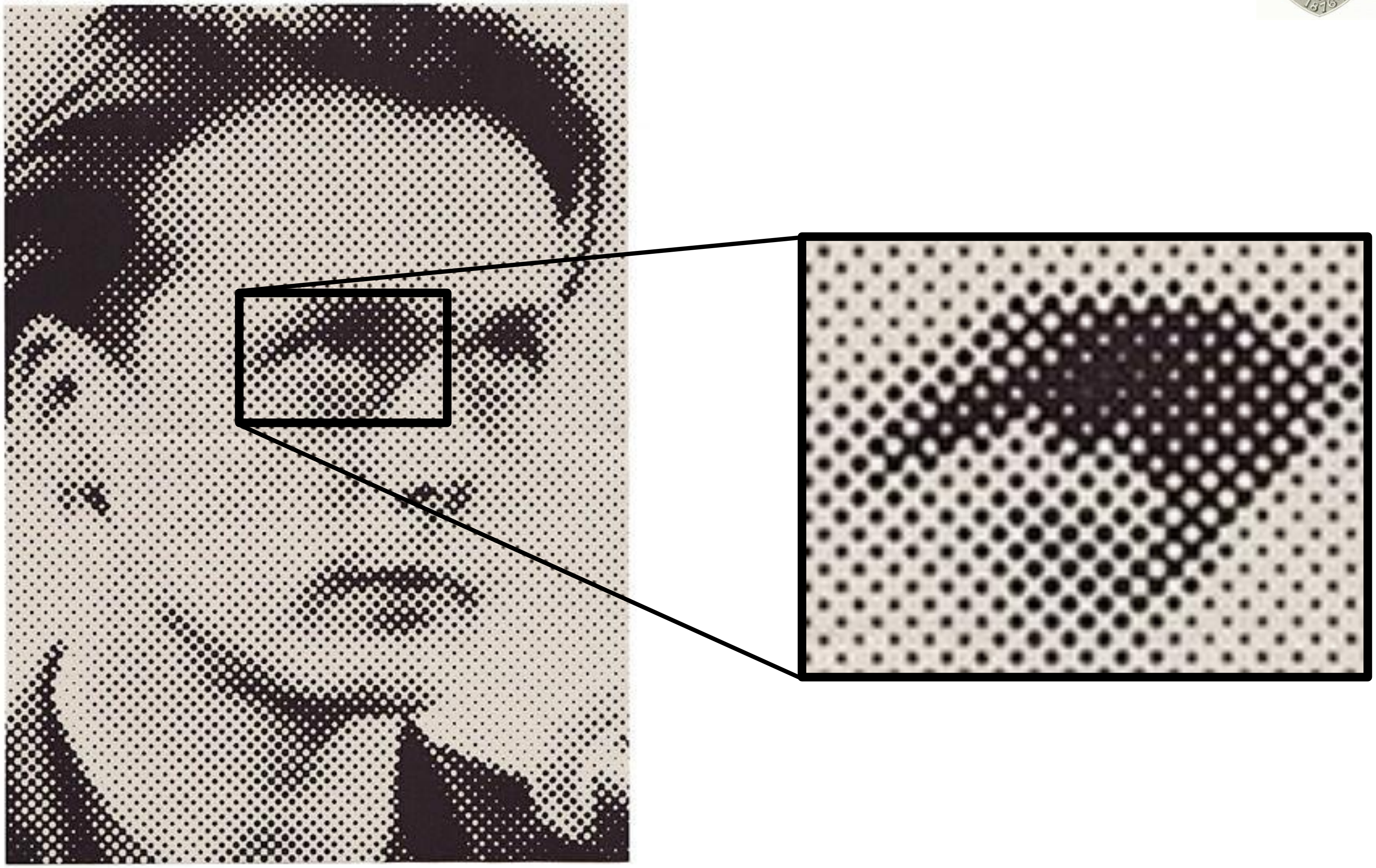
Half-toning:

- Consider the **average** intensity in a region
- Draw varying-size dots representing the average
  - » Area of dots determined by the average intensity in the covered area


$I(x,y)$


$\mathbf{I}(x,y)$

Half-Toning: Historically



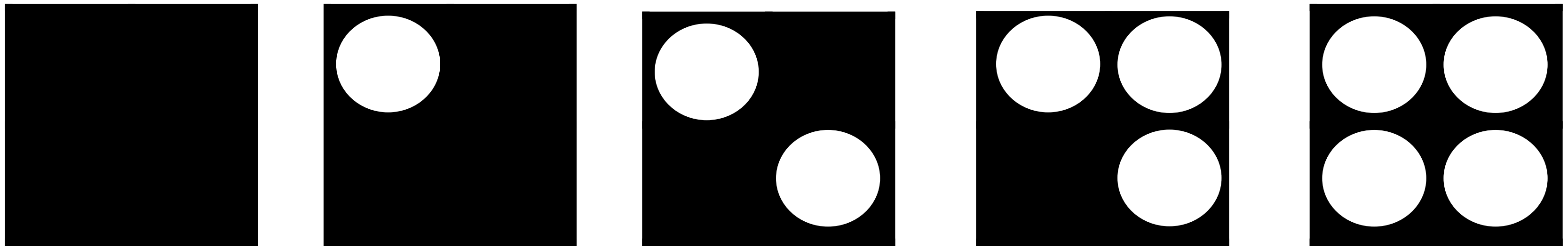
https://i.pinimg.com/736x/e4/18/83/e41883601f5e242a10e71270443be07b.jpg

Half-Toning: Digital

- Consider the **average** intensity in a $k \times k$ block
- Turn on a variable number of the pixels in the block
  - » Number of pixels determined by the average intensity in the covered area

Note:

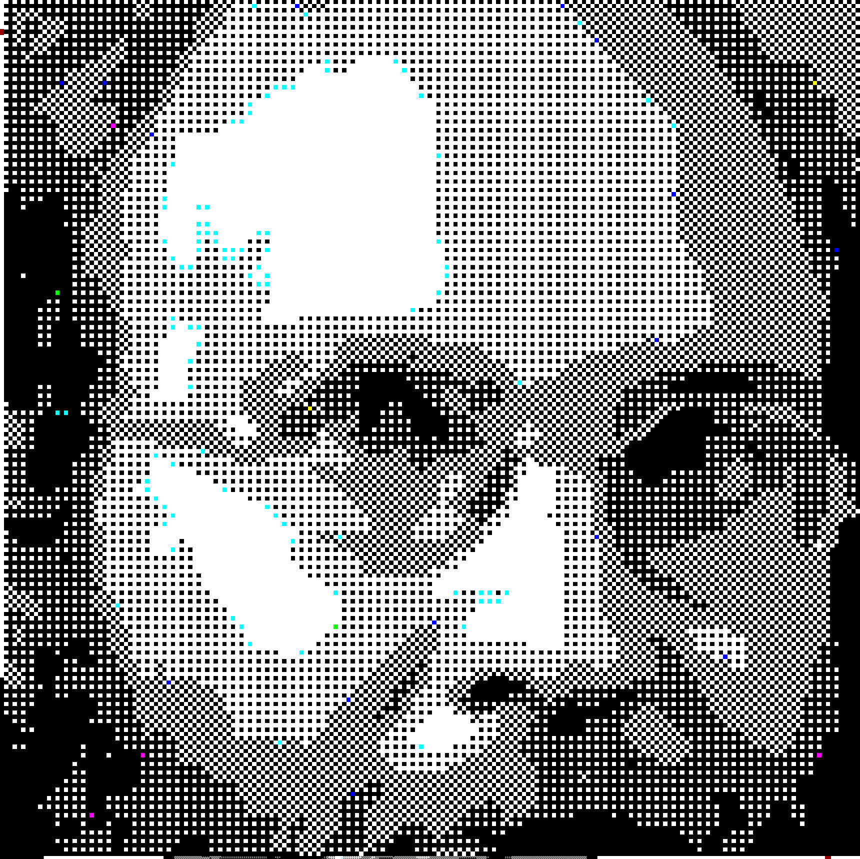
- Half-toning pattern matters
  - » Want to avoid vertical, horizontal lines
- Loss of information
  - » 16 configurations → 5 intensities

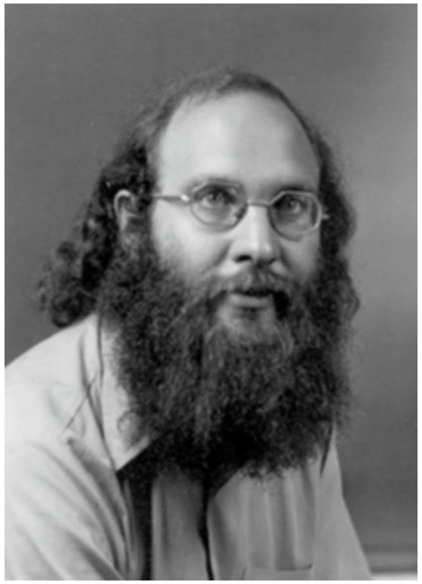


$0 \leq I < 0.2$ $\quad$ $0.2 \leq I < 0.4$ $\quad$ $0.4 \leq I < 0.6$ $\quad$ $0.6 \leq I < 0.8$ $\quad$ $0.8 \leq I < 1$

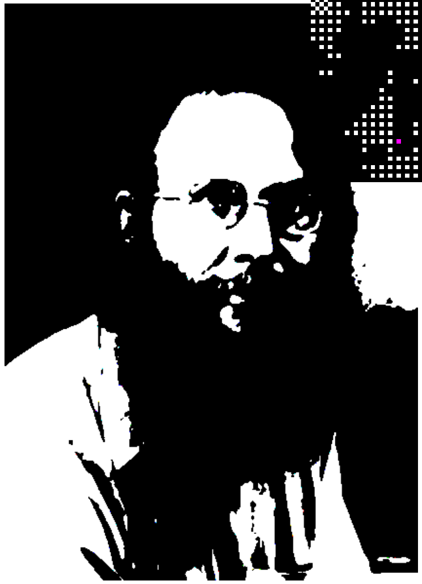
Half-Toning: Digital

- The use of a regular grid still causes contouring.

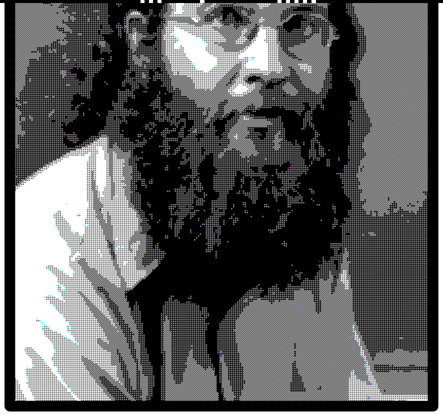




Original
(8 bits)



Quantized
(1 bit)



Half-toned
(1 bit)

Dithering

Per pixel, round to the floor/ceiling value.

- Similar to quantization:
  - » Rounding based on pixel value
- Unlike quantization:
  - » Rounding threshold is not fixed

Ordered Dither (Binary Displays)

Pseudo-random quantization thresholds described by a $k \times k$ matrix D_k with entries in the range $\{1, \ldots, k^2\}$

```
// For a pixel at position (x,y):
//    Locate the index in the block:
          $i = x \bmod k$
          $j = y \bmod k$
//    Get fractional component
          $e = I(x, y)$

//    Round up/down
          if$\left(e > \frac{D_k(i,j)}{k^2+1}\right)$ $\mathbf{I}(x, y) = 1$
          else              $\mathbf{I}(x, y) = 0$
```

$$D_2 = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

Ordered Dither (b-Bit Displays)

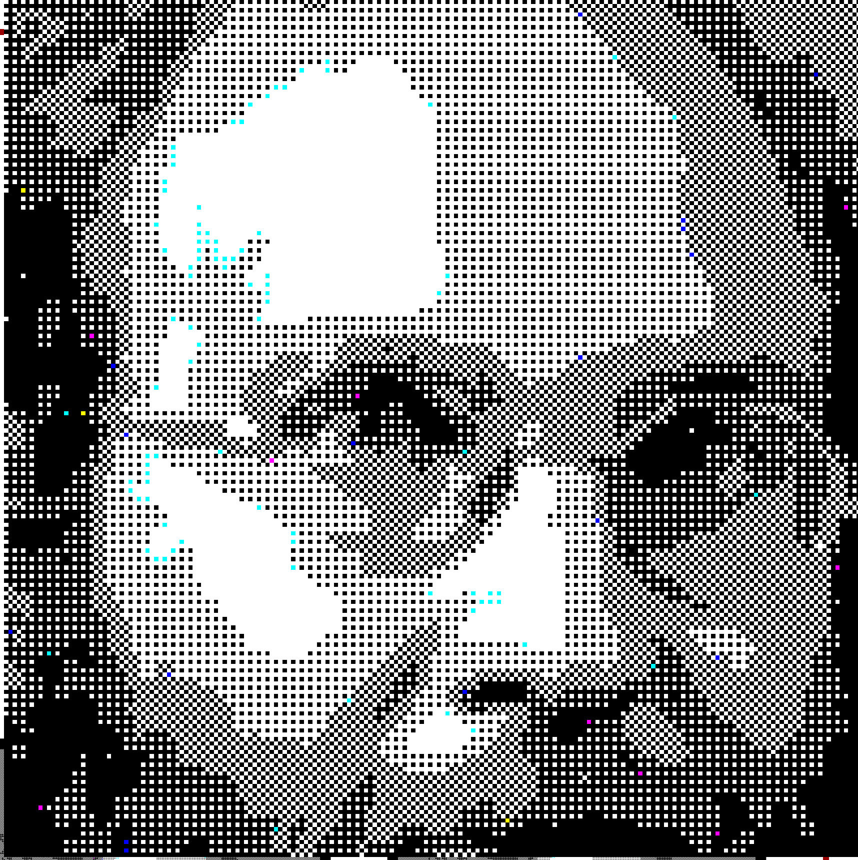
Pseudo-random quantization thresholds described by a $k \times k$ matrix D_k with entries in the range $\{1, \ldots, k^2\}$

```
// For a pixel at position (x,y):
//    Locate the index in the block:
          $i = x \bmod k$
          $j = y \bmod k$
//    Get fractional component
          $c = I(x,y) \cdot (2^b - 1)$
          $e = c - \text{floor}(c)$
//    Round up/down
          if$\left(e > \frac{D_k(i,j)}{k^2+1}\right)$ $\mathbf{I}(x,y) = \text{ceil}(c)$
          else                $\mathbf{I}(x,y) = \text{floor}(c)$
```

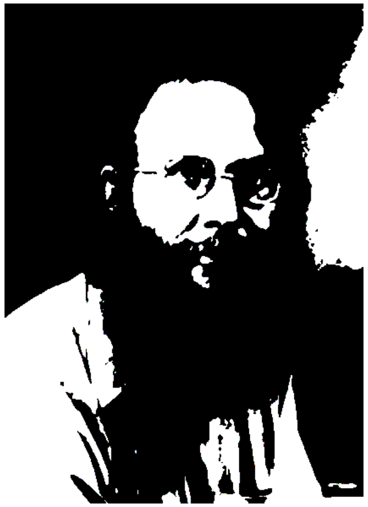
$$D_2 = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

Ordered Dither

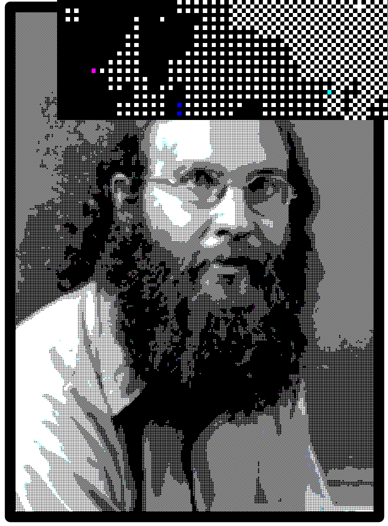
- Similar to half-toning. (And similar issue with contouring due to regularity.)



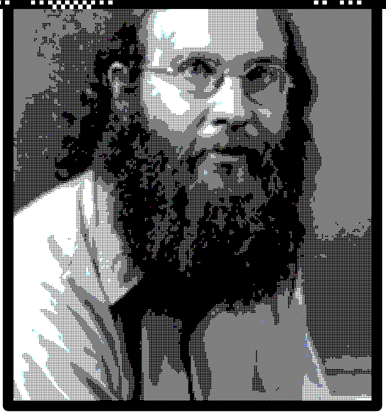
Original
(8 bits)



Uniform
(1 bit)



Half-toned
(1 bit)

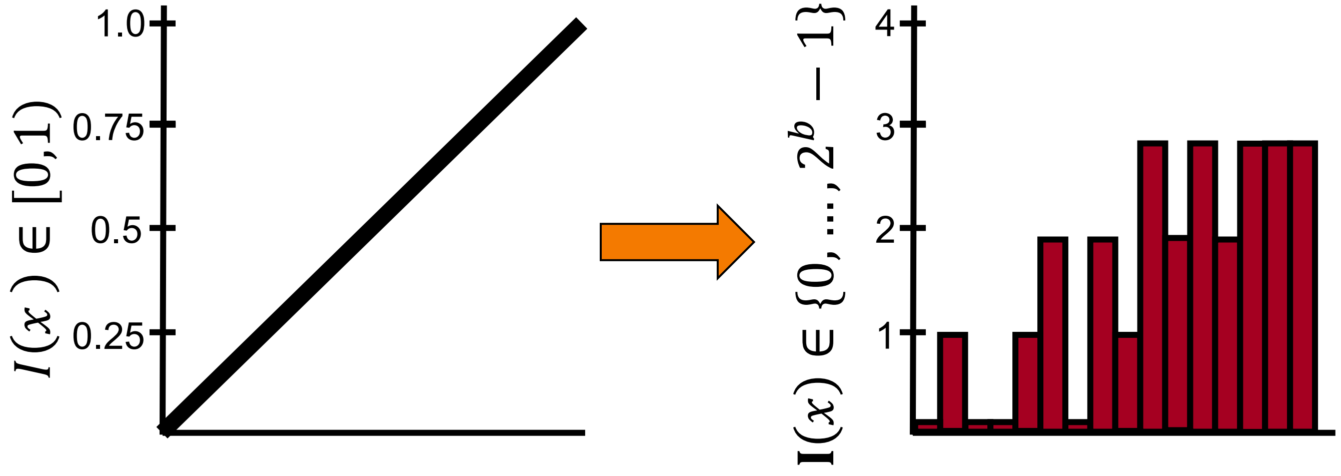


Ordered
(1 bit)

Random Dither

Rounding threshold defined randomly, rather than by a regular grid:

- The local average color is unchanged
- Error appears as noise, which our eye averages out



If a pixel is black, adding random noise to it, you are less likely to turn it into a white pixel then if the pixel were dark gray.

$$\mathbf{I}(x,y) = Q_b\left(I(x,y) + \frac{\text{noise}(x,y)}{2^b}\right)$$

Random Dither

Rounding threshold defined randomly, rather than by a regular grid:

Q: How much noise should we add?

A: Just enough so that we make it to the previous/next intensity value:

$$\text{noise}(x,y) \in (-1.0, 1.0)$$

$\in [0,1)$ 0.

0

ack,

m

u

<u>Warning!</u>

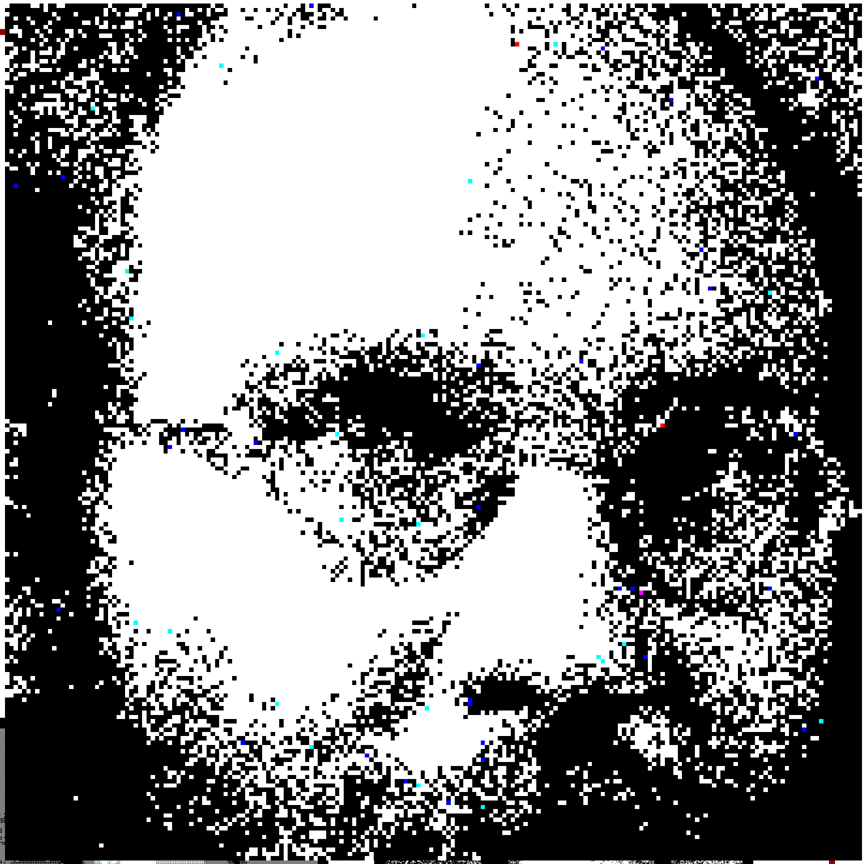
Adding noise may take you out of the $[0,1)$ range.

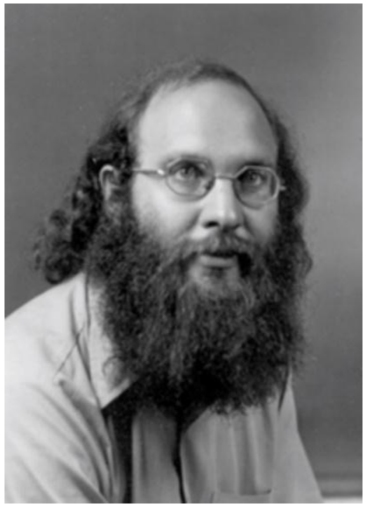
gray.

$$\mathbf{I}(x,y) = Q_b\left(I(x,y) + \boxed{\frac{\text{noise}(x,y)}{2^b}}\right)$$

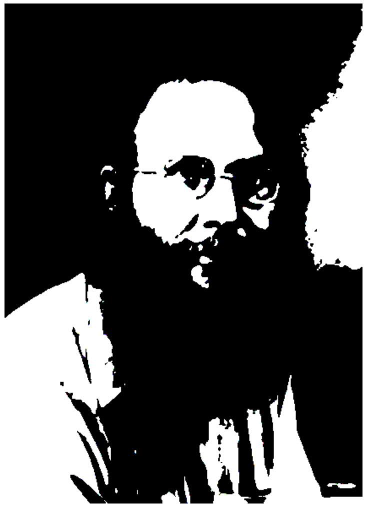
Ordered Dither

- No structure resulting from regular thresholding

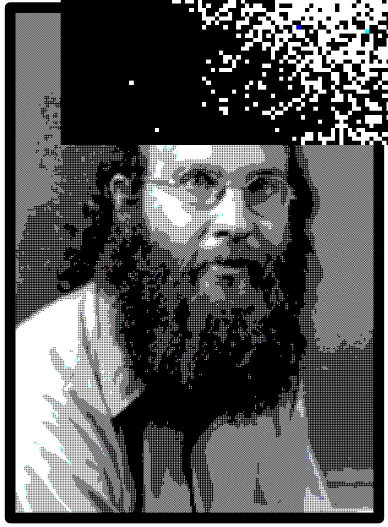




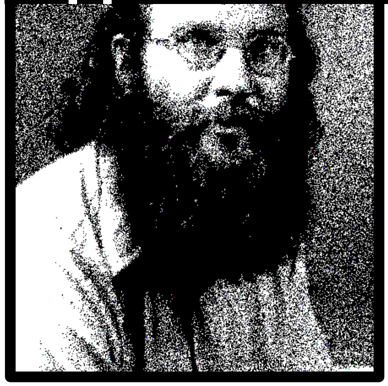
Original
(8 bits)



Uniform
(1 bit)



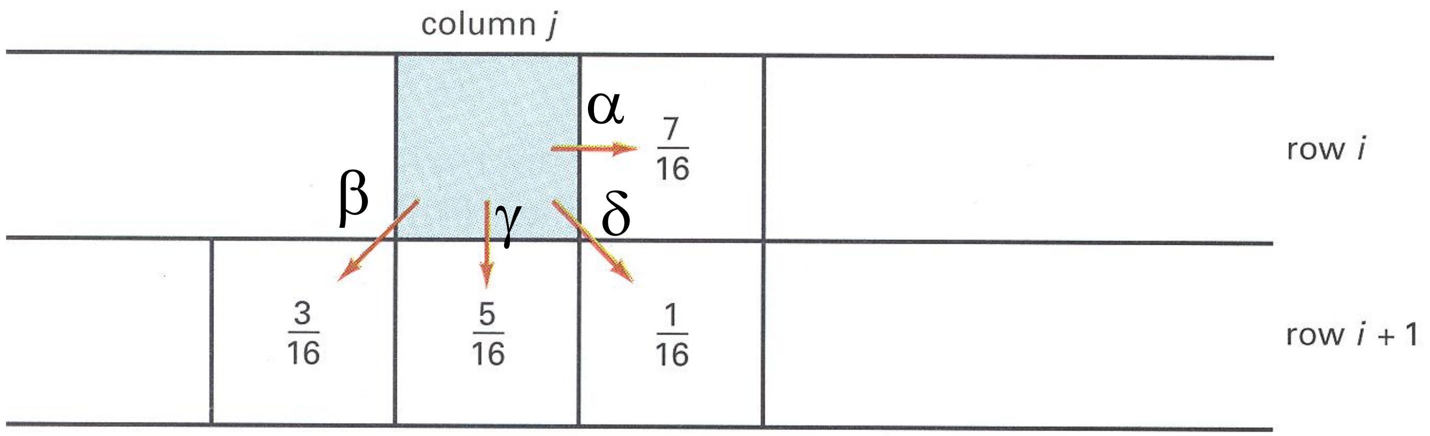
Ordered
(1 bit)



Random
(1 bit)

Error Diffusion Dither

- Process pixels left-to-right and top-to-bottom
- Distribute quantization error over unvisited neighboring pixels
  - Error dispersed to pixels right and below
- Floyd-Steinberg Method



$$\alpha + \beta + \gamma + \delta = 1$$

Figure 14.42 from H&B

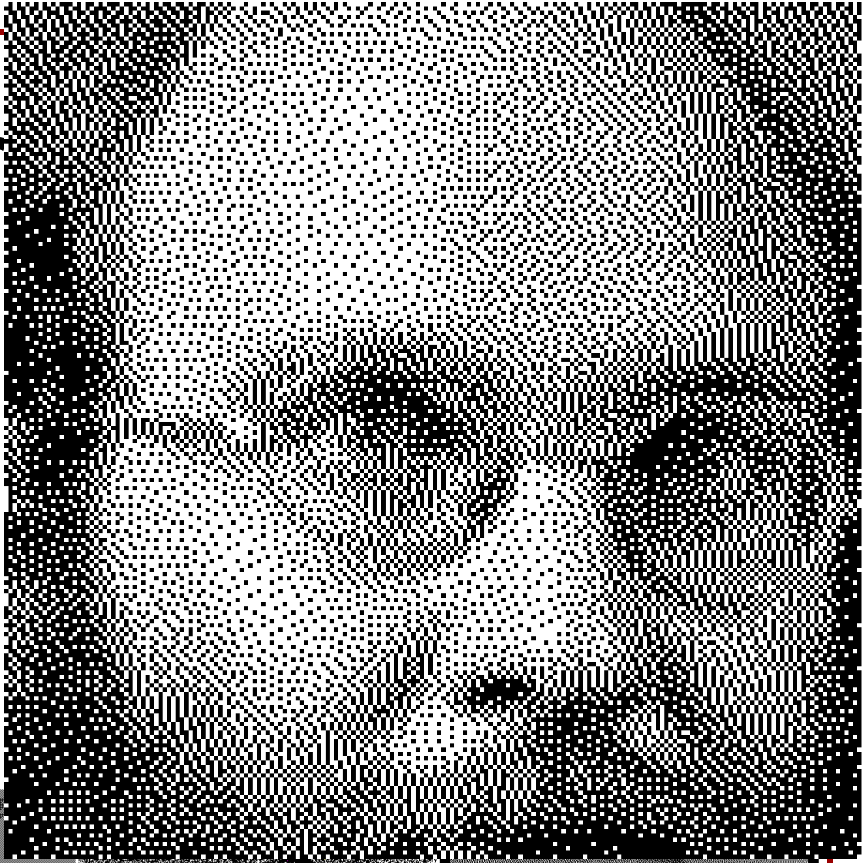
Error Diffusion Dither

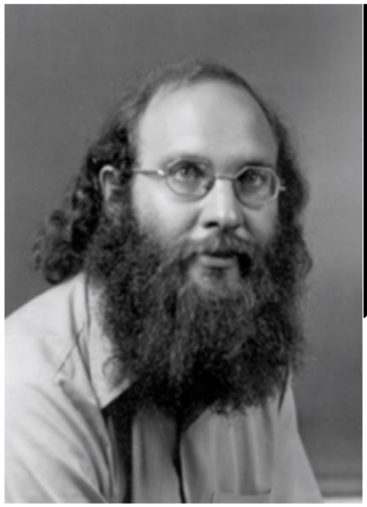
```
for( int j=0 ; j<height ; j++ ) for ( int i=0 ; i<width ; i++ )
{
    $Dest_{i,j}$ = quantize( $Source_{i,j}$ )
    error = $Source_{i,j}$ – $Dest_{i,j}$
    $Source_{i+1,j}$ += $\alpha$ * error
    $Source_{i-1,j+1}$ += $\beta$ * error
    $Source_{i,j+1}$ += $\gamma$ * error
    $Source_{i+1,j+1}$ += $\delta$ * error
}
```

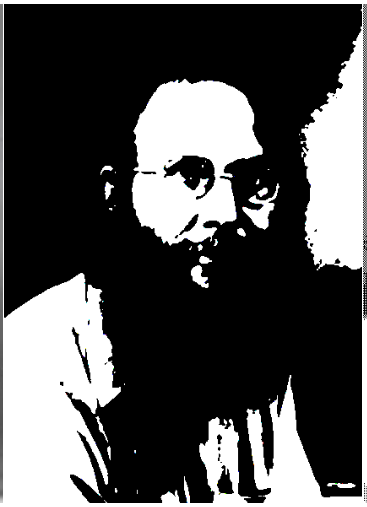
$$\alpha = \frac{7}{16}$$
$$\beta = \frac{3}{16}$$
$$\gamma = \frac{5}{16}$$
$$\delta = \frac{1}{16}$$

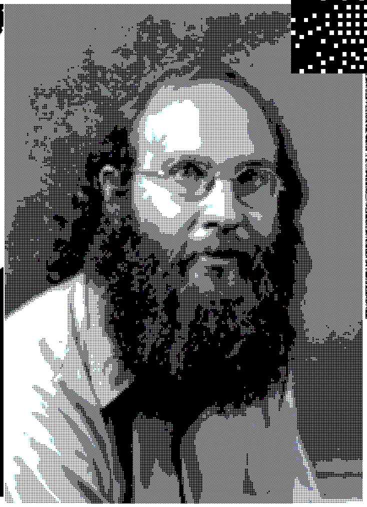
Floyd-Steinberg Dither

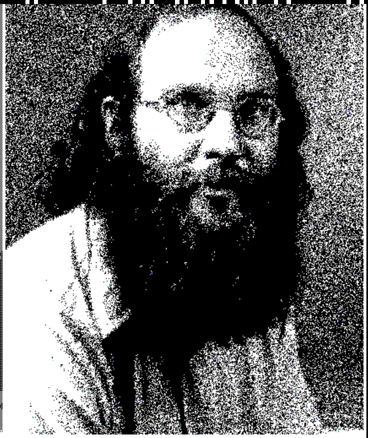
Error Diffusion Dither

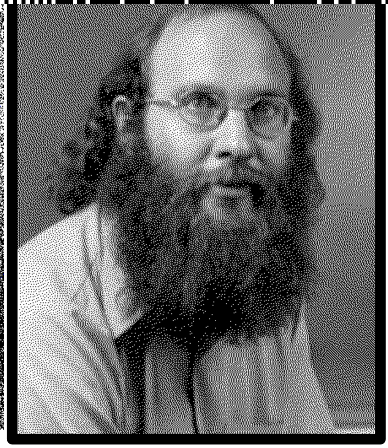



Original
(8 bits)


Uniform
(1 bit)


Ordered
(1 bit)


Random
(1 bit)

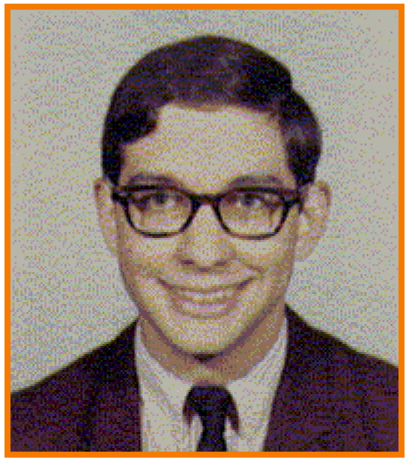

Floyd-Steinberg
(1 bit)

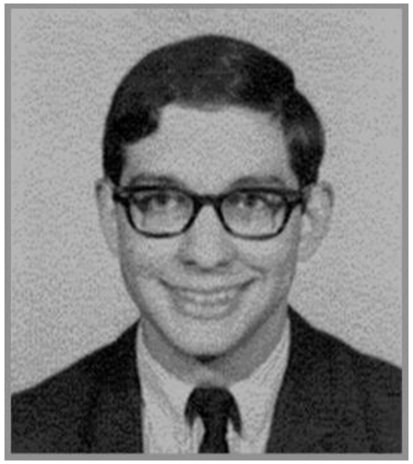
Outline

- Human Vision
- Image Representation
- Reducing Color Quantization Artifacts
- Basic Image Processing
  - Single Pixel Operations

Computing Grayscale

- The human retina perceives red, green, and blue as having different levels of brightness.
- To compute the luminance (perceived brightness) of a pixel, we need to take the <u>weighted</u> average of the RGBs:
  - $\mathbf{L}_p = 0.30 \cdot \mathbf{r}_p + 0.59 \cdot \mathbf{g}_p + 0.11 \cdot \mathbf{b}_p$


Original


Grayscale

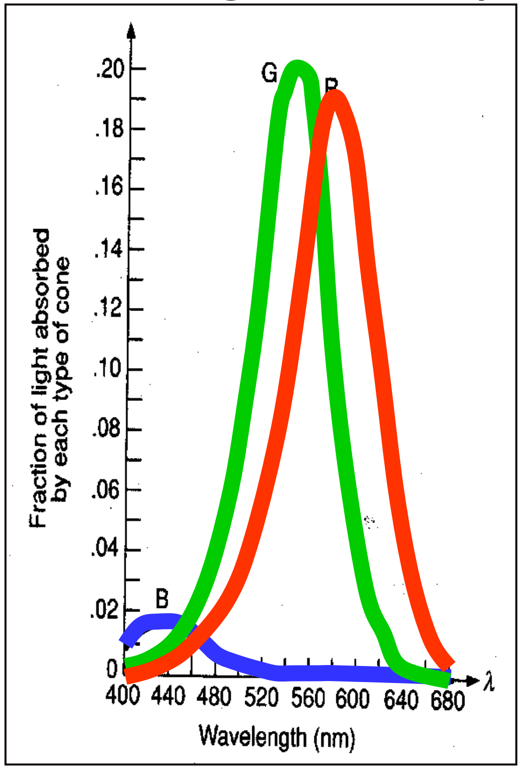
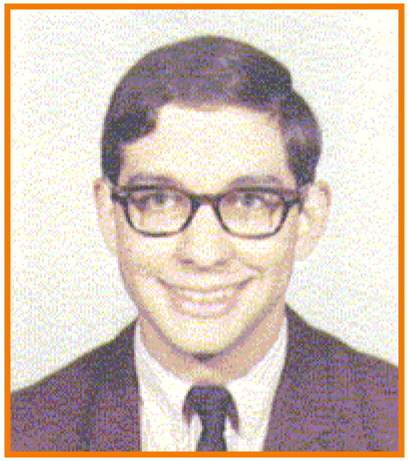


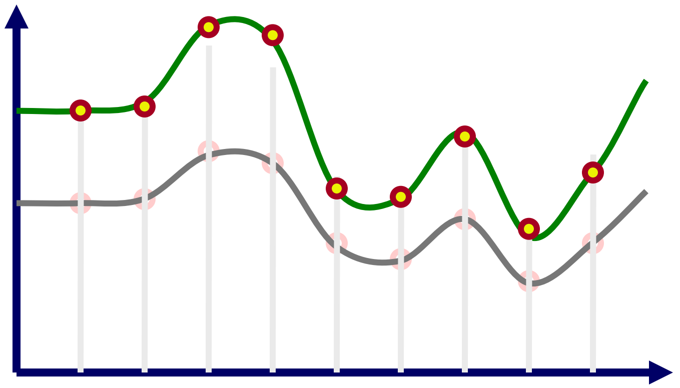
Figure 13.18 from FvDFH

Adjusting Brightness

- Scale pixel components
  - Must clamp to range -- e.g. to [0,255)

Original


Brighter

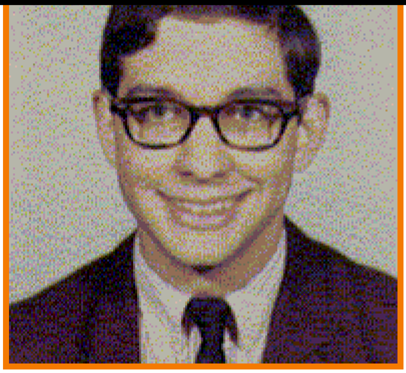


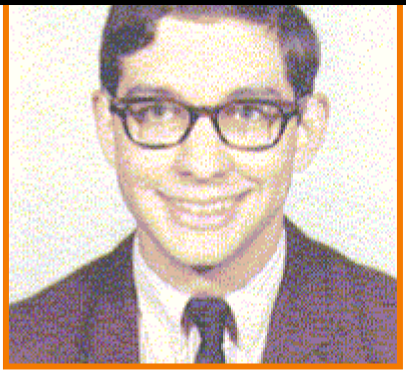
$$\mathbf{I}_p \leftarrow \mathbf{I}_p \cdot \alpha$$

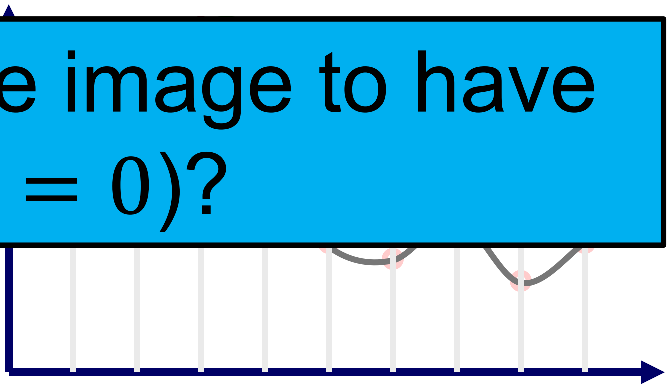
Adjusting Brightness

- Scale pixel components
  - Must clamp to range -- e.g. to [0,255)

What happens if we set the image to have no brightness ($\alpha = 0$)?


Original

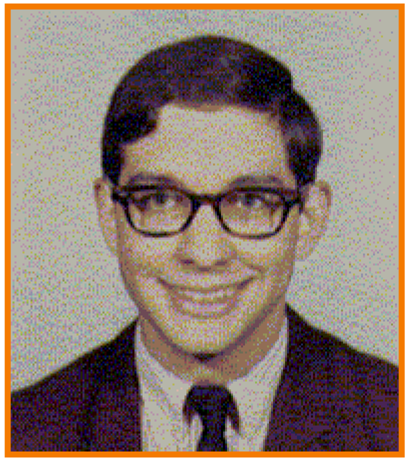

Brighter

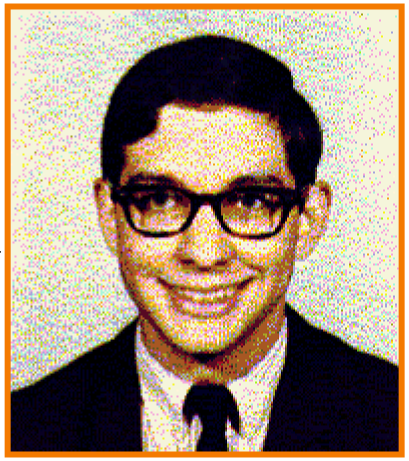


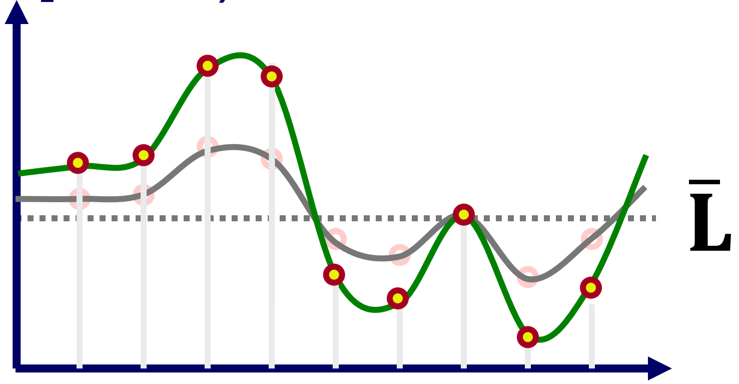
$\mathbf{I}_p \leftarrow \mathbf{I}_p \cdot \alpha$

Adjusting Contrast

- Compute mean image luminance $\bar{\mathbf{L}}$, averaged over all pixels
  - $\bar{\mathbf{L}} = \text{Average}(0.30 \cdot \mathbf{r}_p + 0.59 \cdot \mathbf{g}_p + 0.11 \cdot \mathbf{b}_p)$
- Scale deviation from $\bar{\mathbf{L}}$ for each pixel component
  - Must clamp to range -- e.g. to [0,255)


Original


More Contrast

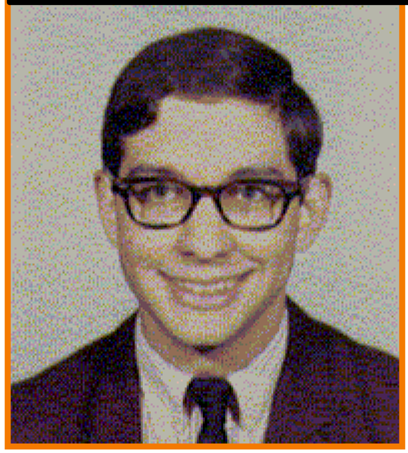


$$\mathbf{I}_p \leftarrow (\mathbf{I}_p - \bar{\mathbf{L}})\alpha + \bar{\mathbf{L}}$$

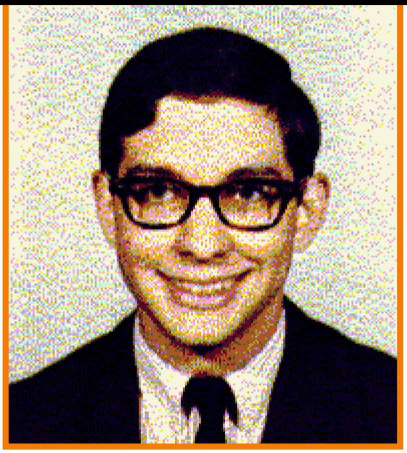
Adjusting Contrast

- Compute <u>mean image</u> luminance $\bar{\mathbf{L}}$, averaged over all pixels
  - $\bar{\mathbf{L}} = \text{Average}(0.30 \cdot \mathbf{r}_p + 0.59 \cdot \mathbf{g}_p + 0.11 \cdot \mathbf{b}_p)$

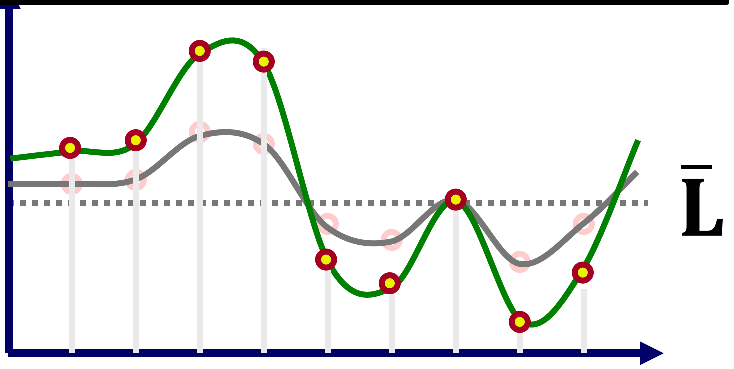
What happens if we set the image to have no contrast ($\alpha = 0$)?



Original



More Contrast

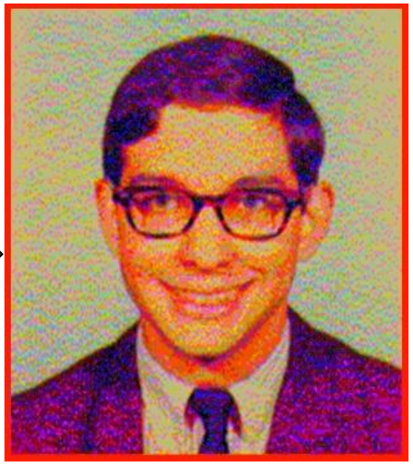


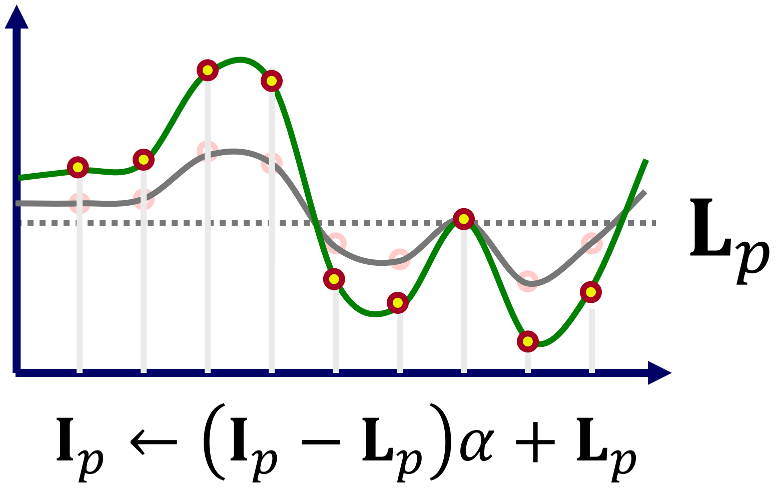
$$\mathbf{I}_p \leftarrow (\mathbf{I}_p - \bar{\mathbf{L}})\alpha + \bar{\mathbf{L}}$$

Adjusting Saturation

- Compute per-pixel luminance $\mathbf{L}_p$
    - $\mathbf{L}_p = 0.30 \cdot \mathbf{r}_p + 0.59 \cdot \mathbf{g}_p + 0.11 \cdot \mathbf{b}_p$
- Scale deviation from $\mathbf{L}_p$ for each pixel component
    - Must clamp to range -- e.g. to [0,255)

Original


More Saturation

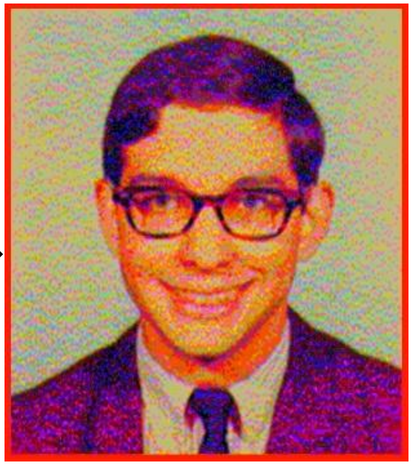

$\mathbf{L}_p$
$\mathbf{I}_p \leftarrow (\mathbf{I}_p - \mathbf{L}_p)\alpha + \mathbf{L}_p$

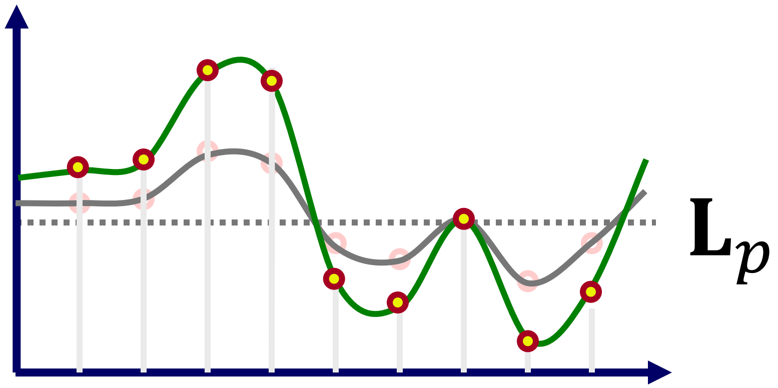
Adjusting Saturation

- Compute per-pixel luminance $\mathbf{L}_p$
  - $\mathbf{L}_p = 0.30 \cdot \mathbf{r}_p + 0.59 \cdot \mathbf{g}_p + 0.11 \cdot \mathbf{b}_p$

What happens if we set the image to have no saturation ($\alpha = 0$)?

Original


More Saturation



$$\mathbf{I}_p \leftarrow (\mathbf{I}_p - \mathbf{L}_p)\alpha + \mathbf{L}_p$$