Surface Reconstruction

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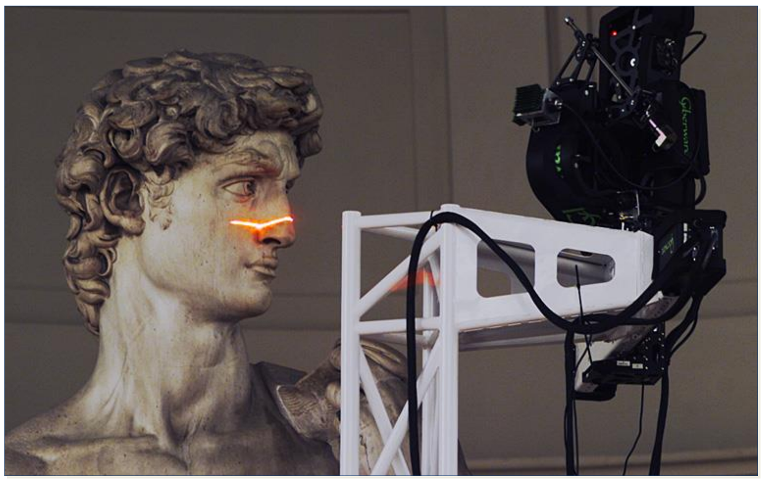
(601.457/657)

Motivation

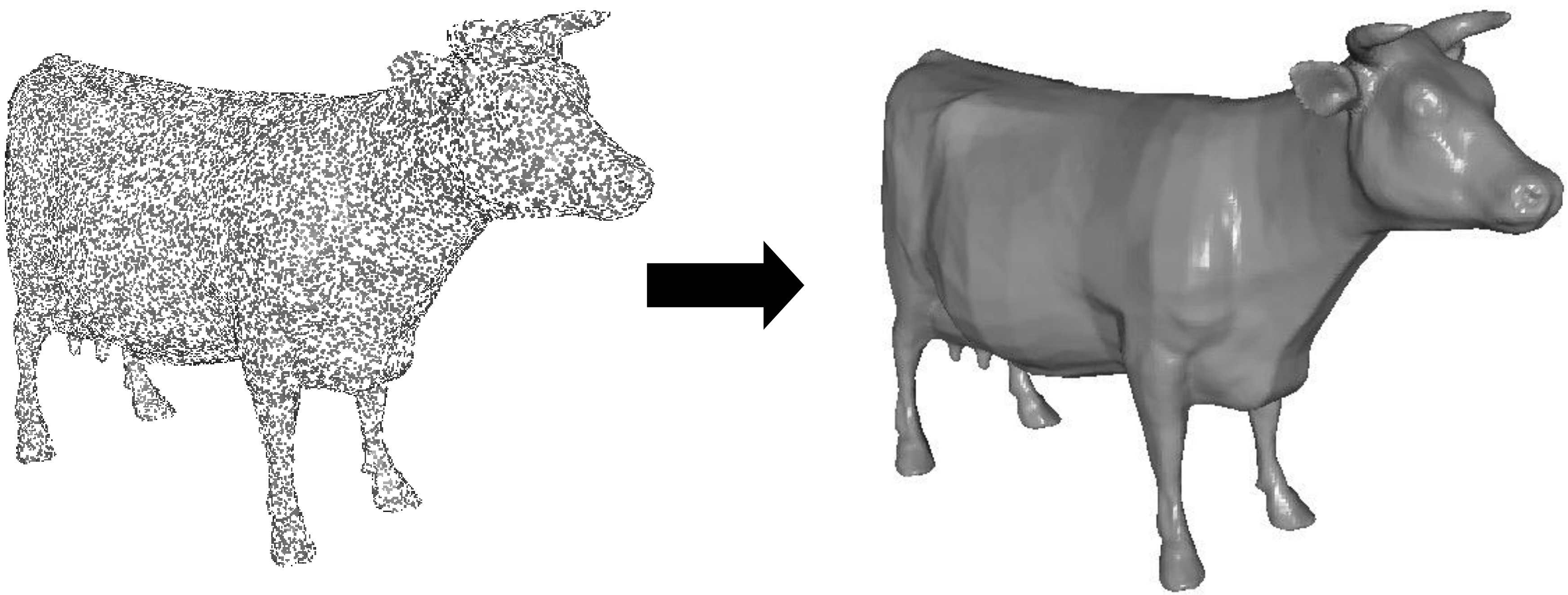
3D Scanners are ubiquitous (and cheap)



[Images courtesy of Rusinkiewicz, Strecha, createdigitalmotion.com, and NextEngine]

Motivation

Merged scans typically consist of un/semi-structured sets of points that need to be connected into a single (water-tight) model.



Related Work

[1] GVU Center Georgia Tech, Graphics Research Grupo, Variational Implicit Surfaces Web site: http://www.cc.gatech.edu/gvu/geometry/implicit/. [6] T. Gentils R. Smith A. Hilton, D. Beresford and W. Sun. Virtual people: Capturing human models to populate virtual worlds. In Proc. Computer Animation, page 174185, Geneva, Switzerland, 1999. IEEE Press. [7] Anders Adamson and Marc Alexa. Approximating and intersecting surfaces from points. In Proceedings of the Eurographics/ACM SIG- GRAPH Symposium on Geometry Processing 2003, pages 230{239. ACM Press, Jun 2003. [8] Anders Adamson and Marc Alexa. Approximating bounded, nonorientable surfaces from points. In SMI '04: Proceedings of Shape Modeling Applications 2004, pages 243{252, 2004. 153 [9] U. Adamy, J. Giesen, and M. John. Surface reconstruction using umbrella ¿ters. Computational Geometry, 21(1-2):63{86, 2002. [10] G. J. Agin and T. O.Binford. Computer description of curved objects. 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…/… [403]

[List courtesy of Alliez]

Related Work

Classification:

- Approach:
  - Computational Geometry
  - Implicit Surfaces
- Input:
  - Oriented vs. Unoriented Points
  - Structured vs. Unstructured Data
- Output:
  - Water-tight vs. Surface with Boundary

Related Work

Classification:

- Computational Geometry (Unoriented Points)
  - Use input to partition space
  - Use a subset of the partition to define the shape
- Implicit Surfaces (Oriented Points)
  - Fit implicit function to the input
  - Extract iso-surface

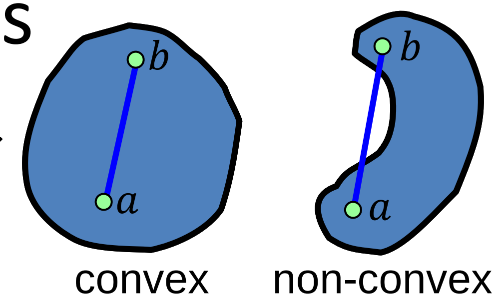
Outline

- Introduction
- Preliminaries
  - Convex Hulls
  - Delaunay Triangulations
  - Voronoi Diagrams
  - Medial Axes
- A sampling of methods
- Why is reconstruction hard?

Computational Geometry

Convex Hulls:

A set S is *convex* if for any two points $a, b \in S$, the line segment between a and b is also in S.



[Notes courtesy of Alliez]

Computational Geometry

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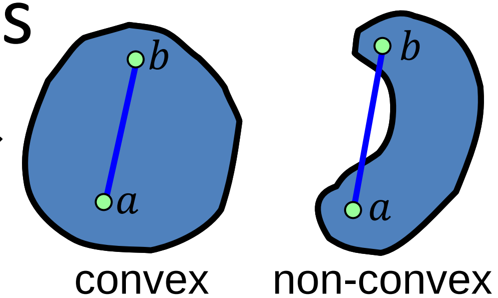
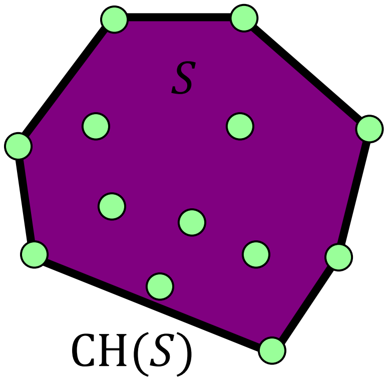


The *convex hull* of a set of points is the smallest convex set containing S.



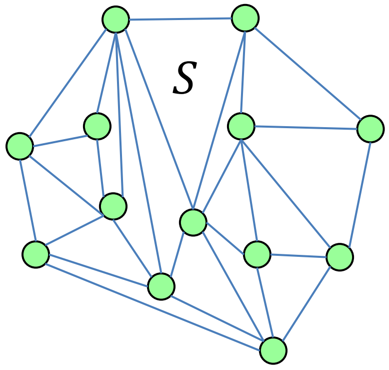
[Notes courtesy of Alliez]

Computational Geometry

Triangulation:

A *triangulation* of a set of sites/points S a decomposition of the convex hull of the points into triangles, whose vertex set is the set of sites/points.

- There are many ways to triangulate the set S.
- Not all are equally "good" (e.g. can have skinny triangles with small angles)

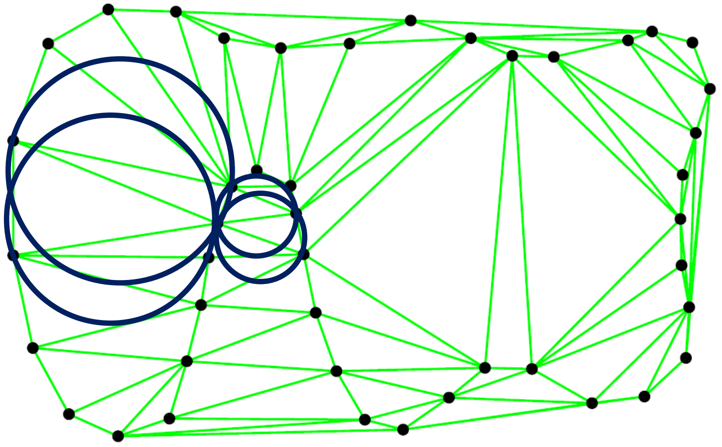


[Notes courtesy of Alliez]

Computational Geometry

Delaunay Triangulation:

A *Delaunay Triangulation* of a set of sites/points S is a triangulation with of S such that the circumscribing circle of any triangle contains no other site in S*.



Compactness Property:

This triangulation maximizes the minimum angle.

[*Assuming general position]

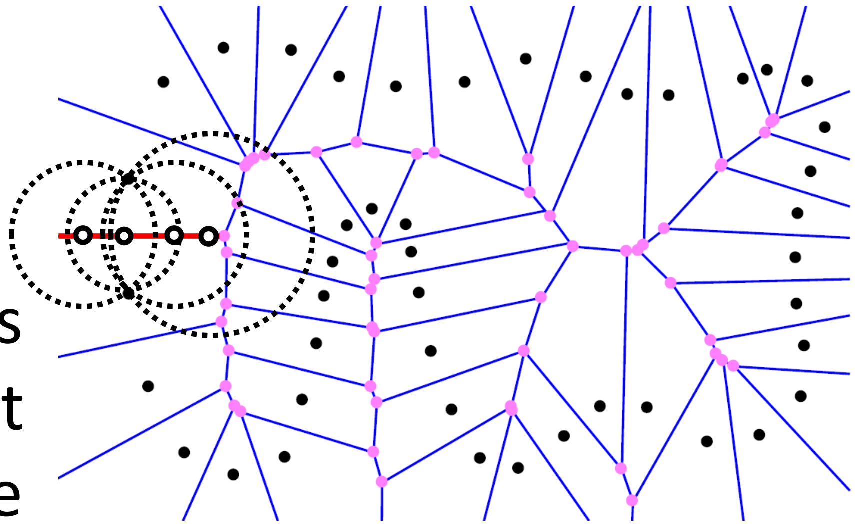
[Notes courtesy of Alliez]

Computational Geometry

Voronoi Diagrams:

The *Voronoi Diagram* of S is a partition of space into regions $\mathrm{VD}(s)$ $(s \in S)$ such that all points in $\mathrm{VD}(s)$ are closer to s than any other sites in S.

- Edges are equidistant from the two sites in the incident cells.
- For each edge point there is an empty circle, centered at the point, only touching the sites in the two incident cells.



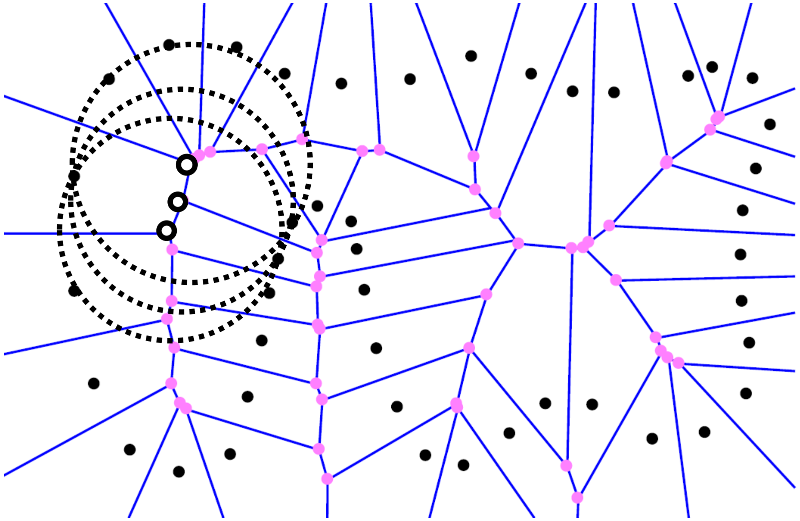
[Notes courtesy of Alliez]

Computational Geometry

<u>Voronoi Diagrams</u>:

The *Voronoi Diagram* of S is a partition of space into regions $\mathrm{VD}(s)$ ($s \in S$) such that all points in $\mathrm{VD}(s)$ are closer to s than any other sites in S.

- Vertices are equidistant from three (or more) sites in the incident cells.
- For a vertex, we can draw an empty circle, centered at the vertex, that just touches the sites in the three (or more) incident cells.



[Notes courtesy of Alliez]

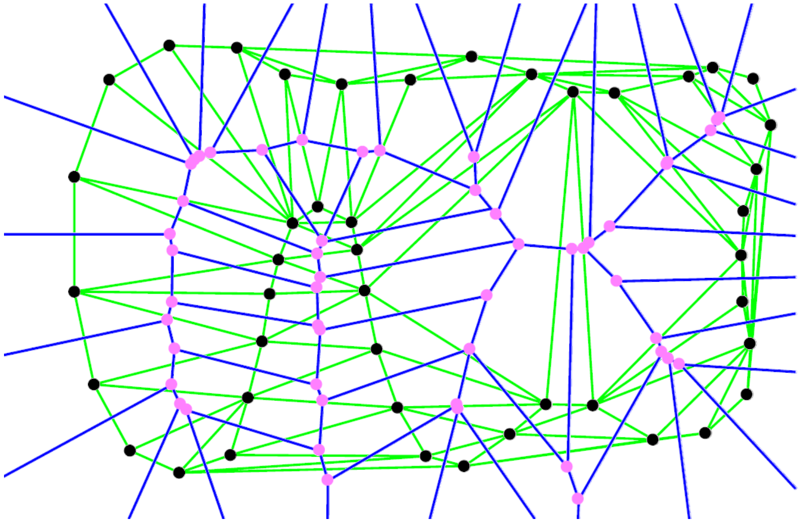
Computational Geometry

Voronoi Diagrams:

The *Voronoi Diagram* of S is a partition of space into regions $\mathrm{VD}(s)$ ($s \in S$) such that all points in $\mathrm{VD}(s)$ are closer to s than any other sites in S.

Duality:

Each Voronoi vertex is in one-to-one correspondence with a Delaunay triangle.

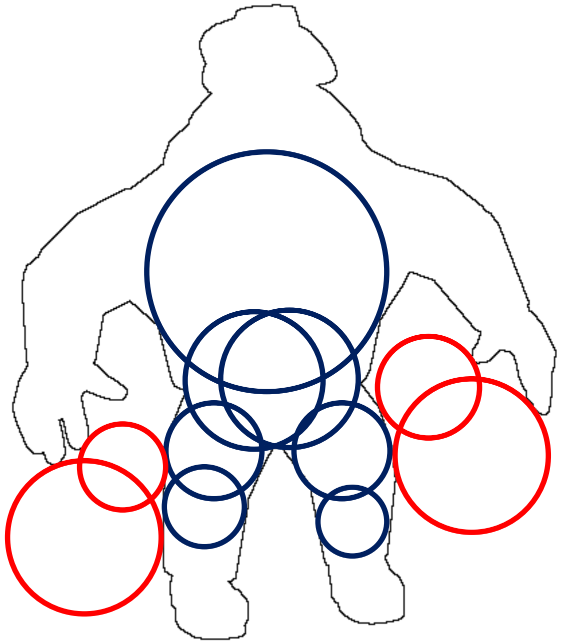


[Notes courtesy of Alliez]

Computational Geometry

Medial Axis:

For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.

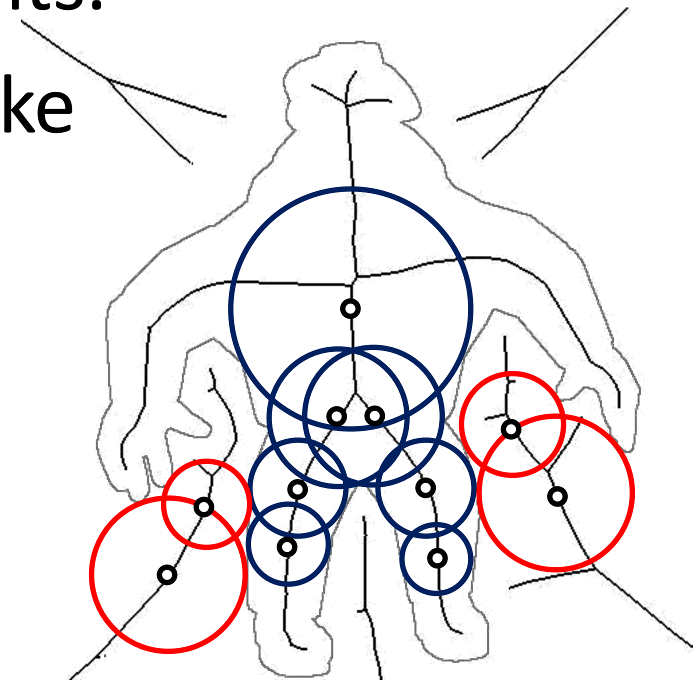


Computational Geometry

<u>Medial Axis</u>:

For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.

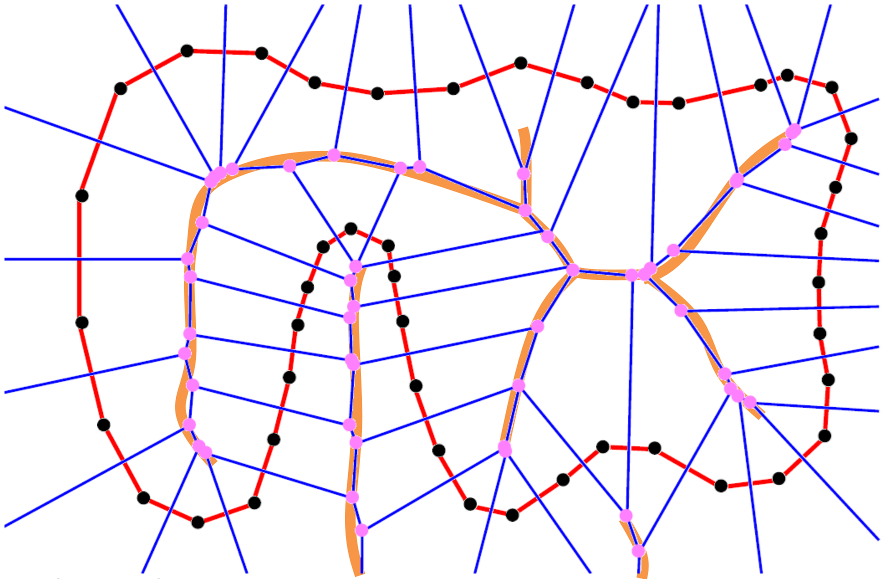
The centers of all such balls make up the *medial axis/skeleton*.



Computational Geometry

Observation in 2D*:

For a reasonable point sample, the medial axis is well-sampled by the Voronoi vertices.



*In 3D, this is only true for a subset of the Voronoi vertices – the *poles*.

Outline

- Introduction
- Preliminaries
- A sampling of methods
  - **Space Partitioning**
  - Crust
  - … from Unorganized Points
  - Poisson Reconstruction
- Why is reconstruction hard?

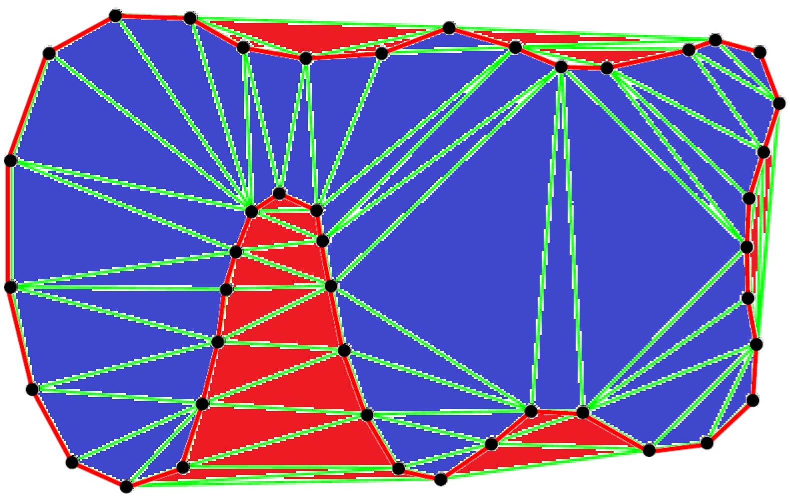
Computational Geometry (Space Partitioning, Crust)

Implicit Surfaces (… from Unorganized Points, Poisson Reconstruction)

Space Partitioning

Given a set of points, we can construct the Delaunay triangulation.

If we could label each triangle as inside/outside, then the surface of interest is the set of edges that lie between inside and outside triangles.

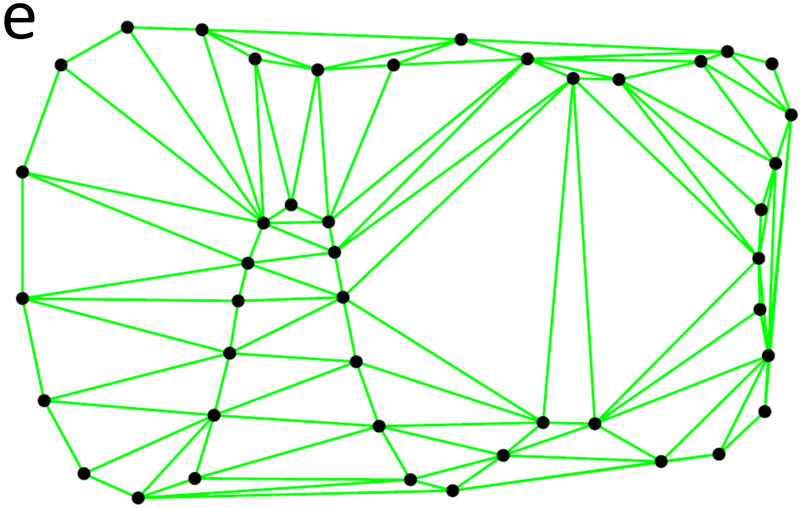


Space Partitioning

Q: How should we assign labels?

A: Spectral Partitioning [Kolluri et al. 2004]

1. Local: Assign a weight to each (interior) edge indicating if the two triangles should have the same label.
2. Global: Evenly partition the triangles minimizing the sum of the weights along partitioning edges.



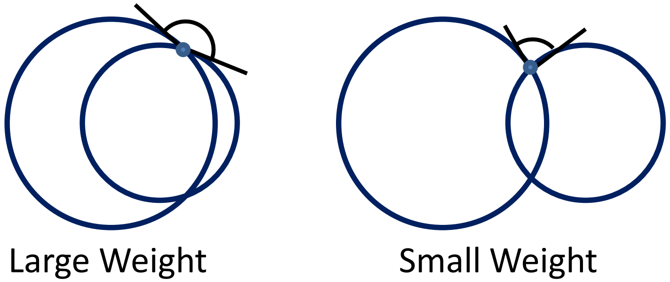
Space Partitioning

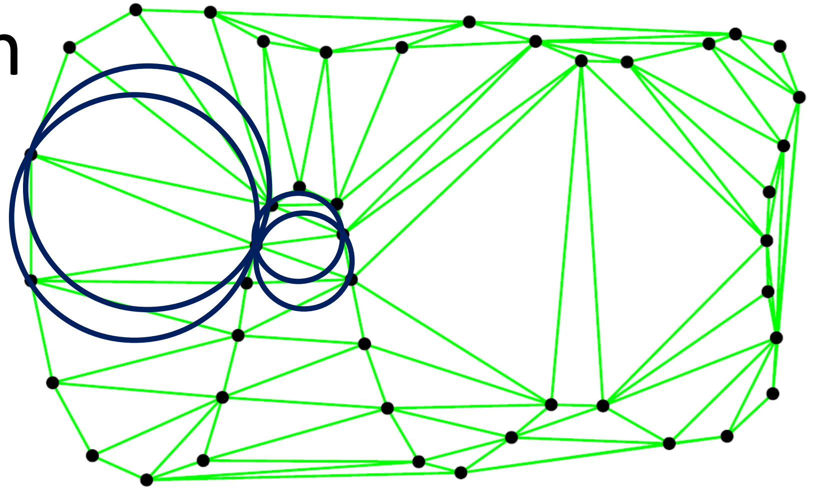
<u>Assigning Edge Weights</u>:

Q: When are triangles on opposite sides of an edge likely to have the same label?

A: If the triangles are on the same side, their circumscribing circles intersect <u>deeply</u>.

Use the angle of intersection to set the weight.





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Computational Geometry (Space Partitioning, Crust)

Implicit Surfaces (… from Unorganized Points, Poisson Reconstruction)

Crust [Amenta et al. 1998]

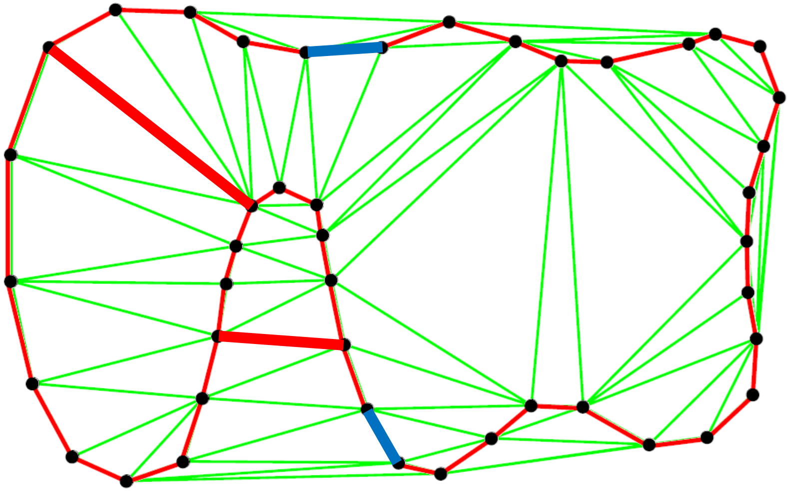
If we consider the Delaunay Triangulation of a point set sampling a curve, the curve should be (approximately) a subset of the Delaunay edges.

Q: How do we determine which edges to keep?

A: Two types of edges:

1. Those connecting adjacent points on the curve
2. Those traversing.

Discard those that traverse.

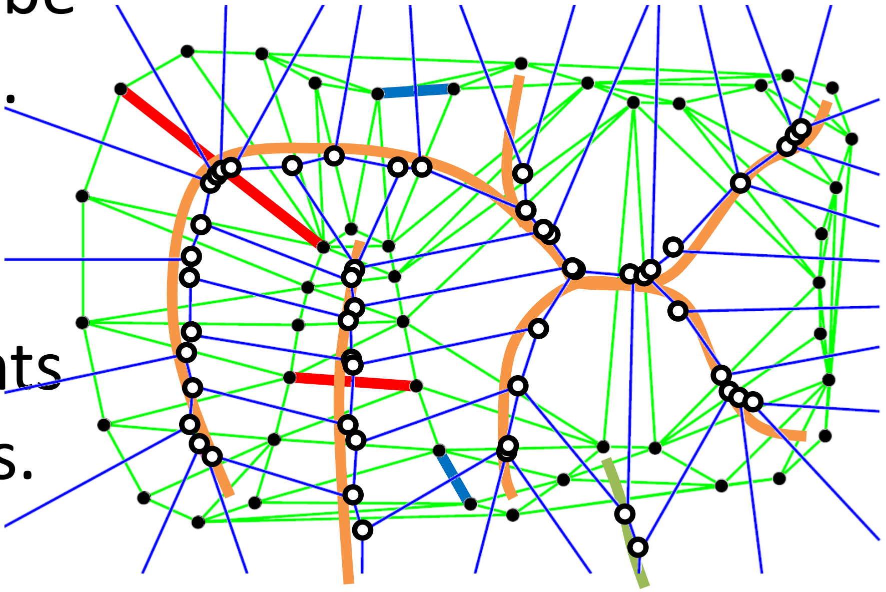


Crust [Amenta et al. 1998]

Observation:

Edges that traverse cross the medial axis.

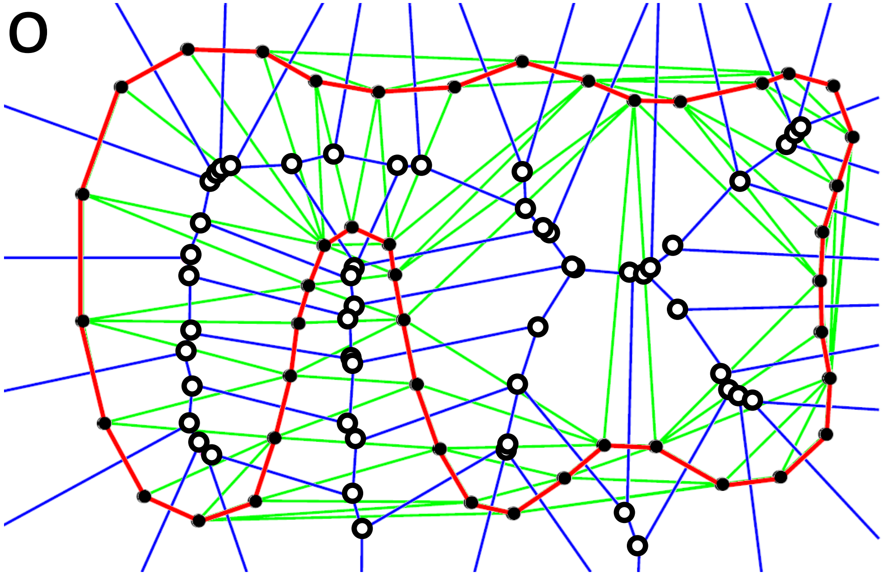
- Although we don't know the medial axis, we can sample it with the Voronoi vertices.
- Edges that traverse must be near the Voronoi vertices.
- We say an edge does not traverse if we can draw a circle through its endpoints empty of Voronoi vertices.



Crust [Amenta et al. 1998]

Algorithm:

1. Compute the Delaunay triangulation.
2. Compute the Voronoi vertices
3. Keep all edges for which there is a circle that contains the edge but no Voronoi vertices.



Note:
As opposed to the previous approach, it is not obvious that this will generate a closed, manifold curve/surface.

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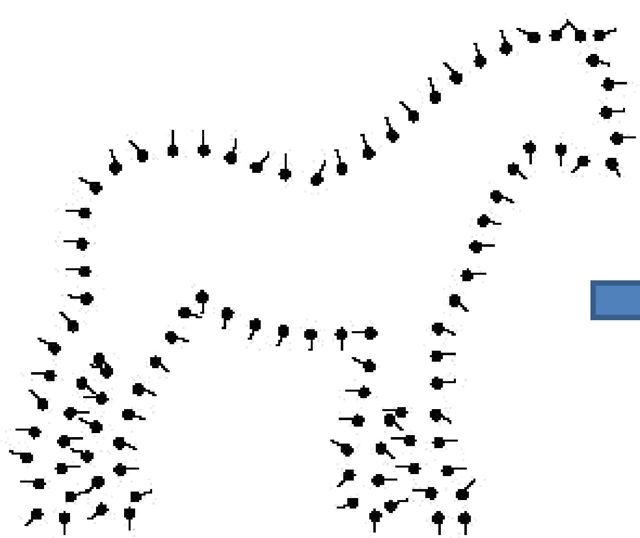
Computational Geometry (Space Partitioning, Crust)

Implicit Surfaces (… from Unorganized Points, Poisson Reconstruction)

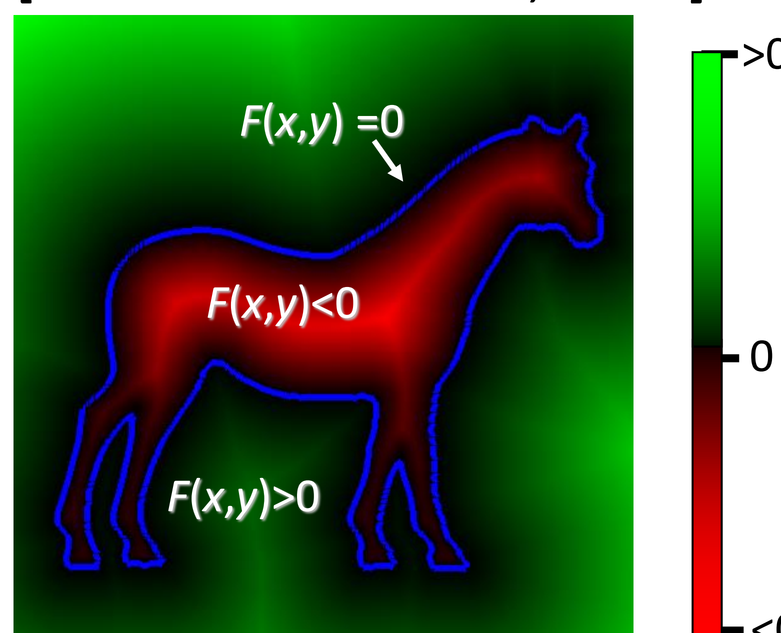
Implicit Surface Reconstruction

<u>Key Idea</u>:

- Use the point samples to define a function whose value at each sample positions is zero.
- Extract the zero level set. [Lorensen and Cline, 1987]



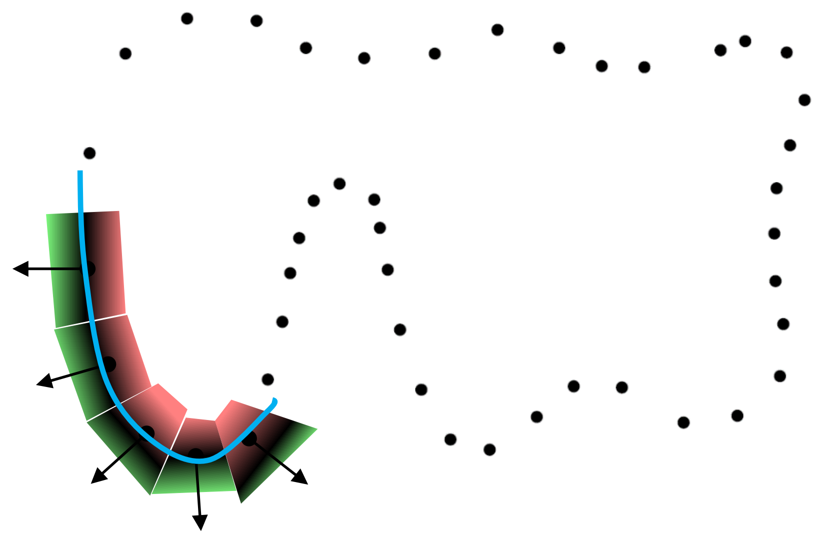
Sample Points



$F(x, y) = 0$

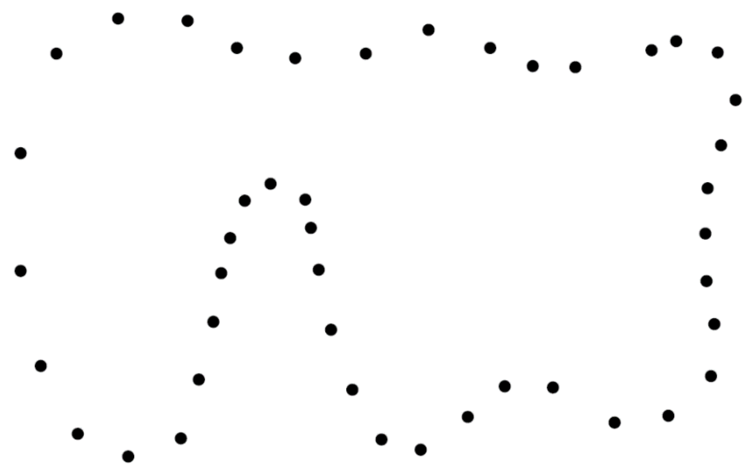
… Unorganized Points [Hoppe et al. 1992]

- Compute a local *signed distance function* by using the sample normals to define a **local** linear approximation to the function.
- Blend the linear approximations.
- Extract the zero level (where defined).



… Unorganized Points [Hoppe et al. 1992]

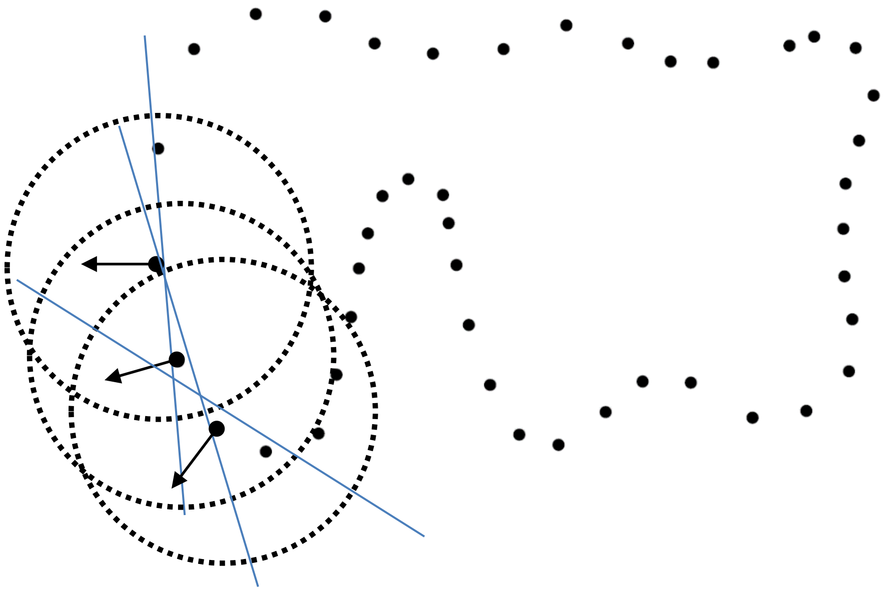
Q: How do we get the normals?



… Unorganized Points [Hoppe et al. 1992]

Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.



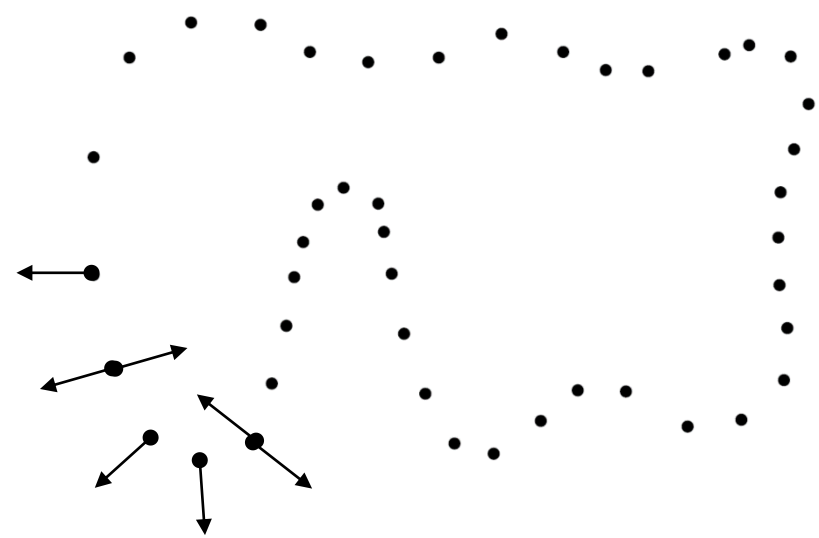
… Unorganized Points [Hoppe et al. 1992]

Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.

This doesn't guarantee a consistent orientation!

For the orientation to be consistent, neighboring points should point in the same direction.

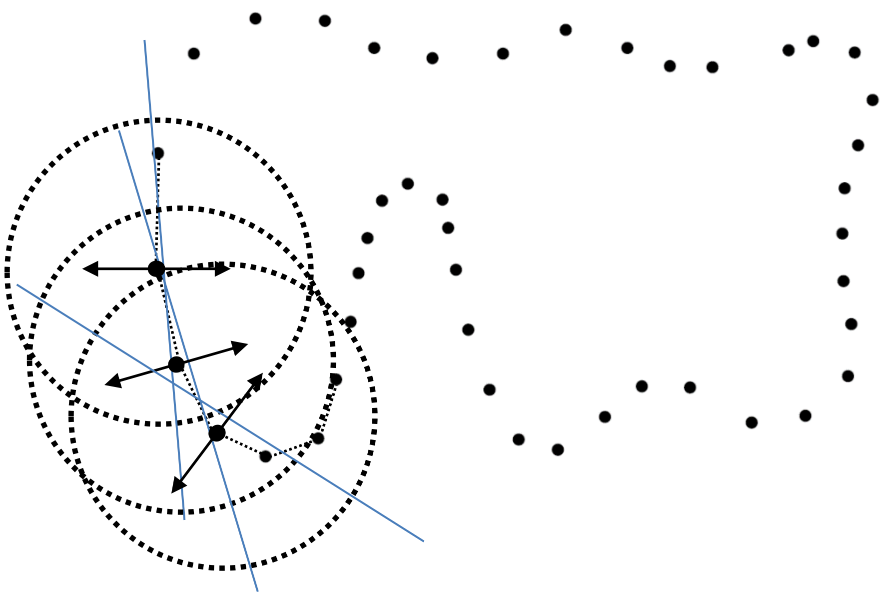


… Unorganized Points [Hoppe et al. 1992]

Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.

A2: Build a (Euclidian) minimal spanning tree and propagate the orientation from a root.



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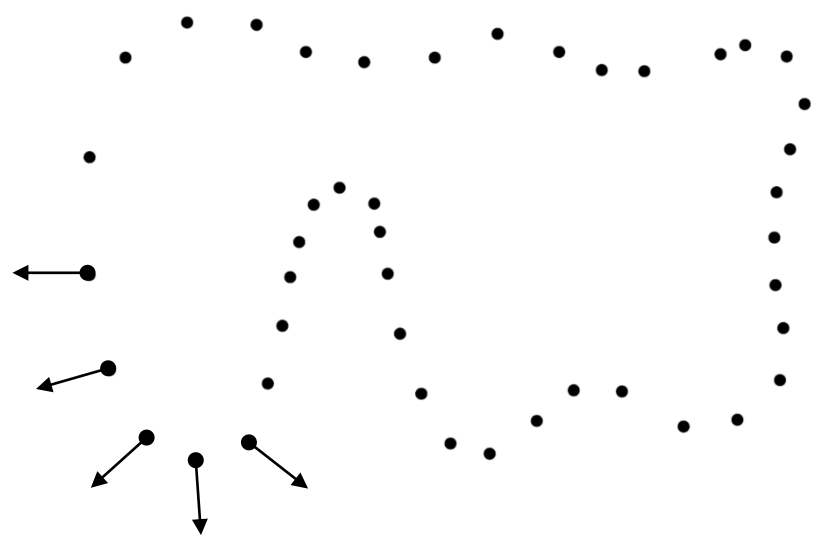

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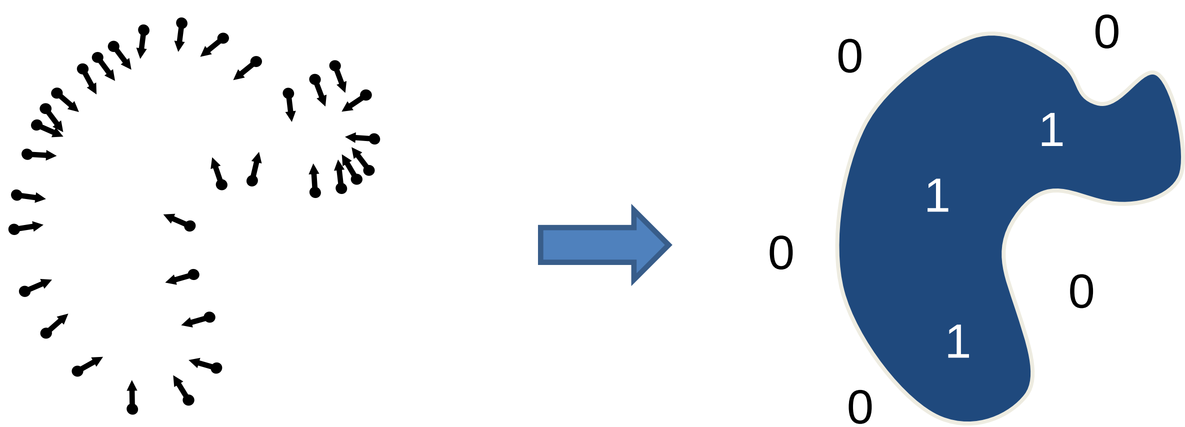
Computational Geometry (Space Partitioning, Crust)

Implicit Surfaces (… from Unorganized Points, Poisson Reconstruction)

Poisson Reconstruction [Kazhdan et al. 2006]

Reconstruct the *indicator function* of the surface and then extract the boundary.

Q: How to fit the function to the samples?



Oriented points

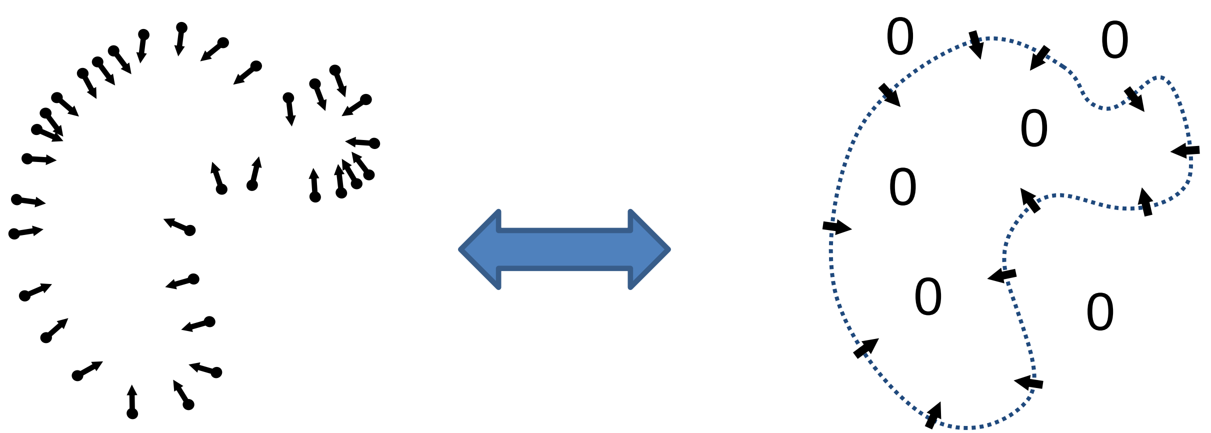
Indicator function

Poisson Reconstruction [Kazhdan et al. 2006]

Reconstruct the *indicator function* of the surface and then extract the boundary.

Q: How to fit the function to the samples?

A: Normals are samples of function's gradients.



Oriented points

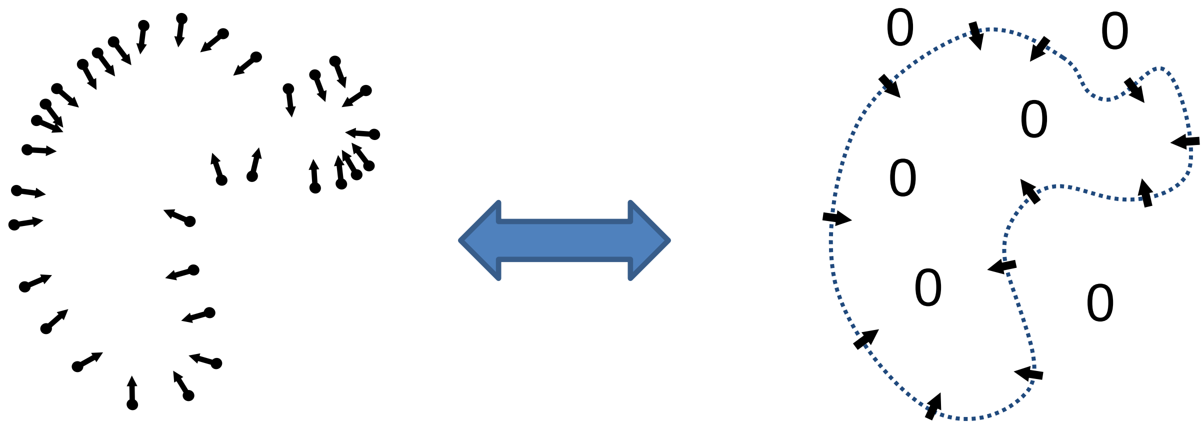
Indicator gradient

Poisson Reconstruction [Kazhdan et al. 2006]

To fit a scalar field F to the gradients $\vec{V}$ solve:

$$\nabla F = \vec{V}$$

- ✘ This is an over-constrained problem, so there is (usually) no solution.



Oriented points

Indicator gradient

Poisson Reconstruction [Kazhdan et al. 2006]

To fit a scalar field F to the gradients $\vec{V}$ solve:

$$\nabla F = \vec{V}$$

✘ This is an over-constrained problem, so there is (usually) no solution.

✓ Solve for the best (least-squares) solution:

$$\arg\min_F \left\| \nabla F - \vec{V} \right\|^2$$

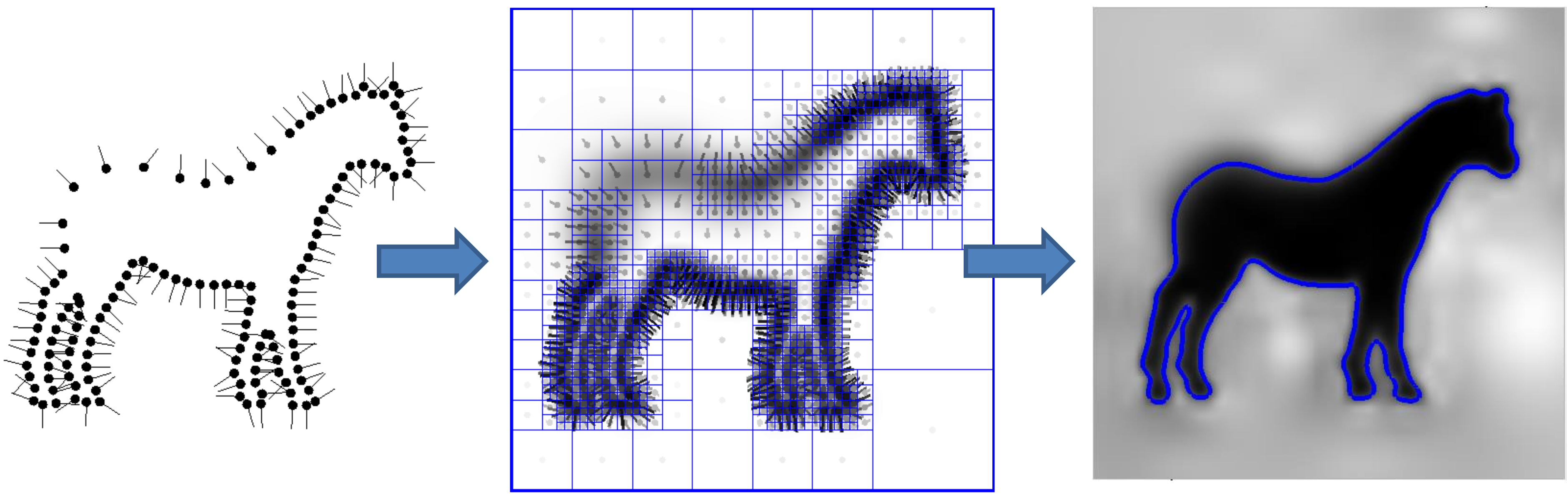
⇒ Taking the divergence, this becomes:

$$\nabla \cdot \left(\nabla F - \vec{V} \right) = 0 \quad \Leftrightarrow \quad \Delta F = \nabla \cdot \vec{V}$$

Poisson Reconstruction [Kazhdan et al. 2006]

Algorithm:

1. Transform samples into a vector field.
2. Fit a scalar-field to the gradients.
3. Extract the isosurface.



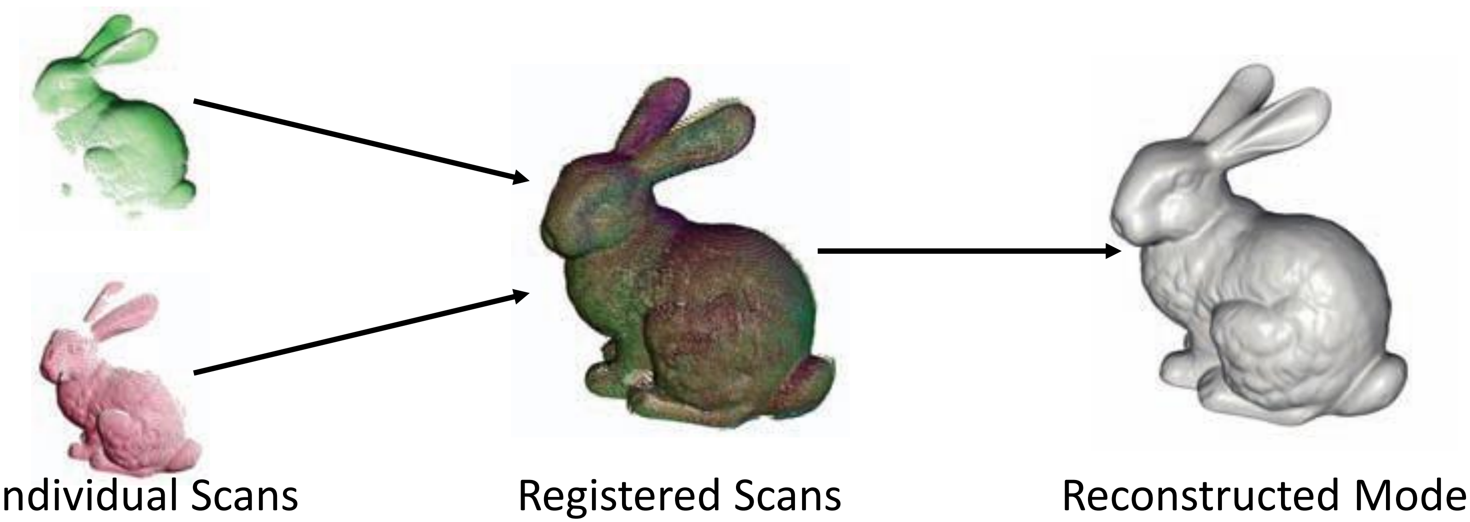
Outline

- Introduction
- Preliminaries
- A sampling of methods
- Why is reconstruction hard?

Why is Reconstruction Hard?

The point-set is often the result of:

- Scanning
- Registering
- Etc.



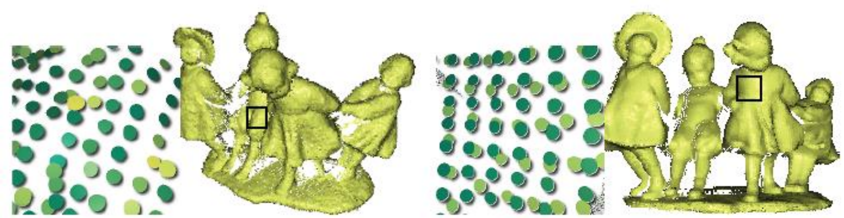
[Image courtesy of Bolitho]

Why is Reconstruction Hard?

Susceptible to:

- Scanning
  - Nonuniform sampling
  - Grazing angles
  - Scanner noise
  - Imprecise estimates
- Registering
  - Misalignment
  - Non-linear camera model

(a) Uniform sampling (b) Nonuniform sampling



(c) Noisy data (d) Misaligned scans

[Image courtesy of Berger *et al*.]

Practical Concerns

- Performance in the presence of bad data
- Interpolating vs. approximating
- Efficiency of reconstruction
- Quality guarantees
- Manifold / water-tight
- Incorporation of prior knowledge