Radiosity

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(601.457/657)
Announcements

• Apparently Apple may longer be supporting more advanced OpenGL functionality.
  ○ Most likely, this will mean that you will not be able to get vertex arrays up and running, which means you will not be able to implement shaders.
  ○ If you are having trouble with segmentation faults, comment out the shader code I provided in the updated source release.

It may be possible to bypass this problem by including `<OpenGL/gl3.h>` in your code. Maybe.
Overview

• Ray Tracing Revisited
• Radiosity
Ray Casting

Ray tracing is based on the Phong lighting model:

- A surface reflects light non-uniformly, with stronger reflection in the specular direction:

\[ I = I_E + \sum_{L} \left[ K_A \cdot I_L^A + \left( K_D \cdot \langle \vec{N}, \vec{L} \rangle + K_S \cdot \langle \vec{V}, \vec{R} \rangle^n \right) \cdot I_L \cdot S_L \right] \]
Ray Tracing

Ray tracing is based on the Phong lighting model:

For the same reason, we only cast secondary rays in the reflected direction – to maximize the contribution to the lighting computation.

\[
I = I_E + \sum_L \left[ K_A \cdot I_L^A + \left( K_D \cdot \langle \vec{N}, \vec{L} \rangle + K_S \cdot \langle \vec{V}, \vec{R} \rangle^n \right) \cdot I_L \cdot S_L \right] + K_S \cdot I_R
\]
Ray Tracing

Advantage:

- It does a good job of capturing the specular properties of materials.
Ray Tracing

Advantage:
- It does a good job of capturing the specular properties of materials.

Disadvantages:
- Difficult to support soft shadows from area lights
- Difficult to support caustics
- Need the ambient term as a hack for the global illumination
Lighting

What do we really want to compute?

The accumulation of light coming in from all directions, modulated by how much the light is reflected in/from that direction.

Typically, one uses Monte-Carlo sampling to generate more reflected rays in directions that contribute more strongly.
Lighting

What do we really want to compute?

The brightness of the light that reaches the camera eye, \( e \), from some point, \( p \), in the scene is the sum of the light:

- Emitted from \( p \), to \( e \), and
- Emanating from every point in the scene scaled by the extent to which it is reflected through \( p \) to \( e \).
Lighting

What do we really want to compute?

Light emitted from \( p \) to \( e \)

Light arriving at \( e \) from \( p \)

Fraction of light arriving from \( s \) that is reflected towards \( e \)

Amount of light arriving at \( p \) from \( q \)

\[
B(p \rightarrow e) = E(p \rightarrow e) + \int_{\Omega} F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p) dq
\]
Lighting

Challenge:

- The integral needs to be computed over many points because the function is discontinuous.
- The function is recursive since the amount of light entering a point depends on the amount of light leaving it.

\[ B(p \rightarrow e) = E(p \rightarrow e) + \int_{\Omega} F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p) dq \]
Radiosity

Make simplifying assumptions about the scene:

- Perceived brightness is equal in all directions

\[ B(p \rightarrow e) = E(p \rightarrow e) + \int_{\Omega} F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p) dq \]
Radiosity

Make simplifying assumptions about the scene:

- Perceived brightness is equal in all directions
  - Lights: modeled by uniformly emissive surfaces

\[ B(p \rightarrow e) = E(p \rightarrow e) + \int_{\Omega} F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p) dq \]
Radiosity

Make simplifying assumptions about the scene:

- Perceived brightness is equal in all directions
  - Lights: modeled by uniformly emissive surfaces
  - Objects: surfaces are Lambertian

\[
B(p \rightarrow e) = E(p \rightarrow e) + \int_{\Omega} F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p) \, dq
\]
Lambertian Lighting

Assuming the receiver $p$ is directed at $q$:

The perceived brightness is independent of the viewer’s position with respect to the surface.

Under our assumptions, point $q$ is equally bright to $p_1$ and $p_2$. 
Lambertian Lighting

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Under our assumptions, point \( q \) is equally bright to \( p_1 \) and \( p_2 \).

However, a patch of size \( A \) about \( p \) gets contributions from a patch of size \( A / \cos \theta \) about \( q \).
Lambertian Lighting

Assuming the receiver $p$ is directed at $q$:

The perceived brightness is independent of the viewer’s position with respect to the surface.

Under our assumptions, point $q$ is equally bright to $p_1$ and $p_2$.

However, a patch of size $A$ about $p$ gets contributions from a patch of size $A / \cos \theta$ about $q$.

Thus, under our assumptions, the amount of light $q$ emits/reflects to a point $p$ falls off as $\cos \theta$. 
Lambertian Lighting

If the receiver $p$ is not directed at $q$:

The brightness emanating from a unit-area patch at $q$ will be spread out across a (larger) patch at $p$.

⇒ The perceived brightness, per unit area, at a point $p$ falls off as $\cos \theta$
Lambertian Lighting

The area of a sphere with radius $r$ is $4\pi r^2$

⇒ If we fix the spherical angle, the area of a patch grows quadratically with the radius.

⇒ Perceived brightness decays with the square of the distance.
Lambertian Lighting

Given surface points $p$ and $q$, if:

- $\theta_{q,p}$ is the angle between the normal at $q$ and the direction from $s$ to $p$
- $\theta_{p,q}$ is the angle between the normal at $p$ and the direction from $p$ to $q$

The brightness from $s$ to $p$ falls off as:

$$\frac{\cos \theta_{q,p} \cdot \cos \theta_{p,q}}{\|q - p\|^2}$$
Radiosity

Make simplifying assumptions about the scene:

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So how does this affect the lighting equation?
Radiosity

So how does this affect the lighting equation?

- The perceived brightness of an emitting surface at $p$ is constant in all directions.

\[
B(p \rightarrow e) = E(p \rightarrow e) + \int_{\Omega} F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p) dq
\]
Radiosity

So how does this affect the lighting equation?

- The perceived brightness of an emitting surface at $p$ is constant in all directions.

\[
B(p \rightarrow e) = E(p) + \int_{\Omega} F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p) \, dq
\]
Radiosity

So how does this affect the lighting equation?

- The perceived brightness of an emitting surface at \( p \) is constant in all directions.
- The perceived brightness of a light is independent of the viewer’s position.

\[
B(p \rightarrow e) = E(p) + \int_{\Omega} F_r(q \rightarrow p \rightarrow e) \cdot B(q \rightarrow p) \, dq
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Radiosity

So how does this affect the lighting equation?

- The perceived brightness of an emitting surface at \( p \) is constant in all directions.
- The perceived brightness of a light is independent of the viewer’s position.

\[
B(p) = E(p) + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) \, dq
\]
Radiosity

The fraction of light from point $q$ that is reflected off of $p$ is determined by:

- The angle: $\theta_{q,p}$
- The angle: $\theta_{p,q}$
- The distance from $q$ to $p$: $\|q - p\|$
- The visibility of $p$ from $q$: $V(q, p)$
- The material properties at $p$: $\rho(p)$

$$F_r(q \rightarrow p) = \rho(p) \cdot V(q, p) \cdot \frac{\cos \theta_{q,p} \cdot \cos \theta_{p,q}}{\|q - p\|^2}$$

$$B(p) = E(p) + \int_{\Omega} F_r(q \rightarrow p) \cdot B(q) \, dq$$
Radiosity

Make simplifying assumptions about the scene:

- Perceived brightness is equal in all directions
  - Lights: modeled by uniformly emissive surfaces.
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The radiosity equation

\[ B(p) = E(p) + \rho(p) \int_{\Omega} V(q, p) \cdot \frac{\cos \theta_{q,p} \cdot \cos \theta_{p,q}}{\|q - p\|^2} \cdot B(q) dq \]
Radiosity

Approximate the solution by decomposing surfaces into patches and doing a discrete summation:

\[ B_i = E_i + \rho_i \sum_{j=1}^{n} F_{j,i} \cdot B_j \]

Form Factor

\[ B(p) = E(p) + \rho(p) \int_{\Omega} V(q, p) \cdot \frac{\cos \theta_{q,p} \cdot \cos \theta_{p,q}}{\|q - p\|^2} \cdot B(q) dq \]

The radiosity equation
Form Factor

The form factor $0 \leq F_{j,i} \leq 1$ is the proportion of the total power leaving patch $P_j$ that is received by patch $P_i$:

- Symmetry: $A_i F_{i,j} = A_j F_{j,i}$
- Definiteness: $F_{ii} = 0$ unless the patch is concave
- Partition of unity: $\sum_i F_{j,i} \leq 1$
Radiosity

Approximate the solution by decomposing surfaces into patches and doing a discrete summation:

\[ B_i = E_i + \rho_i \sum_{j=1}^{n} F_{j,i} \cdot B_j \]

This amounts to solving a linear system of equations:

- \( E_i, \rho_i, \) and \( F_{j,i} \) are given
- \( B_i \) are the unknowns.
Radiosity

Re-ordering terms in the equation gives:

\[
B_i = E_i + \rho_i \sum_{j=1}^{n} F_{j,i} \cdot B_j
\]

\[
E_i = B_i - \rho_i \sum_{j=1}^{n} F_{j,i} \cdot B_j
\]

\[
\begin{pmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{pmatrix} =
\begin{pmatrix}
1 - \rho_1 \cdot F_{1,1} & -\rho_1 \cdot F_{1,2} & \cdots & -\rho_1 \cdot F_{1,n} \\
-\rho_2 \cdot F_{2,1} & 1 - \rho_2 \cdot F_{2,2} & \cdots & -\rho_2 \cdot F_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
-\rho_n \cdot F_{n,1} & -\rho_n \cdot F_{n,2} & \cdots & 1 - \rho_n \cdot F_{n,n}
\end{pmatrix}
\begin{pmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{pmatrix}
\]
Solving the System of Equations

- Challenges:
  - Size of matrix
  - Cost of computing form factors

\[
\begin{pmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n \\
\hat{e}
\end{pmatrix} =
\begin{pmatrix}
1 - \rho_1 \cdot F_{1,1} & -\rho_1 \cdot F_{1,2} & \cdots & -\rho_1 \cdot F_{1,n} \\
-\rho_2 \cdot F_{2,1} & 1 - \rho_2 \cdot F_{2,2} & \cdots & -\rho_2 \cdot F_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
-\rho_n \cdot F_{n,1} & -\rho_n \cdot F_{n,2} & \cdots & 1 - \rho_n \cdot F_{n,n}
\end{pmatrix}
\begin{pmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{pmatrix} = \hat{\vec{e}}.
\]
Solving the System of Equations

- Solution methods:
  - Invert the matrix \( O(n^3) \)
  - Gathering methods \(- O(n^2) \)
  - Shooting methods \(- < O(n^2) \)

\[
\begin{pmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{pmatrix}
= 
\begin{pmatrix}
1 - \rho_1 \cdot F_{1,1} & -\rho_1 \cdot F_{1,2} & \cdots & -\rho_1 \cdot F_{1,n} \\
-\rho_2 \cdot F_{2,1} & 1 - \rho_2 \cdot F_{2,2} & \cdots & -\rho_2 \cdot F_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
-\rho_n \cdot F_{n,1} & -\rho_n \cdot F_{n,2} & \cdots & 1 - \rho_n \cdot F_{n,n}
\end{pmatrix}
\begin{pmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{pmatrix}
\]

\( \hat{e} = A \cdot \vec{b} \)
Gathering Iteration

Initialization:

- For each patch $P_i$, initialize its radiosity to be equal to its emissiveness:

$$B_i = E_i$$

Iteration:

- At each iteration, update the values of each of the $B_i$ based on the values of all the other $B_j$:

$$B_i = E_i + \rho_i \sum_{j \neq i} F_{j,i} \cdot B_j$$
Gathering Iteration

• Geometric interpretation
  ◦ Iteratively gather radiosity from elements
Gathering Iteration

- Geometric interpretation
  - Iteratively gather radiosity from elements

Limitation:
Can spend a lot of time gathering radiosity from patches that don’t contribute much.
Shooting Iteration

- Geometric interpretation:
  - Iteratively shoot “unshot” radiosity from elements
  - Select shooters in order of unshot radiosity
Summary

If we could, we would compute the lighting by recursively reflecting secondary rays in all directions to compute the brightness of a single point.

• Ray-Tracing:
  ◦ Assume that surfaces are specular so that you only need to bounce in a single (specular) direction.

• Radiosity:
  ◦ Assume that surfaces are Lambertian so that they reflect light in the same way in all directions.

• Reality:
  ◦ Surfaces reflect in all directions, but not uniformly.