Texture Mapping

Michael Kazhdan

(601.457/657)

HB Ch. 14.8,14.9
FvDFH Ch. 16.3, 16.4.5, 16.6
Textures

We know how to go from this… to this

[J. Birn]
Textures

But what about this... to this?

[J. Birn]
Textures

• How do we draw surfaces with complex detail?
Textures

• How do we draw surfaces with complex detail?

**Direct:**

• Tessellate finely and then associate the appropriate material properties to each vertex
Textures

• How do we draw surfaces with complex detail?

**Direct:**
• Or, tessellate in a complex manner and then associate the appropriate material properties to each vertex.
Textures

• How do we draw surfaces with complex detail?

Indirect:
• Use a simple tessellation with an auxiliary *texture image*. Use the location of surface points to look up color values from the texture.
Textures

- Advantages:
  - The 3D model remains simple
  - It is easier to design/modify a texture image than it is to design/modify a surface in 3D.
Textures (2 dimensions)

Implementation:

• Associate a *texture coordinate* to each vertex $v$: $(s_v, t_v)$ with $(0 \leq s_v, t_v \leq 1)$

• When rasterizing, *interpolate* to get the texture coordinate to at a pixel: $(s_p, t_p)$

• *Sample* the texture at $(s_p, t_p)$ to get the color at $p$. 
Example: Brick Wall

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Example: Brick Wall

\[(s_v, t_v) = (0,1) \quad (s_v, t_v) = (1,1)\]

\[(s_v, t_v) = (0,0) \quad (s_v, t_v) = (1,0)\]
Textures (2 dimensions)

- Coordinates described by variables $s$ and $t$ and range over interval (0,1)
- Texture elements are called texels
- Often 4 bytes (rgba) per texel
Texture \((d\) \text{ dimensions})

Given:
- A \(d\)-dimensional (multi-channel) texture image
- an assignment of coordinates \((s^1_v, \ldots, s^d_v)\) ranging over interval \((0,1)\) to each vertex

At rasterization:
- Interpolate the texture coordinates to get the texture coordinate at each pixel, \((s^1_p, \ldots, s^d_p)\)
- Sample the texture at \((s^1_p, \ldots, s^d_p)\) to get the value at \(p\)
3D Rendering Pipeline

- 3D Primitives
- 3D Modeling Coordinates
- Modeling Transformation
- 3D World Coordinates
- Viewing Transformation
- 3D World Coordinates
- Lighting
- 3D Camera Coordinates
- Projection Transformation
- 2D Window Coordinates
- Clipping
- 2D Window Coordinates
- Viewport Transformation
- 2D Viewport Coordinates
- Scan Conversion
- 2D Viewport Coordinates
- Image

○ Perform the texture interpolation and look-up while rasterizing the pixels.
Texture Mapping

Recall (Perspective Divide):

When performing scan-line rasterization and interpolating data from vertices, we need to compute the weights in 3D space.

\[
\begin{align*}
R & = R \\
p_0 & = z = 0 \\
p_1 & = z = 1 \\
p_2 & = \\
R & = 
\end{align*}
\]
Texture Mapping

Linear interpolation of texture coordinates in screen space

Correct interpolation of texture coordinates with perspective divide

Hill Figure 8.42
Overview

• Texture mapping methods
  ◦ Parameterization
  ◦ Sampling

• Texture mapping applications
  ◦ Modulation textures
  ◦ Illumination mapping
  ◦ Bump mapping
  ◦ Environment mapping
  ◦ Shadow maps
Map to a Base Domain

- Deform the geometry into a simple surface (with the same topology)
- Define a texture map over the simple surface

✓ Get a continuous map from the surface to the texture map
Map to a Base Domain

- Deform the geometry into a simple surface (with the same topology)
- Define a texture map over the simple surface

✓ Get a continuous map from the surface to the texture map
✗ Tricky, because mapped surface will have severe distortions

It is impossible to parameterize a complex shape to a simple domain so lengths are preserved
Map to a 2D Domain w/ Cuts

- Introduce cuts to give the surface a disk topology
- Map the cut surface to the 2D plane
- Assign texture coordinates in the plane

✓ Good cut placement can reduce distortion
✗ Need to ensure cross-seam continuity
✗ Still have to contend with distortion

[Image of 3D model with cuts applied, mapping to a 2D plane, and final texture mapping, with [Piponi2000] citation]
Texture Atlases

• Decompose the surface into multiple charts
• Map each chart to the 2D plane
• Assign texture coordinates in the plane

✓ Even less distortion in the mapping
✗ Even harder to ensure cross-seam continuity
✗ Need to pack the atlases into 2D

[Sander2001]
Overview

- Texture mapping methods
  - Parameterization
  - Sampling

- Texture mapping applications
  - Modulation textures
  - Illumination mapping
  - Bump mapping
  - Environment mapping
  - Shadow maps
Texture Filtering

Given pixel on a screen:

1. Determine the corresponding surface patch
2. Determine the corresponding texture patch
Texture Filtering

Given pixel on a screen:

1. Determine the corresponding surface patch
2. Determine the corresponding texture patch

While the true shape of the texture patch may be hard to compute, we can obtain a linear approximation by looking at the Jacobian of the map from pixel-space to texture-spaces.
Texture Filtering

Given pixel on a screen:

1. Determine the corresponding surface patch
2. Determine the corresponding texture patch
3. Average texel values over the texture patch
Texture Filtering

- Size of texture patch depends on the deformation
  - Computation is linear in the size of the pixel footprint

- Can pre-filter images for better performance
  - MIP (Multum In Parvo) maps
  - Summed area tables
MIP Maps

- In a **pre-processing** step, compute a hierarchy of successively down-sampled texture images
MIP Maps

- In a **pre-processing** step, compute a hierarchy of successively down-sampled texture images

- At **run-time**, sample the closest MIP map level(s)
  - Easy for hardware
  - Computation is constant in the size of the pixel footprint

```
Screen
```

```
Texture hierarchy
```

Average over a few pixels
MIP Maps

- In a **pre-processing** step, compute a hierarchy of successively down-sampled texture images
  - Storage is only \( \frac{4}{3} \) the size of the input image
MIP Maps

- In a pre-processing step, compute a hierarchy of successively down-sampled texture images
  - Storage is only 4/3 the size of the input image
- At run-time, sample the closest MIP map level(s)
  - This type of filtering is isotropic:
    » Assumes there is the same amount of compression along the vertical and horizontal directions

Again: we’re trading aliasing for blurring!
Summed-Area Tables

Key Idea:

- Approximate the summation/integration over an arbitrary region by a summation/integration over an axis-aligned rectangle:

\[ \text{Sum}([a, b] \times [c, d]) = \int_{a}^{b} \int_{c}^{d} f(x, y) \, dy \, dx \]
Recall

Integration:

Given a function $f(x)$ and an interval $[a, c]$, the integral of $f$ over the interval is denoted:

$$\int_{a}^{c} f(x) \, dx$$
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Given a function $f(x)$ and an interval $[a, c]$, the integral of $f$ over the interval is denoted:

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For any point $b \in [a, c]$ in the interval, we have:

$$\int_{a}^{c} f(x) \, dx = \int_{a}^{b} f(x) \, dx + \int_{b}^{c} f(x) \, dx$$
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Integration:

Given a function $f(x)$ and an interval $[a, c]$, the integral of $f$ over the interval is denoted:

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For any point $b \in [a, c]$ in the interval, we have:

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

This is true even if $c$ is outside the interval since:

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$
Recall

Integration:

In particular, we can write out the integral of the function \( f(x, y) \) over the rectangle \([a, b] \times [c, d]\) as:

\[
\int_{a}^{b} \int_{c}^{d} f(x, y) \, dy \, dx
\]
Recall

Integration:

In particular, we can write out the integral of the function $f(x, y)$ over the rectangle $[a, b] \times [c, d]$ as:

$$\int_a^b \int_c^d f(x, y) \, dy \, dx = \int_a^b \left( \int_0^b f(x, y) \, dy - \int_0^c f(x, y) \, dy \right) \, dx$$
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Summed-Area Tables

Key Idea:

• Approximate the summation/integration over an arbitrary region by a summation/integration over an axis-aligned rectangle.

• Perform the integration quickly by pre-computing integrals and leveraging the formula:

\[
\int_a^b \int_c^d f(x,y) \, dy \, dx = \int_0^b \int_0^d f(x,y) \, dy \, dx - \int_0^b \int_0^c f(x,y) \, dy \, dx - \int_0^a \int_0^d f(x,y) \, dy \, dx + \int_0^a \int_0^c f(x,y) \, dy \, dx
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\]
Summed-Area Tables (Pre-Process)

- Precompute the values of the integral:
  \[ S(a, b) = \int_0^a \int_0^b f(x, y) \, dy \, dx \]

- Each summed-area table texel is the sum of all input texels below and to the left

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1 2 4 0</td>
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<td>0 3 1 1</td>
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Input image:

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1 2 4 0
0 3 1 1
4 2 0 1
1 2 1 3
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Summed area table:

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Summed-Area Tables (Pre-Process)

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<td>1 3 4 7</td>
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![Image of input image and summed area table]
Summed-Area Tables (Pre-Process)

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```plaintext
5
1 3 4 7
```
Summed-Area Tables (Pre-Process)

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<td>1 2 4 0</td>
<td>6 15 21 26</td>
</tr>
<tr>
<td>0 3 1 1</td>
<td>5 12 14 19</td>
</tr>
<tr>
<td>4 2 0 1</td>
<td>5 9 10 14</td>
</tr>
<tr>
<td>1 2 1 3</td>
<td>1 3 4 7</td>
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Summed-Area Tables (Run-Time)

Example:

Compute the average in the rectangle $[1,3] \times [2,3]$. ⇒ Compute the sum and divide by the area
Summed-Area Tables (Run-Time)

Example:

Compute the average in the rectangle \([1,3] \times [2,3]\).

⇒ Compute the sum and divide by the area

\[
\text{Sum}([1,3] \times [2,3]) = S(3,3)
\]
**Summed-Area Tables (Run-Time)**

**Example:**

Compute the average in the rectangle \([1,3] \times [2,3]\).

\[ \Rightarrow \text{Compute the sum and divide by the area} \]

\[ \text{Sum}([1,3] \times [2,3]) = S(3,3) - S(0,3) \]

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**Input image**

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**Summed-area table**
Summed-Area Tables (Run-Time)

Example:

Compute the average in the rectangle \([1,3] \times [2,3]\).

\[ \Rightarrow \text{Compute the sum and divide by the area} \]

\[ \text{Sum}([1,3] \times [2,3]) = S(3,3) - S(0,3) - S(3,1) \]
Summed-Area Tables (Run-Time)

Example:

Compute the average in the rectangle $[1,3] \times [2,3]$. 

$\Rightarrow$ Compute the sum and divide by the area

$$\text{Sum}([1,3] \times [2,3]) = S(3,3) - S(0,3) - S(3,1) + S(0,1)$$
Summed-Area Tables (Run-Time)

Example:

Compute the average in the rectangle $[1,3] \times [2,3]$.  
$\Rightarrow$ Compute the sum and divide by the area

$\text{Sum}([1,3] \times [2,3]) = S(3,3) - S(0,3) - S(3,1) + S(0,1)$

$= 26 - 6 - 14 + 5 = 11$

Average($[1,3] \times [2,3]$) = $\frac{\text{Sum}([1,3] \times [2,3])}{\text{Area}([1,3] \times [2,3])} = \frac{11}{6}$

Input image

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<td>1</td>
<td>3</td>
<td>4</td>
<td>7</td>
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</table>
Summed-Area Tables (Run-Time)

- Precompute the values of the integral
  ✓ Constant time averaging, regardless of rectangle size
  ✗ If the input image has values in the range $[0,255]$ (i.e. one byte per channel), the summed area table can have values in the range $[0,255 \cdot \text{width} \cdot \text{height}]$

<table>
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<th>2</th>
<th>4</th>
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</table>

Input image  Summed-area table
Overview

• Texture mapping methods
  ◦ Parameterization
  ◦ Sampling

• Texture mapping applications
  ◦ Modulation textures
  ◦ Illumination mapping
  ◦ Bump mapping
  ◦ Environment mapping
  ◦ Shadow mapping
Modulation textures

Map texture values to scale factor

Modulation

\[ I = T(s, t) \left( I_E + K_A I_{AL} + \sum_i \left( K_D \langle \vec{N}, \vec{L}_i \rangle I_i + K_S \langle \vec{V}, \vec{R}_i \rangle^n I_i \right) \right) \]
Illumination Mapping

Map texture values to a material parameter

Modulation

Diffuse

\[ I = I_E + K_A I_{AL} + \sum_i \left[ (T(s,t) \langle \vec{N}, \vec{L}_i \rangle I_i + K_S \langle \vec{V}, \vec{R}_i \rangle^n I_i \right] \]
Illumination Mapping

Map texture values to a material parameter

Modulation  Diffuse

Note that we need to evaluate the texture at each pixel but can still use the interpolated lighting values $\langle \vec{N}, \vec{L}_i \rangle$

This requires the graphics card to separately store the diffuse component of the lighting at each vertex

$$I = I_E + K_AI_AL + \sum_i \left( T(s,t) \langle \vec{N}, \vec{L}_i \rangle I_i + K_S \langle \vec{V}, \vec{R}_i \rangle^n I_i \right)$$
Illumination Mapping

Map texture values to a material parameter

Modulation
Diffuse
Specular

\[
I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \vec{N}, \vec{L}_i \rangle I_i + T(s, t) \langle \vec{V}, \vec{R}_i \rangle^n I_i \right)
\]
Illumination Mapping

Map texture values to a material parameter

Modulation  Diffuse  Specular

Again, we can use the interpolated lighting values

\[ \langle \hat{V}, \hat{R}_i \rangle^n \]

\[ I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \hat{N}, \hat{L}_i \rangle I_i + T(s,t) \langle \hat{V}, \hat{R}_i \rangle^n I_i \right) \]
Bump Mapping

- Recall that many parts of our lighting calculation depend on surface normals

\[
I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \vec{N}, \vec{L}_i \rangle I_i + K_S \langle \vec{V}, \vec{R}_i \rangle^n I_i \right)
\]
Bump Mapping

**Phong** shading performs per-pixel lighting calculations with the interpolated normal

\[ \downarrow \]

approximates a smoothly curved surface

Bump maps encode the normals in the texture

\[ \downarrow \]

approximates a more complex undulating surface
Bump Mapping
Bump Mapping

Note that bump mapping does not change object silhouette
Environment Mapping

Simulate reflective materials through pre-computed texture maps representing the environment around a shape.
Environment Mapping

- Generate a spherical/cubic map of the environment around the model.
- Texture coordinates are computed dynamically through reflection.
Environment Mapping

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- Generate a spherical/cubic map of the environment around the model.
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Set the texture coordinates based on the direction of the reflected view direction.
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At the same triangle, changing the position of the camera changes the texture coordinates.
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Set the texture coordinates based on the direction of the reflected view direction.

At the same triangle, changing the position of the camera changes the texture coordinates.
Environment Mapping

Texture coordinates are computed dynamically through reflection of the view direction through the surface normal.
Shadow Mapping (Williams 1978)

Test if surface is visible to the light when computing the contribution the lighting equation.

Shadow Mapping (Williams 1978)

Q: Is the point that is seen by the camera visible (i.e. not in shadow) to the light?

A: The point is visible if:

- The is nothing on the line segment between the point and the light.

  ⇒ **In the light’s coordinate system**, the point’s \( z \)-coordinate equals the \( z \)-coordinate of the primitive seen by the light.

  ⇒ Rendering the scene from the light’s perspective, the point’s \( z \)-coordinate equals the value stored in the \( z \)-buffer.
Shadow Mapping (Williams 1978)

Test if surface is visible to the light when computing the contribution the lighting equation.

- Render the scene from the light’s perspective and read back the $z$-buffer/shadow map.

Shadow Mapping (Williams 1978)

Test if surface is visible to the light when computing the contribution the lighting equation.

- Render the scene from the light’s perspective and read back the $z$-buffer/shadow map.
- For each pixel in the camera view, compute its coordinates relative to the light
  - If it's further back than the value in the shadow map, it's in shadow
  - Otherwise, it's illuminated

Shadow Mapping (Williams 1978)

Test if surface is visible to the light when computing the contribution the lighting equation.

- The projection used for rendering from the light-source depends on the type of light:
  - Directional → Parallel
  - Point → Perspective

- Need to use multiple shadow maps if there are multiple lights in the scene

Shadow Mapping (Williams 1978)

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- **Perspective Aliasing:**
  Stair-stepping due to limited shadow map resolution (particularly at grazing angles)

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Test if surface is visible to the light when computing the contribution the lighting equation.

- **Perspective Aliasing:**
  Stair-stepping due to limited shadow map resolution (particularly at grazing angles)

- **Shadow Acne:**
  Self-shadowing due to limited depth resolution

Images courtesy of