Feature Preserving Point Set Surfaces based on Non-Linear Kernel regression

1. Briefly summarize the paper's contributions. Does it address a new problem? Does it present a new approach? Does it show new types of results?

- **[AS]**
  This paper presents a new point set surface representation by combining implicit moving least squares (IMLS) with robust statistics. In particular, they explicitly derive IMLS in the local kernel regression (LKR) framework, forming the basis of a new robust moving least squares (MLS) surface representation that can handle sparse sampling, preserves details, and handles sharp features.
- **[DS]**
  The paper presents a method that applies a robust statistics methods to MLS implicit surface. They use a robust kernel regression, which results in a representation that can handle sparse sampling, generates a continuous surface that preserves fine details better, and can handle sharp features with controllable sharpness. It can handle outliers and high frequency features, it is efficient and easy to implement, and it uses local computations so no preprocessing is needed. Improves representation of sharp features and details of any frequency, and can deal with high order corners and peaks.
- **[FP]**
  The authors implicitly define the surface using a Robust Local Kernel Regression technique. The approach uses a first order estimation of the Local Kernel, which is reduced to a zero order estimation by approximating the gradient with the normal at each sample point. The obtained Local Kernel is equivalent to Kolluri's IMLS implicit function. The main contribution is the introduction of an efficient and robust technique which is resistant to both spatial and normal outliers. This robust approach allows the reconstruction of sharp features without needing to decompose or tag a local neighborhood.
- **[JD]**
- **[LF]**
- **[MK]**
  The paper proposes interpreting the (implicit) moving least squares approach in the context of local kernel regression. This allows for designing a more robust algorithm that minimizes an energy that is less sensitive to outliers.
  Additionally, the authors show that by effectively incorporating a bilateral term that penalizes normal variation, their method can be made to better adapt to sharp features.

2. What is the key insight of the paper? (in 1-2 sentences)

- **[AS]**
  The key insight of this paper is that MLS surfaces can be expressed in terms of LKR, which is a method in statistics to estimate the conditional expectation of a random variable.
- **[DS]**
  The key insight is the fact that MLS surfaces can be expressed in terms of local kernel regression (LKR), so that a variety of techniques can now be applied to surfaces. The representation is a combination of robust kernel regression techniques with implicit MLS surfaces (IMLS).
- **[FP]**
  Iterative reweighted least squares provide an efficient and robust technique for function fitting. Weights are given by positional error (residual) and the normal error (gradient).
- **[JD]**
- **[LF]**

- **[MK]**
  The key idea is the observation that MLS is essentially minimizing a (weighted) sum of squares energy, which is known to be sensitive to outliers. Using a different function to measure the fit, one can obtain a reconstruction method that is less sensitive to outliers.

3. What are the limitations of the method? Assumptions on the input? Lack of robustness? Demonstrated on practical data?

- **[AS]**
  One limitation of this method is that it depends on the choice of the starting point.
- **[DS]**
  Under certain sampling conditions and extreme parameter settings, the reconstructions degrade (e.g. discontinuities). The results depend on starting point.
- **[FP]**
  As in any local fitting technique, dense sampling is an assumed condition.
  The method heavily relies on accurate normal since they are used as a weighting criteria in the iterative least square approach. To alleviate this problem they use a preprocessing stage to filter noisy normal.
  Another limitation of the problem is the strong dependence of the implicit function evaluation on the selection of the initial conditions (weights) for the iterative least square fitting. In practice the authors choose equal weight for the entire neighborhood to guarantee continuity. But it is not clear if this still provides robustness.
- **[JD]**
- **[LF]**
- **[MK]**
  I would imagine that the method is sensitive to parameter choices, making it difficult to correctly identify edges in noisy data. In particular, because there is no global structure enforced on the crease edges, the method may only identify bits and pieces of the crease curve as such, resulting in a reconstruction that is qualitatively worse than just smoothing across the crease.
  Additionally, since the energy is no longer quadratic, it cannot be solved directly, and requires solving a sequence of iteratively reweighted systems. As such, the approach is quite sensitive to selection of the initial guess. This also makes it harder to compute the gradients of the implicit function on the zero level-set. (However, this iterative reweighting has been well studied, and was shown to converge to a local minimum.)
  The input requires normals that are consistently oriented.

4. Are there any guarantees on the output? (Is it manifold? does it have boundaries?)

- **[AS]**
  The output surface representation is a continuous surface which can preserve fine detail and sharp features.
- **[DS]**
  The method sharpifies the initially over-smoothed solution to a more faithful approximation and keeps the surface differentiable.
- **[FP]**
  Due to the MLS nature of the approach the output should be infinitely smooth. Since the initial conditions of the iterative least square method change continuously, it is expected for the implicit function to be continuous and smooth. For infinitely many iterations the implicit function should still be continuous but may be non-smooth.
  Since the method provides an implicit definition, it is expected that the zero level set is closed (i.e., no boundaries).
- **[JD]**
- **[LF]**

- **[MK]**
  The method defines the surface as the zero level set of an implicit function so the surface should be manifold without boundaries (assuming non-vanishing gradient). Clearly, because the method seeks to reproduce sharp creases, the output surface is not guaranteed to be smooth.

5. What is the space/time complexity of the approach?

- **[AS]**
  The time complexity of this approach is O(*i*N*k* logN), where *i*is the number of refitting iterations, N is the number of points, and *k*is the size of the neighborhood over which *f*(x) is evaluated as the weighted average for each of the N points. The space complexity is O(N), since storing the points in a k-D tree requires O(N) space.
- **[DS]**
  Similar order as fastest MLS methods. Neighbor search part is main computational cost. Can fit into real-time upsampling and rendering framework of algebraic PSS.
- **[FP]**
  In order to compute the value of the implicit function at any point, the method relies in zero order estimation. Just one parameter, the constant term of the Local Kernel, is fitted per least square iteration. At each iteration we only require to update the weights of the least square problem and these are given by the residuals and the normal error, both easily computed in (n). Therefore, implicit function evaluation can be done in (n) (assuming a constant number of iterations), and really small amount of operations.
  The normal filtering method proposed is also obtained using iterative minimization and is (n).
- **[JD]**
- **[LF]**
- **[MK]**
  Assuming a good data-structure for obtaining nearest neighbors and assuming that the ϕ_i functions are compactly supported with a fixed number of points in their support, the method should be linear.

6. How could the approach be generalized?

- **[AS]**
  This approach can be extended to produce even better MLS reconstructions by changing the order of regression and constraints, and by using unsupervised kernel regression.
- **[DS]**
  Find more robust alternatives so that choice of starting point does not affect results. Interpreting MLS point sets with respect to LKR allows for the adoption of regression methods to solve problems in the geometric setting.
- **[FP]**
  The order of approximation of the Local Kernel and the weights used for the iterative least square methods are two features that are prone to be adapted /modified. The authors chose a zero order approximation since they are computational economic, but is not clear if any notorious improvement could be attained using a high order approximation. In the case of weights they use Gaussian (for the residual) and Euclidian (for the normal) weights but some other choices can be adaptively done.
  As the authors claim, the iterative method used in their approach strongly relies on initial conditions. Robust methods to choose that initial conditions would complement the current work.

- **[JD]**
- **[LF]**
- **[MK]**
  The current implementation focuses on using robust IMLS with piecewise linear fits. The method can be extended to use higher order fits like the APSS approach. (However, this would require a slight philosophical modification in the approach. The current implementation assumes that every point sample contains enough information to provide a first-order approximation, i.e. positions and normals. Since it is unlikely that points will carry enough information for estimating second order quantities, i.e. curvature, these will need to be explicitly estimated from the data.)

7. If you could ask the authors a question (e.g. "can you clarify" or "have you considered") about the work, what would it be?

- **[AS]**
  The paper mentions that smooth regions only require one iteration until convergence, although sharp features or regions containing high frequency detail require slightly more iterations. Could you clarify what this slight increase in iterations is? That is, how does the number of iterations increase as sharpness or high frequency detail increases.
- **[DS]**
  In what ways can the choice of starting point affect the representation? How critical are the choices of w and phi (Eq 9) and could different choices improve the results (or make them worse)?
- **[FP]**
  The authors propose uniform weights (w=1) in the point neighborhood as initial condition for the iterative least square fitting. Is this selection robust in the case of undersampling?
- **[JD]**
- **[LF]**
- **[MK]**
  It seems kludgey to try and fit the IMLS into a framework where parameters are locally estimated at each point. (In particular, it's weird that the 0^{th} order approximation uses multiple different approximations of the gradient of $f(x)$, defined in terms of the gradients at the different x_i.) Couldn't the robust frame-work be formulated by interpreting the IMLS as the locally weighted average of the evaluation of functions defined at each of the x_i? That is, the value at the point x can be viewed as the minimizer of the energy: $E(f_x) = \sum \phi_i(x) \rho\big(f_x - f_i(x)\big)$ where f_x is the estimated value at the point x and $f_i(x)$ is the function fit to the point x_i. Then, the same iteratively reweighted framework could be used to estimate f_x robustly.