Survey of Methods in Computer Graphics:

Spectral Surface Reconstruction From Noisy Point Clouds

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SGP 2004.

Article and presentation slides material from:
http://graphics.berkeley.edu/papers/Kolluri-SSR-2004-07/

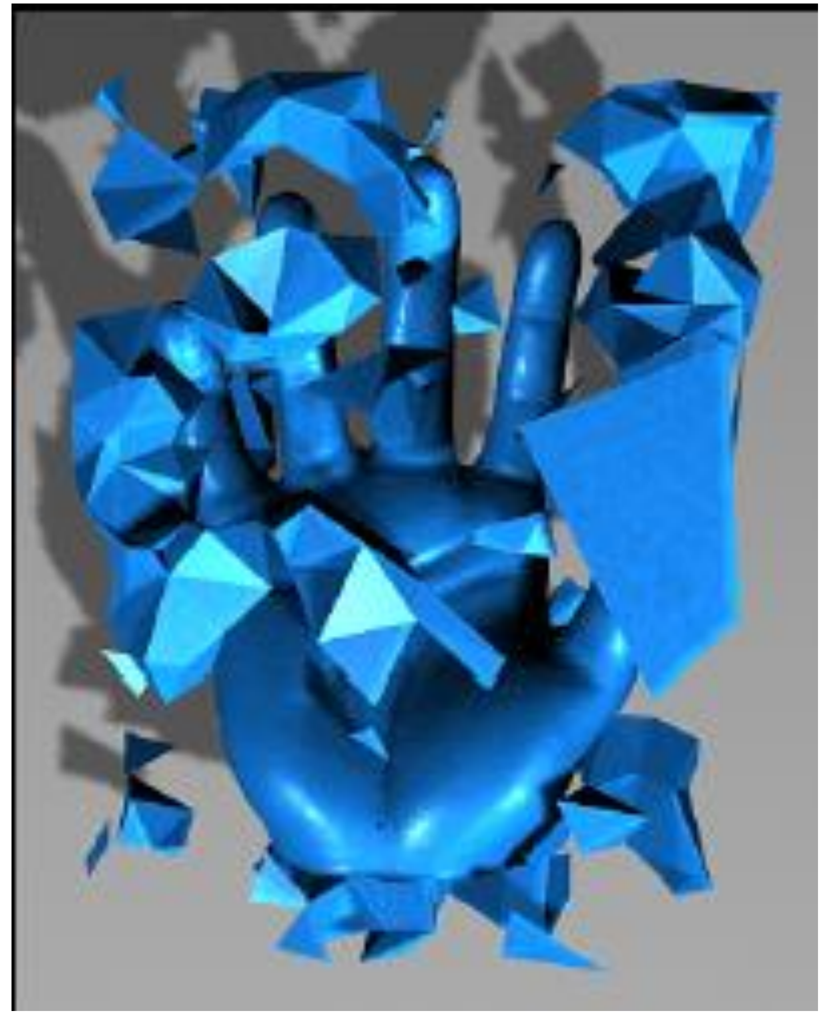
What is New ?

- Previous reconstruction methods based only in local decisions are weak to outliers, noise and undersampling.

- *Spectral Surface Reconstruction* propose an optimization framework that provides a global view of model. The obtained method is specially robust to outliers.

Spectral Reconstruction (Eigencrust)

Tight-Cocone



1200 Outliers

Method Pipeline

Input:

- Cloud of Points.
- Orientation **not** required.

Output:

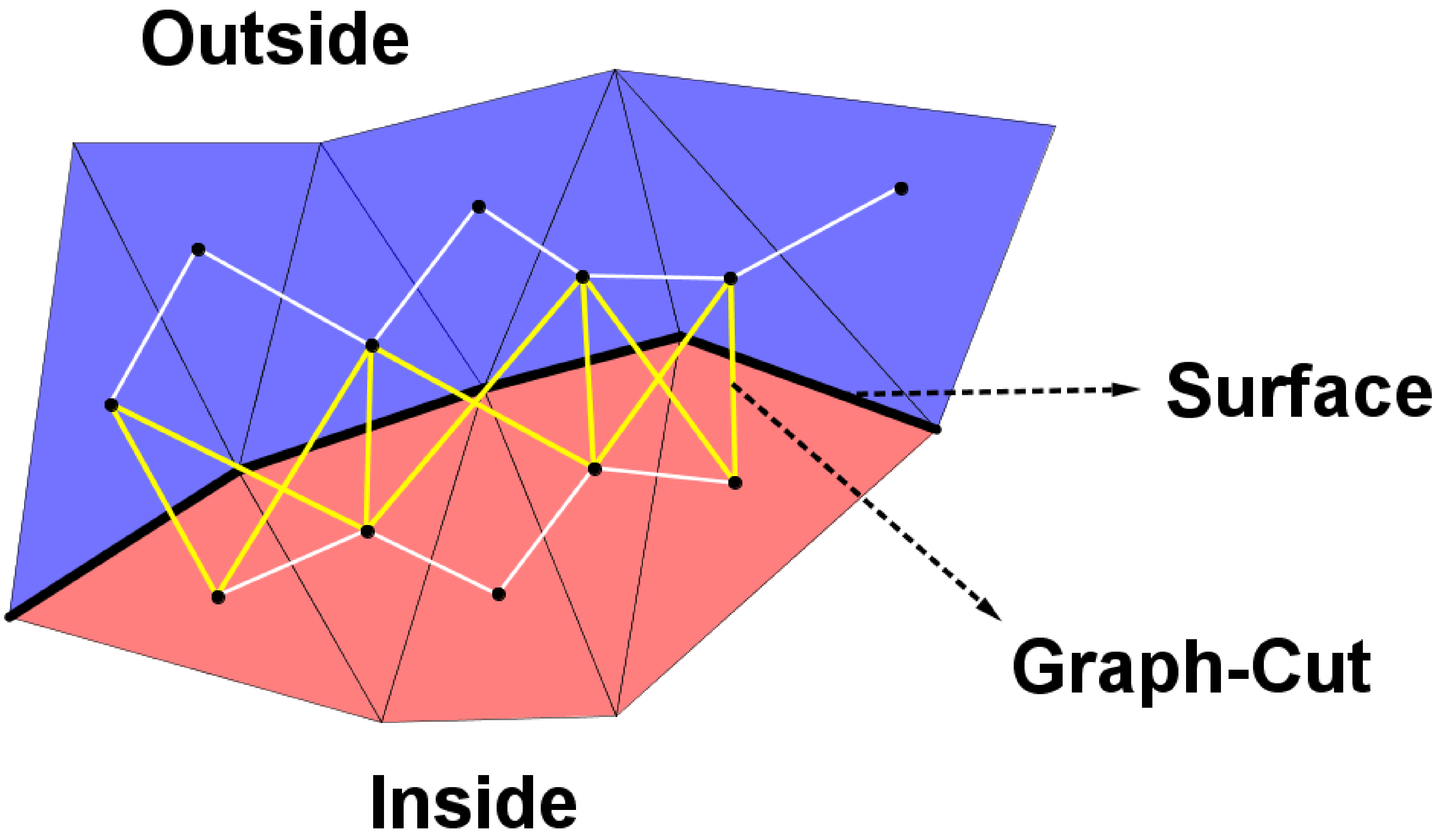
- Triangular Mesh.
- No boundary edges.

Guarantees:

- Always Watertight
- Manifold after post-processing.

Key Insight

Interpret the reconstruction problem as a segmentation of interior- exterior structures. Solve the segmentation problem using a graph-cut approach.

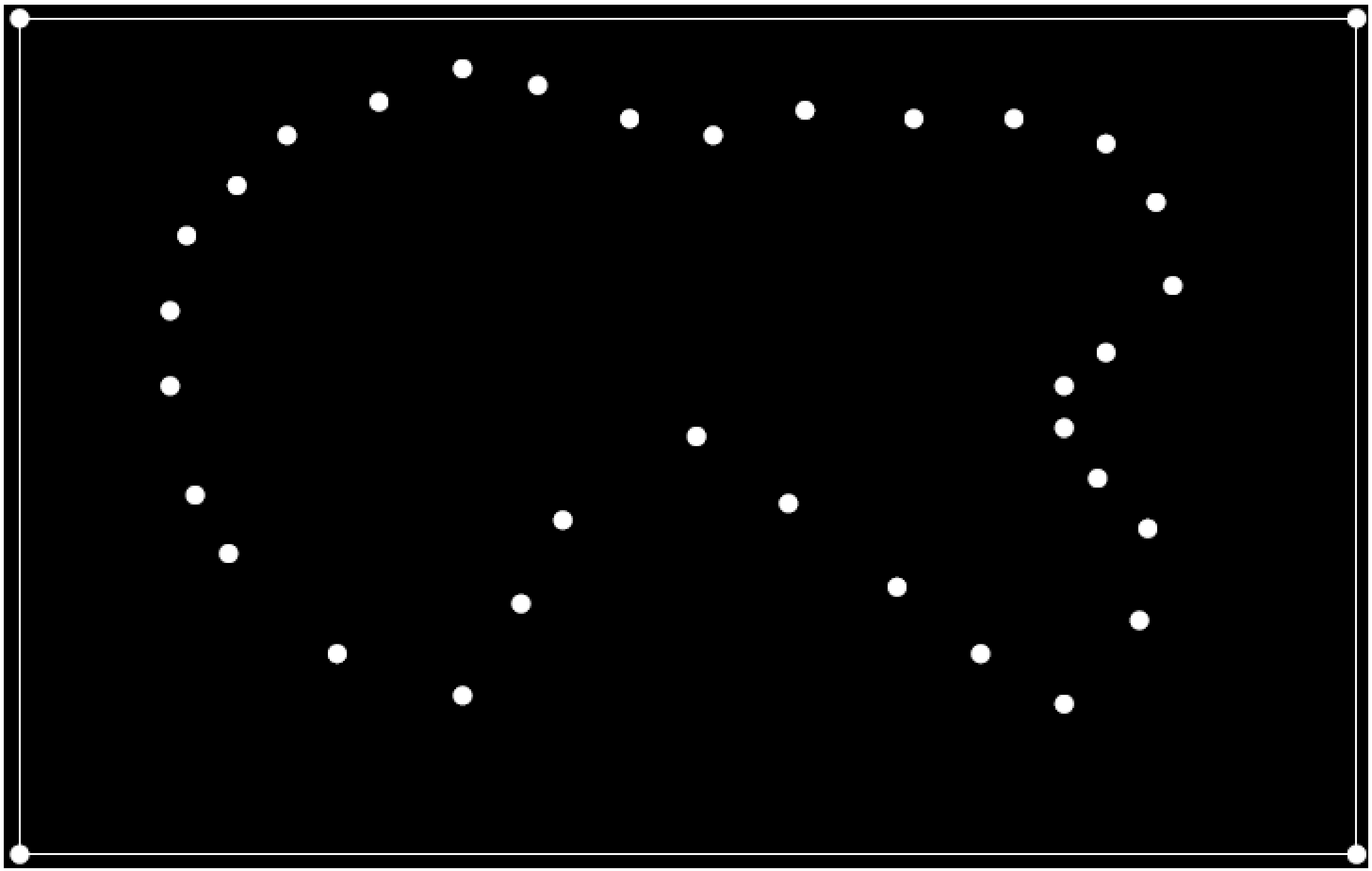

Outside
Surface
Graph-Cut
Inside

Method Sketch

1. Define the space partition structures. They will be classified as Interior or Exterior.
2. Set a graph structure over these elements. Edge weights measures likelihood of belonging to a same component.
3. Find an approximate MIN CUT. This cut defines the structure set classification.
4. Post process the segmentation result to guarantee Manifold properties and reduce genus.

Space Partition

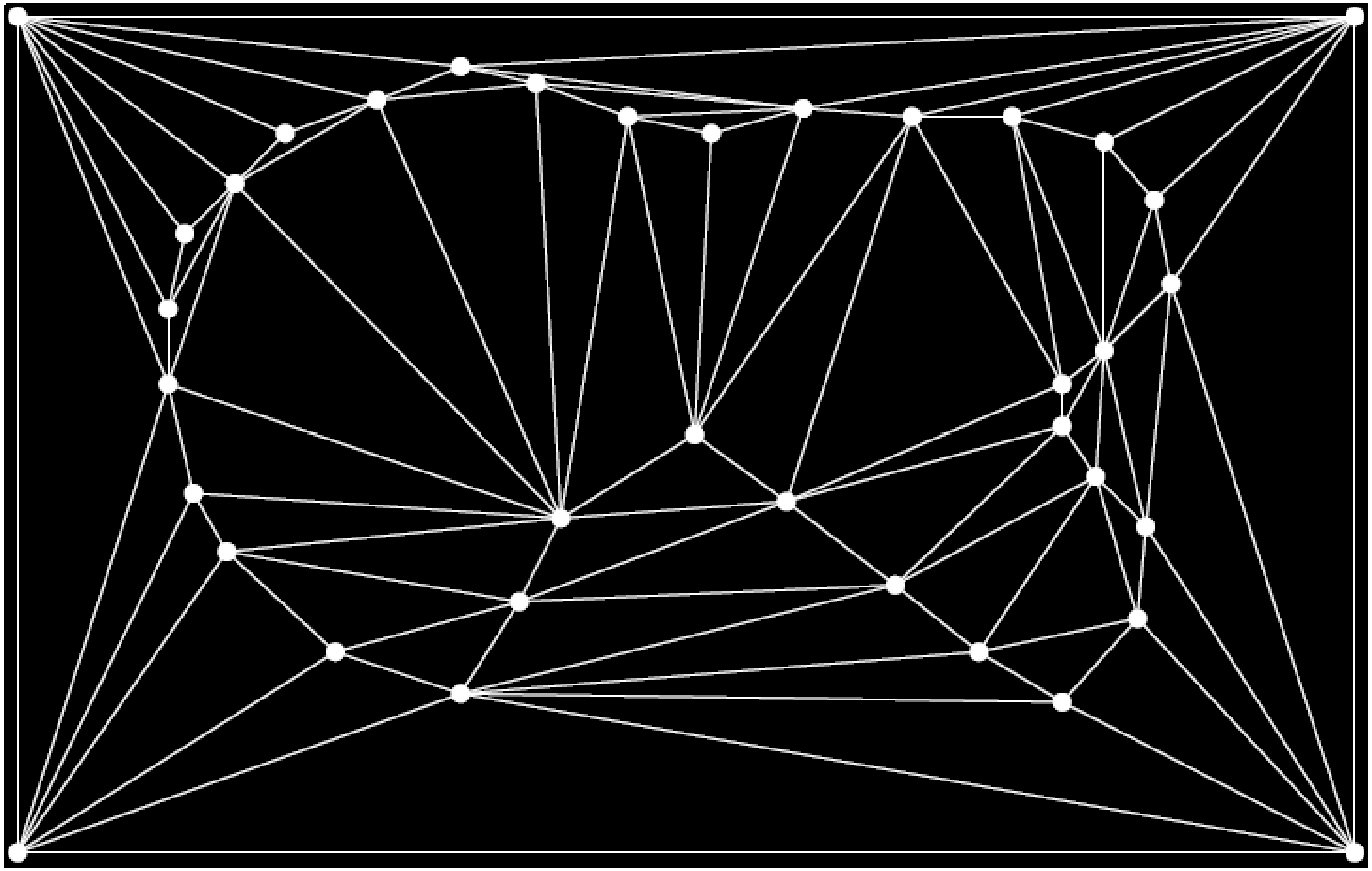
Delanauy Tetrahedralization



[1] Power Cells is another alternative to space partition.

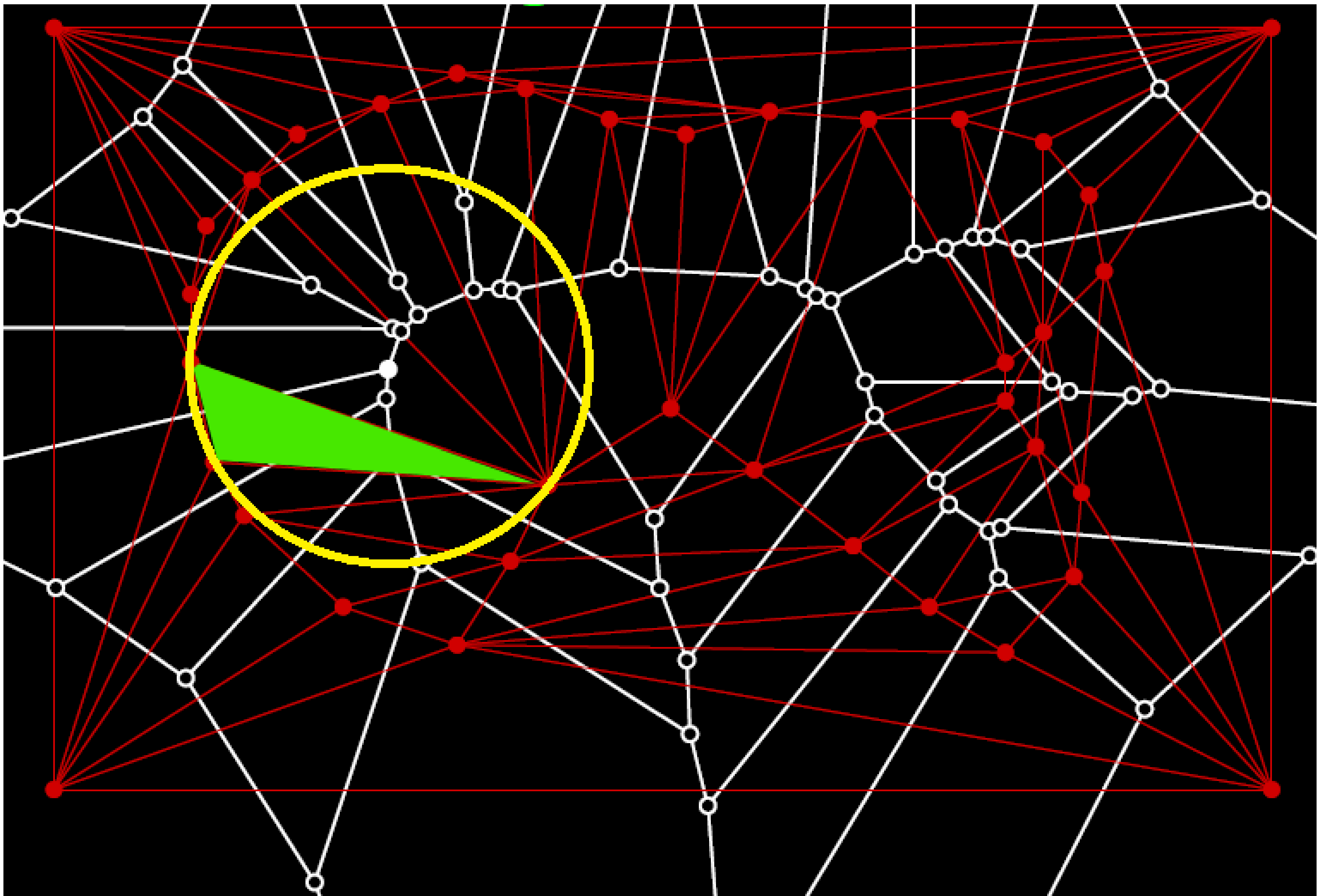
Space Partition

Delanauy Tetrahedralization



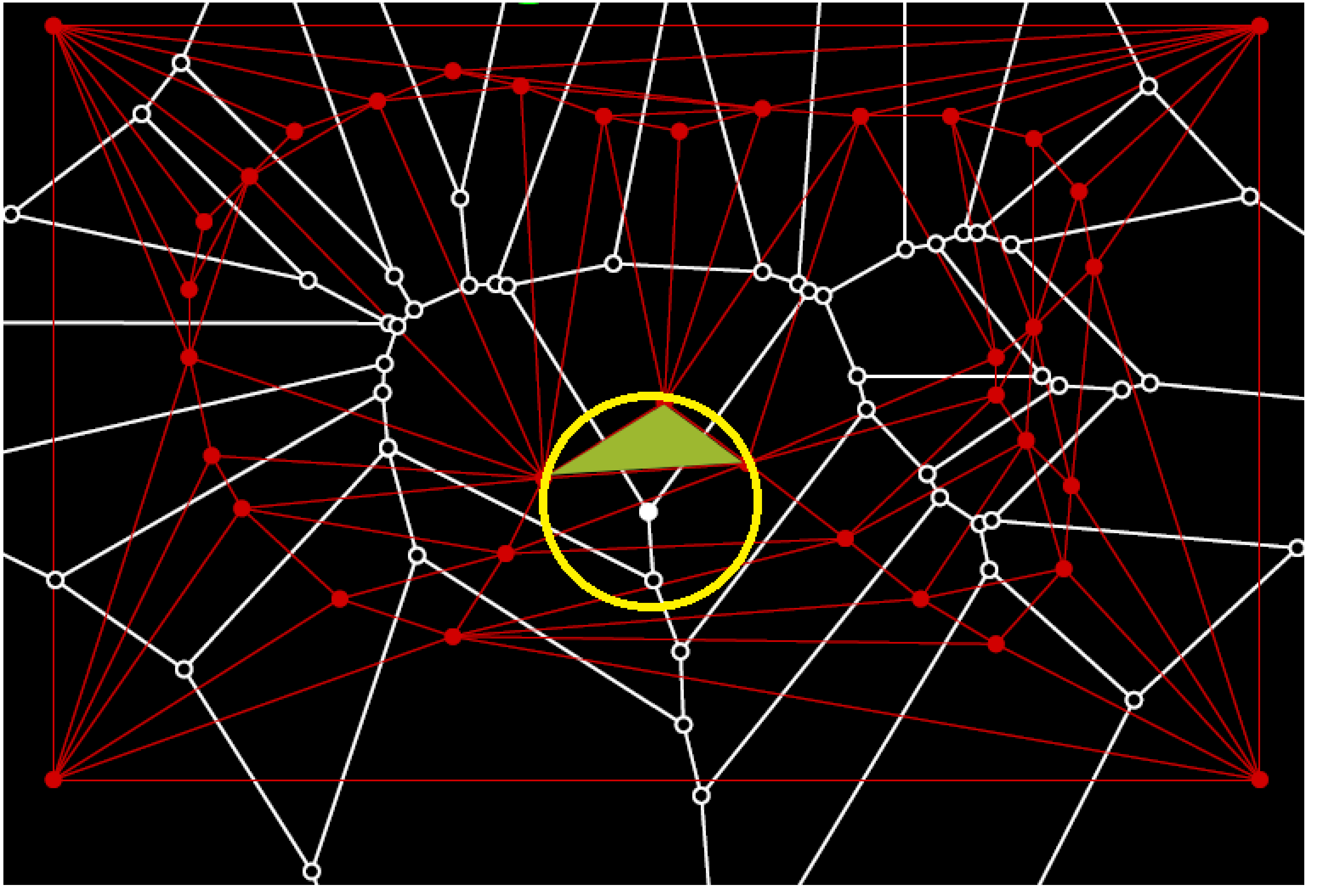
Identifying Reliable Structures

Circumcenters of **reliable** tetraheadra lay near medial axis



Identifying Reliable Structures

Circumenters of **ambiguous** tetrahedra lay near surface



Tetrahedra Classification Approach

- **First**: Classify reliable tetrahedra.

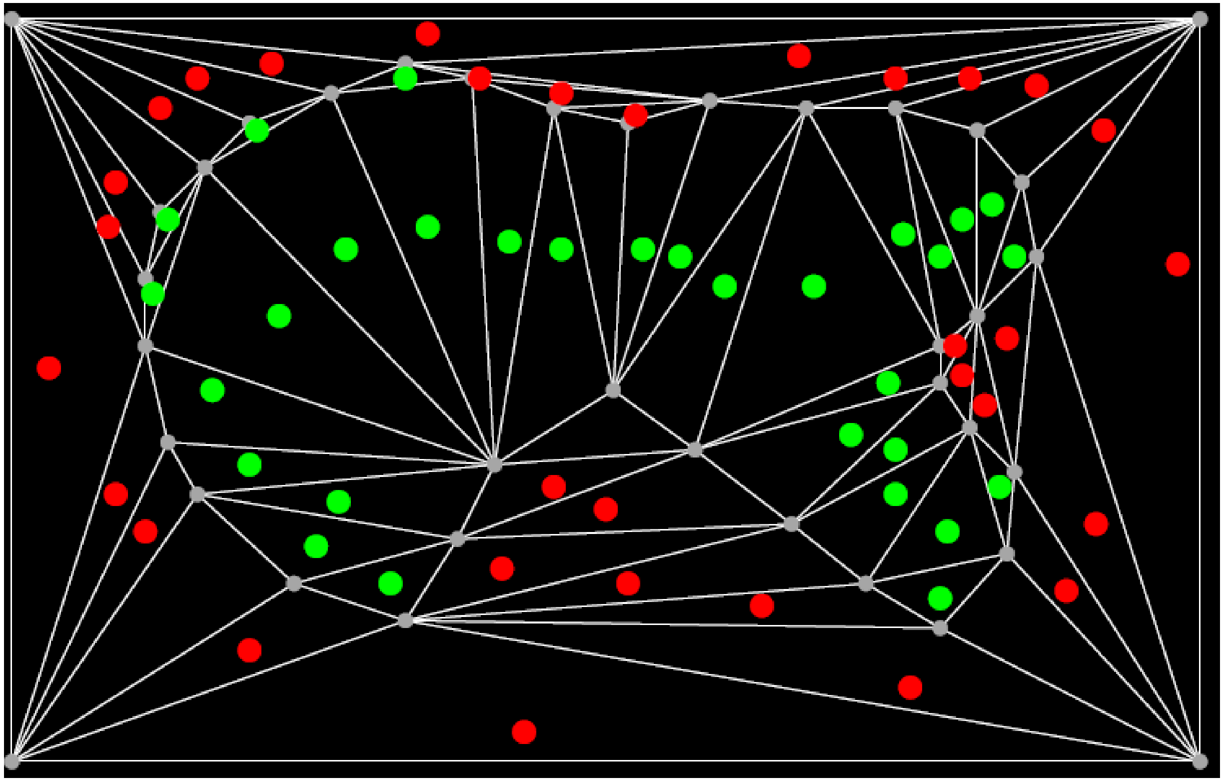
The Pole Graph G

- **Second**: Classify ambiguos tetrahedra.

Complement Graph H

The Pole Graph: Vertices

Poles of Voronoi cells usually lays near medial axis. Therefore tetrahedra associated to poles are usually reliable.

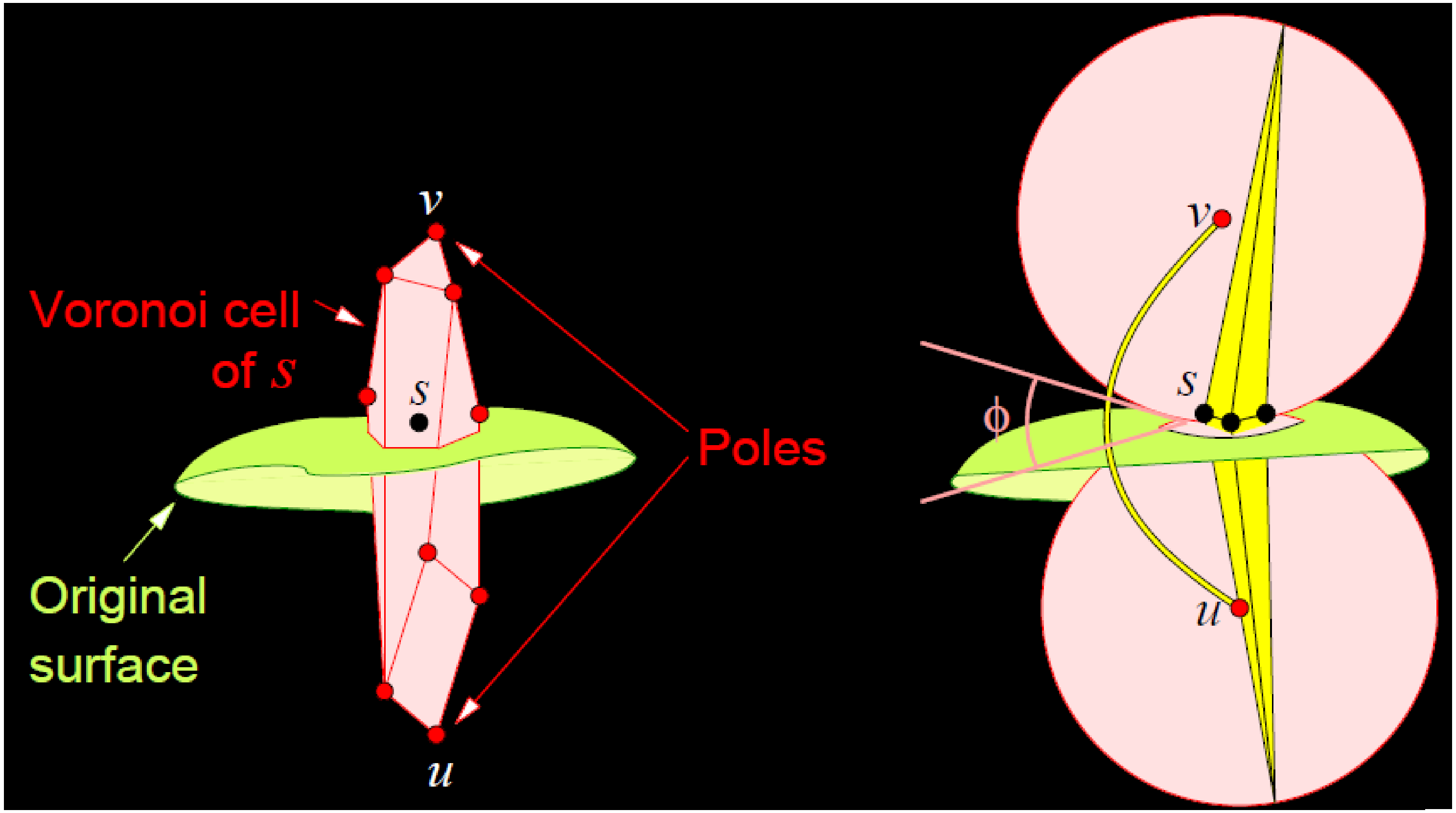


For each of such tetrahedra define a vertex.

Ignore the colors for now!.

The Pole Graph: Edges

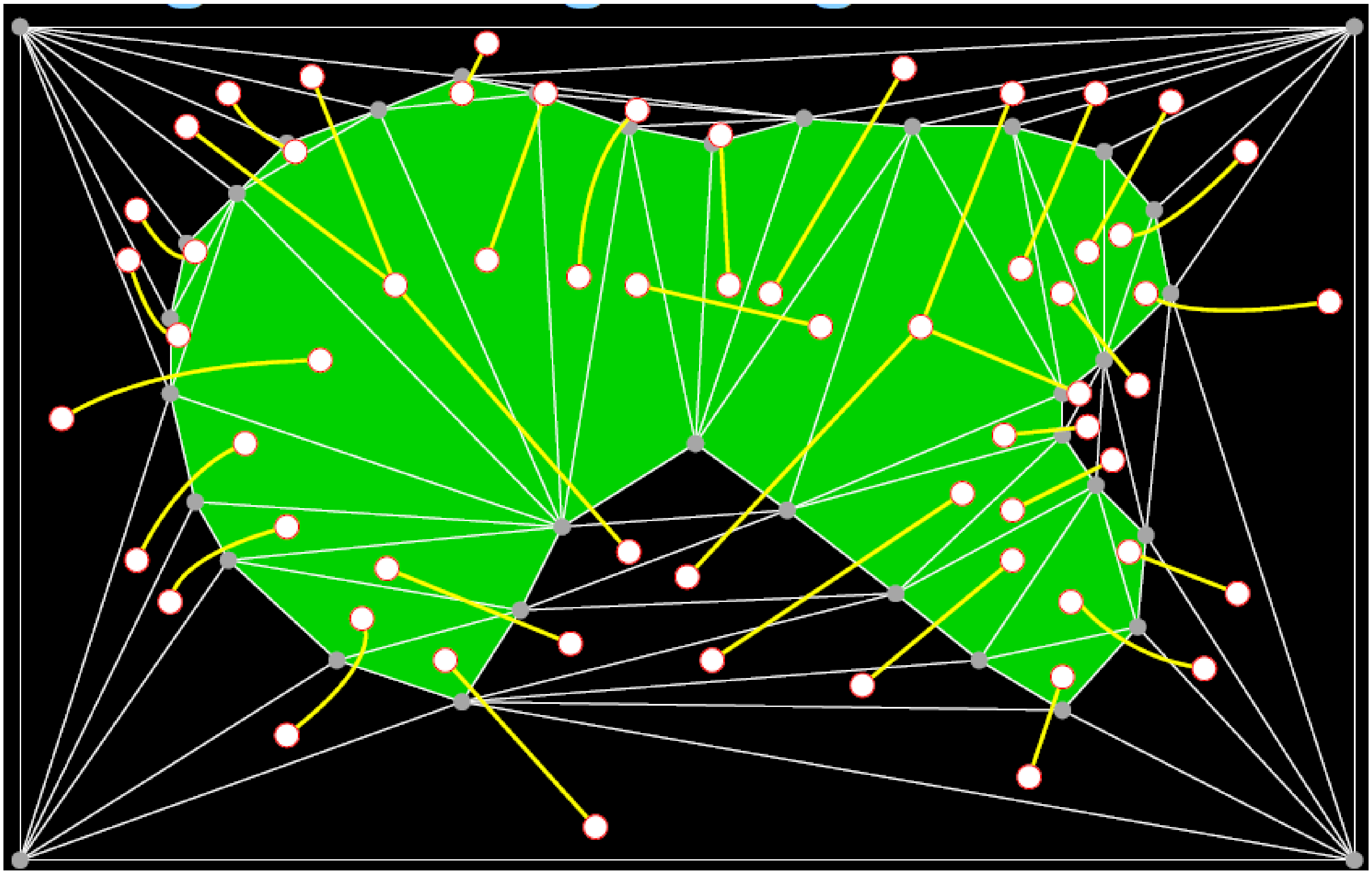
Tipe I : Define egdes for tetrahedra associated to poles of a same Voronoi cell.



This kind of tetrahedra usually lay in opposite sides of the surface. Assign a negative (repelling) edge between the graph vertices of weight: $-e^{4+4cos\phi}$

The Pole Graph: Edges

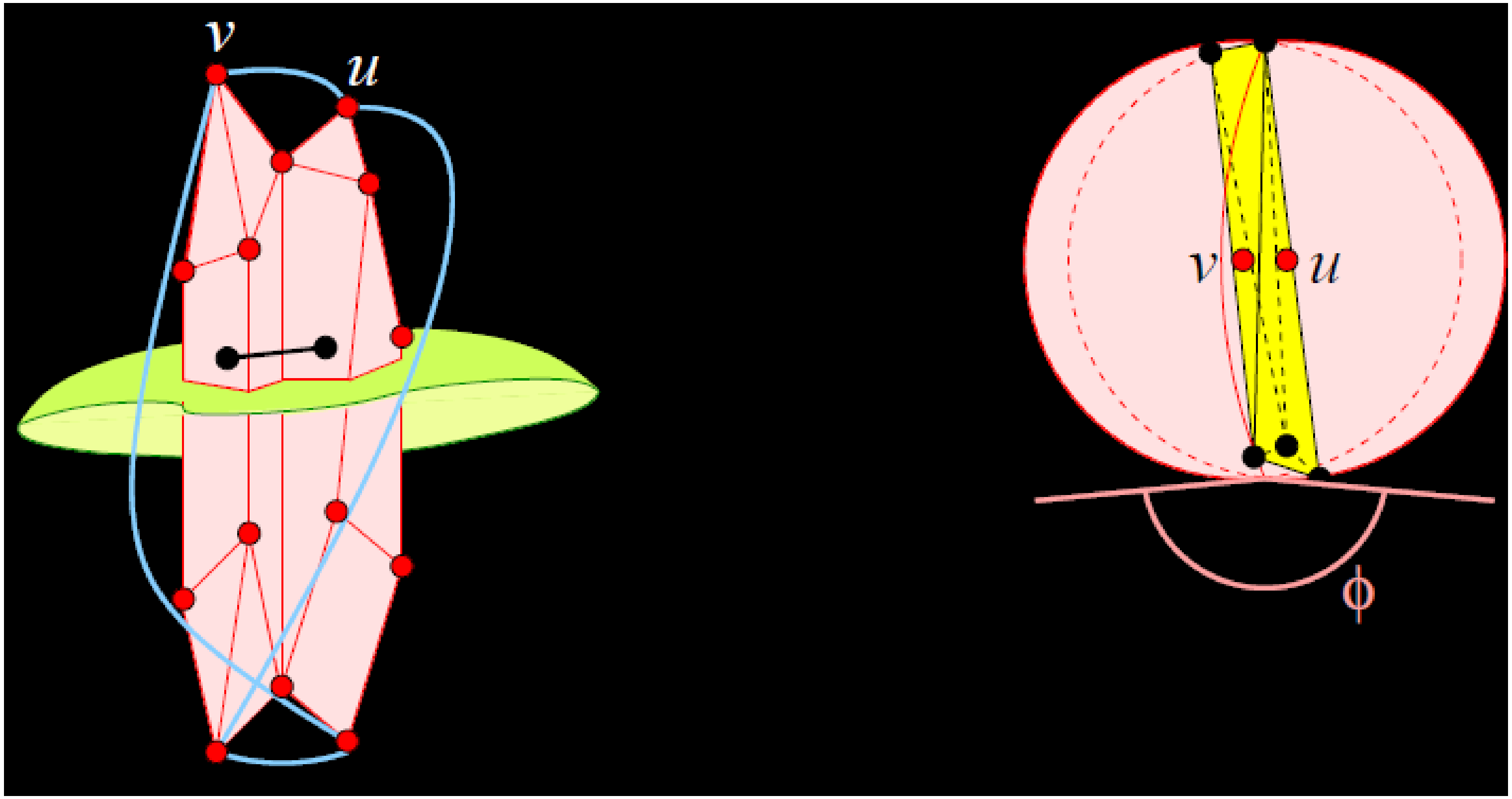
Edges of **Type I** are shown in yellow. These are negative (repelling) edges.



Ignore the colors for now!.

The Pole Graph: Edges

Tipe II : Define egdes for tetrahedra associated to poles of a adjacent Voronoi cells.

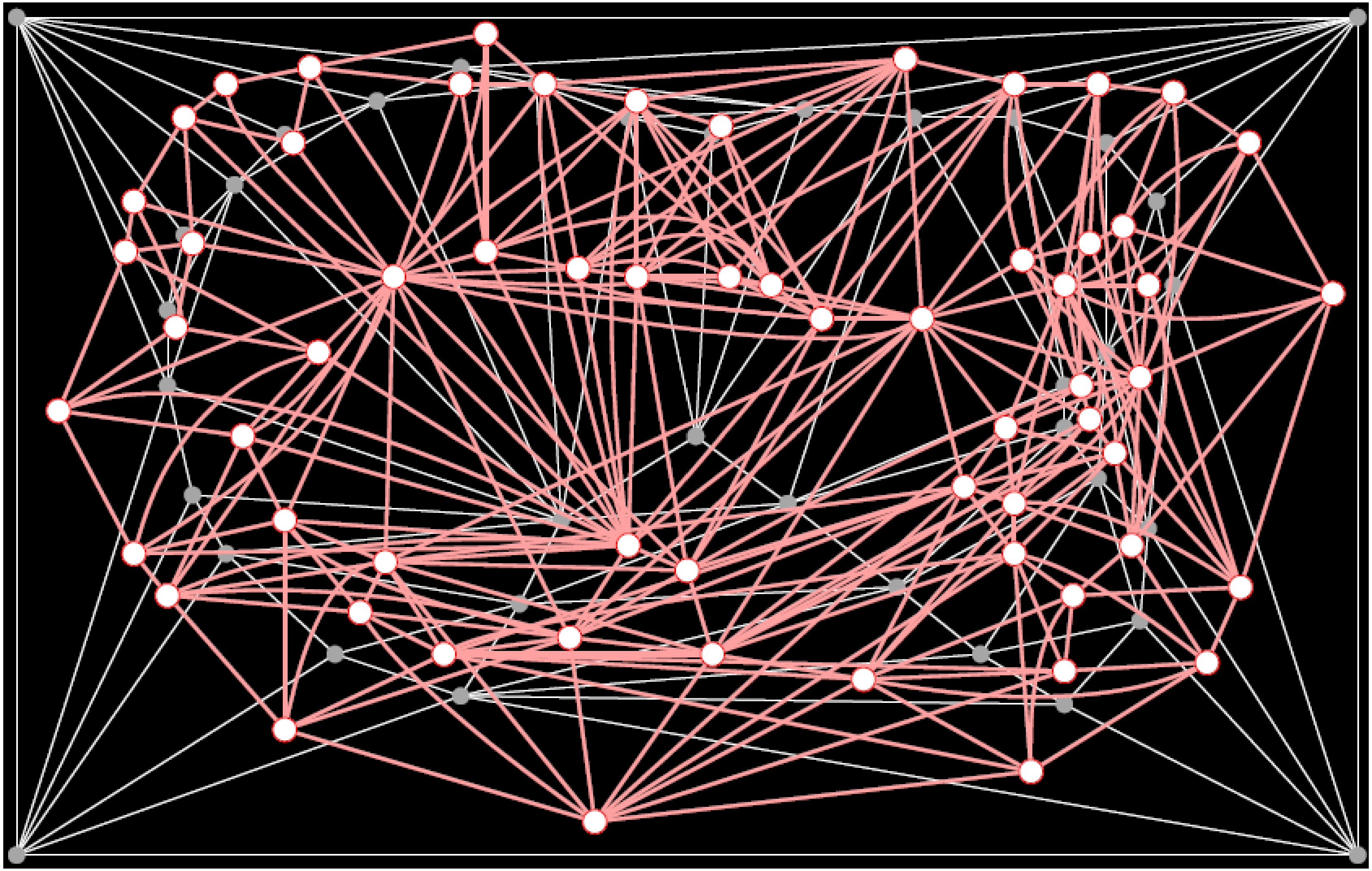


Pair of tetrahedra laying in identical sides usually have large circumsphere intersection. In this case we assign a positive (attractor) edge of weight: $e^{4-4cos\phi}$

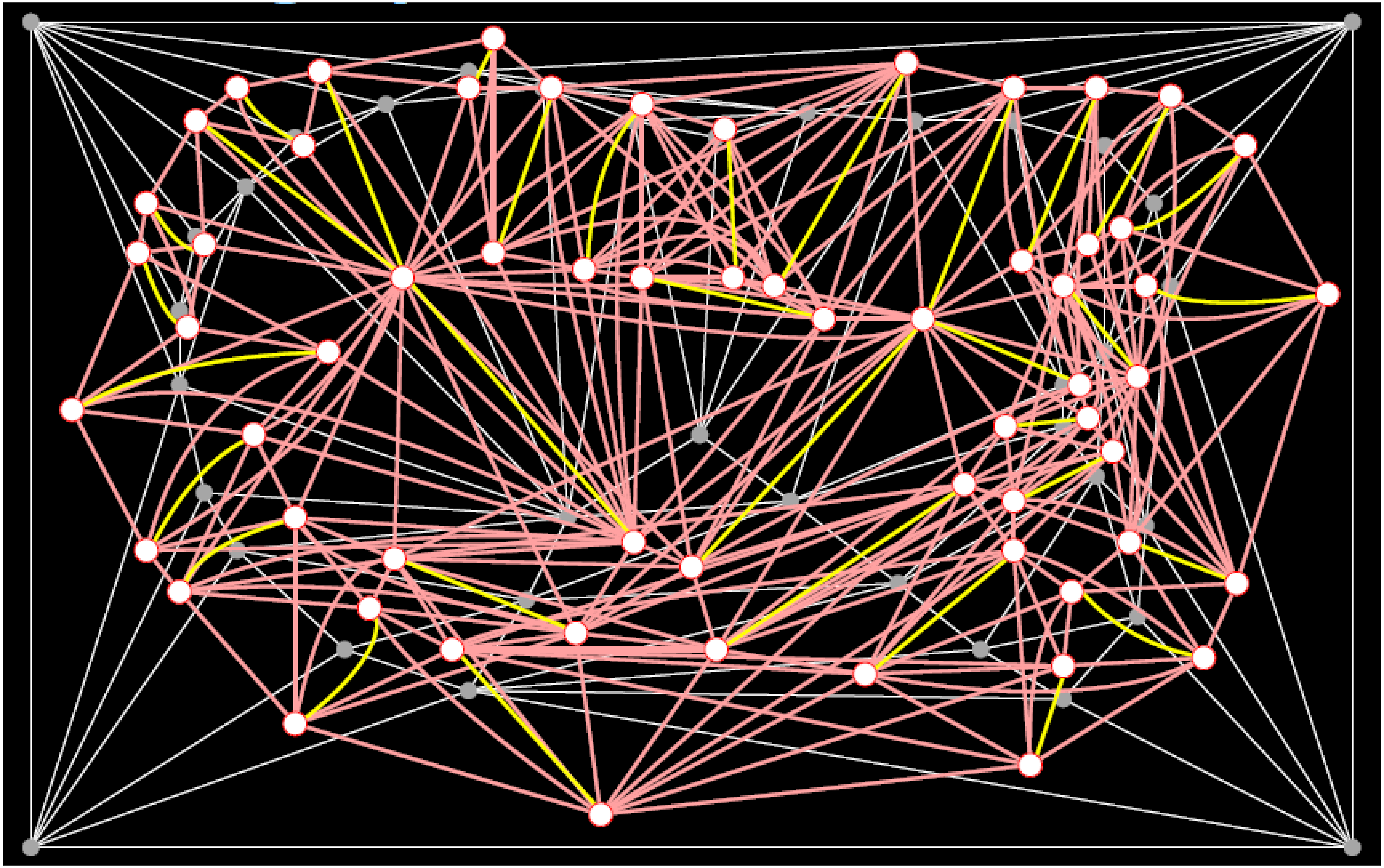
If the circumspheres do not intersect the edge is omitted.

The Pole Graph: Edges

Edges of **Type II** are shown in pink. These are positve (attractor) edges.



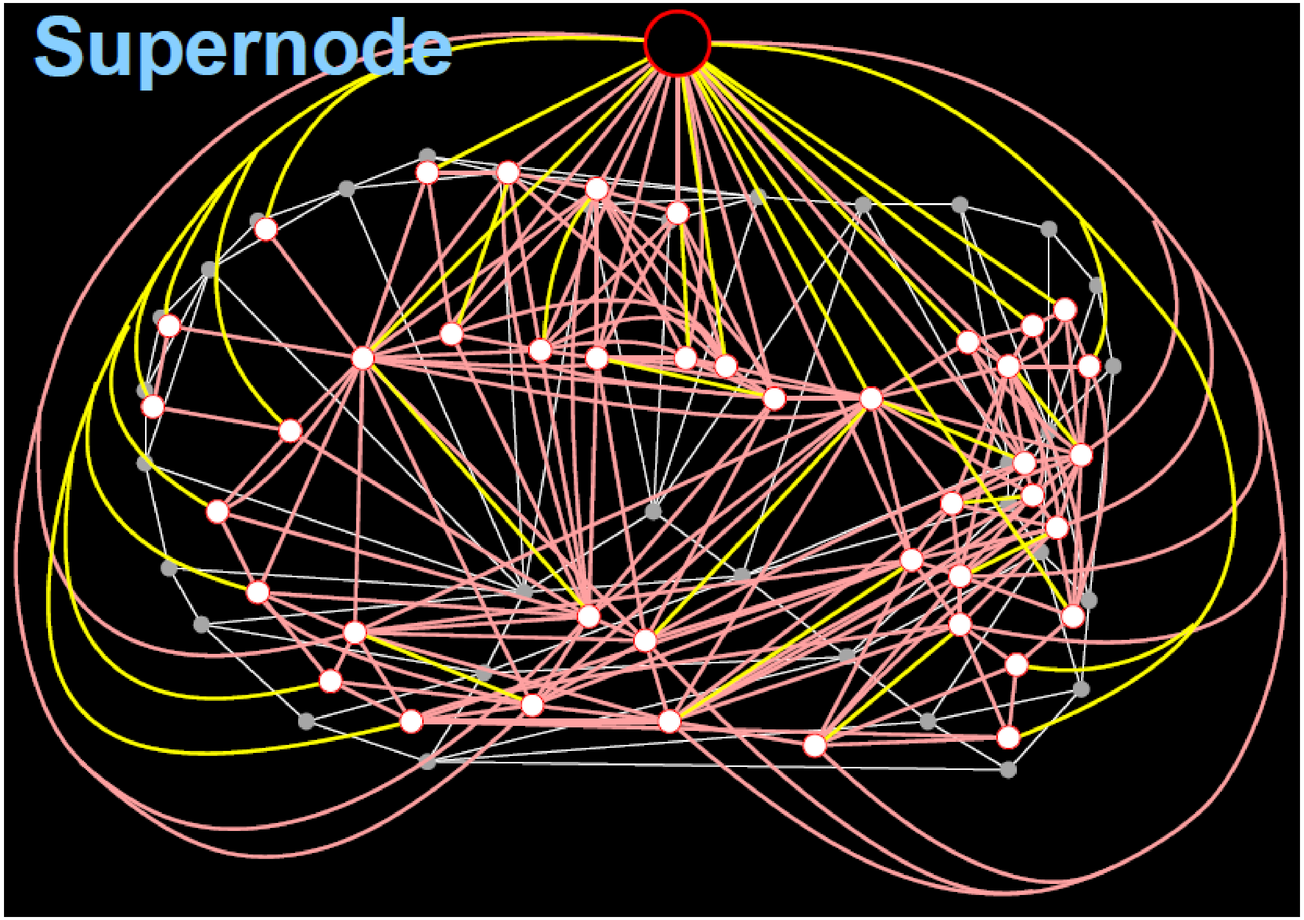
The Pole Graph



Ignore the colors for now!.

The Pole Graph

Tetrahedra with vertices at the bounding box are exterior. Collapse them in a single node.



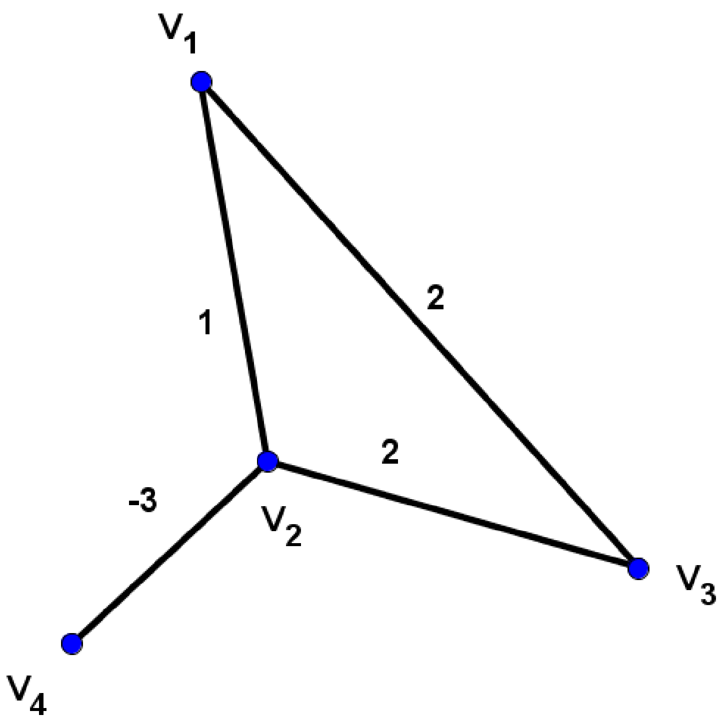
Additional vertices can be collapsed to the exterior (or interior node) by manual seeding.

Graph Cut: Spectral Partitioning

Modified Laplacian Matrix

$$L_{ij} = -w_{ij}$$

$$L_{ii} = \sum_{j \neq i} |L_{ij}|$$



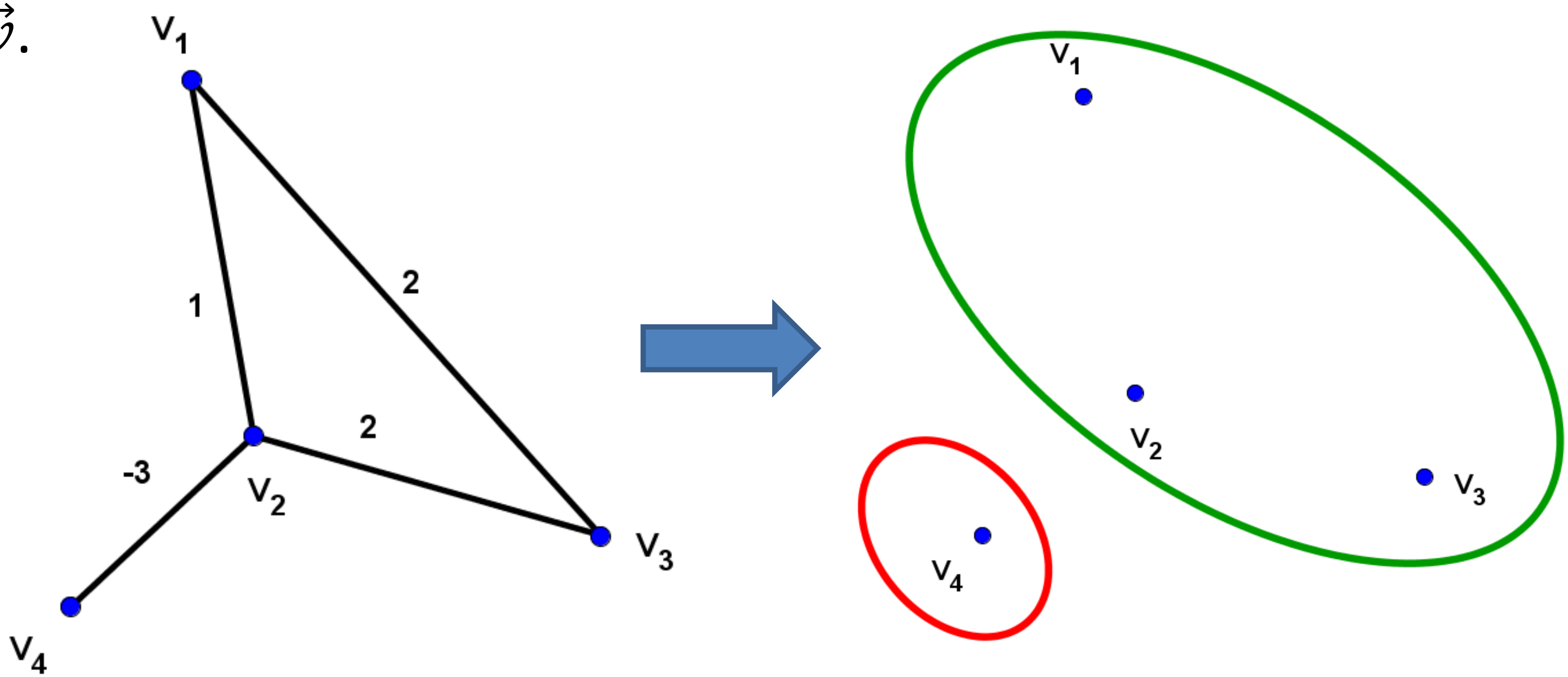
$$\begin{pmatrix} 3 & -1 & -2 & 0 \\ -1 & 6 & -2 & 3 \\ -2 & -2 & 4 & 0 \\ 0 & 3 & 0 & 3 \end{pmatrix}$$

Graph Cut: Spectral Partitioning

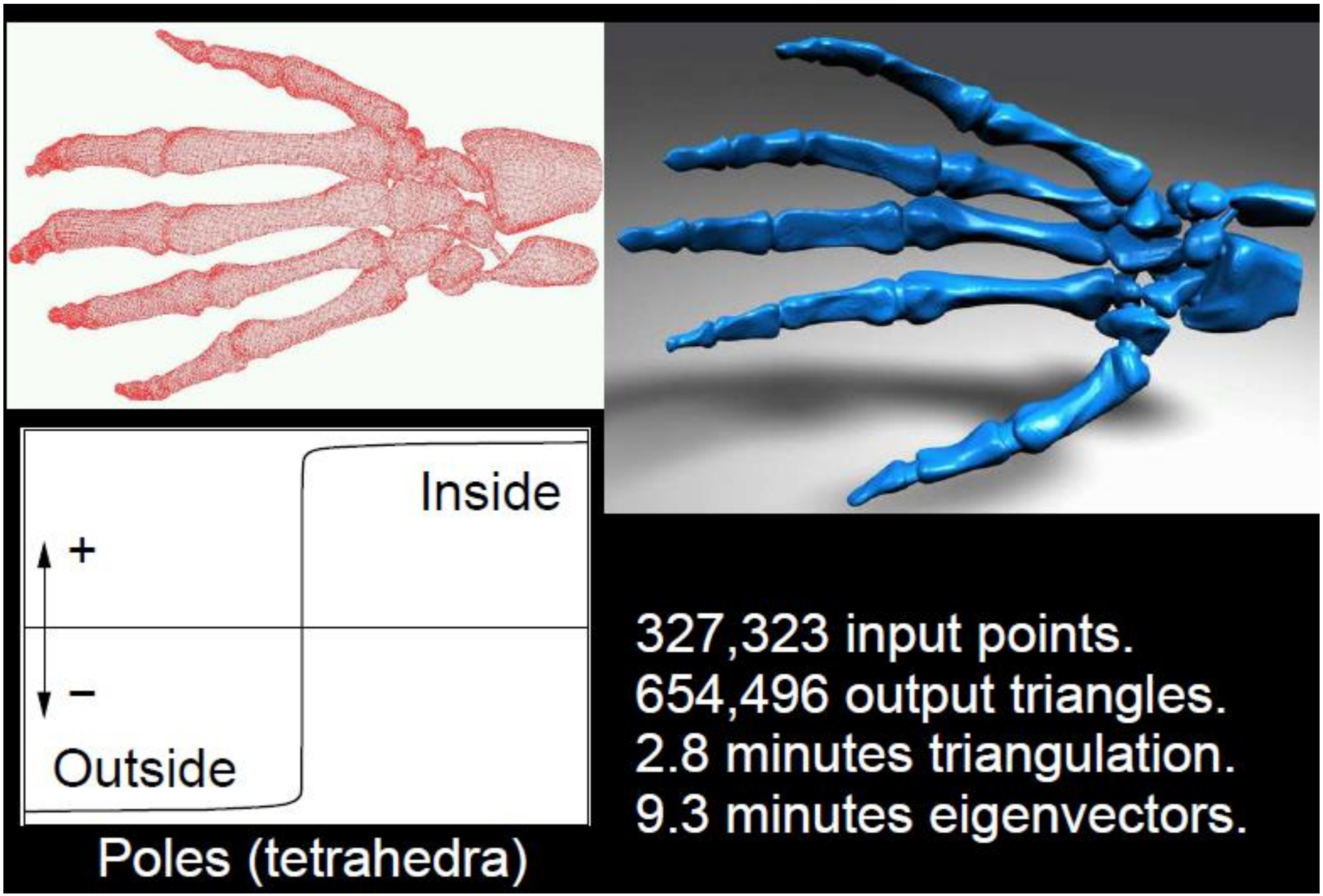
1) Compute an eigenvector $\vec{v}$ of the smallest eigenvalue of L.

$$\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}$$

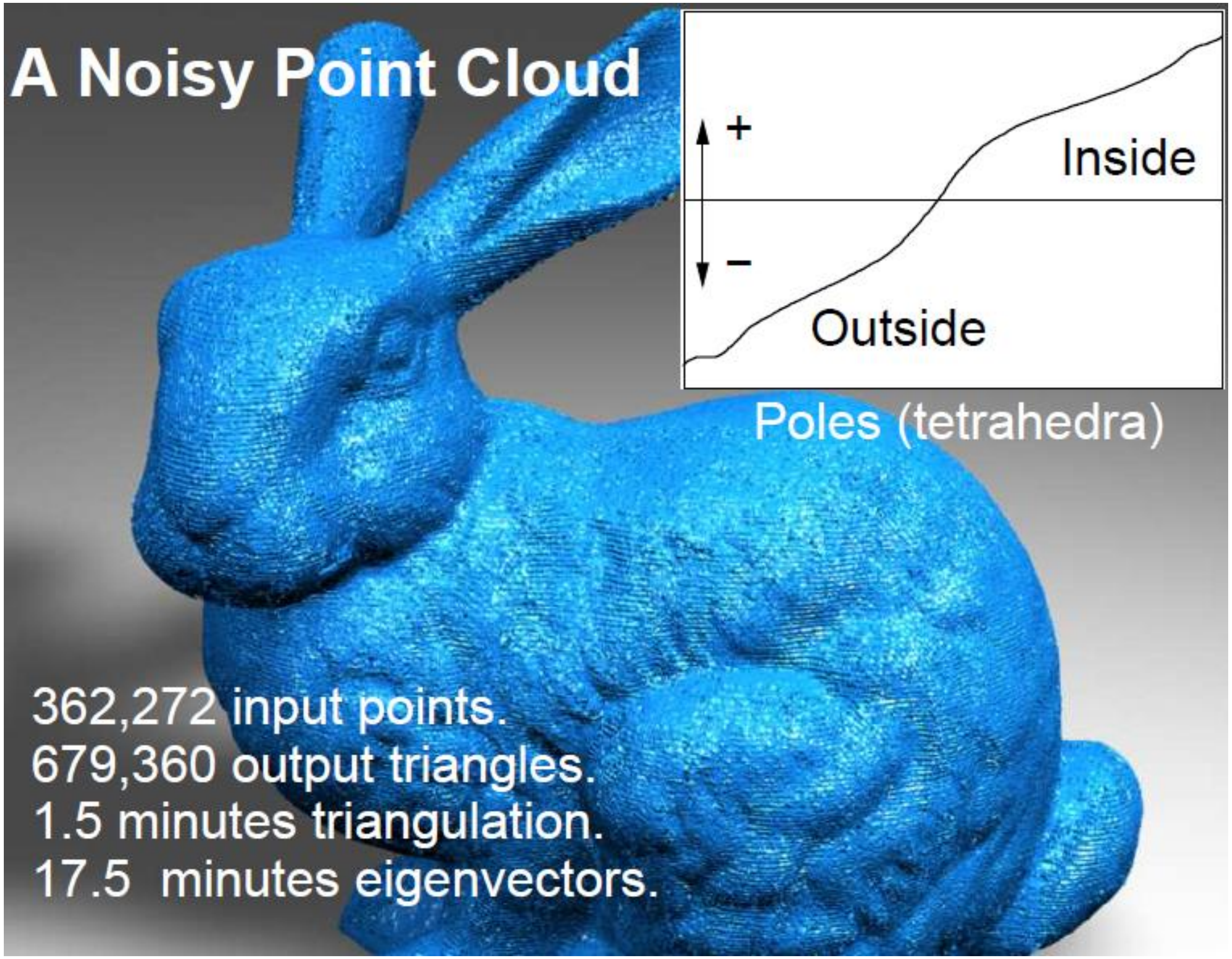
2) Partition the graph according to the sign of respective coordinates in $\vec{v}$.



Noise Free Example



Noisy Example



Why Spectral Graph Partitioning works?

1) It resembles a system of spring and masses. At lowest frequency the "inside masses are usually found vibratring out of phase with the outside masses". [1]

2) It can also be interpreted by transforming the CUT problem to a Total Weight problem.

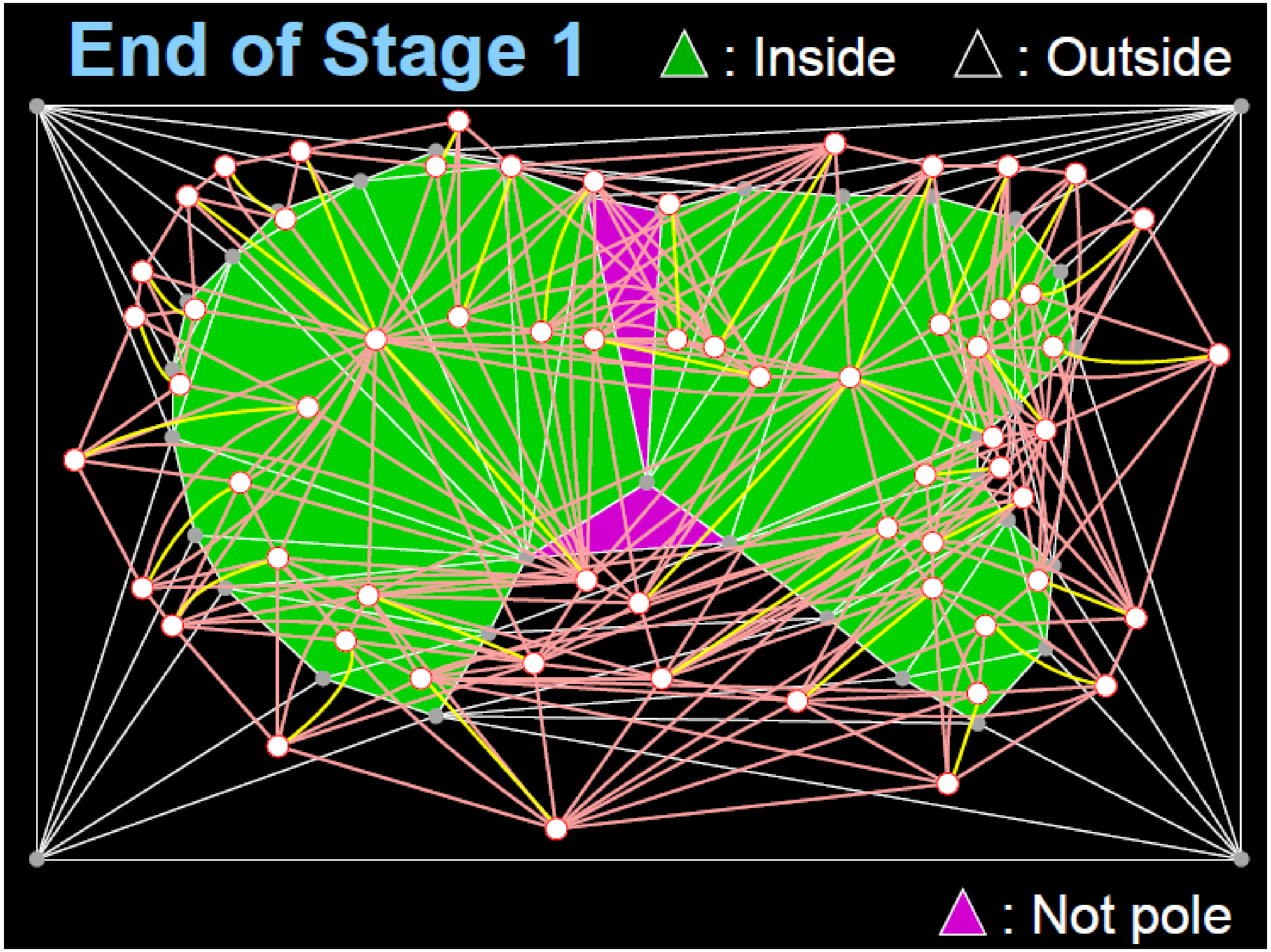
$$\min \sum_{i,j} w_{ij}(1 - v_i v_j) \iff \min \sum_{i,j} -w_{ij} v_i v_j$$

3) The computation of the objective eigenvector can be done using Lanczos iterative solver, which is $O(n\sqrt{n})$.

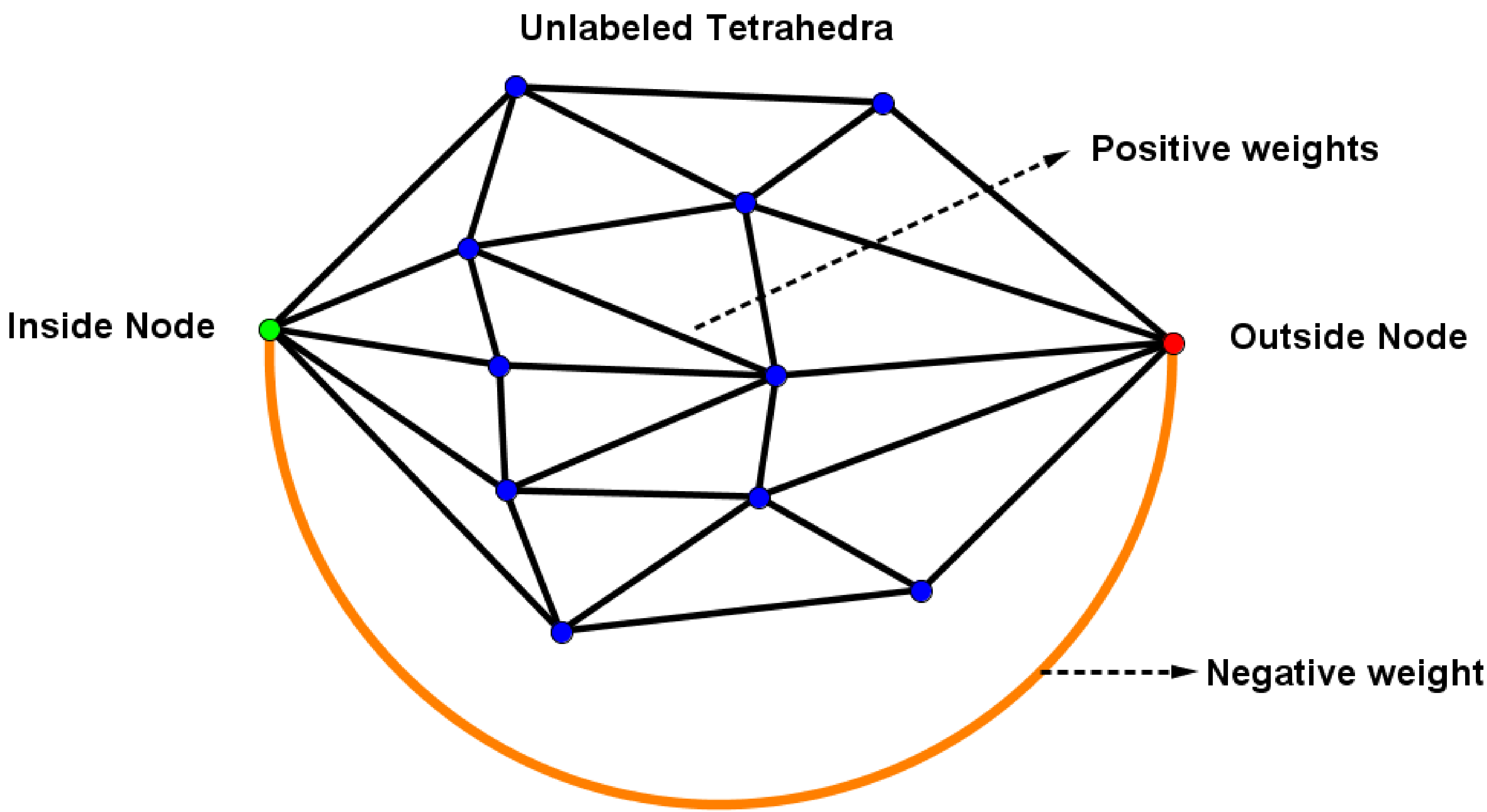
[1] See the link www.cs.berkeley.edu/~jrs/papers/partnotes.pdf for a deeper insight on this interpretation.

Complement Graph

The previous step just labeled tetrahedra associated to poles.
Now we have to decide a label for the remaining tetrahedra.

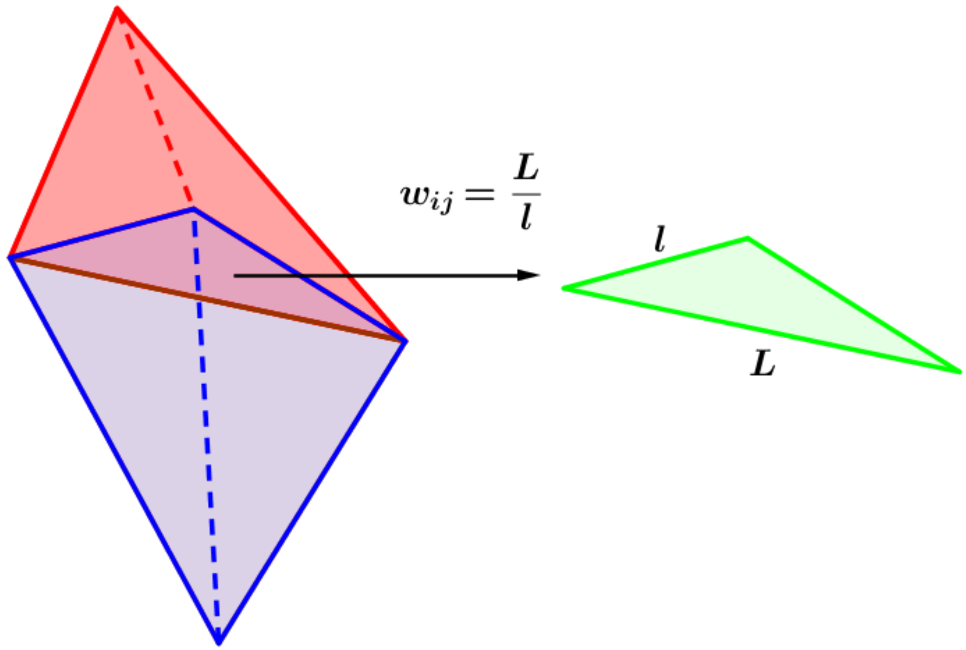


Complement Graph: Vertices and Edges



Cut the graph using spectral partitioning.

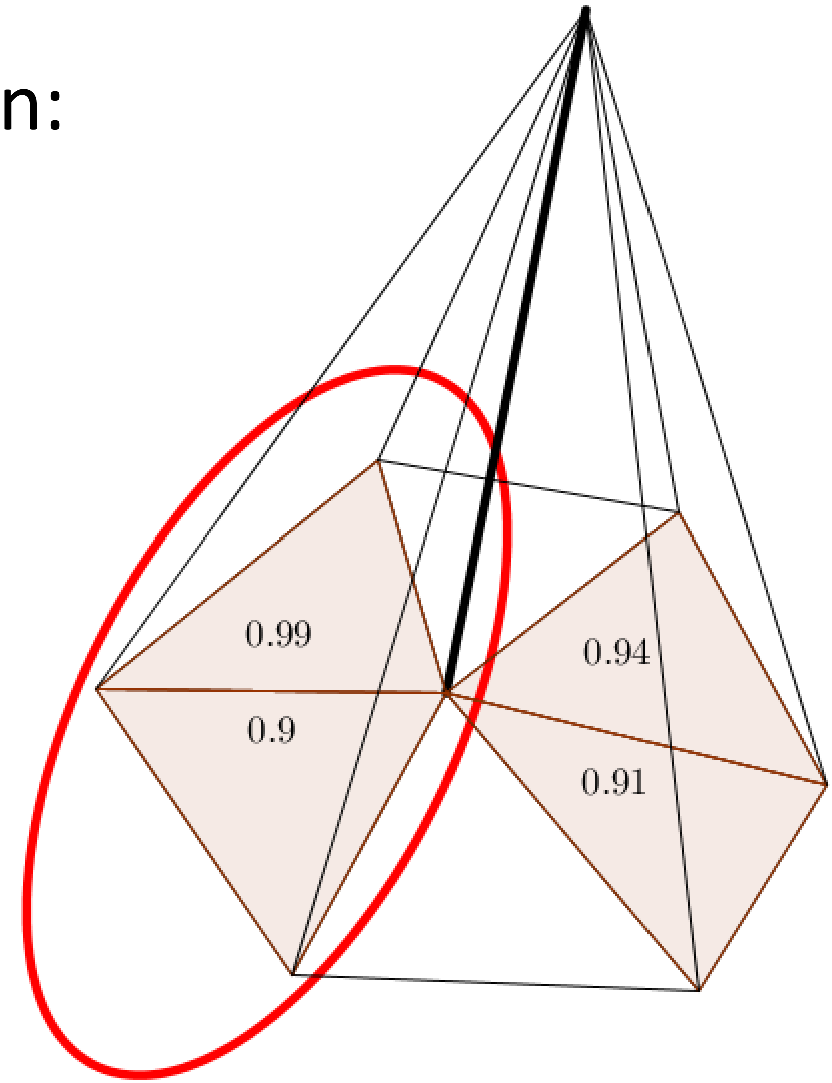
Complement Graph: Weights



1) These kind of weights produced the best results in terms of low genus.
2) This also favorates a surface with regular shaped triangles.
3) The two supernodes are joined by a negative weighted edge that is the negative of the sum of all other edge weights.

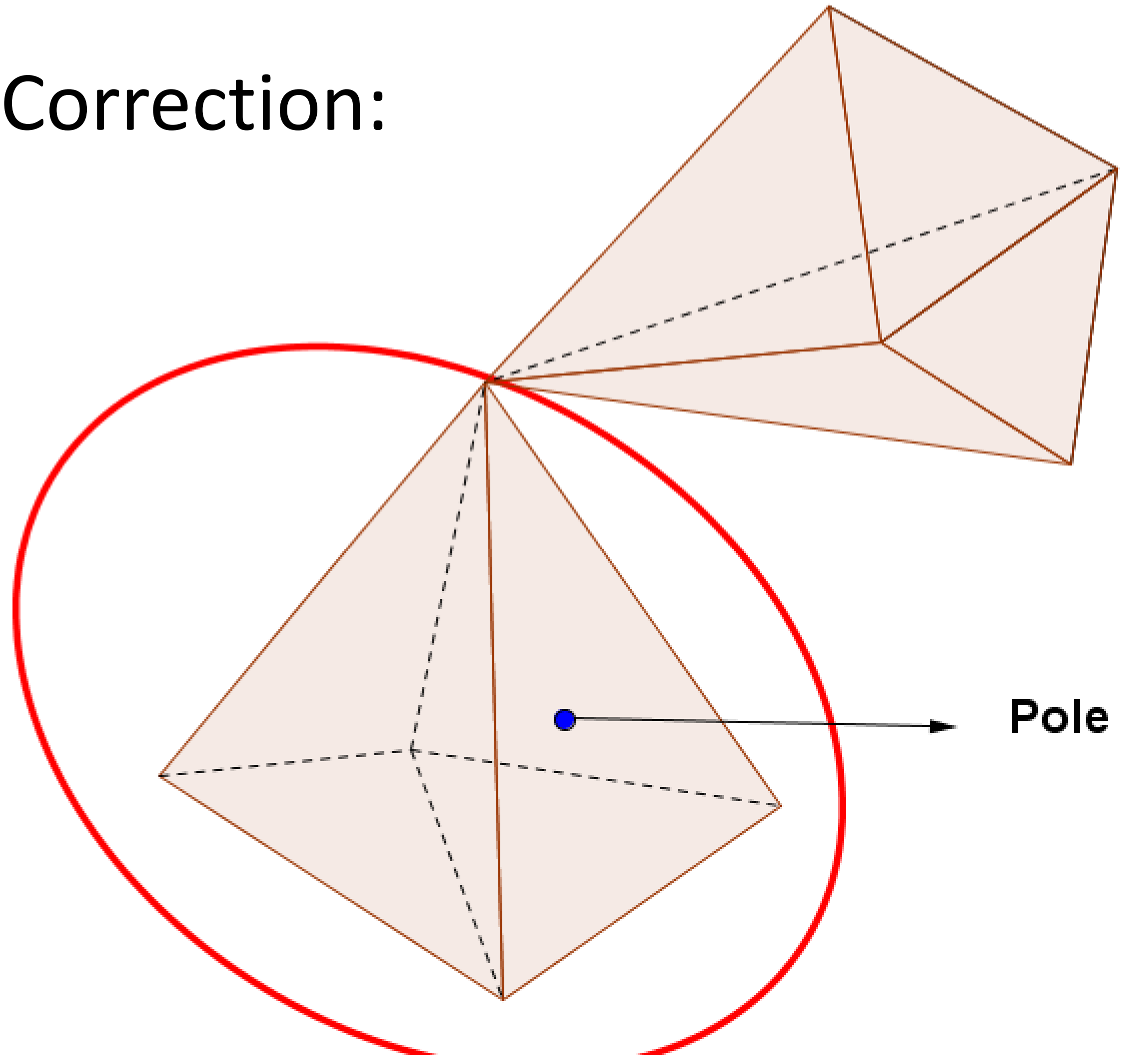
Postprocessing: Make it Manifold

1) Edge Correction:

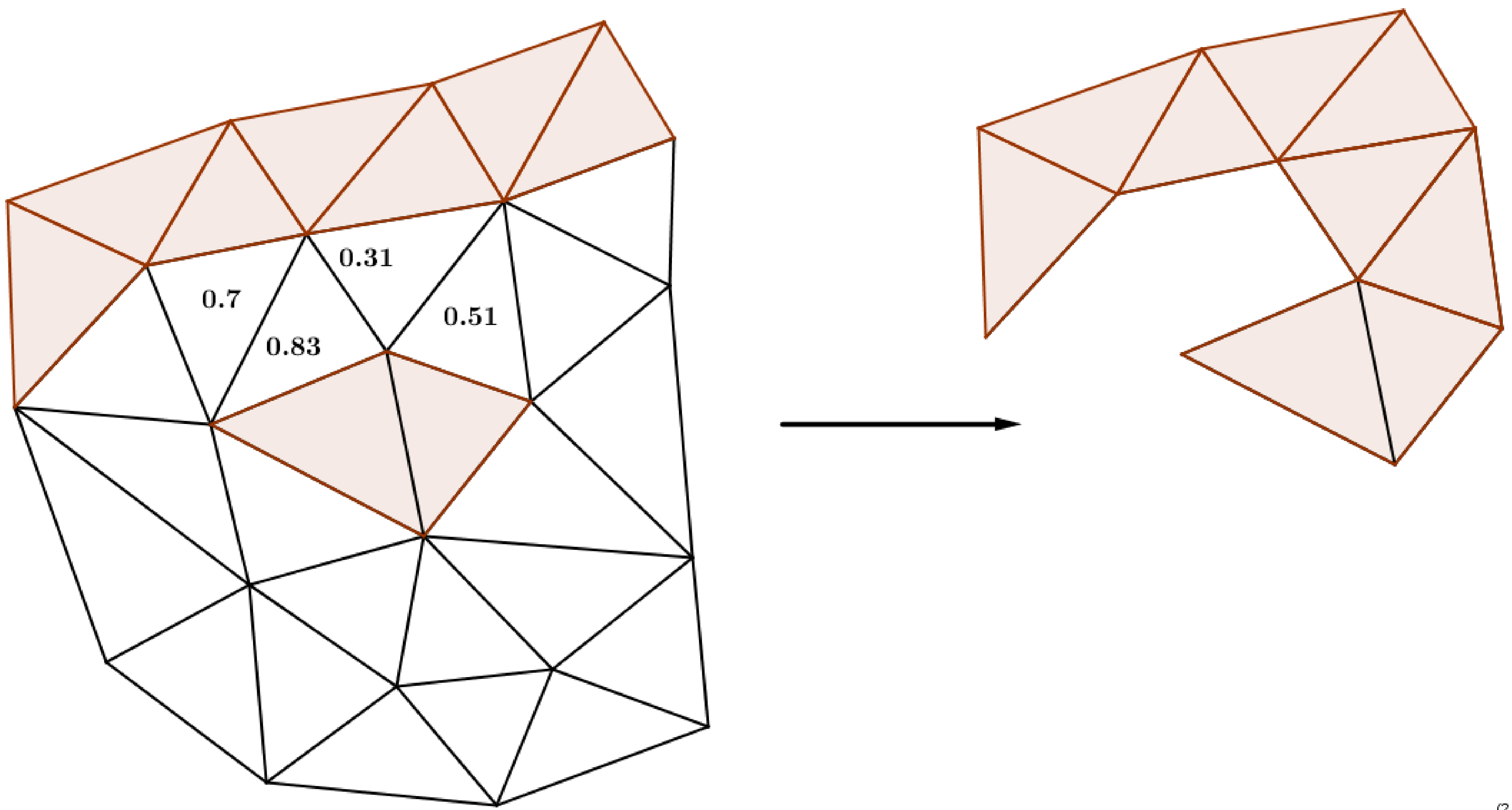


Postprocessing: Make it Manifold

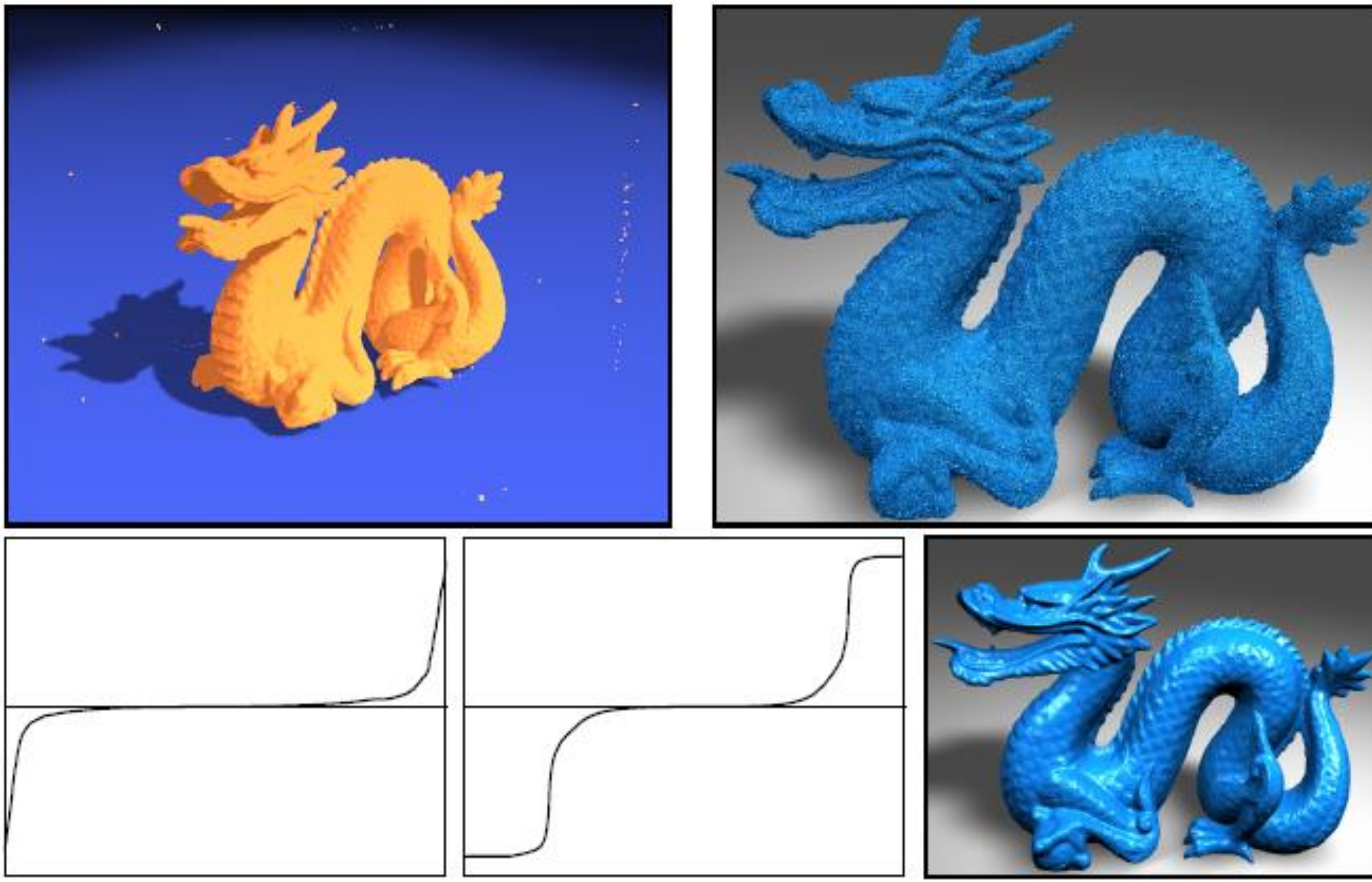
2) Vertex Correction:



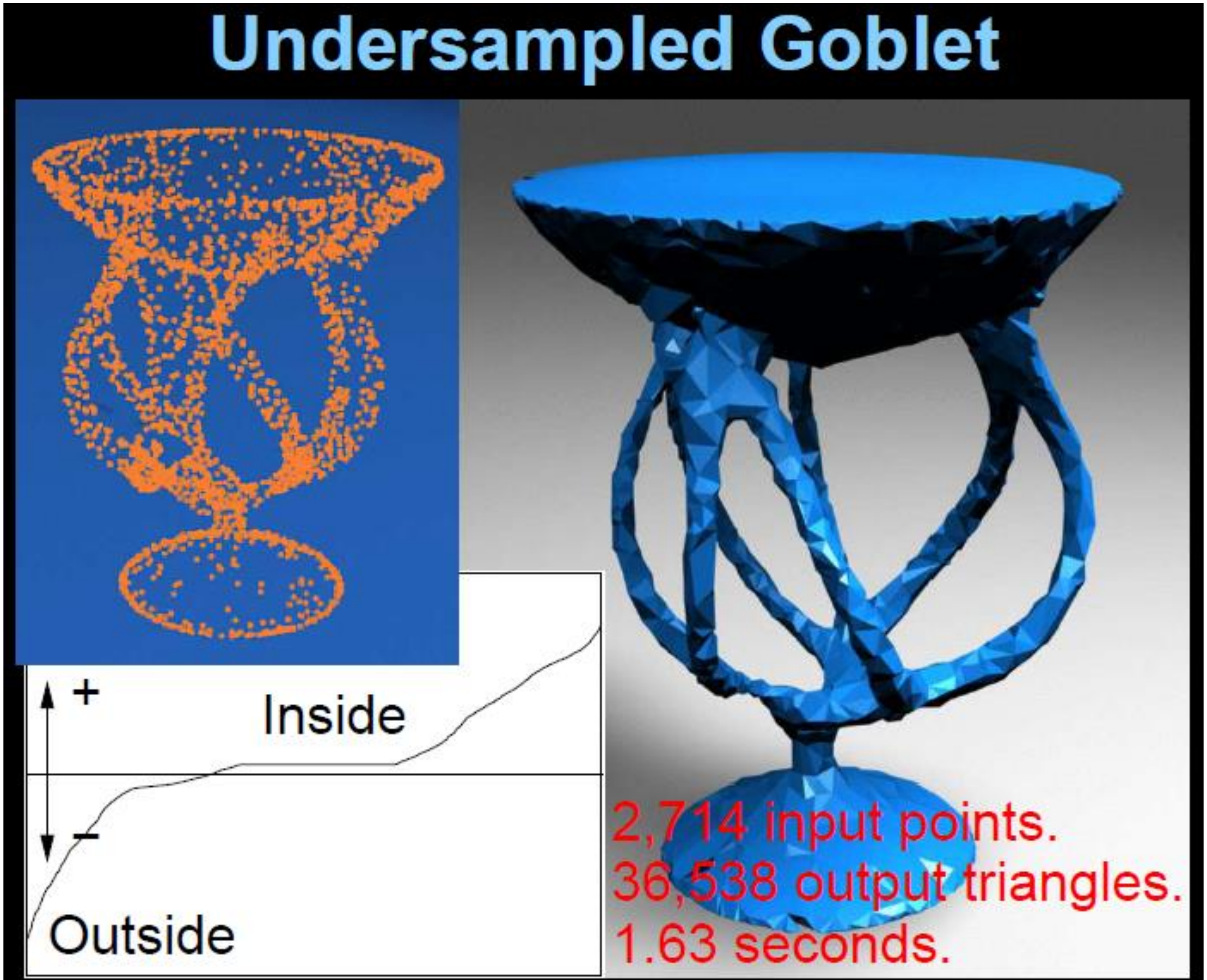
Postprocessing: Genus Reduction



Results



Results


Undersampled Goblet
+
Inside
–
Outside
2,714 input points.
36,538 output triangles.
1.63 seconds.

Authors Observations:

1) “It occasionally creates unwanted handles”.
2) “Eigenvector computation is slow”.
3) “Tetrahedra labeling algorithms do not reconstruct sharp cornes well”.