Survey of Methods in Computer Graphics:

Surface Reconstruction from Unorganized Points

H. Hoppe, T. DeRose, T. Duchamp, J. McDonald, W. Stuetzle

SIGGRAPH 1992.

Article and Additional Material at:

http://research.microsoft.com/en-us/um/people/hoppe/proj/recon/

Method Pipeline

Input:

- Cloud of Points.
- Orientation **not** required.

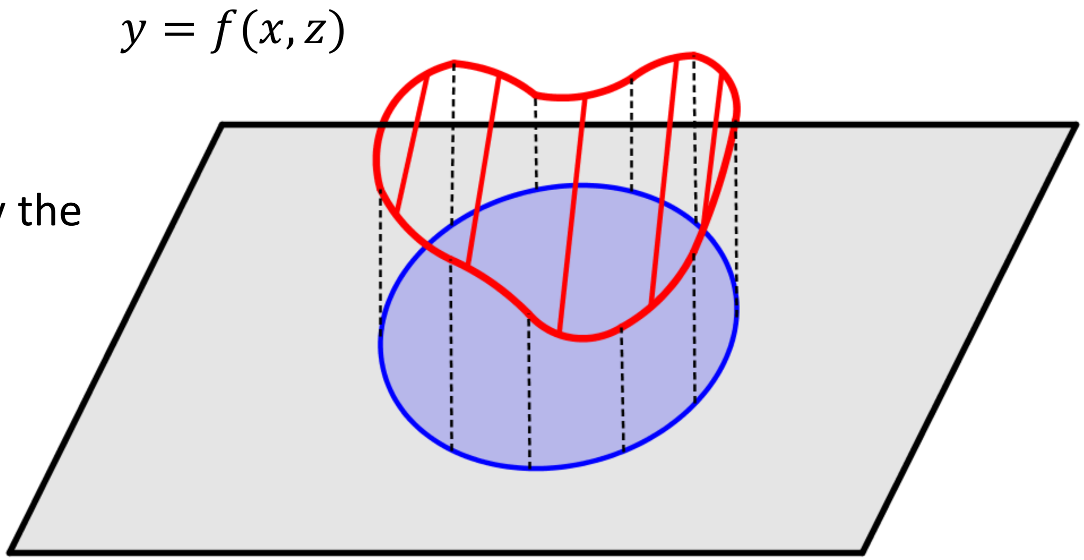
Output:

- Triangular Mesh.
- **Possible boundary edges.**

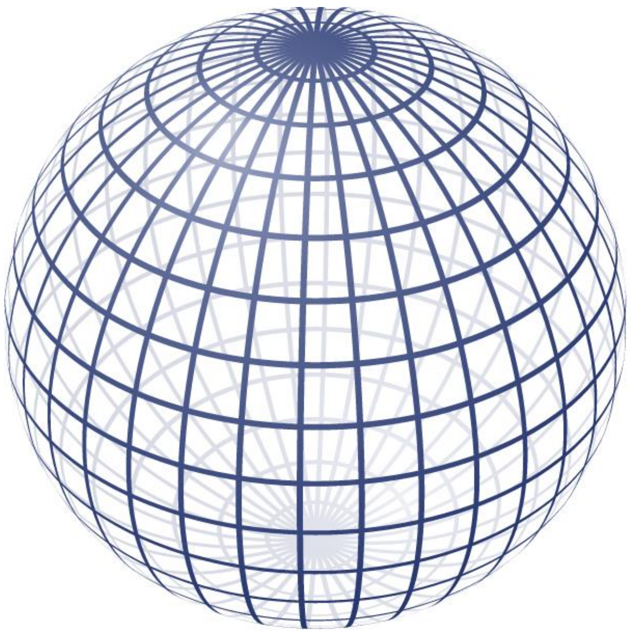
Guarantees:

- Manifold.

Surface Reconstruction vs Function Reconstruction

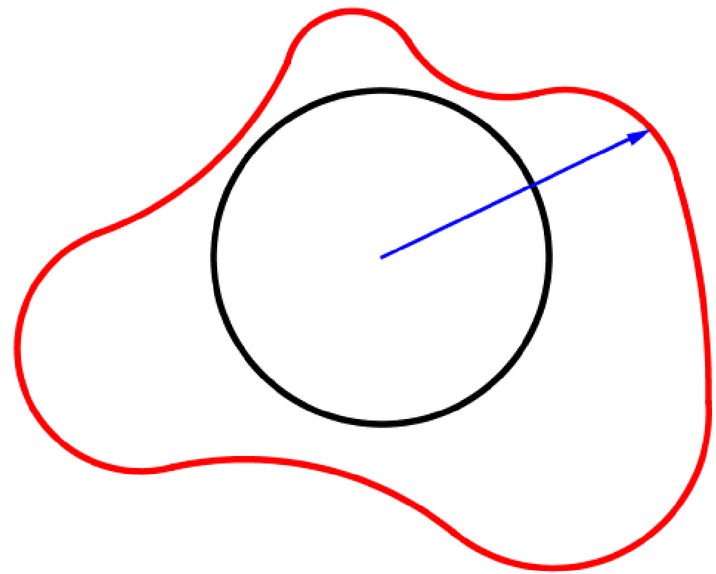


By **definition**, every surface is locally the graph of a function.

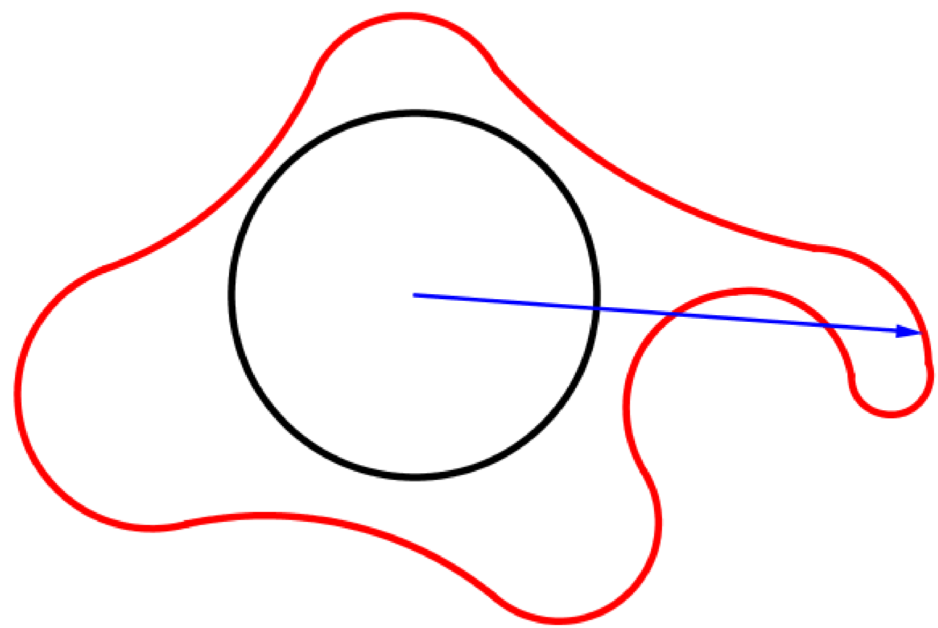


Not all surfaces admit a representation as the graph of a unique function (chart). Instead, we should represent the surface as a collection of such charts (atlas) .

Surface Reconstruction vs Function Reconstruction

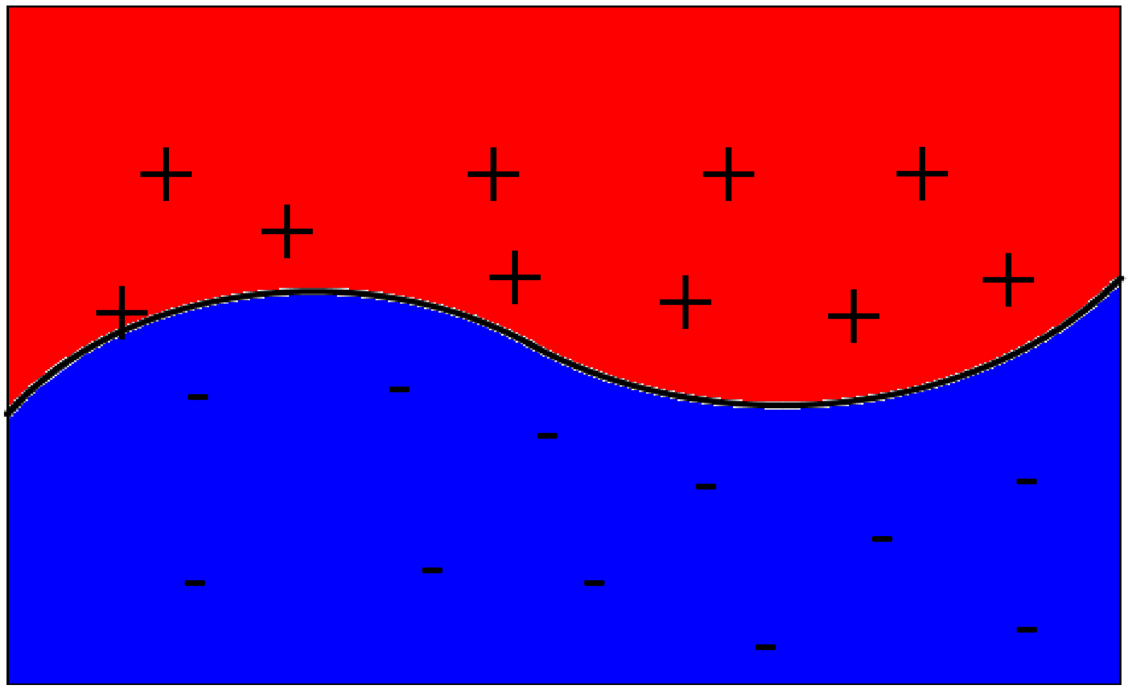


Even surfaces homeomorphic to spheres may not be respresented as the graph of a function over a sphere.



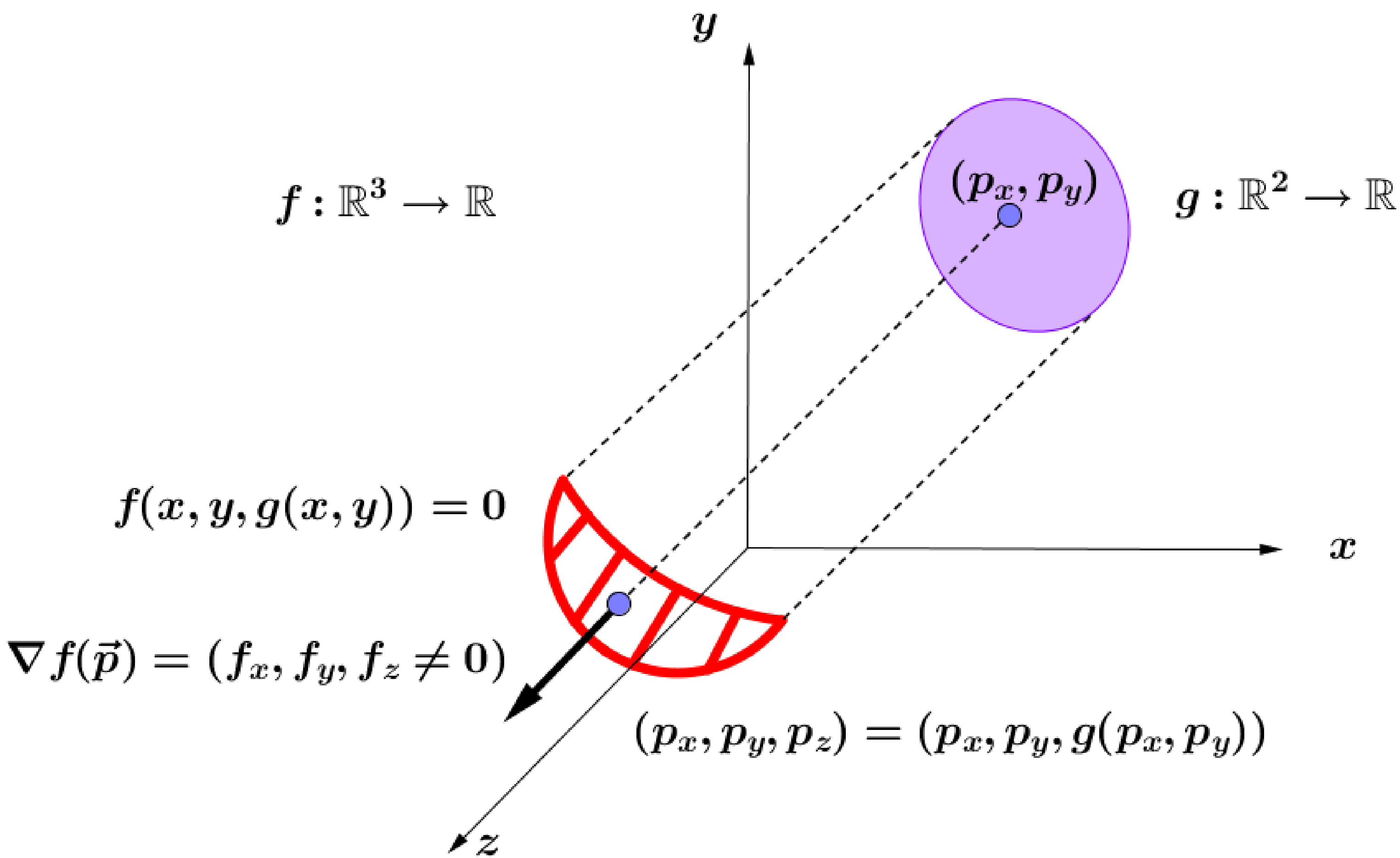
Implicit Surfaces : Regular Value

- We would like to represent a function as the zero set of a function $f: R^3 \rightarrow R$.



- We say that zero is a regular value of f if $\nabla f(p) \neq 0$ for all points such that $f(p) = 0$.

Implicit Function Theorem



If zero is a regular value → The zero set is locally the graph of a function → The zero set is a surface!

Computation of the Signed Distance Function

1) For each sample fit a tangent plane using its k- Nearest Neighbours.

2) Define a coherent orientation for the tangent plane of all sample points.

3) For any $p \in R^3$ the signed distance function is given by its closest (oriented) tangent plane.

Tangent Plane Fitting

Find a plane that fits, in the Least Squares sense, to its K- Nearest Neighbours:

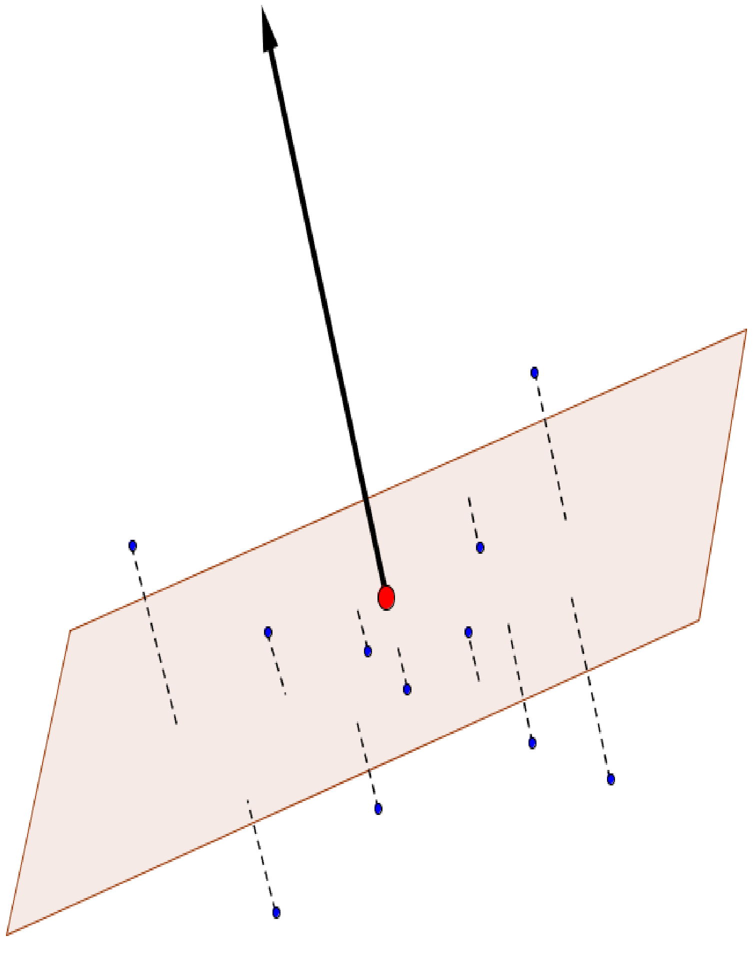
$$\min_{\vec{n}\in S^2, a\in R} \sum_{i=1}^{k} (x_i \cdot \vec{n} - a)^2$$

Properties:

1) This plane passes trhough the baricenter of the neighbours $o = \frac{1}{k}\sum_{i=1}^{k} x_i$

2) The normal direction is given an eigenvector of smallest eigenvalue of the covariance matrix

$$\sum_{i=1}^{k} (x_i - o)(x_i - o)^T$$



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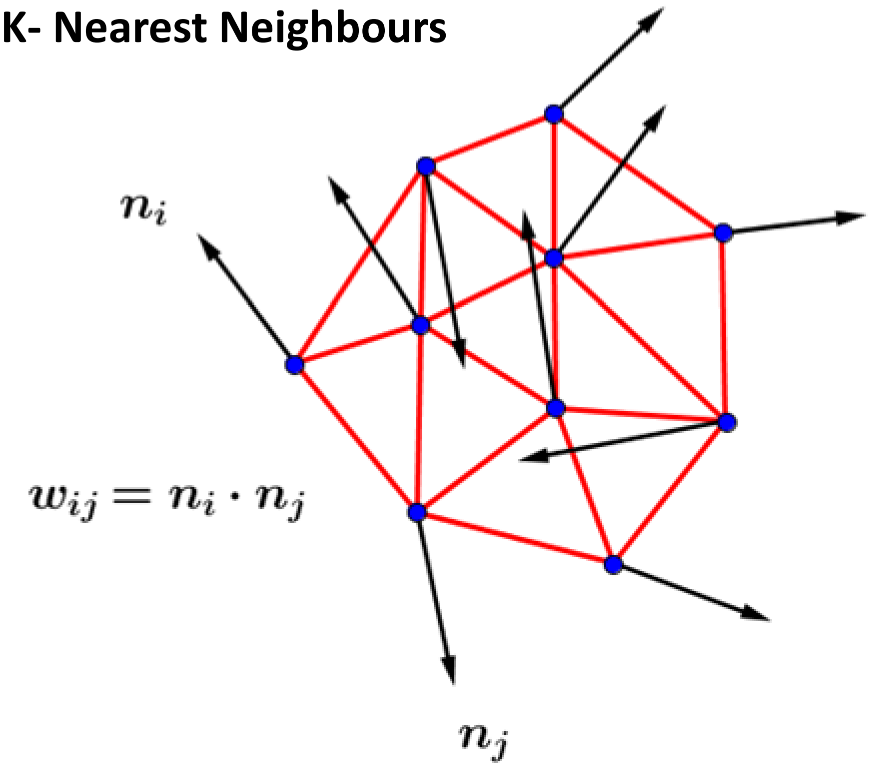
Global Optimization

Riemannian Graph

K- Nearest Neighbours



Energy Function

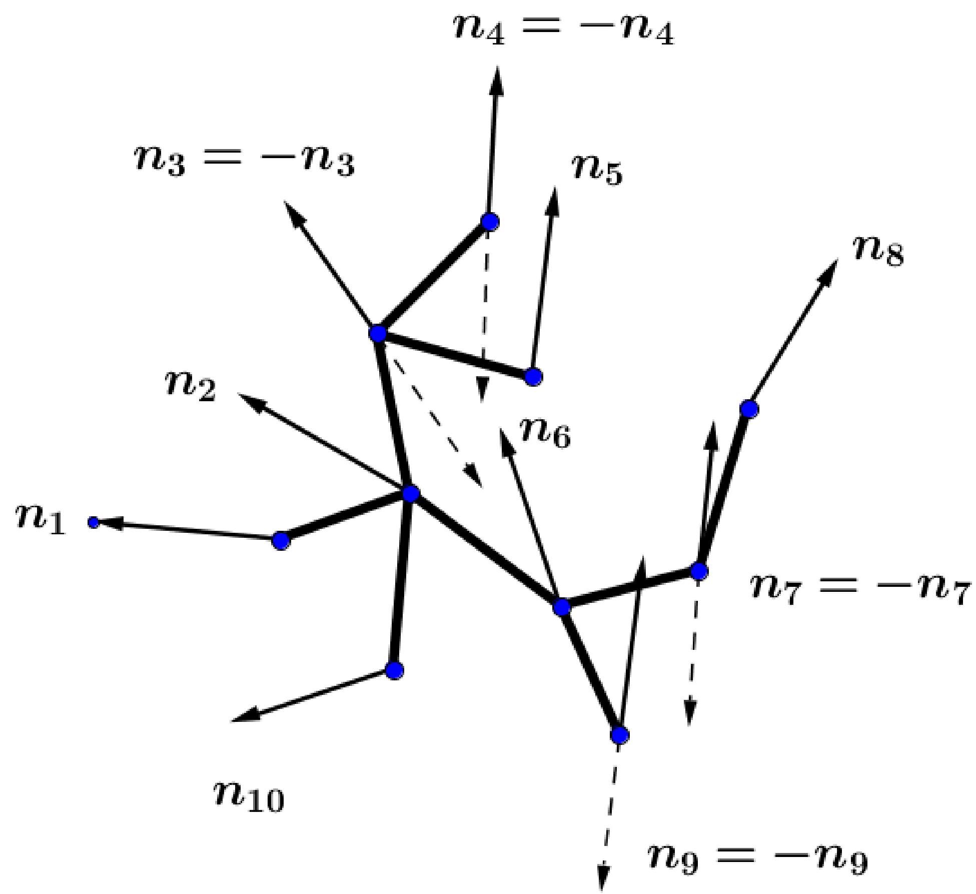
$$\Phi : V \rightarrow \{-1, 1\}$$

$$\max \sum_{(i,j) \in E} w_{ij} \Phi_i \Phi_j$$

NP-hard!

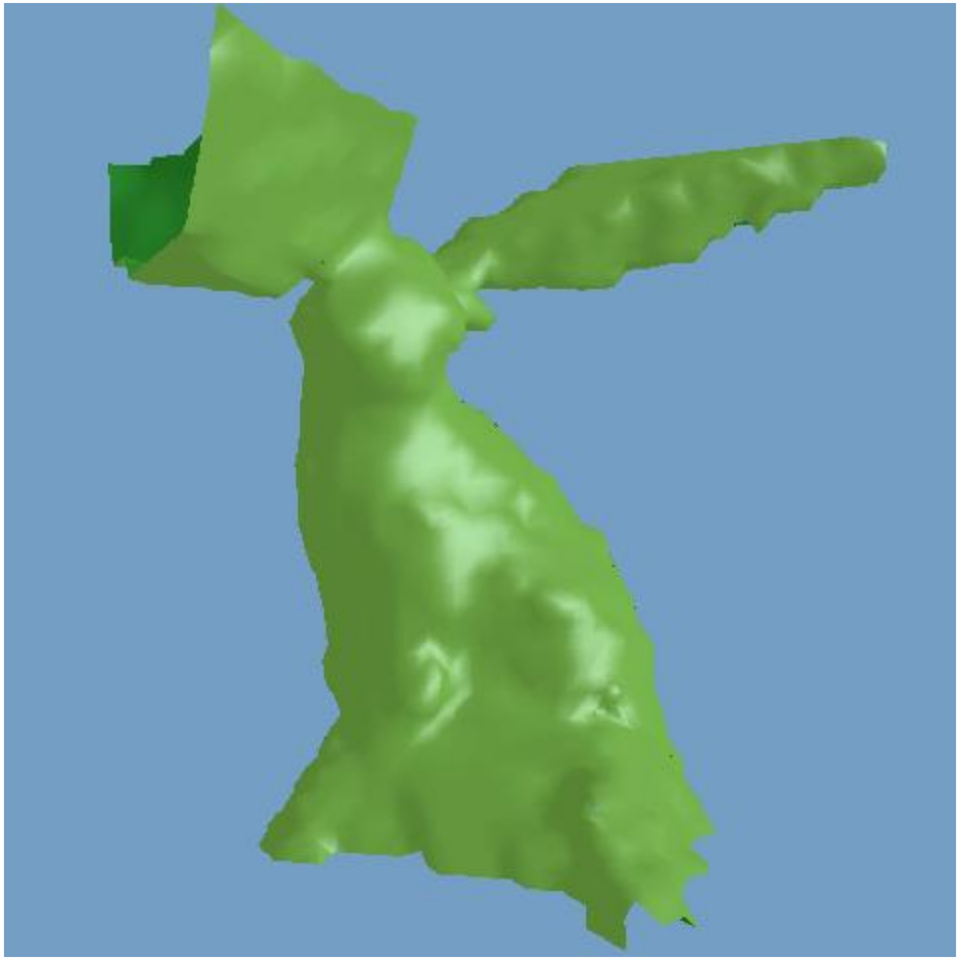
Normal Propagation By Geometric Proximity

Euclidean Minimum Spanning Tree



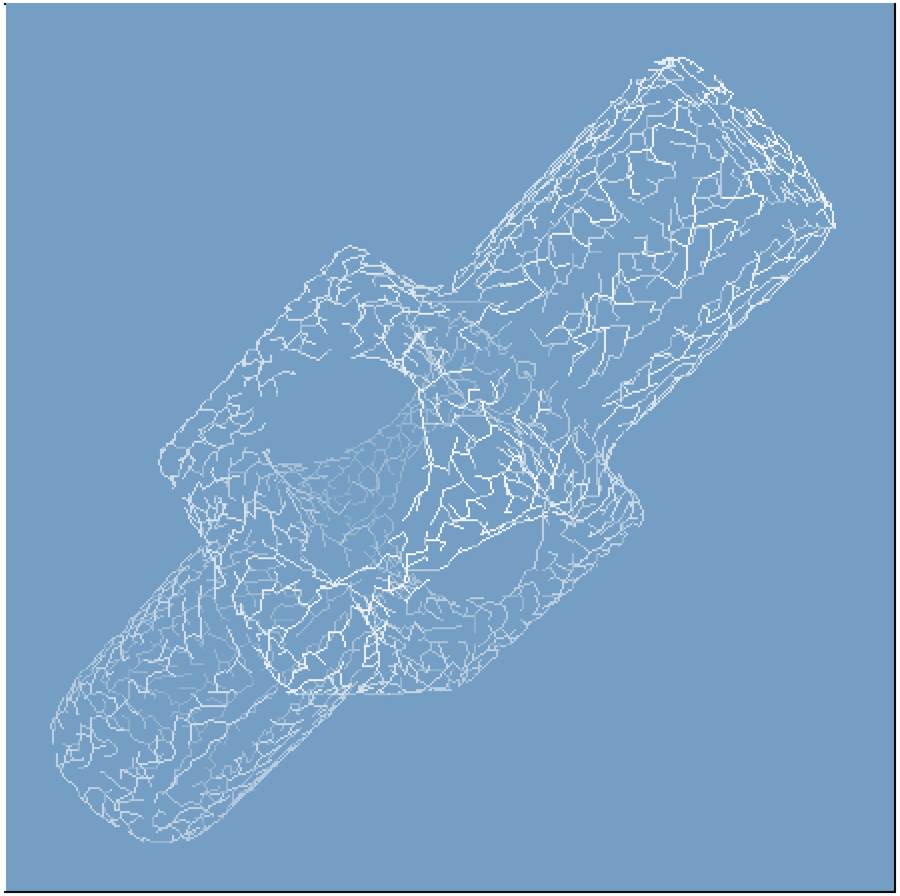
Take $n_j = -n_j$ if $n_i \cdot n_j < 0$

Normal Propagation By Geometric Proximity

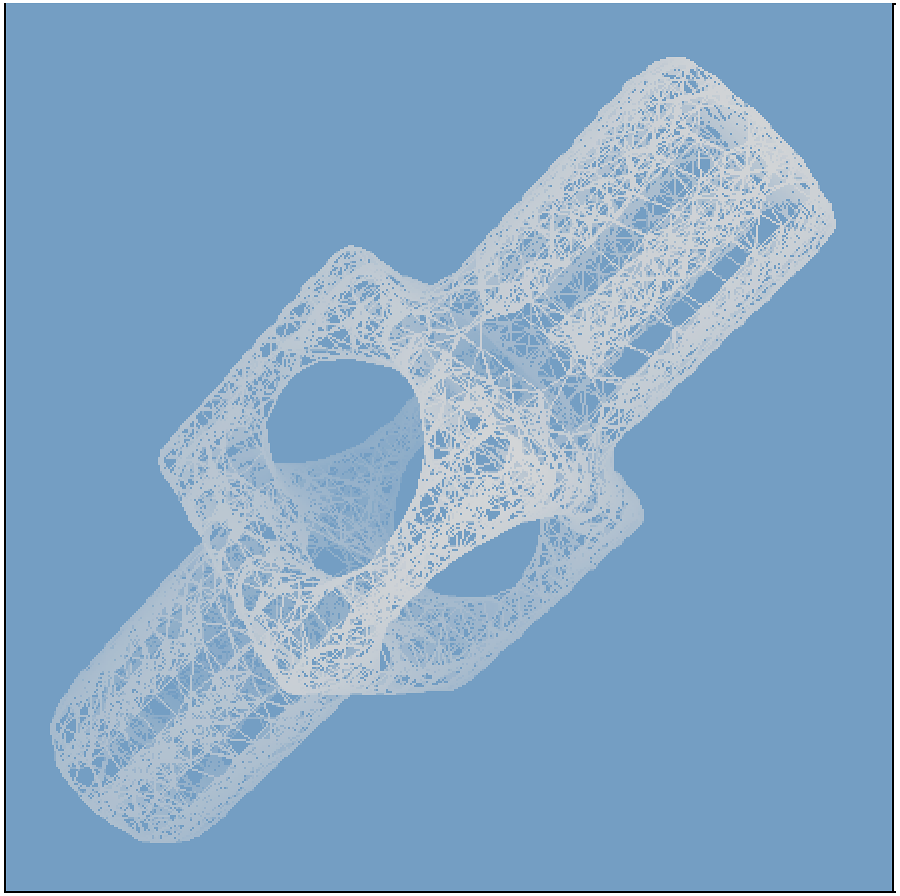


Geometric proximity is not a good criteria for normal propagation.

EMST



Riemmanian Graph

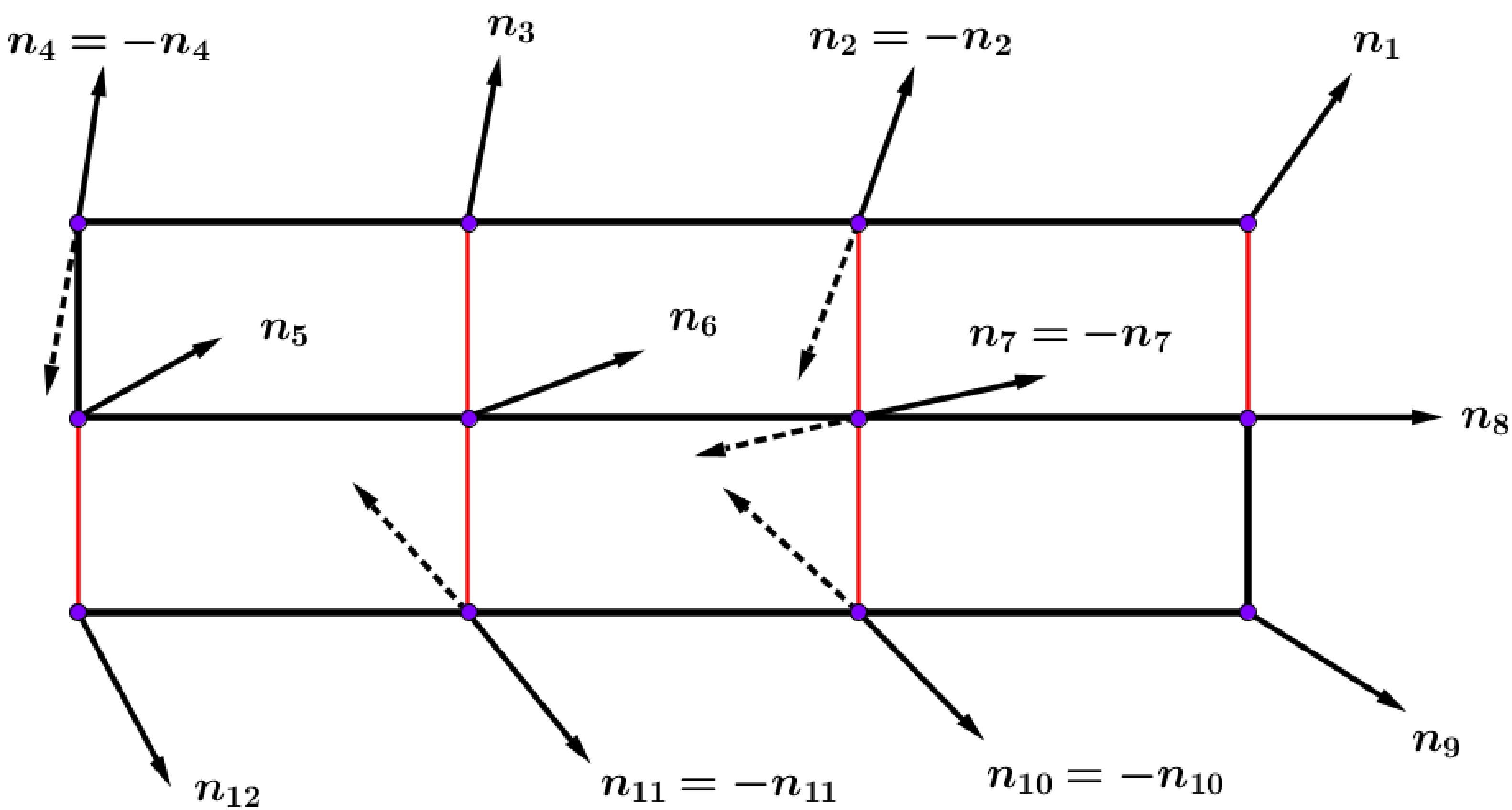


EMST is connected but not dense enough in edges.

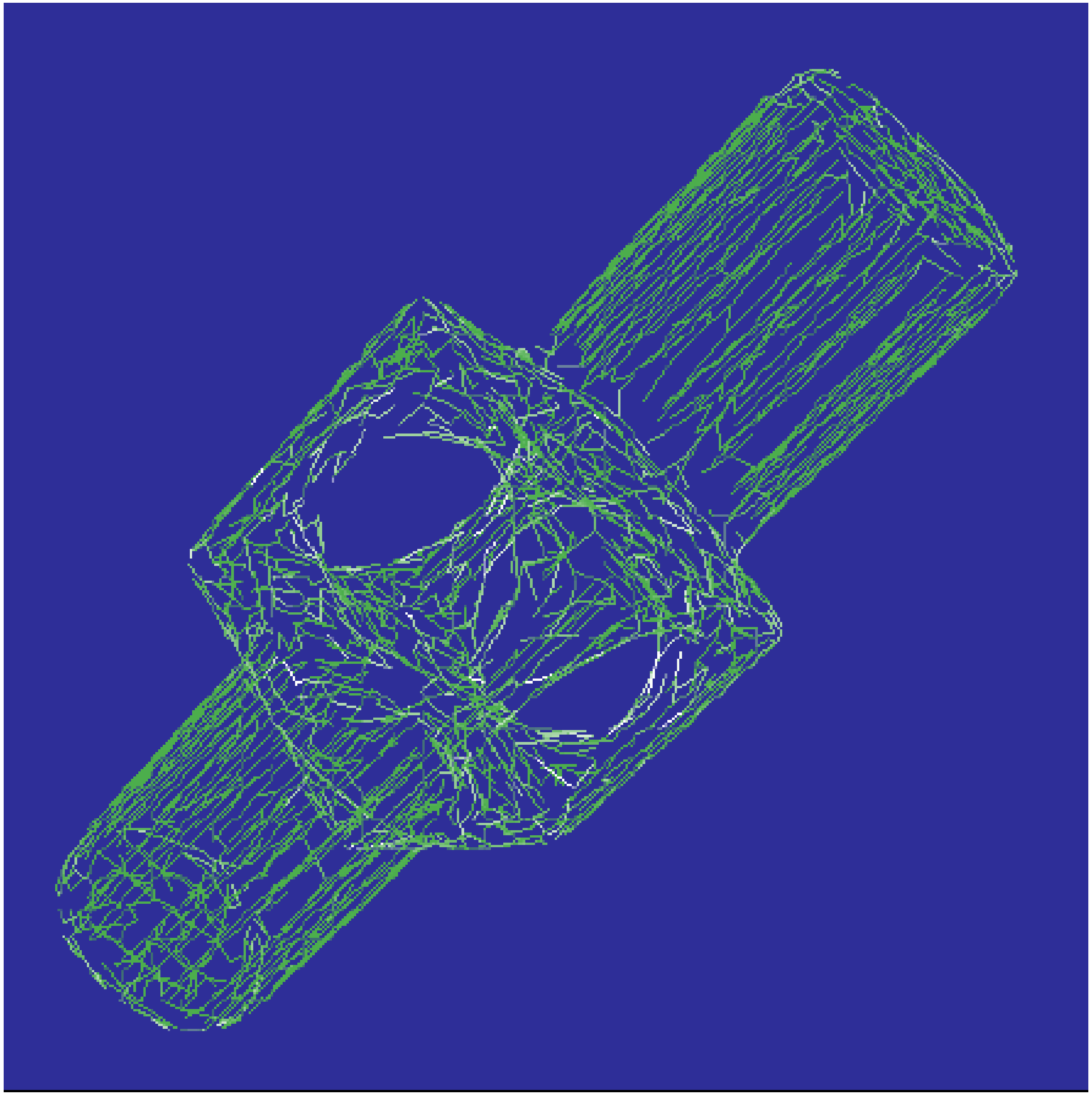
Normal Propagation by Plane Parallelism

1) Construct a Riemmanian graph over the plane centers (o_i's) and edge weights $w_{ij} = 1 - |n_i \cdot n_j|$.

2) Propagate normals along the Minimun Spanning Tree of this graph.

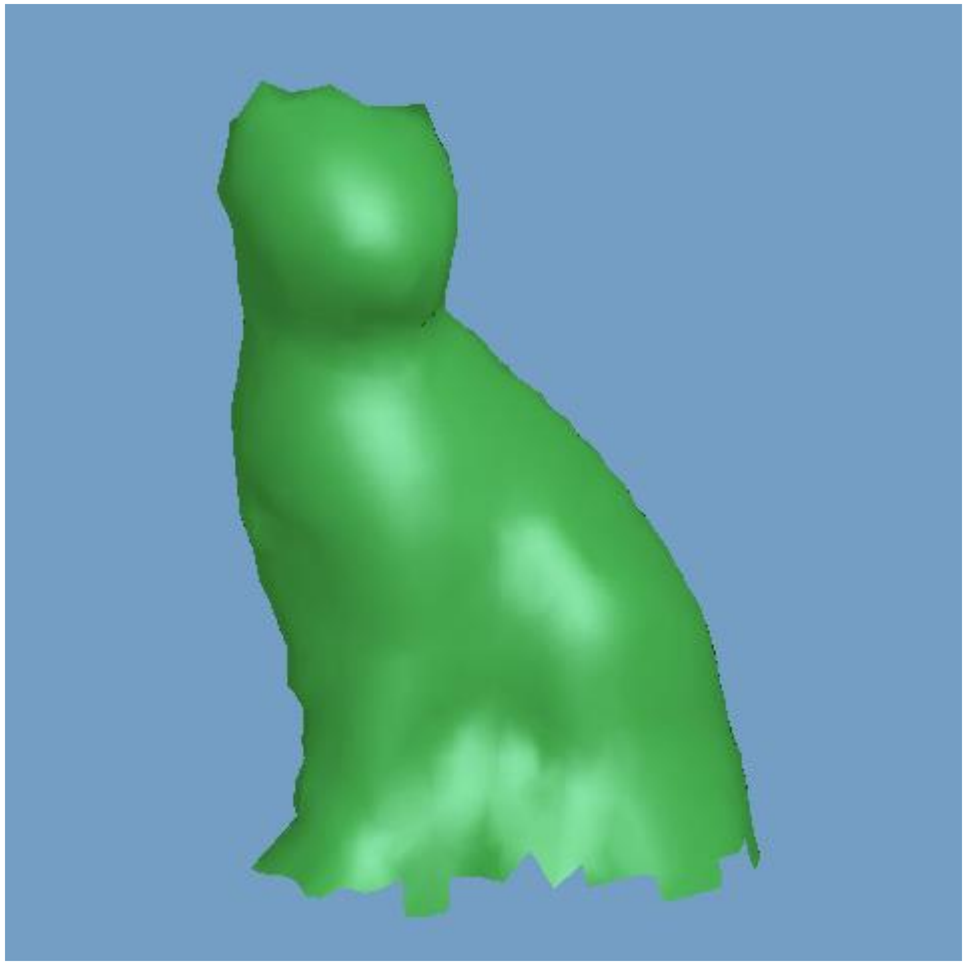


Normal Propagation by Plane Parallelism



Favorates normal proagation along low curvature regions.

Normal Propagation by Plane Parallelism



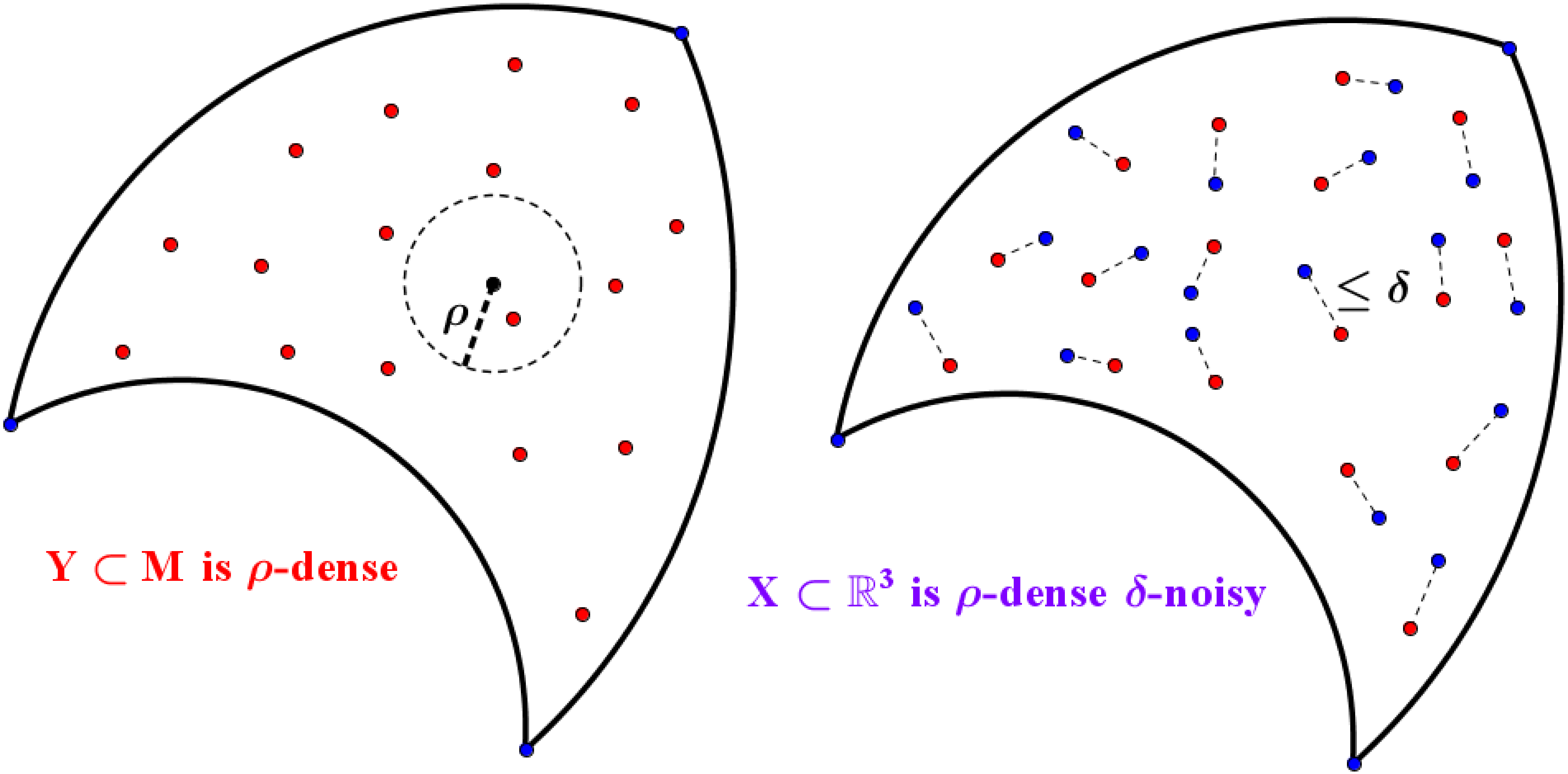
Computation of the Signed Distance Function

1) For each sample fit a tangent plane using its k- Nearest Neighbours.

2) Define a coherent orientation for the tangent plane of all sample points.

3) For any $p \in R^3$ the signed distance function is given by its closest (oriented) tangent plane.

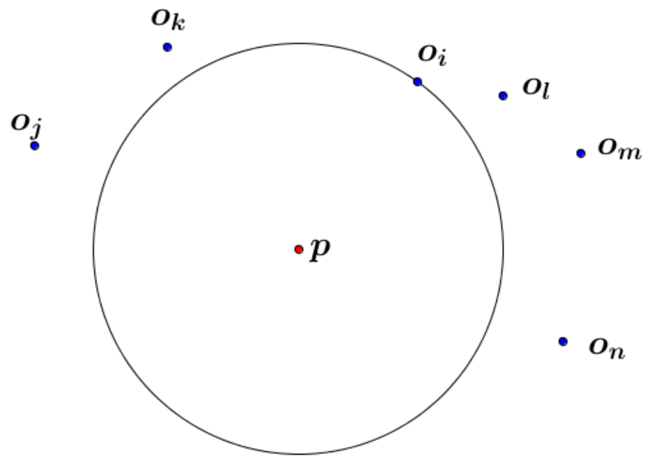
Sampling Assumptions



For any point in the surface the closest sample point in **X** is at most $\rho + \delta$ apart.

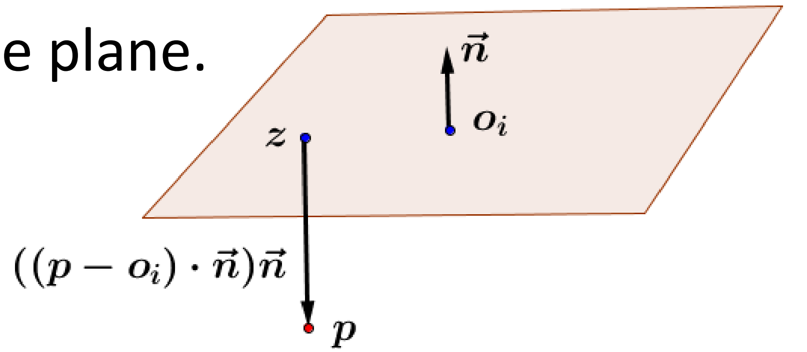
Signed Distance Function

Compute $f(p)$:



1) Find the closest center to p.

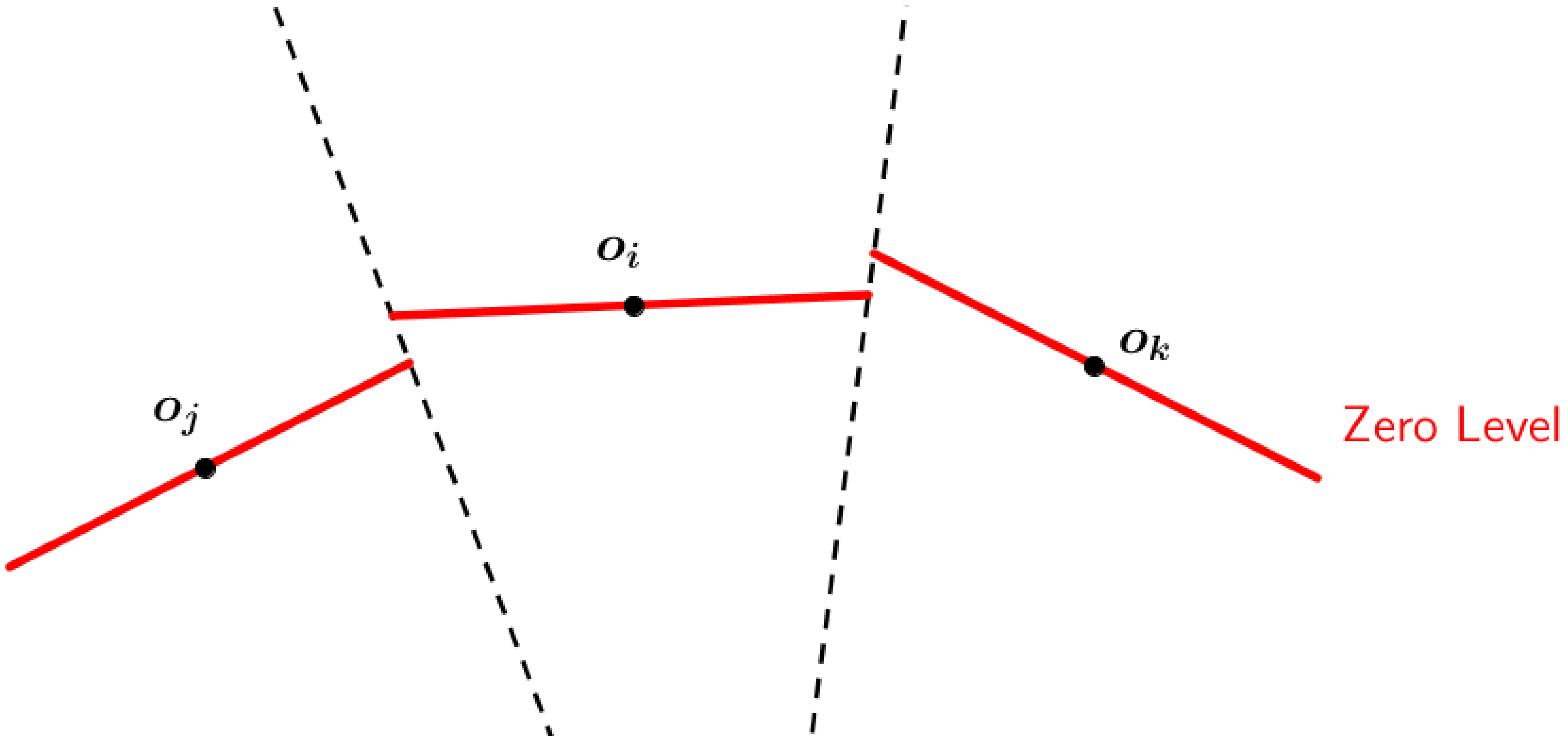
2) Computed the signed distance to the plane.



3) If $d(z, X) < \rho + \delta$ then $f(p) = ((p - o_i) \cdot \vec{n})\vec{n}$.
Otherwise, $f(p)$ is undefined.

Remark

Signed distance function is not continuous.



Still it provides a good approximation to reconstruct the surface.

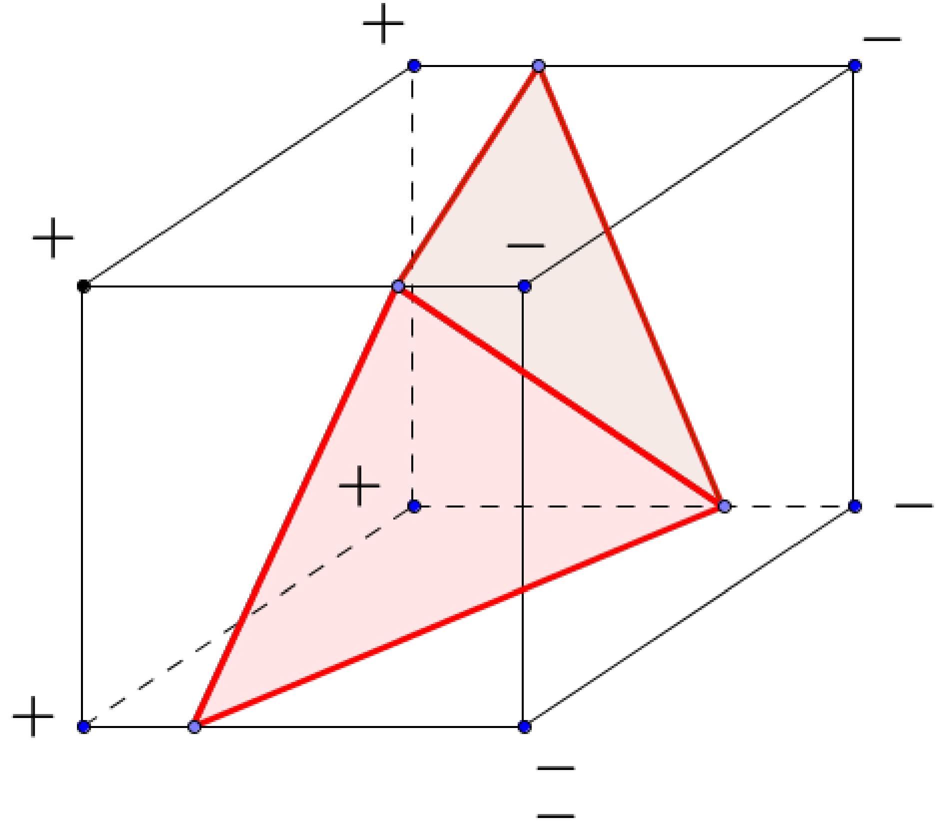
Surface Reconstruction: Marching Cubes

1) Define a cube partition of the space. The edge of each cube should be less than $\rho + \delta$.

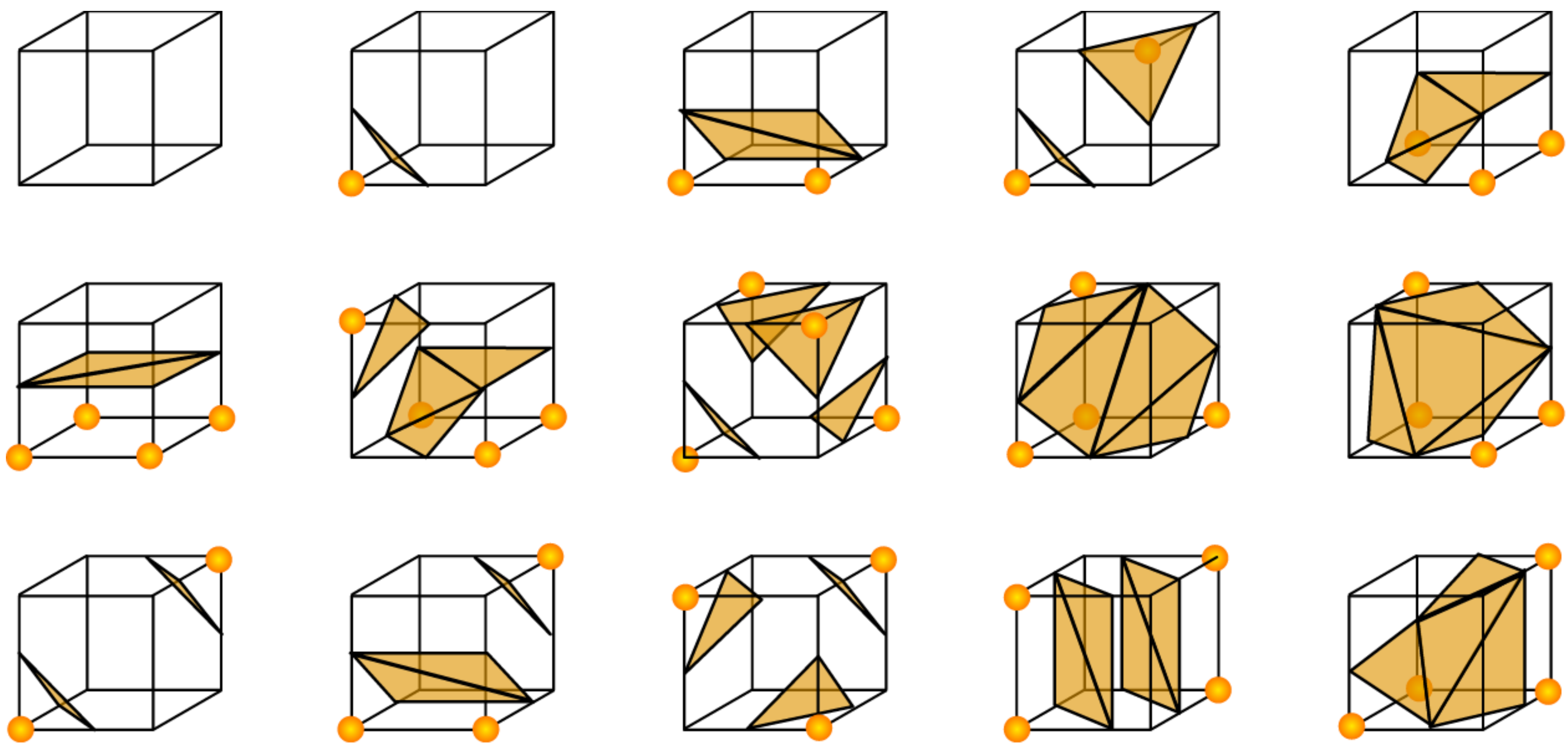
2) Compute the signed distance function on the cube vertices

3) Interpolate zero values (i.e., surface intersections) at changing sign edges.

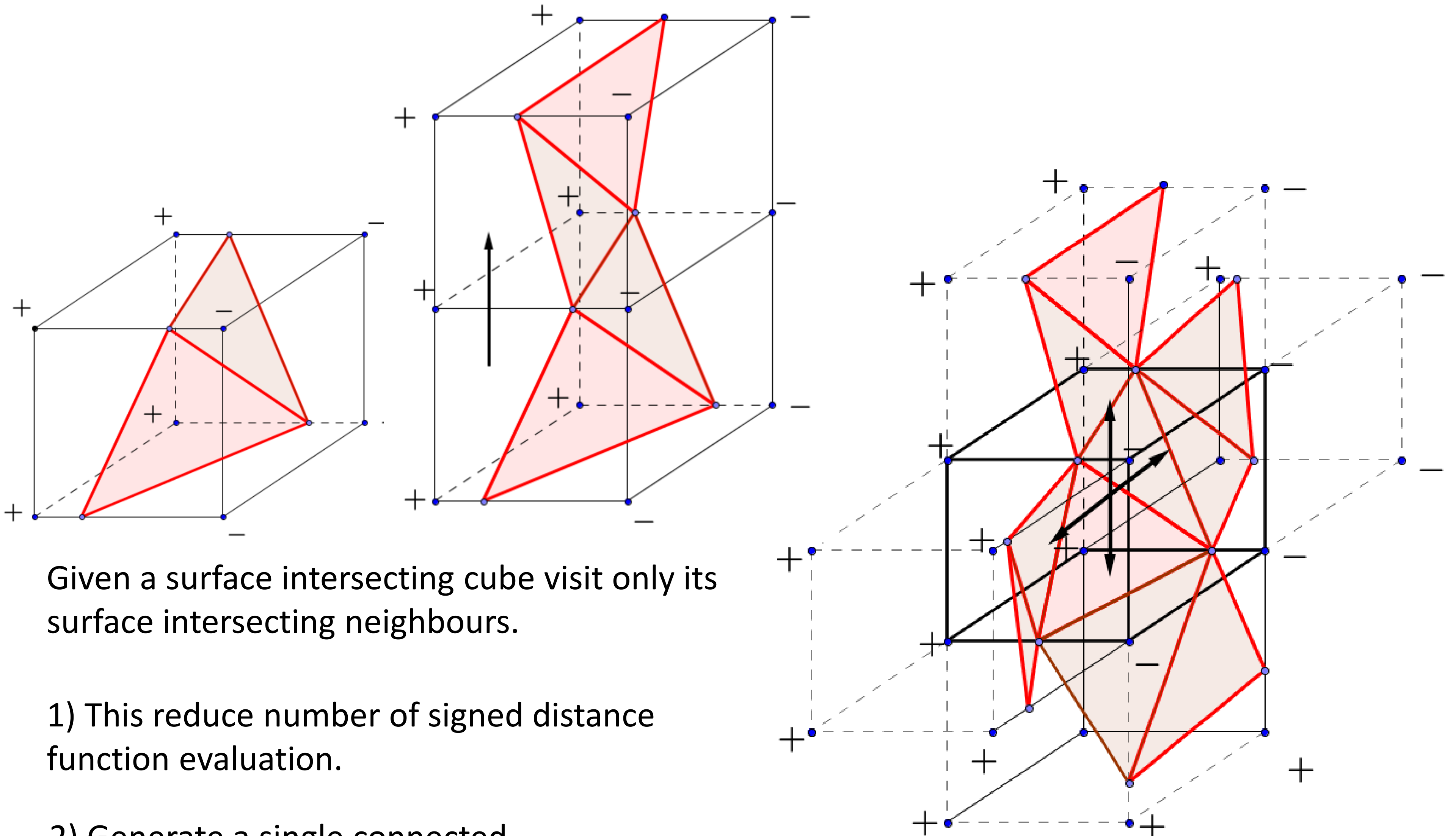
4) Find a trinagulation with vertices at zero values.



Surface Reconstruction: Marching Cubes



Surface Reconstruction: Marching Cubes

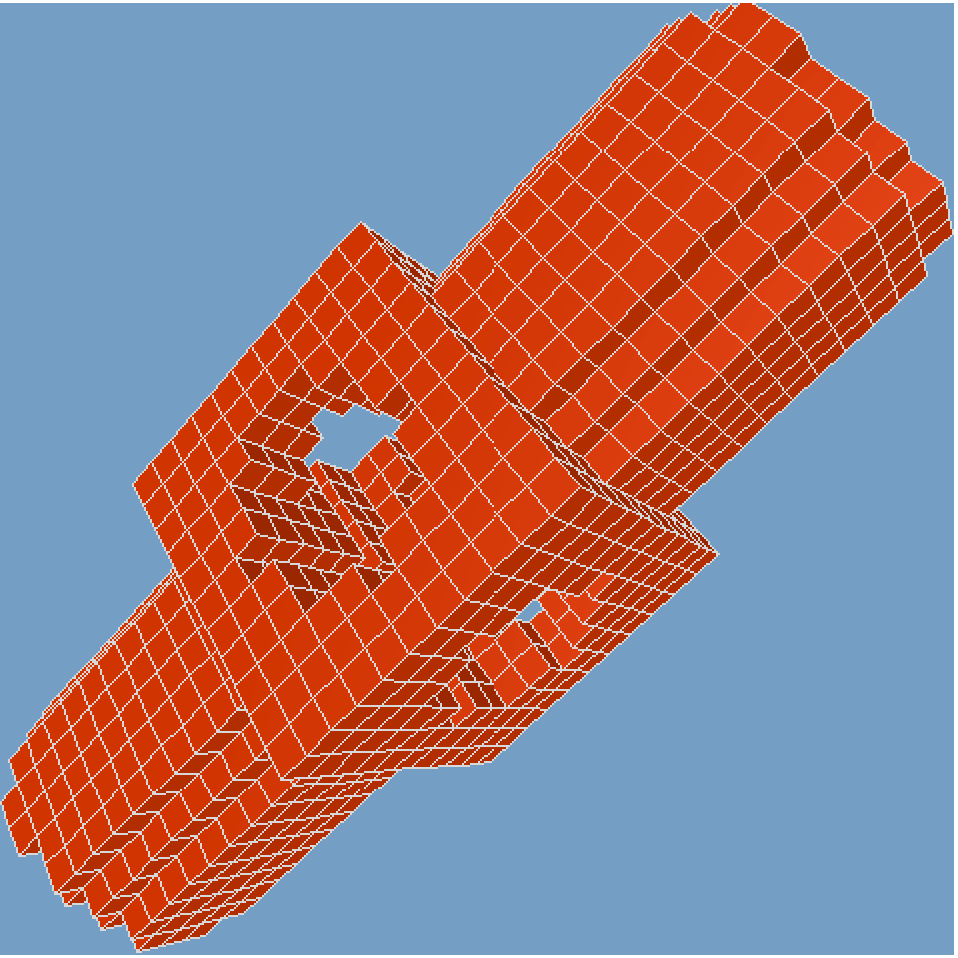


Given a surface intersecting cube visit only its surface intersecting neighbours.

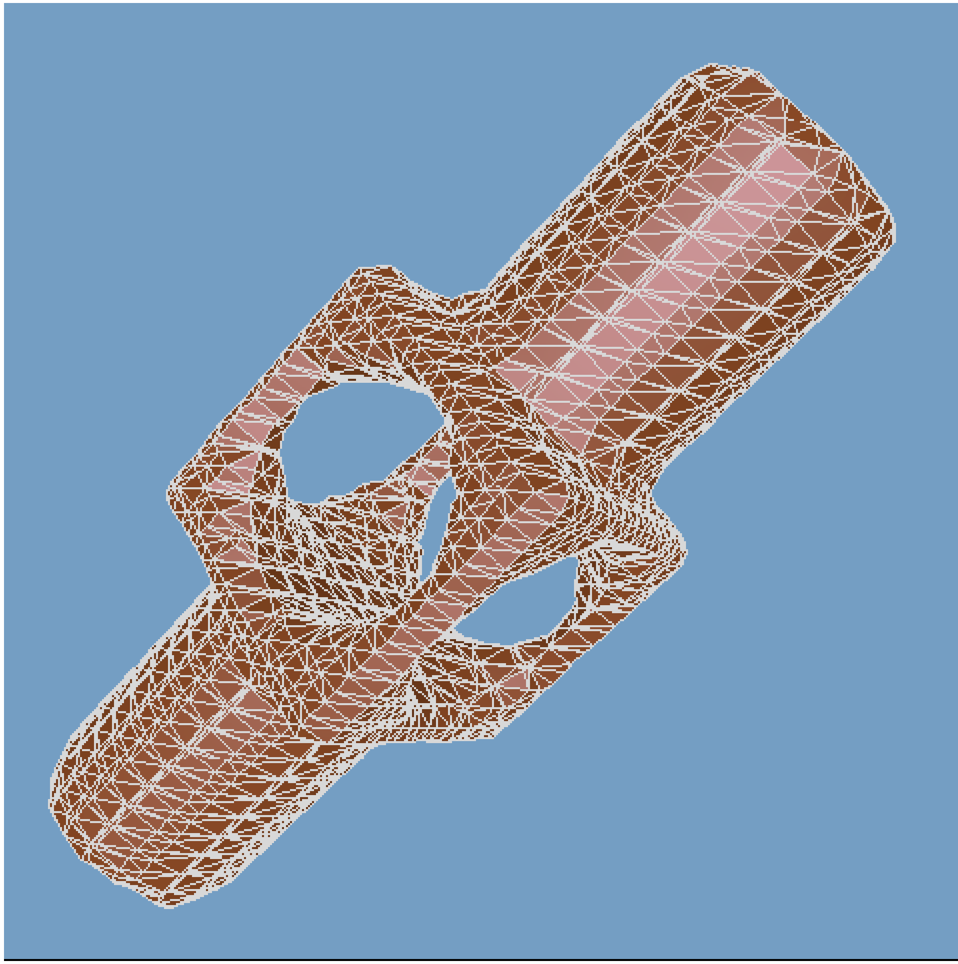
1) This reduce number of signed distance function evaluation.

2) Generate a single connected component.

Surface Reconstruction: Marching Cubes



Cubes visited

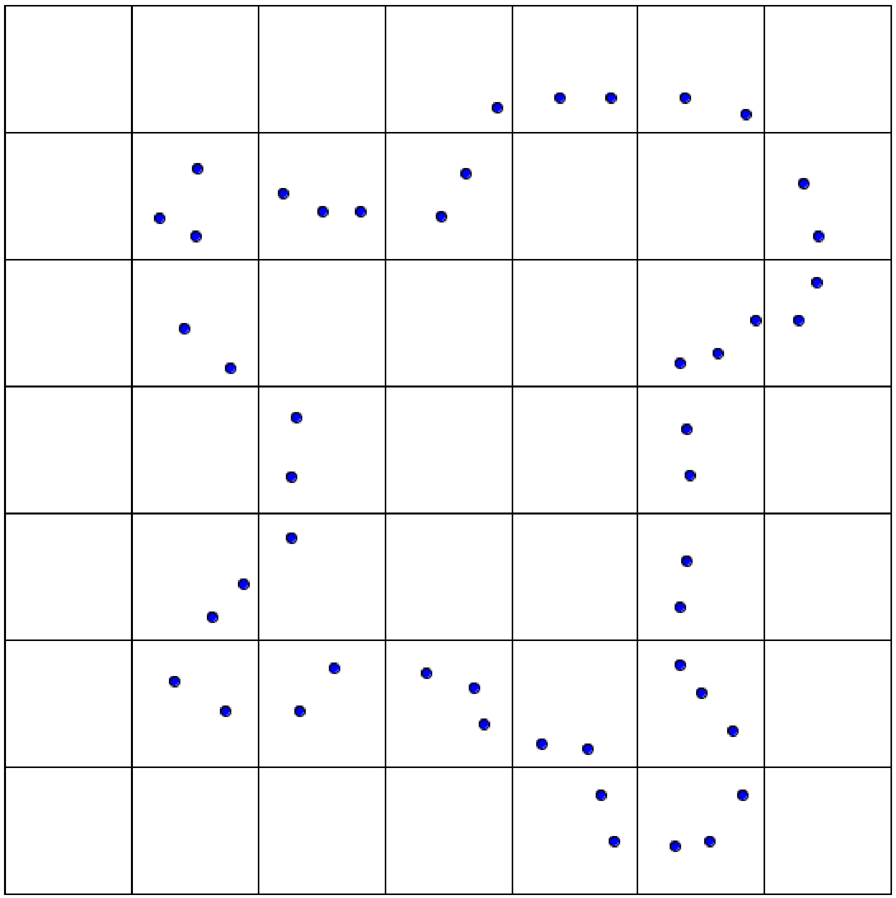


Triangle Mesh

Data Structure

Group samples by voxels

Constant number of samples per voxel if sampling is unifom.



Complexities:

1) Riemmanian Graph construction: $O(nk)$

2) MST: $O(n \log n)$

3) Normal Propagation: $O(n)$

4) Distance Function Evaluation: $O(1)$

Results