Tight Cocone

DIEGO SALUME

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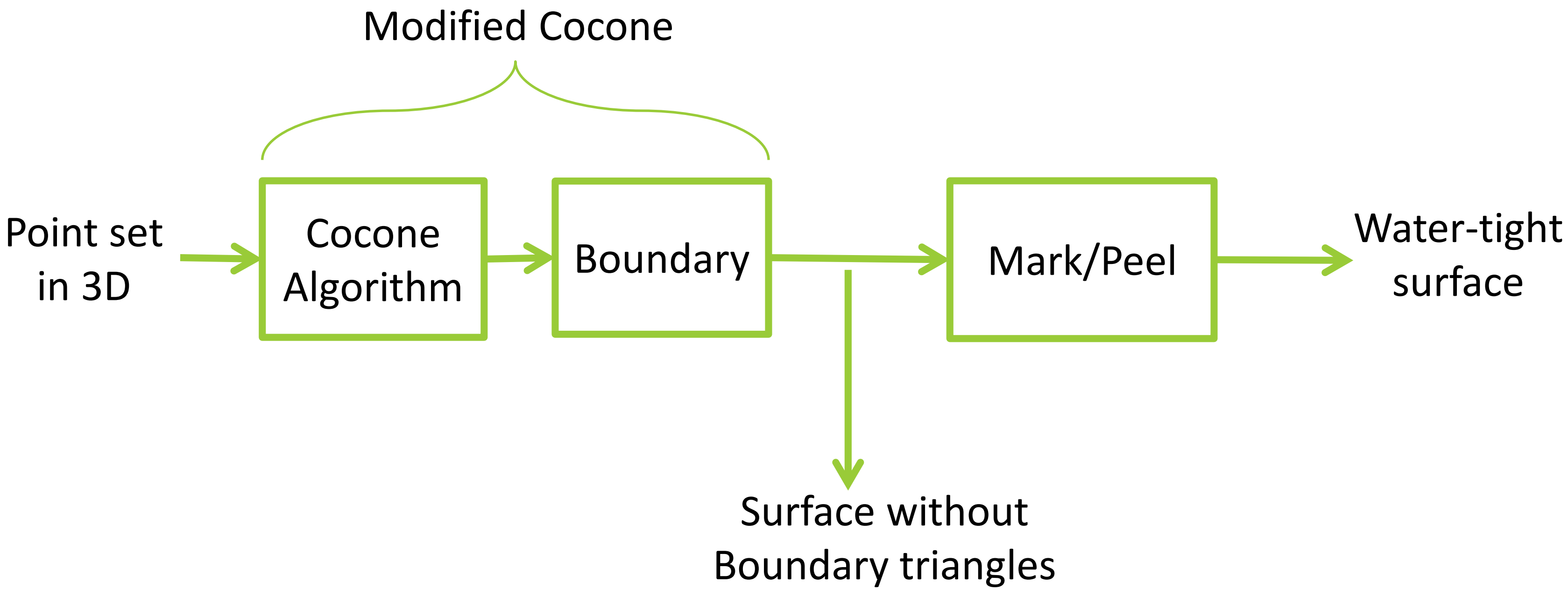
Summary

- Problem: The Cocone algorithm produces surfaces with holes and artifacts in undersampled areas in the data.
- Solution: Extend the Cocone algorithm to output water-tight surfaces by filling all holes without introducing extra points.
- Key insight: Mark and Peel algorithm introduced to seal reparable holes.

Summary

- Limitations:
  - Algorithm relies on the principle of locality.
  - Some noise handling.
- Guarantees a water-tight surface.
- $O(n^2)$ time and space complexity. Governed by the Voronoi diagram computation.
  - In practice, Delaunay triangulation is most time-consuming step.
- Method can be modified to handle internal voids.

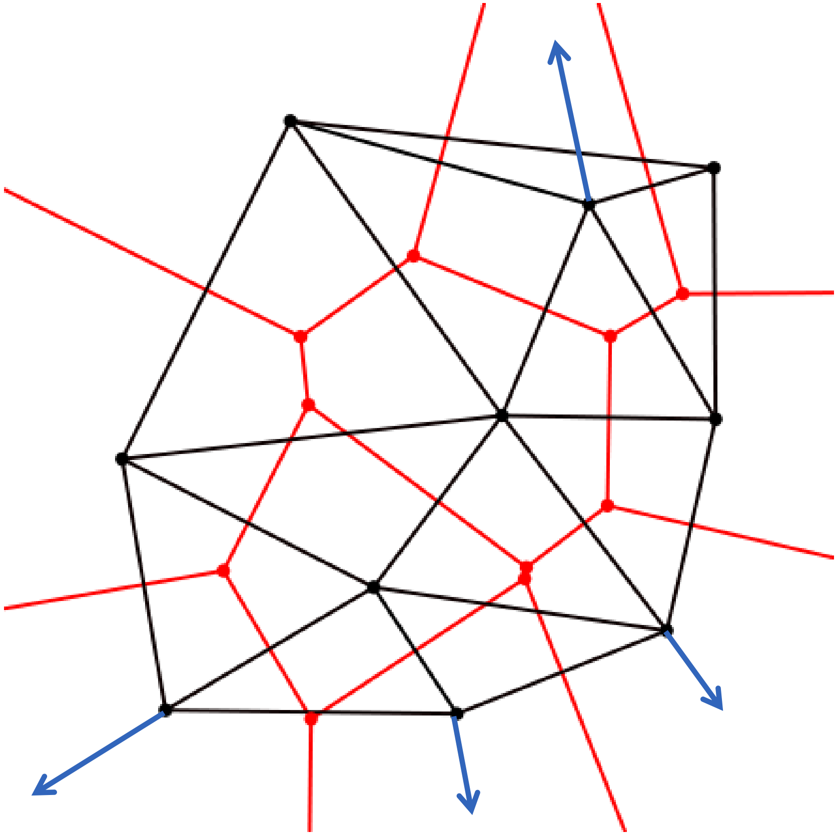
Tight Cocone Algorithm Structure



$$v_p = p^+ - p$$

Modified Cocone

- Cocone algorithm:
  - Find pole vector v_p (approx. surface normal at point p).



$$v_p = p^+ - p \qquad C_p = \{y \in V_p : \angle((y-p), v_p) \geq \frac{3\pi}{8}\}$$

Modified Cocone

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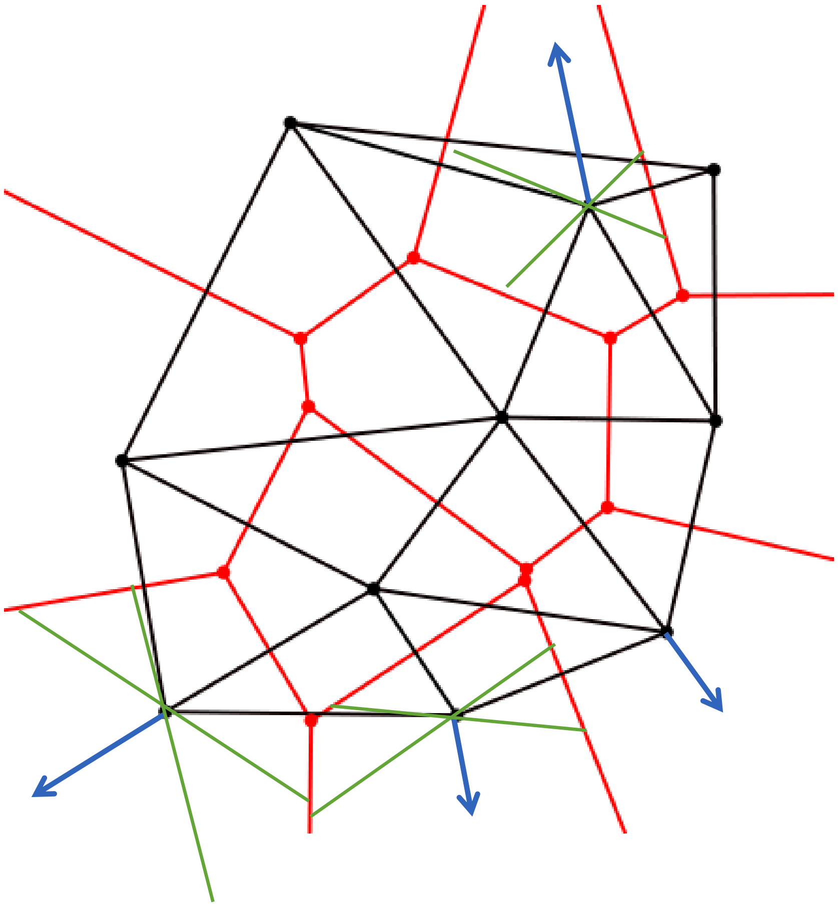
  - Compute Cocone on Voronoi cell associated with point.



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- Cocone algorithm:
  - Find pole vector v_p (approx. surface normal at point p).
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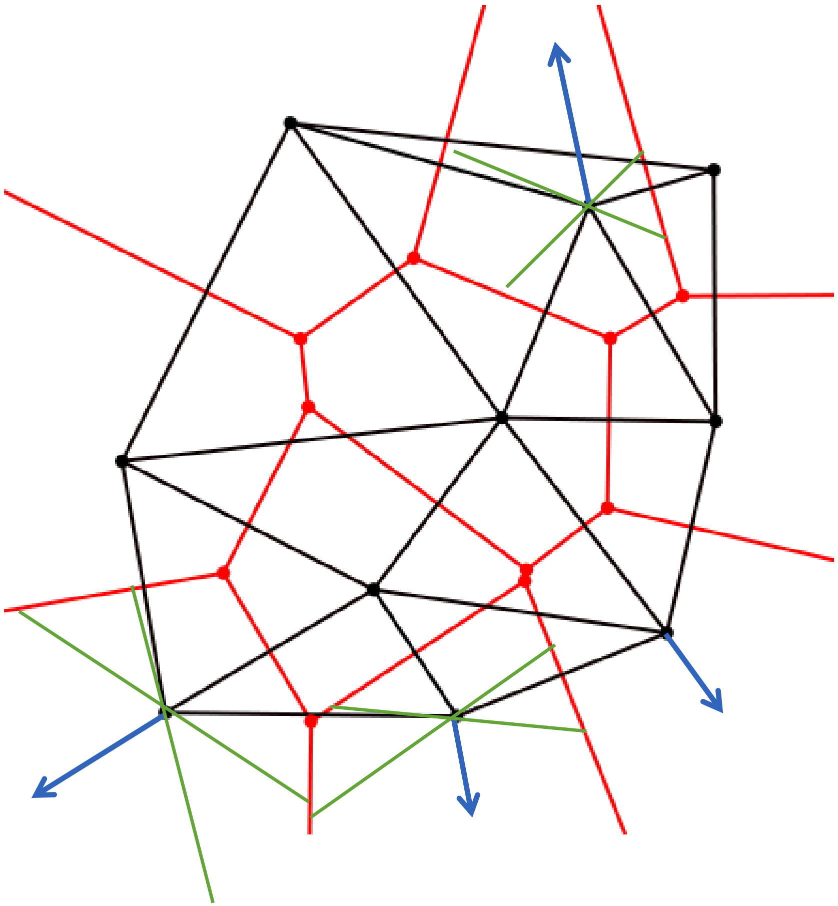
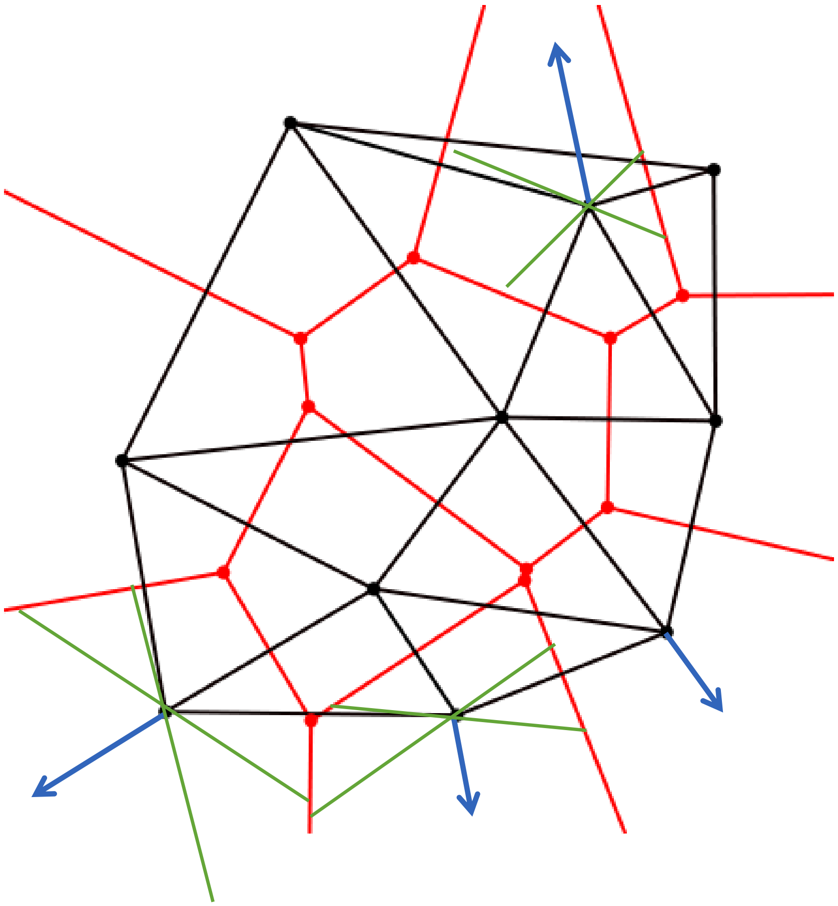
  - Consider Voronoi edges that intersect the cocone.



Modified Cocone

- Cocone algorithm:
  - Find pole vector v_p (approx. surface normal at point p).
  - Compute Cocone on Voronoi cell associated with point.
  - Consider Voronoi edges that intersect the cocone.
  - Triangles made up by these edges are candidate triangles, which lie close to original surface.



Boundary Algorithm

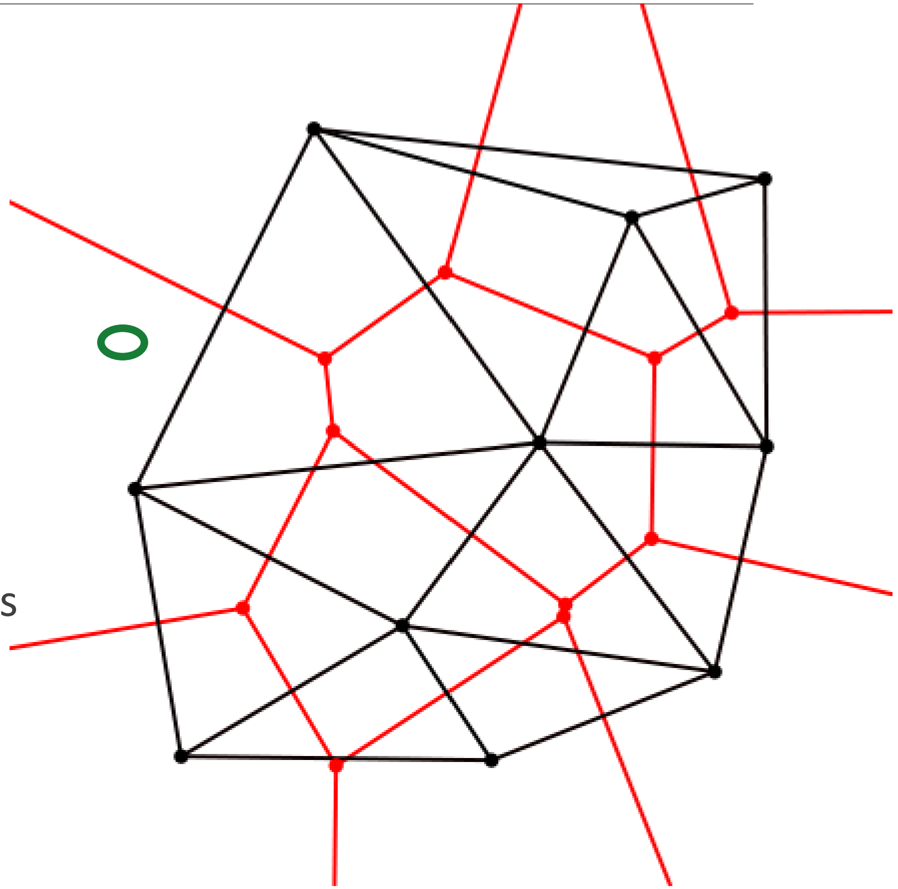
- Boundary algorithm:
  - Normals are unreliable in undersampled areas.
  - Boundary algorithm detects these areas.
  - Ignore the candidate triangles for these points.

Tight Cocone

- Label Delaunay tetrahedral as “in” or “out” based on the initial surface approximation obtained with the Cocone algorithm.
- Peel off “out” tetrahedral.
- Compute boundary of union of “in” tetrahedra.
- Requirement
  - Principle of locality: undersampling is local. Cocone algorithm computes most of the surface except for reparable holes.

Tight Cocone: Marking

- Surface triangles incident to “good” points form a topological disk called umbrella.
- Marking is done by walking through Delaunay triangulation.
- For every good vertex, we know an “out” tetrahedron.
  - Start with a good point on the convex hull and label an infinite tetrahedron as “out”.



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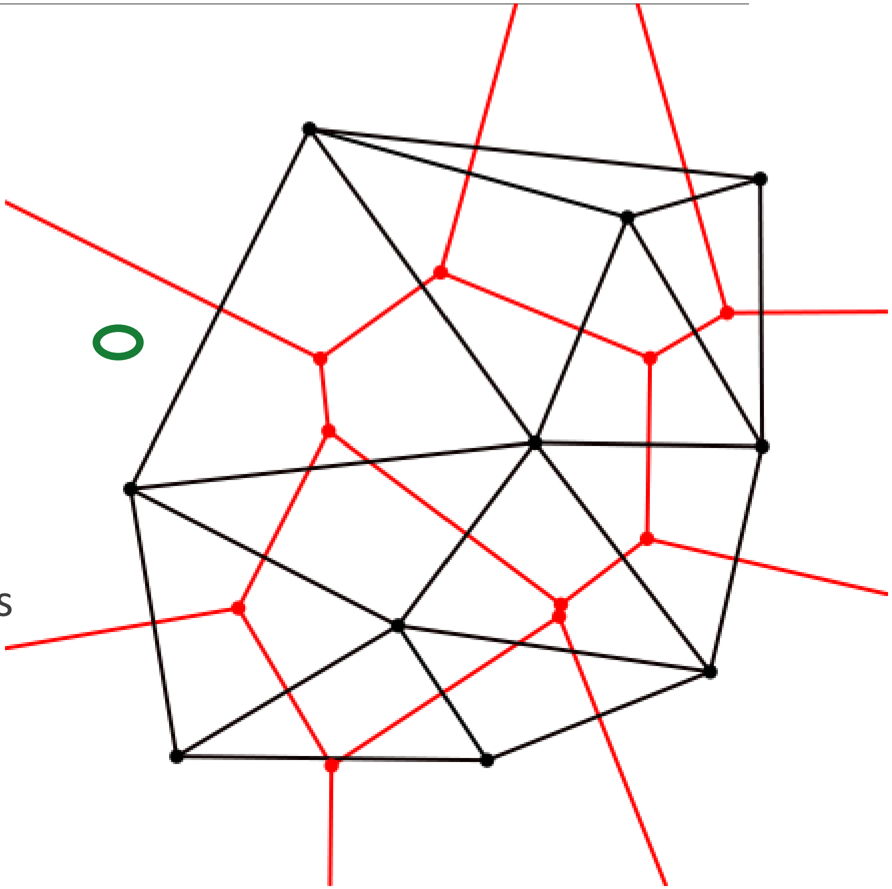

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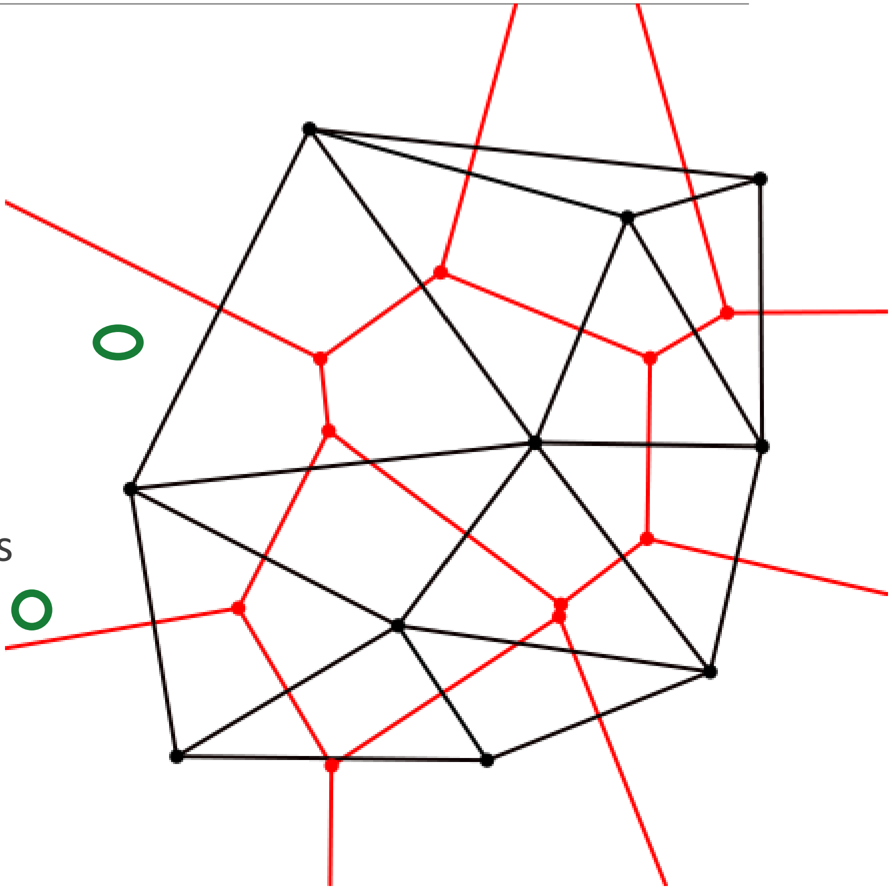
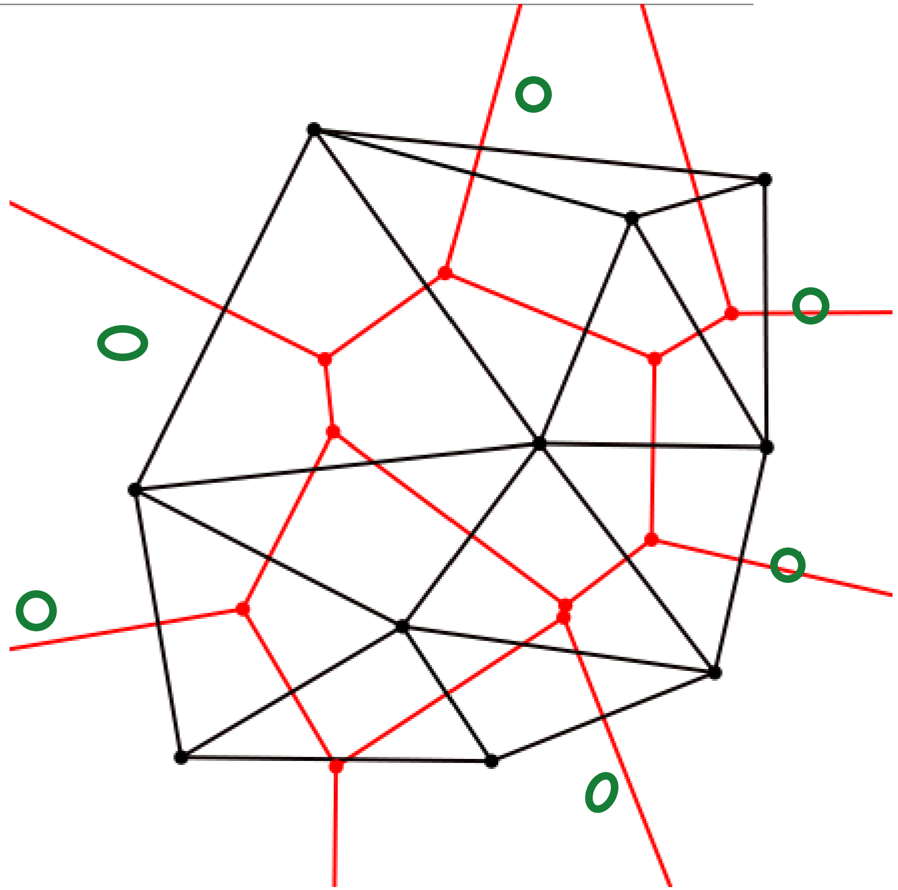
  - Compute its umbrella.
  - Every tetrahedron on the same side is also labeled "out".
  - Otherwise, it's labeled "in".



Tight Cocone: Peeling

- Peel off tetrahedral marked as “out”.
- Walk along surface triangles that form boundary of peeled tetrahedral.
  - Initiate with all convex hull triangles.
  - Every triangle is incident to an “out” tetrahedron.
    - If the other incident tetrahedron is “out”, this is not a surface triangle.
    - If the other incident tetrahedron is “in”, then this is a surface triangle.



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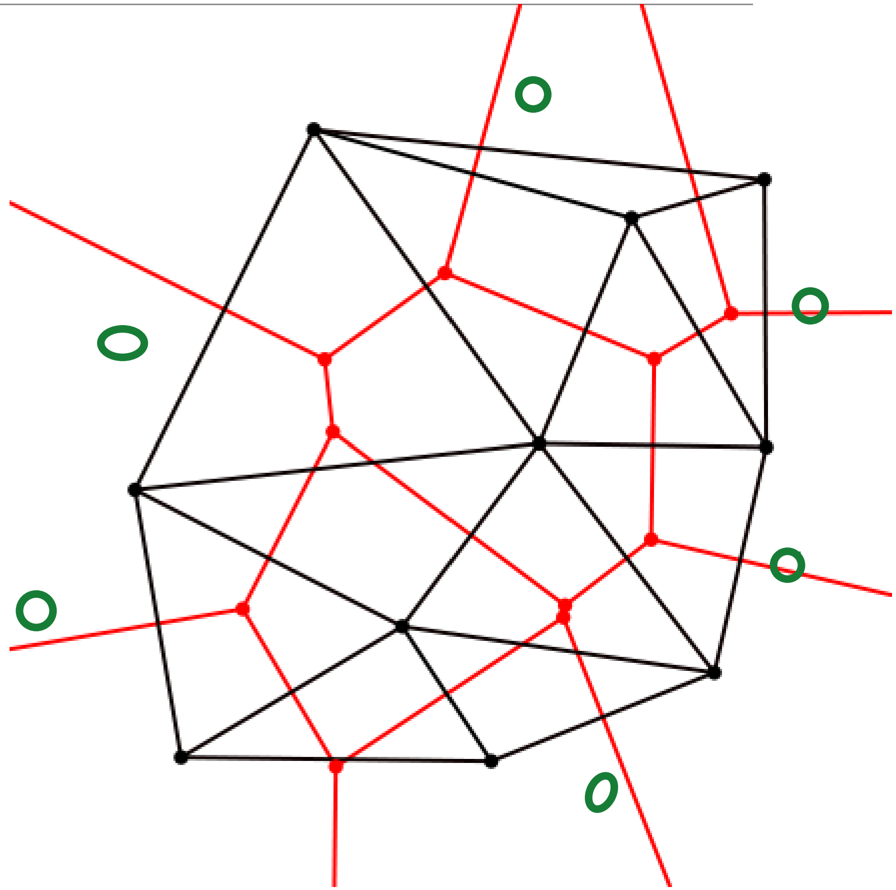

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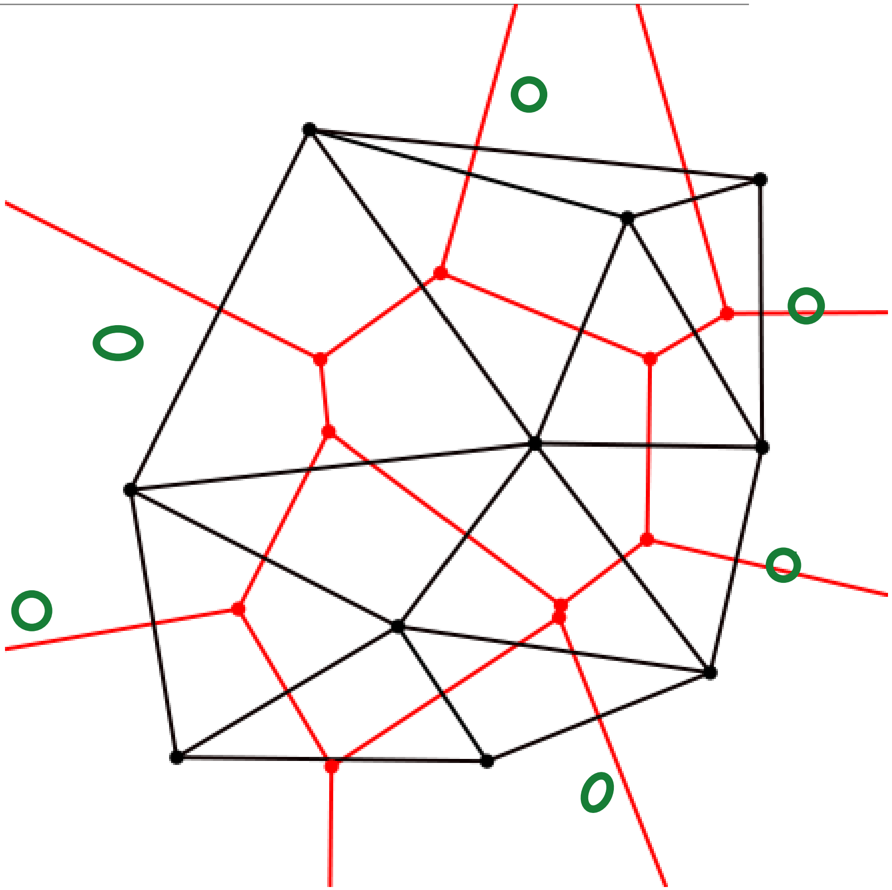

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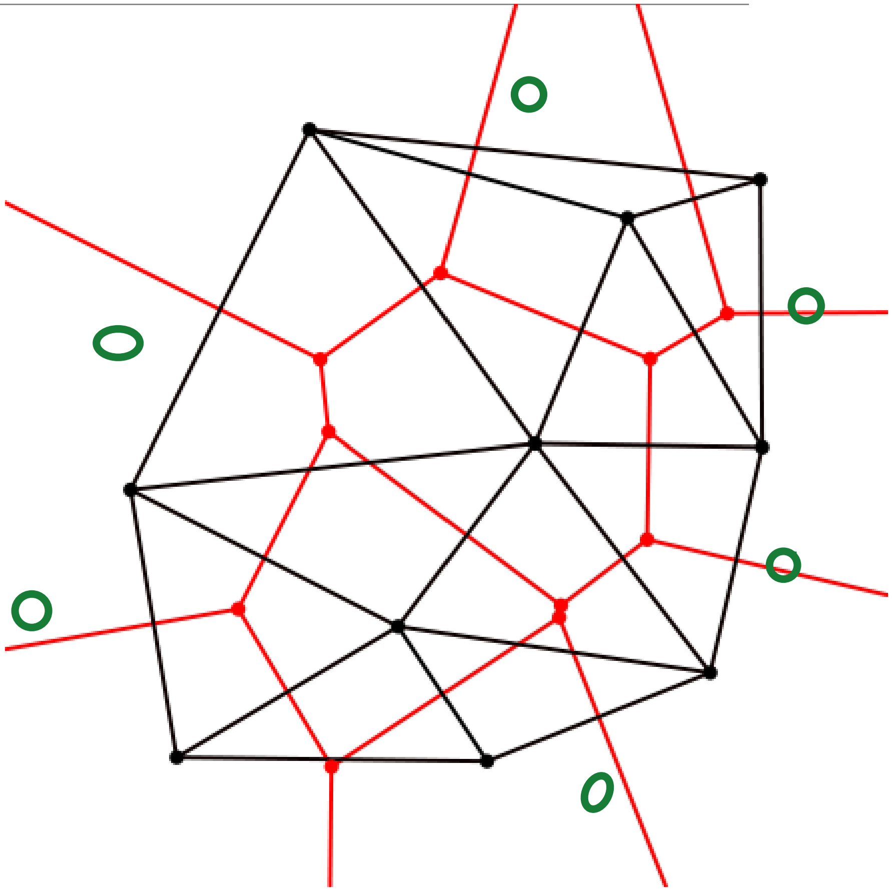

Poor Tetrahedra

- Tetrahedra with all poor points are poor tetrahedra.
- Small poor tetrahedra are marked as “in”.
- Big poor tetrahedra could be “in” or “out”. Don’t label them.
  - By principle of locality, we have a small triangle (least circumradius).
  - Inner poor tetrahedra must be reached through small triangle.
- During peeling, we are only allowed to move into a poor tetrahedron if it’s not through the small triangle to avoid peeling inner tetrahedral.
  - If it’s an “out” tetrahedron, it’ll be reached through one of the big triangles.
  - If it’s an “in” tetrahedron, it will never be reached and won’t be peeled.