Reconstruction and Representation of 3D Objects with Radial Basis Functions

1. Briefly summarize the paper's contributions. Does it address a new problem? Does it present a new approach? Does it show new types of results?

- **[AS]**
  This paper presents a fast approach to reconstruct smooth, manifold surfaces from point clouds and to repair incomplete meshes using polyharmonic Radial Basis Functions (RBFs), which were previously thought to be too slow to compute to be scalable.
- **[DS]**
  The paper presents a way to use polyharmonic Radial Basis Functions (RBFs) to reconstruct smooth, manifold surfaces from point-cloud data. RBFs are fitted and evaluated using fast methods without which this was computationally infeasible to do on large point sets and complex surfaces with arbitrary topology.
  The fast method used for RBF evaluation is the Fast Multipole Method (FMM). The RBF center reduction method is used to fit the RBF function to a point set. This has smaller memory requirements and faster evaluation times without a loss of accuracy.
  The resulting implicit function can be easily manipulated through Boolean operations. It also allows for arbitrary accuracy. RBFs slo allow for mesh repair and surface extrapolation.
- **[FP]**
  Surface is implicitly defined from a global function given by the sum of biharmonics at sample positions and a linear shift. The nature of the radial basis functions used by the authors, $|x - xi|$, contrast with the type of functions used in previous approaches (e.g., Othake) : instead of being monotonic decreasing these are monotonic increasing functions. According to the authors, this selection of basis functions provides the smoothest interpolation. This characteristic makes the method robust to undersampling and useful for mesh repair.
- **[JD]**
- **[LF]**
- **[MK]**
  The paper presents a new approach for surface reconstruction that fits RBF to the point, seeking the coefficients that evaluate to zero at the input samples. To avoid the trivial solution, the authors also add off-surface constraints, adding constraint points at a distance of *d* from the samples along the normal, and constraining the function to be $\pm d$ at these points (depending on whether they are inside or outside the surface.)

2. What is the key insight of the paper? (in 1-2 sentences)

- **[AS]**
  One key insight of this paper is the use of non-compactly supported basis functions, which are independent of sampling density, hence making their system scale independent and suitable for irregular, non-uniformly sampled data. However, such basis functions lead to systems which are not sparse, and hence solving such systems is slow and not scalable. The other key insight of this paper is that RBFs can be evaluated quickly using the Fast Multipole Method (FMM), requiring $O(N \log N)$ time to solve for a system with N^2 non-zero entries, making it computationaly possible to represent complicated topology using RBFs.

- **[DS]**
  The key insight is the introduction of fast methods to evaluate and fit RBFs, which provide computational and space efficiency that makes this method feasible.
- **[FP]**
  Smoothest interpolation is implicitly defined using biharmonic functions. Approximation and evaluation of the global implicit function can be efficiently done using the Fast Multipole Method.
- **[JD]**
- **[LF]**
- **[MK]**
  The key insight behind this approach is that by formulating the problem in terms of global RBF fitting, with biharmonic radial functions, the method can incorporate smoothness and interpolation constraints without having to define a notion of scale, thereby allowing the method to be implemented without ever having to estimate (local) sampling density, neighbors, etc.

3. What are the limitations of the method? Assumptions on the input? Lack of robustness? Demonstrated on practical data?

- **[AS]**
  In order to append off-surface points into the input data such that each off-surface point, at a distance d_i, is closest to the on-surface point that generated it, we need some knowledge of the sampling density of the point cloud. Therefore, this step of the process is scale dependent.
- **[DS]**
  Since the RBF representation is global, then there are drawbacks when manipulating part of the model or doing ray tracing.
- **[FP]**
  The normal may not be required at all points but is still necessary in a representative set of points in order to define auxiliary interior and exterior samples.
  Due to the interpolative nature of the approach the method won't be robust in the presence of noise and outliers. Also, since it provides a smooth reconstruction it should not be adequate to reconstruct objects with sharp features (unless a dense sampled model is provided).
- **[JD]**
- **[LF]**
- **[MK]**
  The method assumes that signed normals can be robustly estimated at a reasonable subset of the points (e.g. using local plane fitting and then taking the dot-product with the scanner's view direction).
  As observed by the authors, one of the limitations of using the biharmonic RBFs is that the functions increase with distance so that the system is dense and solutions may not be stable.

4. Are there any guarantees on the output? (Is it manifold? does it have boundaries?)

- **[AS]**
  The output is a smooth surface, which when extracted using a method like marching cubes will almost always produce a manifold mesh.
- **[DS]**
  Implicitly reconstructs smooth, manifold surfaces. Repairs incomplete meshes.

- **[FP]**
  Due to the implicit formulation the reconstructed surface must be watertight (no boundaries).
  Since the radial basis functions are smooth almost everywhere (except at the centers), the implicit function should be smooth almost everywhere, and the same should hold for the reconstructed surface.
  Reconstruction could not be manifold. If points were densely sampled from a non-manifold structure, the implicit defined surface should also be non-manifold.
- **[JD]**
- **[LF]**
- **[MK]**
  Assuming that the gradient is non-vanishing at the zero-level-set, the surface will be a manifold without boundary (possibly non-compact). However, this is a somewhat tricky proposition since the zero-level-set is constrained to pass through the RBF centers, and these are precisely the positions at which the gradient of the RBF is undefined. Independently, and this is a problem with many of the implicit reconstruction schemes, the definition of a robust zero-level-set also requires that the gradients not have small magnitude near the zero-level-set.

5. What is the space/time complexity of the approach?

- **[AS]**
  The time complexity for this approach is $O(N \log N)$, since the system can be solved in $O(N \log N)$ time, and then each evaluation takes constant time. The space complexity is $O(N)$.
- **[DS]**
  The fast methods for fitting are O(N) in space and O(NlogN) in time.
  For evaluation, they are O(NlogN) for the setup and O(1) after.
- **[FP]**
  The approximation and evaluation of the implicit function is possible due to Fast Multipole Method. According to the authors, this method provide and approximation of the implicit function coefficients in $(n\log n)$, and evaluation of the implicit function is $O(1)$ after an initial setup which requires $O(n\log n)$. These computational complexities contrast a lot with exact solvers and exact function evaluation which require higher order polynomial times.
  Despite these simplifications, both fitting and reconstruction are still very expensive procedures, as can be observed from the computation time provided by the authors.
- **[JD]**
- **[LF]**
- **[MK]**
  Naively, the storage would be $O(N^2)$, since the matrix is dense and the running time is $O(N^3)$ since the matrix needs to be inverted. However, using the fast multipole method, the authors show that this can be done in $O(N)$ storage and $O(N \log N)$ time. (Surprisingly, this implies that the system can be solved sufficiently accurately without ever having to evaluate all the matrix coefficients).

6. How could the approach be generalized?

- **[AS]**
  This global approach can be generalized to a piecewise representation of implicit surface patches to facilitate local manipulation and ray-tracing.

- **[DS]**
  Improving the performance of the RBF center reduction algorithm.
  Being able to decompose a global RBF to a piecewise mesh of implicit surface patches to facilitate local manipulation and ray tracing.
  Computational improvements by parallelizing the algorithm.
- **[FP]**
  The method presented by the authors specialize on biharmonic functions, but certainly any kind of radial or piecewise polynomial functions can be used instead. It would be interesting to see the kind of properties that different functional basis produce on the defined implicit surface.
  The method formulation is not specific to 3D space, so it can be used in any finite dimensional space. In particular it can also be used for planar curve reconstruction.
- **[JD]**
- **[LF]**
- **[MK]**
  Rather than using off-surface constraints to avoid the trivial solution, the authors could consider explicitly adding gradient constraints to have the gradient of the implicit function match the estimated normals at the sample points. (Though perhaps this won't work because of the previous observation that gradients of the implicit function are undefined at the sample positions.) More on this next.

7. If you could ask the authors a question (e.g. "can you clarify" or "have you considered") about the work, what would it be?

- **[AS]**
  When polygonizing the surface, a small subset of centers is used to seed the surface, one for each distinct surface section or surface patch, in the case of a disconnected surface. How do you guarantee that patches are not missed?
- **[DS]**
  Why are RBFs the best choice for implicit function fitting? Where do they/their properties come from? How does the choice of basic function phi affect the results?
- **[FP]**
  In the evaluation of the implicit function, $(x)=p(x)+\Sigma \alpha i|x-xi|i$, the farthest points to x have a larger impact on the value of $f(x)$ than the closest points (totally different to previous local fitting approach). Therefore, evaluation of (x) seems to be expensive and hard to do with good precision. How is the implicit function evaluation done in (1) using FMM?
- **[JD]**
- **[LF]**
- **[MK]**
  One of the (all-be-it) small disappointments with this work is that it does not actually end up being scale-independent. In particular, the implementation requires a choice of offset distance which can affect the solution. (One could try to get around this by constraining gradients, i.e. using infinitesimally small offsets, but this would require choosing a weight to balance value and gradient constraints, which would be scale dependent.)
  To minimize the cost of evaluating the implicit function, the authors perform iso-surface extraction using a variant of Bloomenthal's Polygonizer, only evaluating the function near the samples. The problem with this approach (in general) is that if there are spurious surfaces created away from the samples, they won't appear in the reconstruction, hiding potential problems with the implicit function away from the samples.