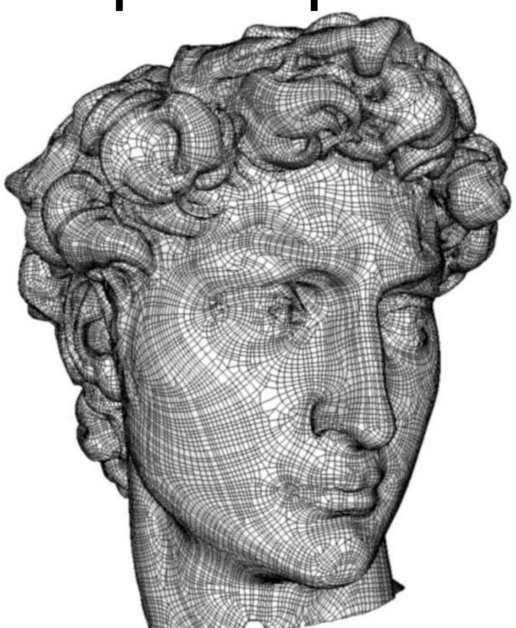
600.657: Mesh Processing

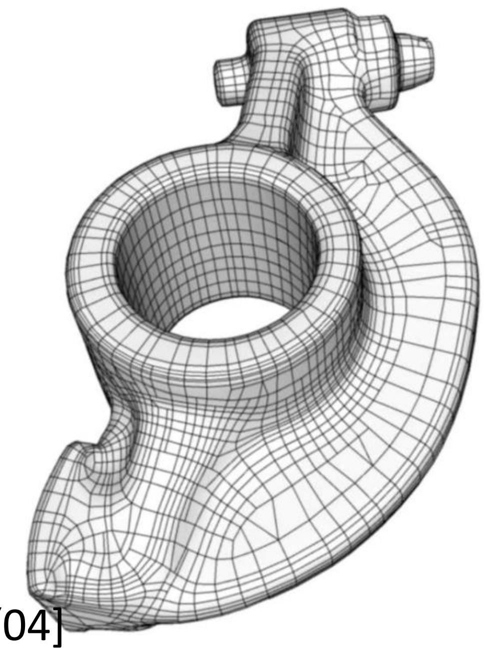
Chapter 6

Quad-Dominant Remeshing

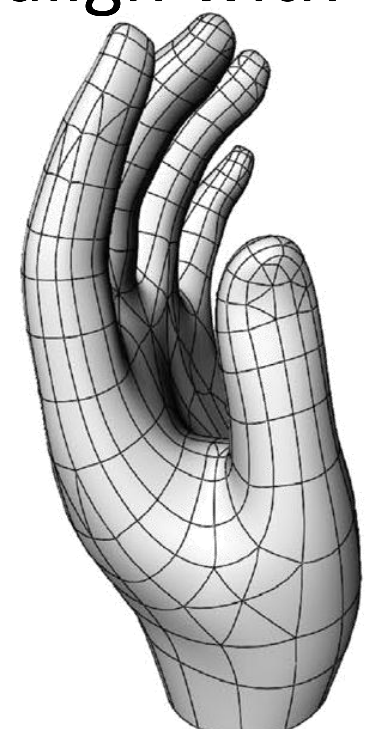
Goal:

Generate a remeshing of the surface that consists mostly of quads whose edges align with the principal curvature directions.





[Marinov *et al*. ‘04]

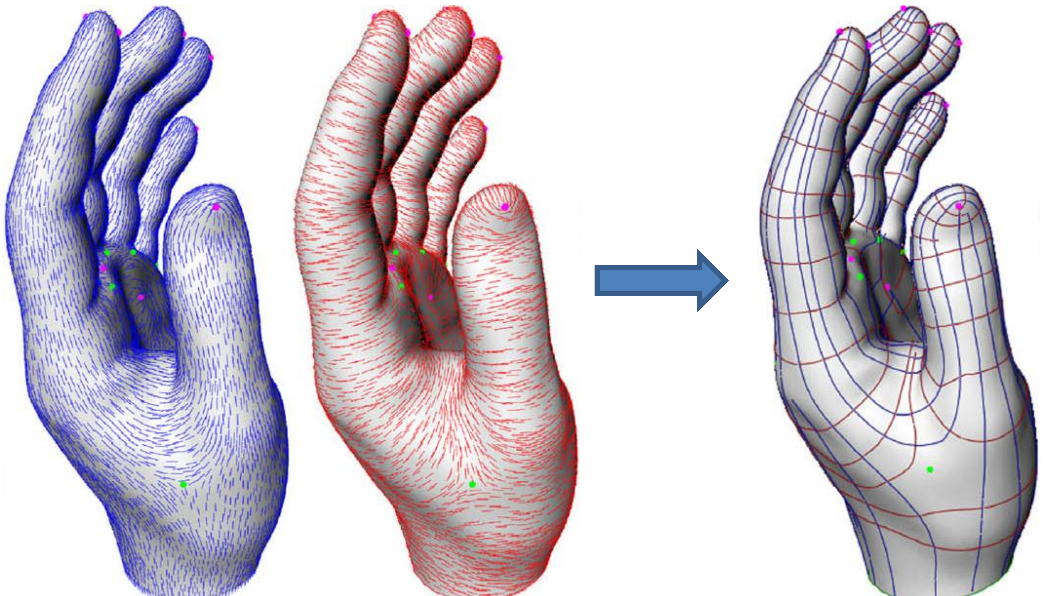


[Alliez *et al*. ‘03]

Quad-Dominant Remeshing

Approach:

Where we can, trace lines of minimal/maximal curvature. Since these are orthogonal, their intersections should give quads.



[Alliez *et al*. ‘03]

Quad-Dominant Remeshing

Challenges:

1. What are the principal curvature directions?
2. What are the principal curvature lines?
3. Where do we place the lines and how long should they be?
4. What happens when principal directions are not well-defined?

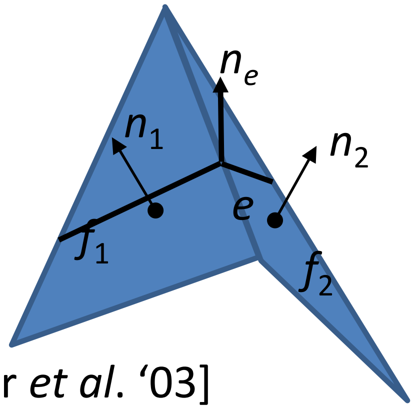
Principal Curvature Directions

Recall:

We can define curvatures at an edge *e* in terms of the angle $\beta(e)$ between curve segments*:

– The min/max curvature is 0, with principal curvature direction along *e*.

– The max/min curvature is equal to the dihedral angle ($\beta(e)=\angle n_1 n_2$), with principal curvature direction along $n_e x e$.



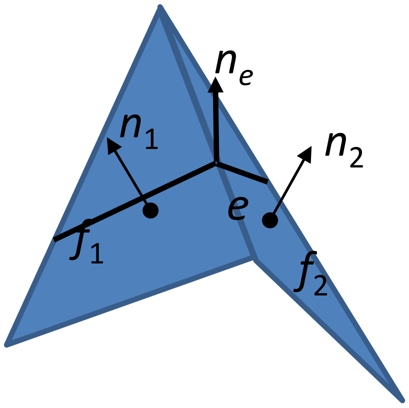
*This definition follows the definition of H_v rather than $\widetilde{H}_v$ *in* [Cohen-Steiner *et al*. '03]

Principal Curvature Directions

Recall:

This allows us to define a 3x3 curvature tensor along the edge *e* as the symmetric matrix with eigenvalue β(*e*) in the direction across *e* and eignvalues of 0 in perpendicular directions:

$$\boldsymbol{C}(p \in e) = \beta(e)\frac{1}{\left\|n_e \times e\right\|^2}\left(n_e \times e\right)\left(n_e \times e\right)^t$$

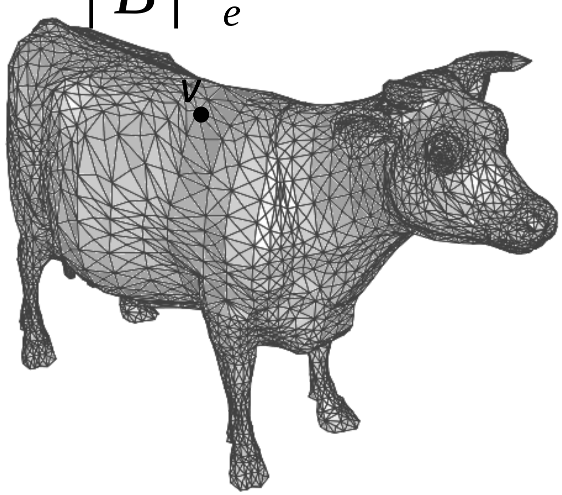


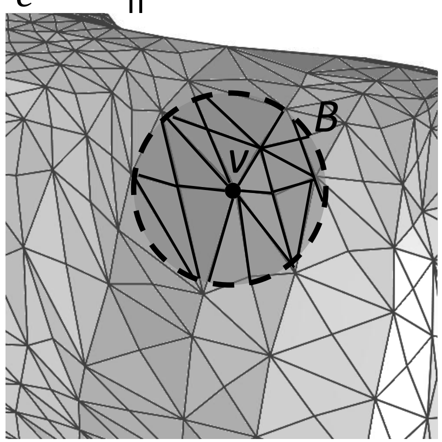
Principal Curvature Directions

Recall:

This, in turn, allows us to define the curvature tensor around a vertex *v*, average over a neighborhood *B* around *v*:

$$\boldsymbol{C}(v) = \frac{1}{|B|}\sum_e |B \cap e|\,\beta(e)\,\frac{1}{\|n_e \times e\|^2}(n_e \times e)(n_e \times e)^t$$





Principal Curvature Directions

$$\boldsymbol{C}(v) = \frac{1}{|B|} \sum_e |B \cap e| \beta(e) \frac{1}{\|n_e \times e\|^2} (n_e \times e)(n_e \times e)^t$$

Recall:

Computing the eigen-decomposition of the curvature tensor we get an estimate of:

- The normal: The eigenvector with smallest absolute eigenvalue.
- The principal directions and values: The other two eigenvectors and their associated eigenvalues.

Principal Curvature Directions

$$\boldsymbol{C}(v) = \frac{1}{|B|} \sum_e |B \cap e| \beta(e) \frac{1}{\|n_e \times e\|^2} (n_e \times e)(n_e \times e)^t$$

Note:

When the two principal directions have the same principal curvature values, the principal directions are not well defined.

Principal Curvature Directions

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Note:

When the two principal directions have the same principal curvature values, the principal directions are not well defined.

Such points are called umbilical points.

Principal Curvature Lines

<u>What are the principal curvature lines?</u>

Assuming that we are away from the umbilical points, we can define two vector fields:

1. v_{min}: Aligns with the min. curvature
2. v_{max}: Aligns with the max. curvature

Principal Curvature Lines

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1. v_{min}: Aligns with the min. curvature
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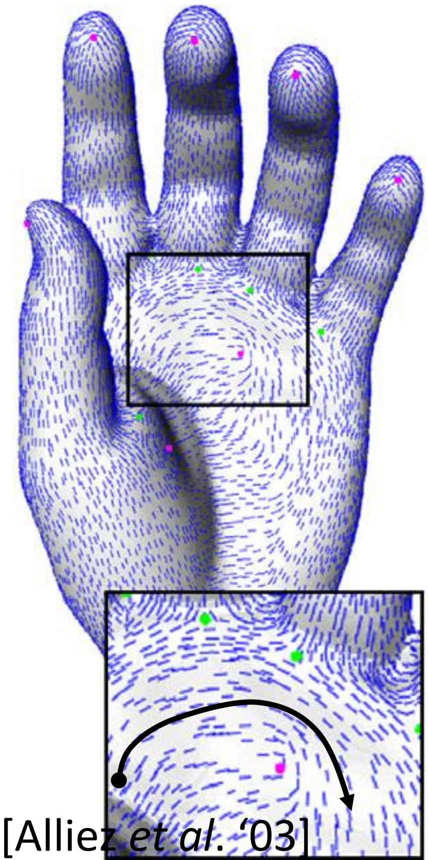
Given a starting *p*, solve the diff. eq.:

$$\gamma_{\min/\max}'(t) = v_{\min/\max}(\gamma(t)) \quad \text{s.t.} \quad \gamma(0) = p$$



[Alliez *et al.* '03]

Principal Curvature Lines

<u>How far should we integrate</u>?

We should integrate the min/max curves until they are within a prescribed density:

1. Accuracy of the remesh
2. Local curvature

Principal Curvature Lines

Q: If the user wants the remeshed surface to be within a distance of ε from the original surface, how far should the minimal/maximal curvature lines be from each other?

Principal Curvature Lines

A: Consider the surface between two lines of minimal/maximal curvature:

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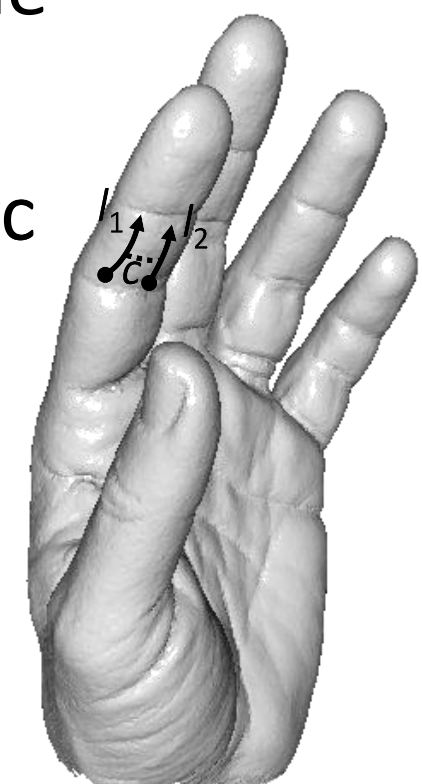
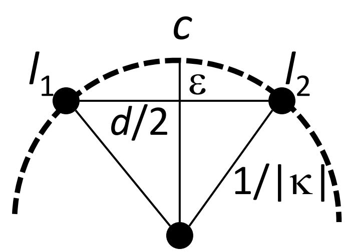
The curve between them will follow the maximal/minimal curvature direction.

The curve will be, roughly, a circular arc with radius equal to one over the maximal/minimal curvature.



Principal Curvature Lines

Looking at this in cross section, we choose the distance *d* between the curves so that the distance to the surface is below a threshold ε.



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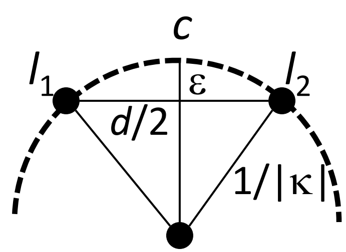

Denoting the distance by ε we get:

$$\left(\frac{d}{2}\right)^2 + \left(\frac{1}{|\kappa|} - \varepsilon\right)^2 = \left(\frac{1}{|\kappa|}\right)^2$$

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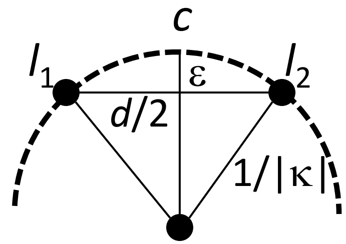


Denoting the distance by ε we get:

$$\left(\frac{d}{2}\right)^2 + \left(\frac{1}{|\kappa|} - \varepsilon\right)^2 = \left(\frac{1}{|\kappa|}\right)^2$$

$$d = 2\sqrt{\varepsilon\left(\frac{2}{|\kappa|} - \varepsilon\right)}$$

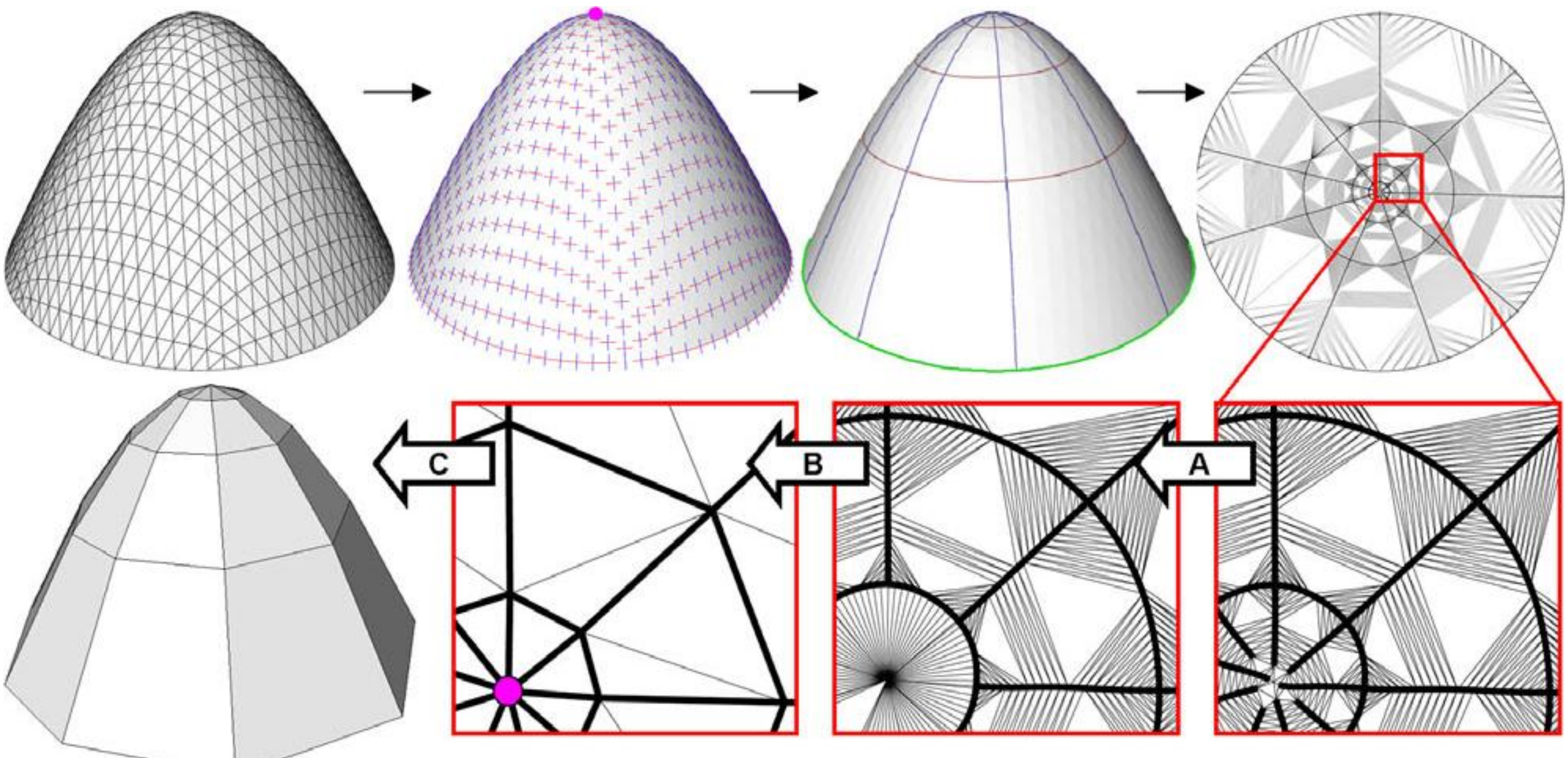
Variations on a Theme

[Alliez *et al*. '03]:

1. Compute a conformal parameterization of the surface.
2. Identify high curvature umbilicals and start growing curvature lines, adding candidate seed points into queue as the lines are grown.
3. Uniformly sample umbilicals in isotropic areas.
4. Use the quads in the anisotropic areas and use the edges of a constrained Delaunay triangulation to triangulate the isotropic points.

Variations on a Theme

[Alliez *et al*. ‘03]:



[Alliez *et al*. ‘03]

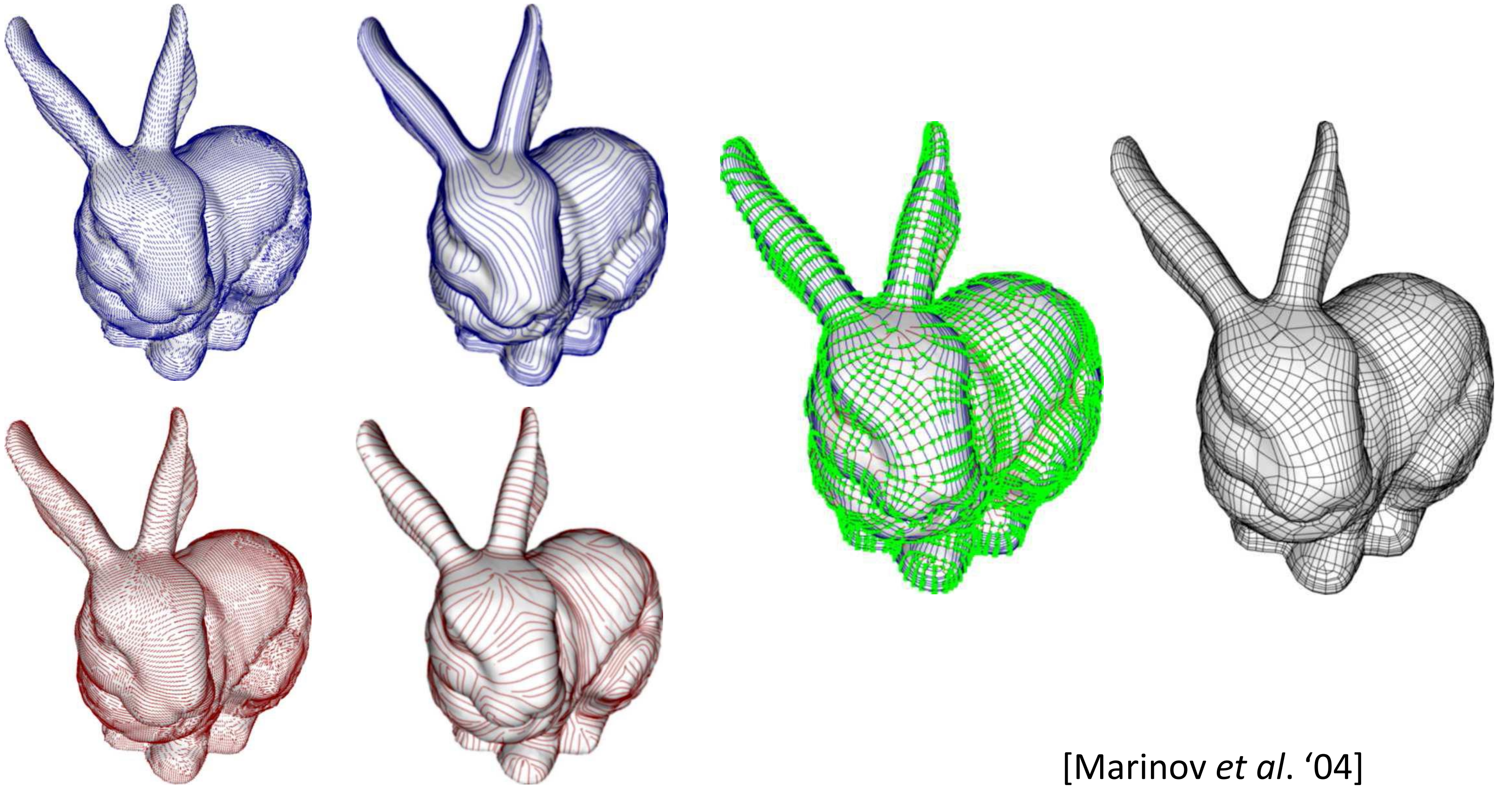
Variations on a Theme

[Marinov *et al*. '04]:

1. Work directly on the mesh
2. Estimate per-triangle confidences for the curvature tensors by looking at the consistency of the minimal curvature directions over the three vertices.
3. Grow curves from regions of high confidence, using the principal curvature direction in confident areas and continuing on along a straight line in regions of low confidence.
4. Compute intersections and polygonize.

Variations on a Theme

[Marinov *et al*. ‘04]:



[Marinov *et al*. ‘04]

Variations on a Theme

Distinctions:

- Parameterization
    - [Alliez *et al*. ’03]: Conformal parameterization, limiting the approach to either disk-like objects or patching.
    - [Marinov *et al*. ’04]: None

Variations on a Theme

Distinctions:

– Smoothing

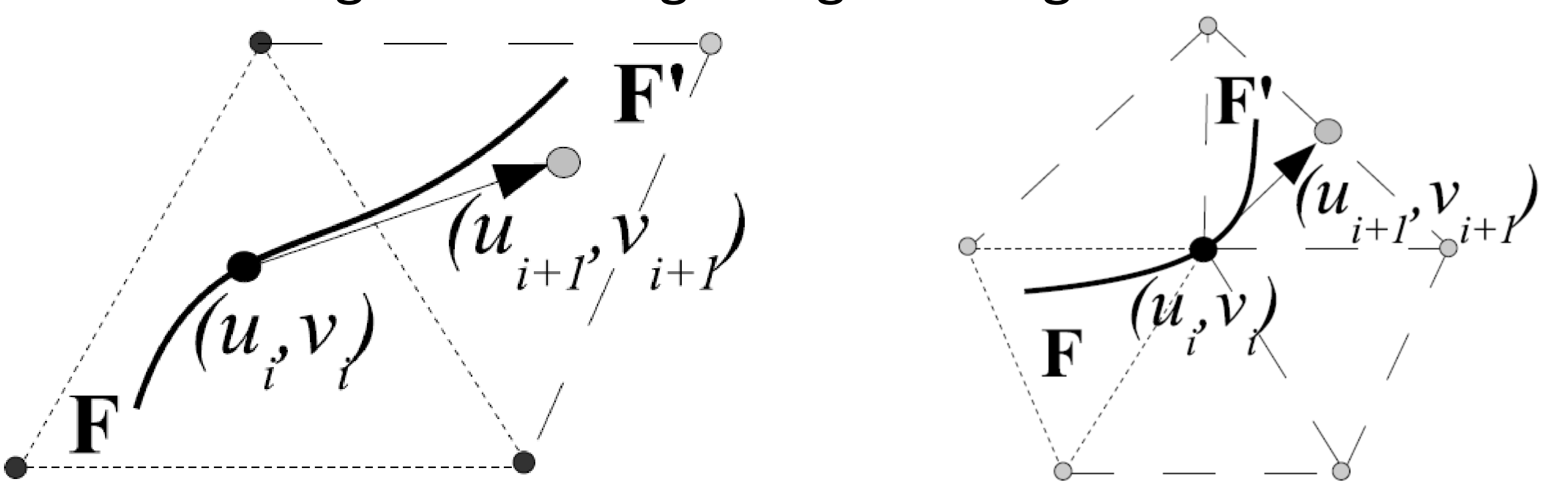
- [Alliez *et al*. ’03]: Gaussian smoothing with spatially varying radius.
- [Marinov *et al*. ’04]: Confidence-weighted Laplacian smoothing.

Variations on a Theme

<u>Distinctions</u>:

– Integration

- [Alliez *et al*. ’03]: Performed in the 2D parameterization domain.
- [Marinov *et al*. ’04]: Performed on mesh by locally flattening and walking along a “straight” line.



[Marinov *et al*. ‘04]

Variations on a Theme

Distinctions:

– Proximity Queries

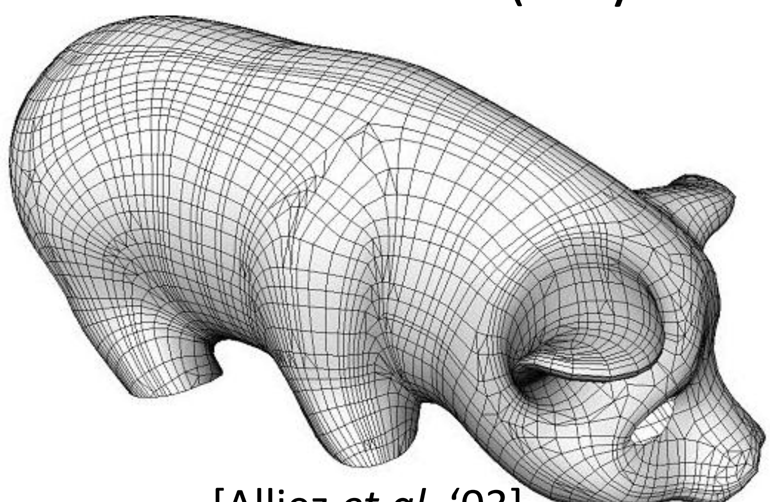
- [Alliez *et al*. ’03]: Maintain (and update) a 2D Constrained Delaunay Triangulation as new edge segments are introduced.
- [Marinov *et al*. ’04]: Hash curve edges with associated triangles, find adjacent triangles and exhaustively test edges.

Variations on a Theme

Distinctions:

– Isotropic Regions

- [Alliez *et al*. ’03]: Change from quadrangulation to triangulation.
- [Marinov *et al*. ’04]: Attempt to continue going in the same direction (may switch curves from minimal to maximal).



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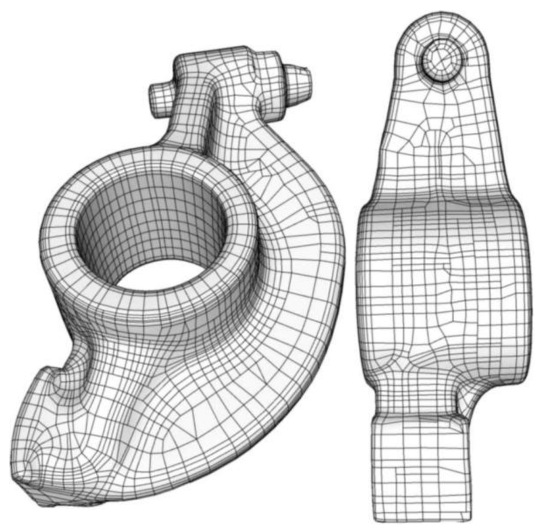

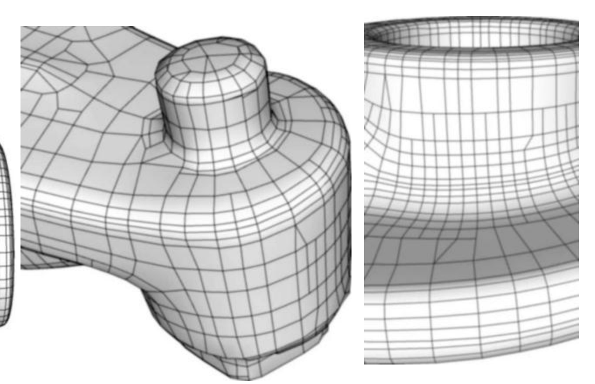

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