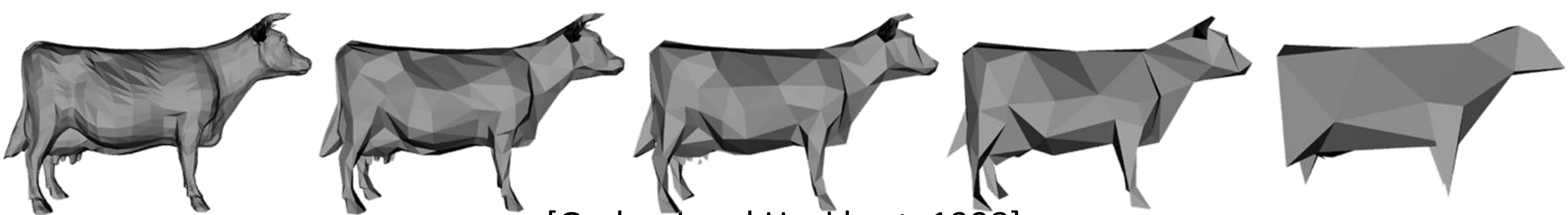
600.657: Mesh Processing

Chapter 7

Surface Simplification

Goal:

Compute lower-complexity representation(s) of a surface capturing the detail(s): $M_0 \rightarrow M_1 \rightarrow \ldots$



[Garland and Heckbert, 1998]

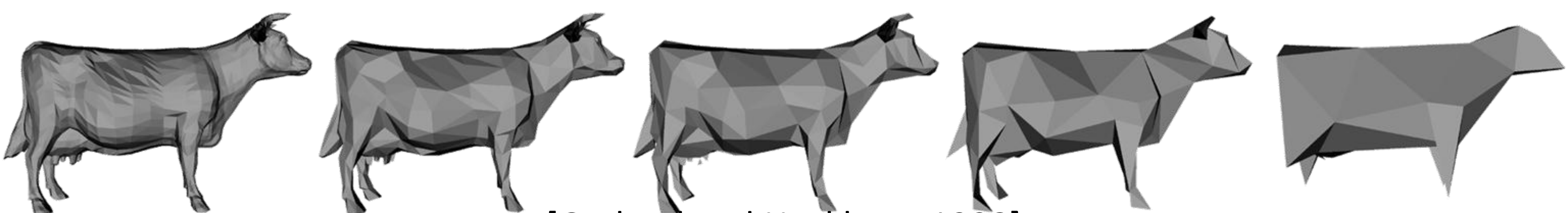
Surface Simplification

Goal:

Compute lower-complexity representation(s) of a surface capturing the detail(s): $M_0 \rightarrow M_1 \rightarrow \ldots$

Considerations:

- Topology preservation
- Surface accuracy



[Garland and Heckbert, 1998]

Surface Simplification

How:

- Vertex Clustering
- Iterative Contraction
  - Vertex collapse
  - Edge collapse
  - Vertex pair contraction

Surface Simplification

Vertex Clustering:

Partition space:

$$M_0 = \bigcup_i R_i$$

Surface Simplification

<u>Vertex Clustering</u>:

Partition space:

$$M_0 = \bigcup_i R_i$$

- Merge vertices within a partition

$$\{v \in M_0 \mid v \in R_i\} \rightarrow v_i \in M_1$$

Surface Simplification

<u>Vertex Clustering</u>:

Partition space:

$$M_0 = \bigcup_i R_i$$

- Merge vertices within a partition

$$\{v \in M_0 \mid v \in R_i\} \to v_i \in M_1$$

- Add edges and triangles

$$(v_i, v_j) \in M_1 \text{ if } \exists v \in R_i, w \in R_j \text{ s.t. } (v, w) \in M_0$$

Surface Simplification

Vertex Clustering:

Partition space:

$$M_0 = \bigcup_i R_i$$

- Merge vertices within a partition

$$\{v \in M_0 \mid v \in R_i\} \rightarrow v_i \in M_1$$

- Add edges and triangles

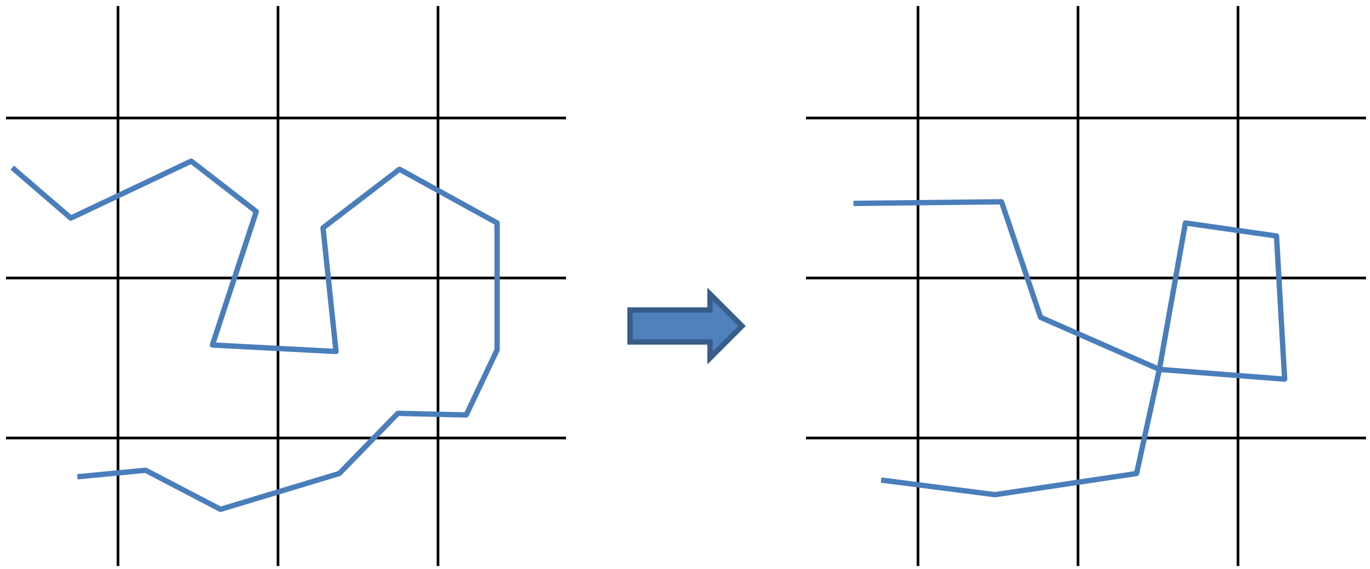
$$(v_i, v_j) \in M_1 \text{ if } \exists v \in R_i, w \in R_j \text{ s.t. } (v,w) \in M_0$$

$$(v_i, v_j, v_k) \in M_1 \text{ if } \exists u \in R_i, v \in R_j, w \in R_j \text{ s.t. } (u,v,w) \in M_0$$

Surface Simplification

<u>Vertex Clustering</u>:

Note that this method is not guaranteed to preserve the topology of the mesh.

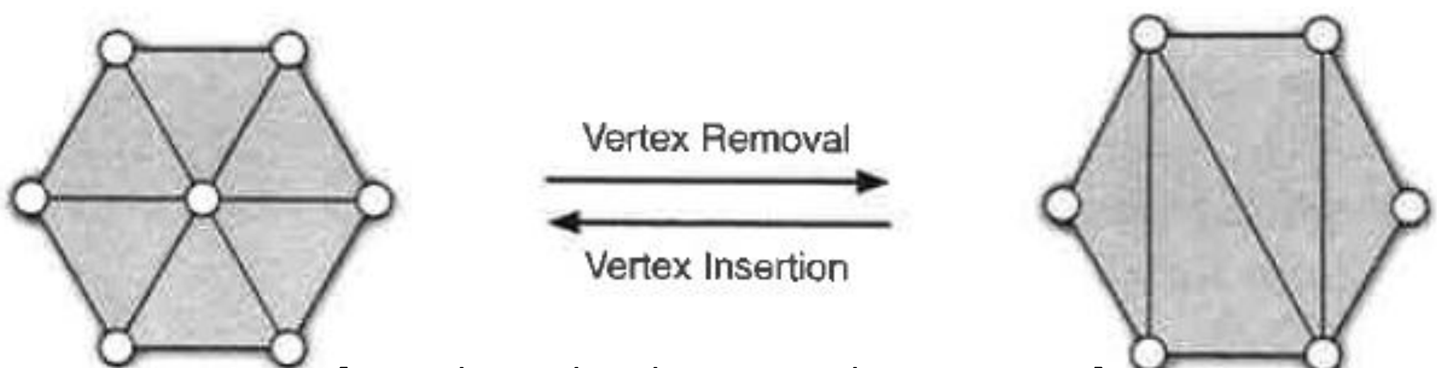


Surface Simplification

Vertex Collapse:

Collapse the one-ring of a vertex to a point:

- Remove the vertex
- Triangulate the new face



[Botsch *et al*, Polygon Mesh Processing]

Surface Simplification

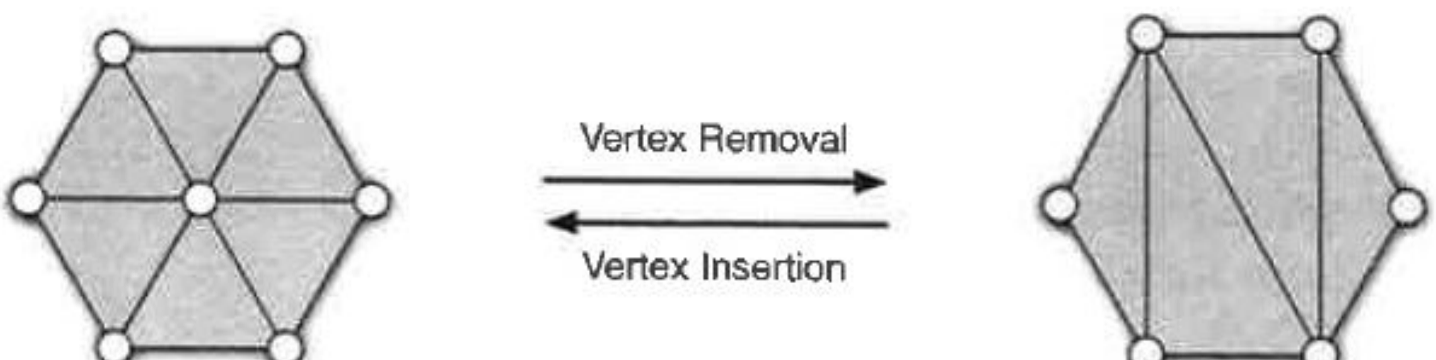
Vertex Collapse:

Collapse the one-ring of a vertex to a point:

– Remove the vertex

– Triangulate the new face

This method will not preserve topology when the one-ring is not homeomorphic to a disk.



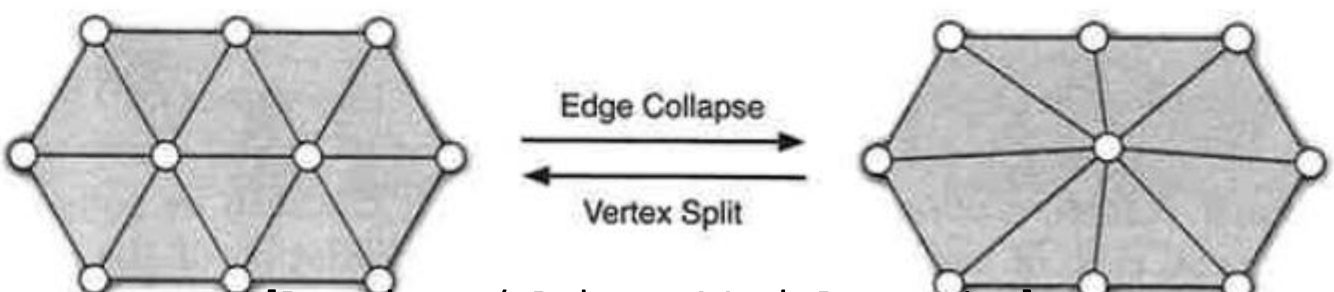
[Botsch *et al*, Polygon Mesh Processing]

Surface Simplification

<u>Edge Collapse</u>:

Collapse an edge to a vertex:

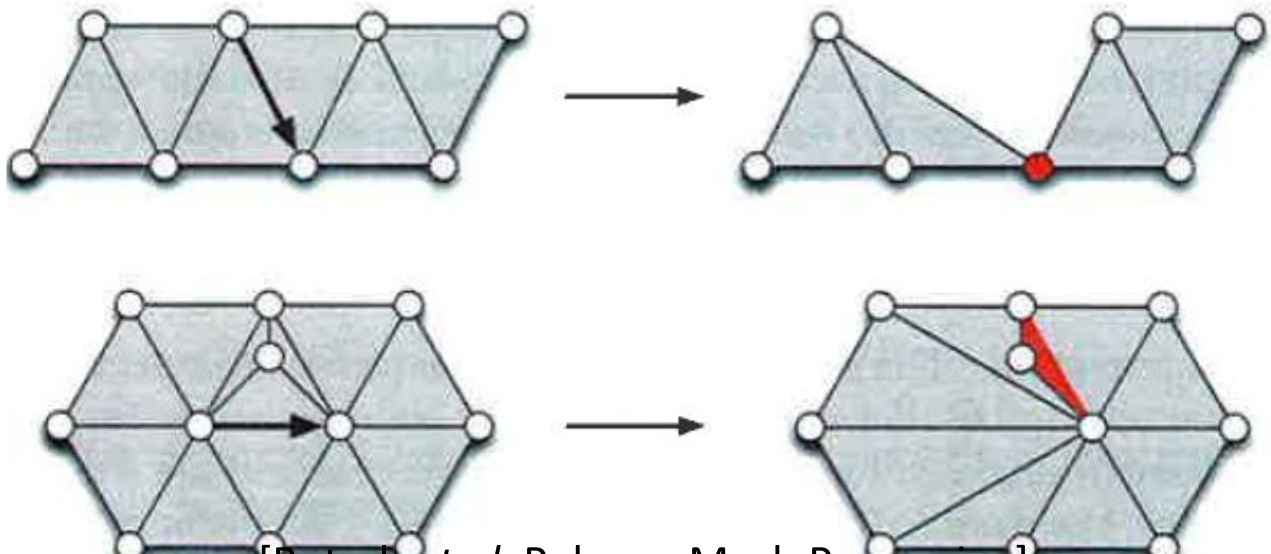
- Merge the vertices
- Remove the edge
- Remove the triangles
- Merge the wing edges



[Botsch *et al*, Polygon Mesh Processing]

Surface Simplification

<u>Edge Collapse</u>:



[Botsch *et al*, Polygon Mesh Processing]

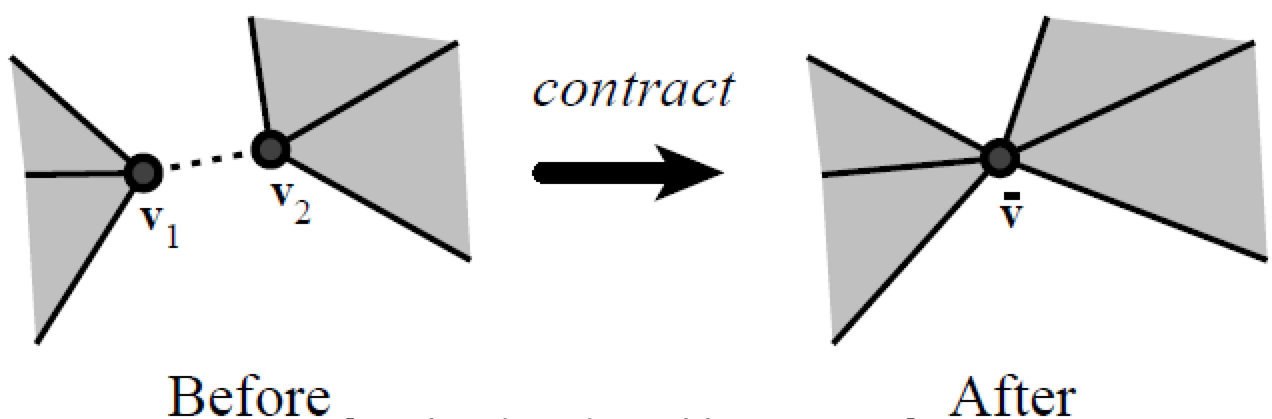
This method will not preserve topology if:

- Edges are on separate boundaries
- The one-rings intersect in more than two points

Surface Simplification

<u>Vertex Pair Contraction</u>:

Like an edge collapse but the vertices don't have to share an edge.



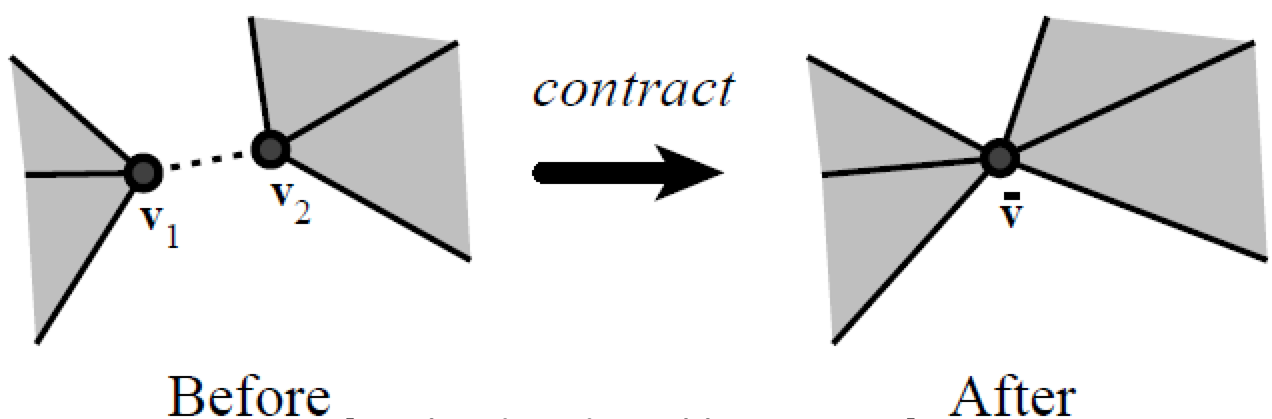
[Garland and Heckbert, 1998]

Surface Simplification

Vertex Pair Contraction:

Like an edge collapse but the vertices don't have to share an edge.

(Certainly does not preserve topology.)



[Garland and Heckbert, 1998]

Surface Simplification

Accuracy:

We would like the simplified surface to be close to the original surface.

Surface Simplification

<u>Definition</u>:

The *Hausdorff distance* from set *S* to *T* is the maximum of the distances of points on *S* to the nearest points on *T*:

$$H(S,T) = \sup_{p \in S} \inf_{q \in T} \|p - q\|$$

Surface Simplification

<u>Definition</u>:

The *Hausdorff distance* from set *S* to *T* is the maximum of the distances of points on *S* to the nearest points on *T*:

$$H(S,T) = \sup_{p \in S} \inf_{q \in T} \|p - q\|$$

Although we would like to use this distance to guide how we simplify, it tends to be too inefficient in practice.

Surface Simplification

Key Idea:

Instead of searching for the closest point on the original surface, track the original surface through the simplification.

Surface Simplification

Key Idea:

Instead of searching for the closest point on the original surface, track the original surface through the simplification.

For each $t \in M_i$, maintain a list of vertices:

$$V_i^t \subset M_0$$

that are closest to t.

Surface Simplification

<u>Key Idea</u>:

Instead of searching for the closest point on the original surface, track the original surface through the simplification.

For each $t \in M_i$, maintain a list of vertices:

$$V_i^t \subset M_0$$

that are closest to *t*.

If edge (*u*,*v*) is contracted, redistribute the vertices tied to the triangles containing (*u*,*v*).

Surface Simplification

Key Idea:

Instead of searching for the closest point on the original surface, track the original surface through the simplification.

Challenge:

Keeping track of everything can be difficult to do correctly/quickly.

Quadratic Error Metric

Using homogenous coordinates, points on the plane $P=\{(x,y,z)|ax+by+cz+d=0\}$ can be represented by the vector (a,b,c,d).

Quadratic Error Metric

Using homogenous coordinates, points on the plane $P=\{(x,y,z)\,|\,ax+by+cz+d=0\}$ can be represented by the vector (a,b,c,d).

For a point $q=(x,y,z)$, the square distance from q to the plane P is*:

$$d(P,q) = \|ax+by+cz+d\|^2$$
$$= \|(a,b,c,d)^t(x,y,z,1)\|^2$$

*Assuming that (a,b,c) is a unit-vector.

Quadratic Error Metric

Using homogenous coordinates, points on the plane $P=\{(x,y,z) \mid ax+by+cz+d=0\}$ can be represented by the vector (a,b,c,d).

For a point $q=(x,y,z)$, the square distance from q to the plane P is*:

$$d(P,q) = \left\| (a,b,c,d)^t (x,y,z,1) \right\|^2$$

Setting $\overline{p}=(a,b,c,d)$ and $\overline{q}=(x,y,z,1)$ gives:

$$d(P,q) = \left\| \overline{p}^t \overline{q} \right\|^2 = \overline{q}^t \, \overline{p}\overline{p}^t \overline{q}$$

*Assuming that (a,b,c) is a unit-vector.

Quadratic Error Metric

More generally, if $\bar{p}_i=(a_i,b_i,c_i,d_i)$ are the coordinates of a set of planes $\{P_i\}$, the sum-of-squared distances from q to the planes is:

$$d(\{P_i\},q) = \sum_i \bar{q}^t \bar{p}_i \bar{p}_i^t \bar{q} = \bar{q}^t \left(\sum_i \bar{p}_i \bar{p}_i^t \right) \bar{q}$$

Quadratic Error Metric

<u>Observation</u>:

For vertex *v* in a mesh, *v* is in the intersection of the planes containing the neighboring triangles.

Quadratic Error Metric

Observation:

For vertex *v* in a mesh, *v* is in the intersection of the planes containing the neighboring triangles.

That is, if $\bar{p}_i$ are the coordinates of the planes containing the triangles, then:

$$\bar{v}^t \left(\sum_i \bar{p}_i \bar{p}_i^{\,t} \right) \bar{v} = 0$$

Quadratic Error Metric

Observation:

For vertex *v* in a mesh, *v* is in the intersection of the planes containing the neighboring triangles.

That is, if $\bar{p}_i$ are the coordinates of the planes containing the triangles, then:

$$\bar{v}^t \left(\sum_i \bar{p}_i \bar{p}_i^{\,t} \right) \bar{v} = 0$$

Thus, we can represent the support planes of *v* by the symmetric 4x4 matrix:

$$Q_v = \sum_i \bar{p}_i \bar{p}_i^{\,t}$$

Quadratic Error Metric

<u>Note</u>:

Given a vertex pair (*u*,*v*) and a target position for contraction, *w*, we can measure the quadric error of the contraction as:

$$\begin{aligned} E\big((u,v)\rightarrow w\big) &= \overline{w}^t Q_u \overline{w} + \overline{w}^t Q_v \overline{w} \\ &= \overline{w}^t \big(Q_u + Q_v\big)\overline{w} \end{aligned}$$

Quadratic Error Metric

Note:

Given a vertex pair (*u*,*v*) and a target position for contraction, *w*, we can measure the quadric error of the contraction as:

$$E\big((u,v)\rightarrow w\big) = \overline{w}^t Q_u \overline{w} + \overline{w}^t Q_v \overline{w}$$
$$= \overline{w}^t \big(Q_u + Q_v\big)\overline{w}$$

So, when *u* and *v* are collapsed into *w*, we can track the support planes by associating the unions of the support planes to *w*:

$$Q_w = Q_u + Q_v$$

Quadratic Error Metric

<u>Note</u>:

Given a vertex pair (*u*,*v*) and a target position for contraction, *w*, we can measure the quadric error of the contraction as:

$$\begin{aligned} E\big((u,v)\rightarrow w\big) &= \overline{w}^t Q_u \overline{w} + \overline{w}^t Q_v \overline{w} \\ &= \overline{w}^t \left(Q_u + Q_v\right)\overline{w} \end{aligned}$$

Moreover, given a vertex pair (*u*,*v*), we can solve for the position of *w* minimizing the error.

Quadratic Error Metric

<u>Approach</u>:

– Initialize by assigning the symmetric 4x4 matrix:

$$Q_v = \sum_i \bar{p}_i \bar{p}_i^t$$

to each vertex *v*.

– Compute candidate vertex contraction pairs.

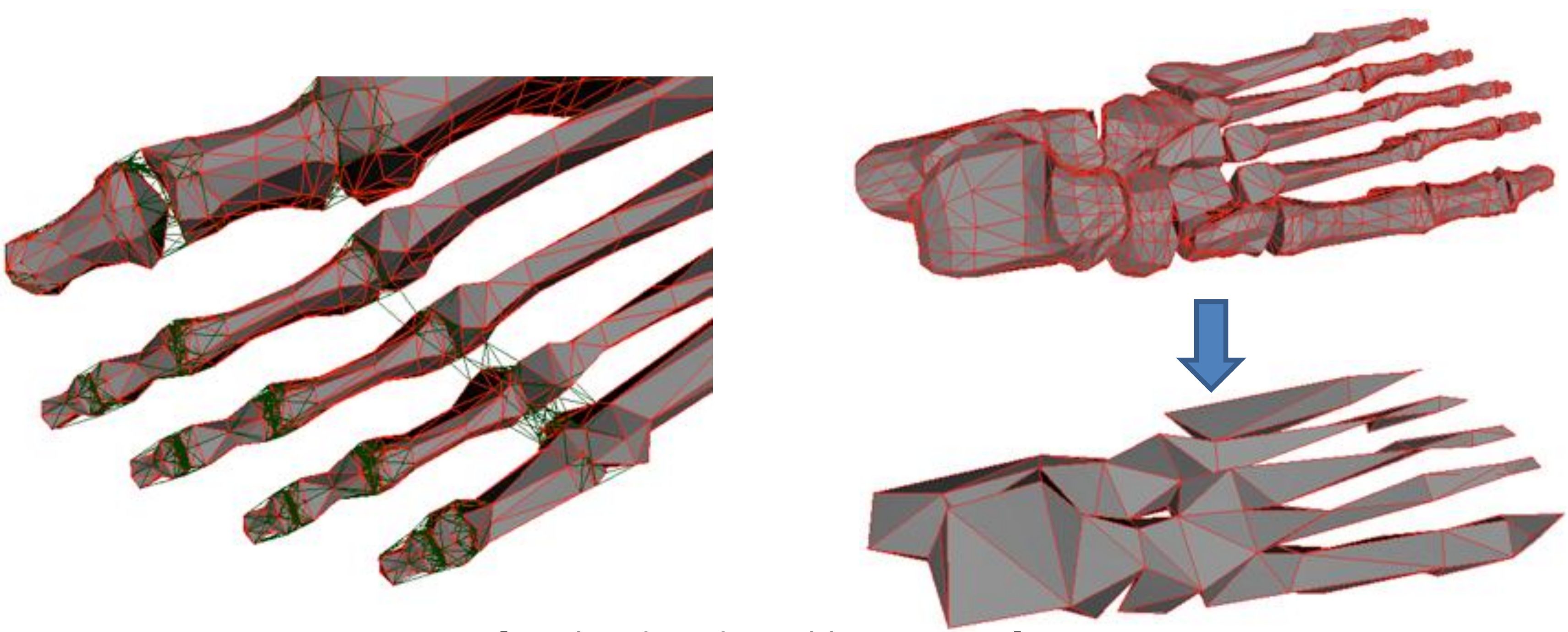
– While not done:

- For each (u,v) compute w minimizing contraction error.
- Choose $(u,v) \rightarrow w$ with minimal contraction error.
- Contract, setting $Q_w = Q_u + Q_v$.
- Update candidate vertex contraction pairs.

Quadratic Error Metric

How do we choose candidate contraction pairs?

- Add all edges of the mesh.
- Add all pairs (u,v) with $|u-v|<\varepsilon$.

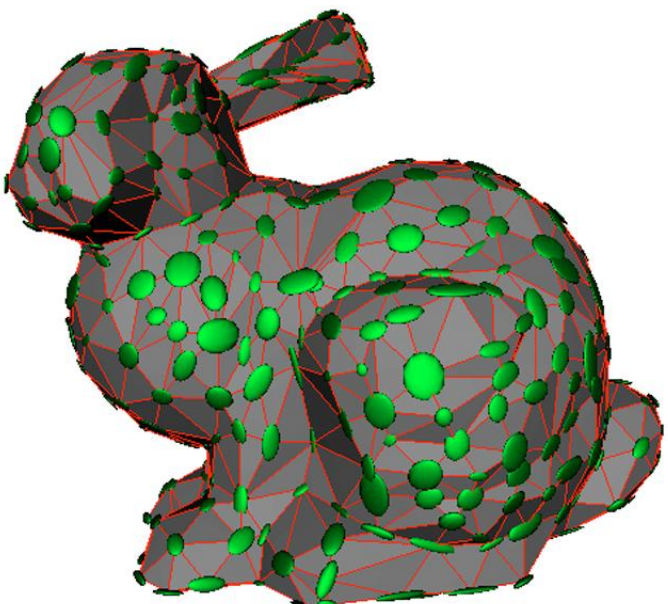


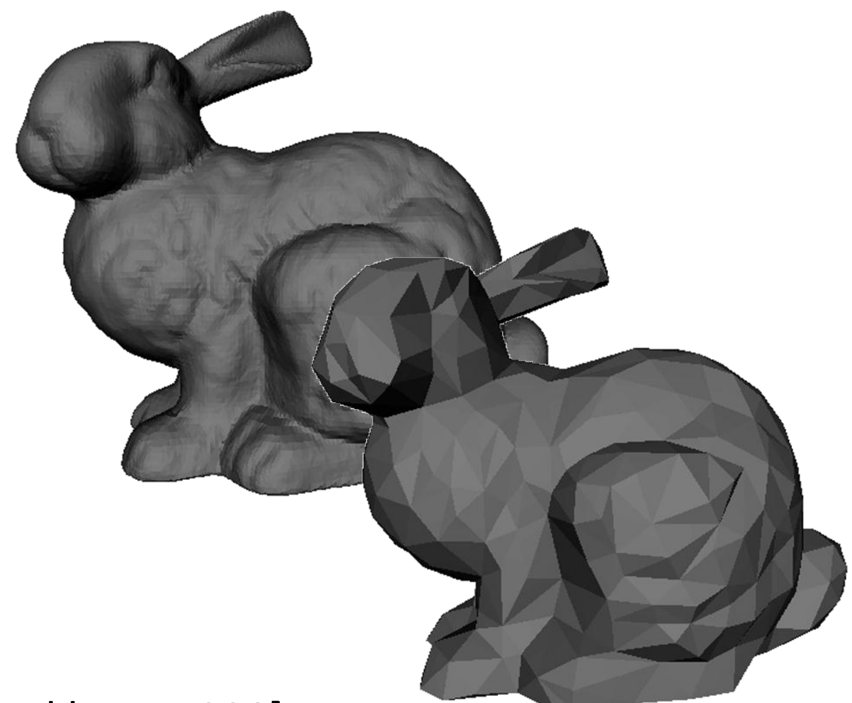
[Garland and Heckbert, 1998]

Quadratic Error Metric

<u>Note</u>:

At a given step in the algorithm, every vertex in the simplified mesh is associated with a quadric characterizing the fit error.





[Garland and Heckbert, 1998]

Quadratic Error Metric

Advantages:

- Tracking of support planes is easy.
- Closed-form solution for best contraction position.
- Topology is not preserved.

Quadratic Error Metric

Advantages:

- Tracking of support planes is easy.
- Closed-form solution for best contraction position.
- Topology is not preserved.

Disadvantages:

- Tracking vertex contraction pairs may be messy.
- Point-to-plane is not point-to-triangle.
- Support planes may be double/triple counted.
- Topology is not preserved.