600.657: Mesh Processing

Chapter 1

Mesh Representations

- <u>Parameteric</u>
  - Represent a surface as (continuous) injective function from a domain $\Omega \subset \boldsymbol{R}^2$ to $S \subset \boldsymbol{R}^3$.

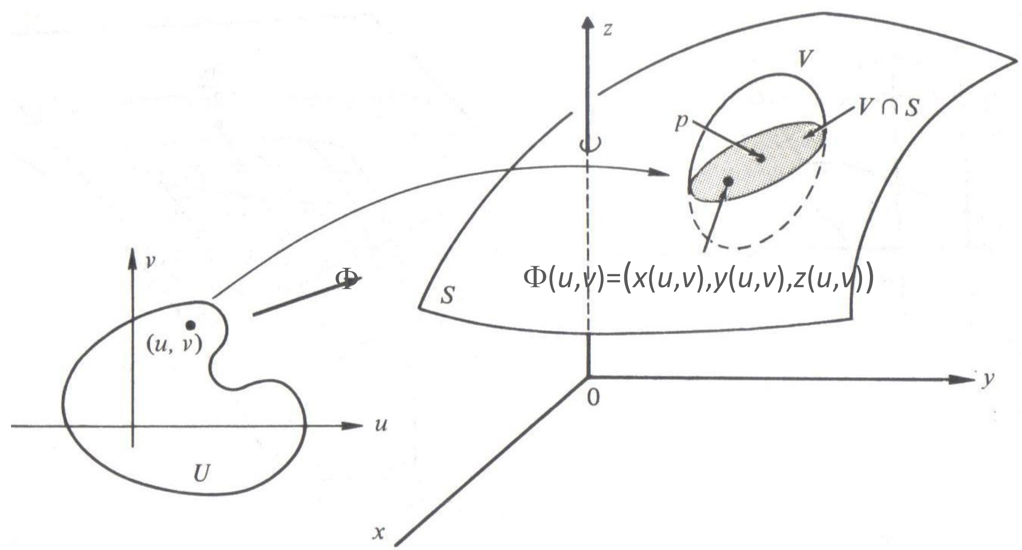
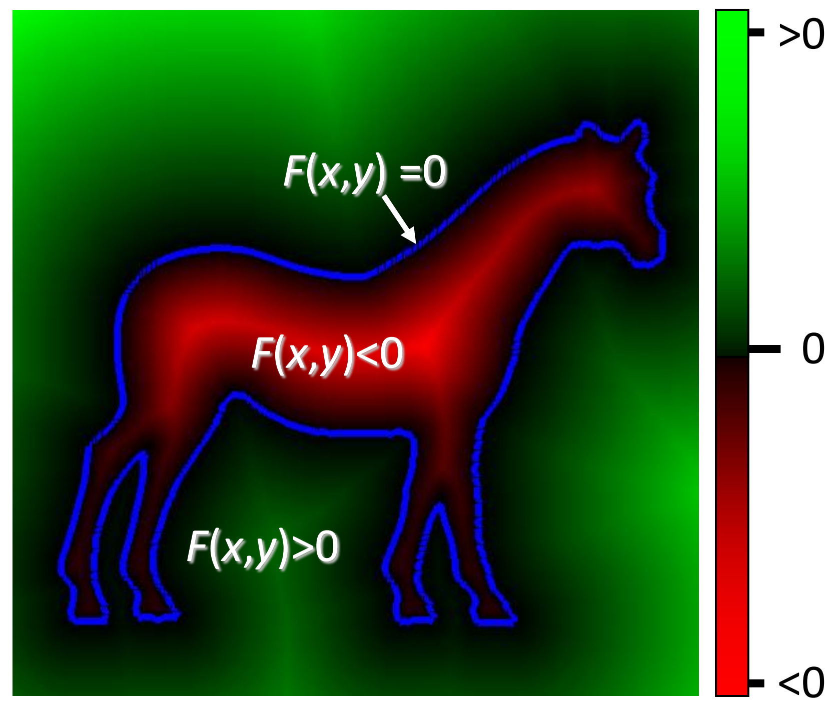


Figure 2-1

Mesh Representations

- Parameteric
  - Represent a surface as (continuous) injective function from a domain $\Omega \subset \boldsymbol{R}^2$ to $S \subset \boldsymbol{R}^3$.
- Implicit
  - Represent a surface as the zero set of a (regular) function defined in $\boldsymbol{R}^3$.



$F(x,y)=0$

Mesh Representations

- <u>Parameteric</u>
  - Represent a surface as (continuous) injective function from a domain $\Omega \subset \boldsymbol{R}^2$ to $S \subset \boldsymbol{R}^3$.

  In practice, it's not easy to find a single function that parameterizes the surface.

  So instead, we represent a surface as a collection of functions (charts) from (simple) 2D domains into 3D.

Mesh Representations

- <u>Parameteric</u>
  - Represent a surface as (continuous) injective function from a domain $\Omega\subset\boldsymbol{R}^2$ to $S\subset\boldsymbol{R}^3$.

  Given a set of charts, we say that the manifold S is "smooth" if for any two charts $\phi_1:\Omega_1\rightarrow S$ and $\phi_2:\Omega_2\rightarrow S$, the map $\phi_2^{-1}\circ\phi_1$ is smooth.

Mesh Representations

- Implicit
  - Represent a surface as the zero set of a (regular) function defined in $\boldsymbol{R}^3$.

1. Why is the condition that the function be regular (i.e. have non-vanishing derivative) necessary?
2. How smooth is the surface?

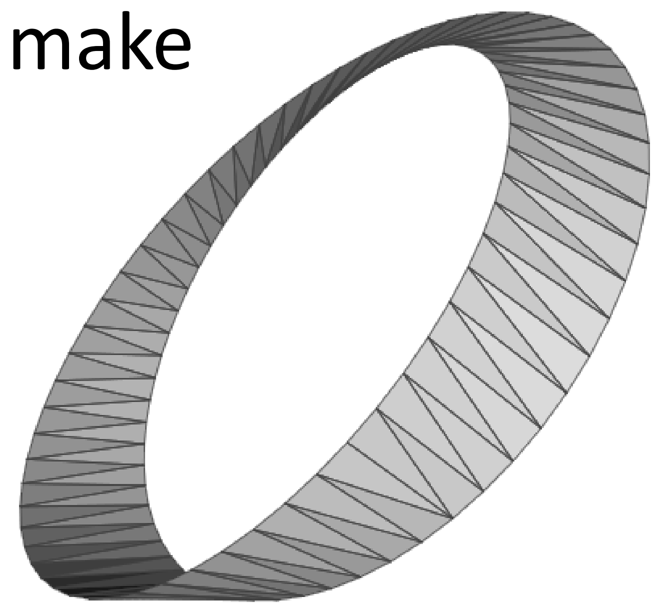
Mesh Representations

- Parameteric
  - Easy to enumerate points on the surface
  - Easy to find neighbors
- Implicit
  - Easy to determine if you are inside or outside
  - CSG operations are easy
  - Easy to modify the topology of the surface

Parameteric Representations

<u>Triangle Meshes</u>:

- *Geometry*: The positions of the vertices on the mesh.
- *Topology*: The connectivity of the vertices (e.g. which triplets of vertices make up the triangles).



Parameteric Representations

Precision:

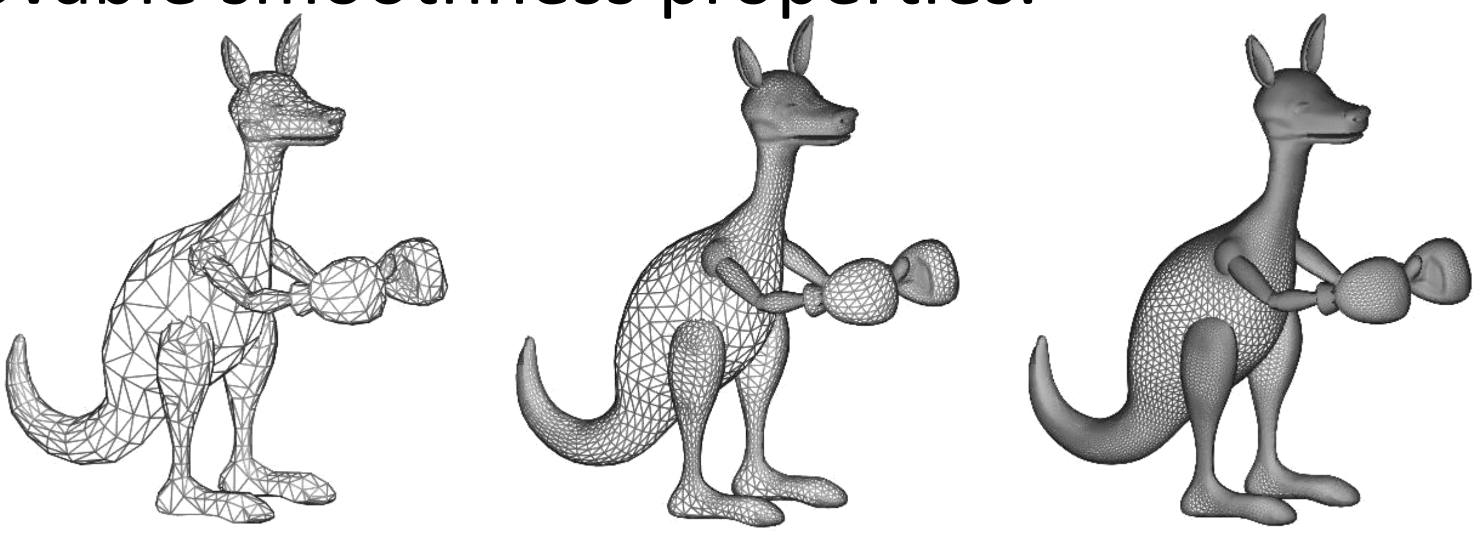
- Though accuracy improves with refinement, the error of the approximation depends on the variation (curvature) of the surface.

Parameteric Representations

<u>Subdivision Surfaces</u>:

Given a base (triangle) mesh and a set of rules for refining the geometry.

Repeated subdivision results in a surface with provable smoothness properties.



Triangle Meshes

1. What properties does the mesh have to satisfy to be manifold?
2. How continuous is the mesh?
3. What are the charts that define the manifold?

Euler's Formula

For a closed, connected, water-tight mesh with genus g, the number of vertices (V), edges (E), and faces (F) satisfy:

$$V - E + F = 2 - 2g$$

For a triangle mesh:

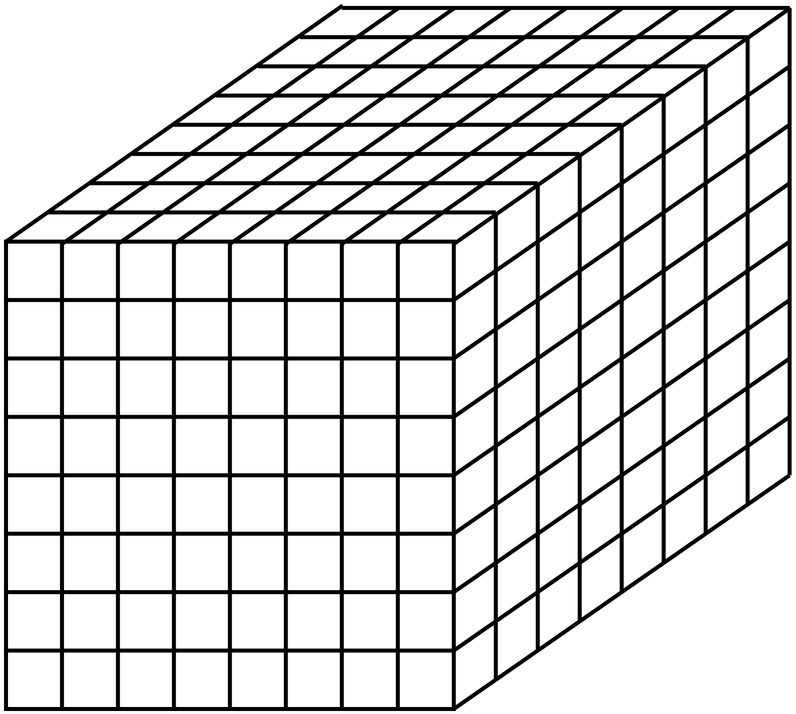
- What is the ratio of triangles to vertices?
- What is the ratio of edges to vertices?
- What is the average vertex valence?

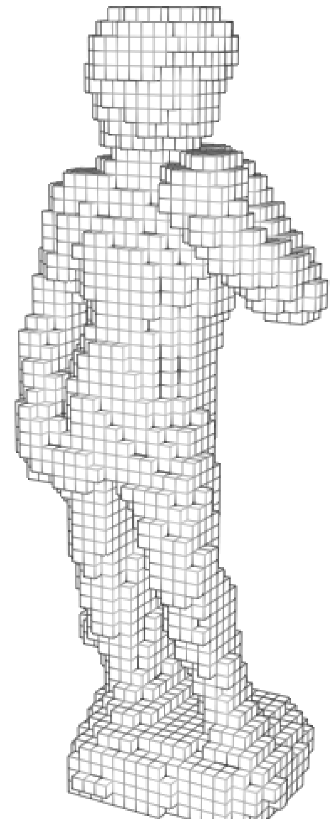
How about for a quad (dominant) mesh?

Implicit Representations

Voxel Grids:

Represented by the values of the function on a regular grid.





Implicit Representations

Voxel Grids:

Represented by the values of the function on a regular grid.

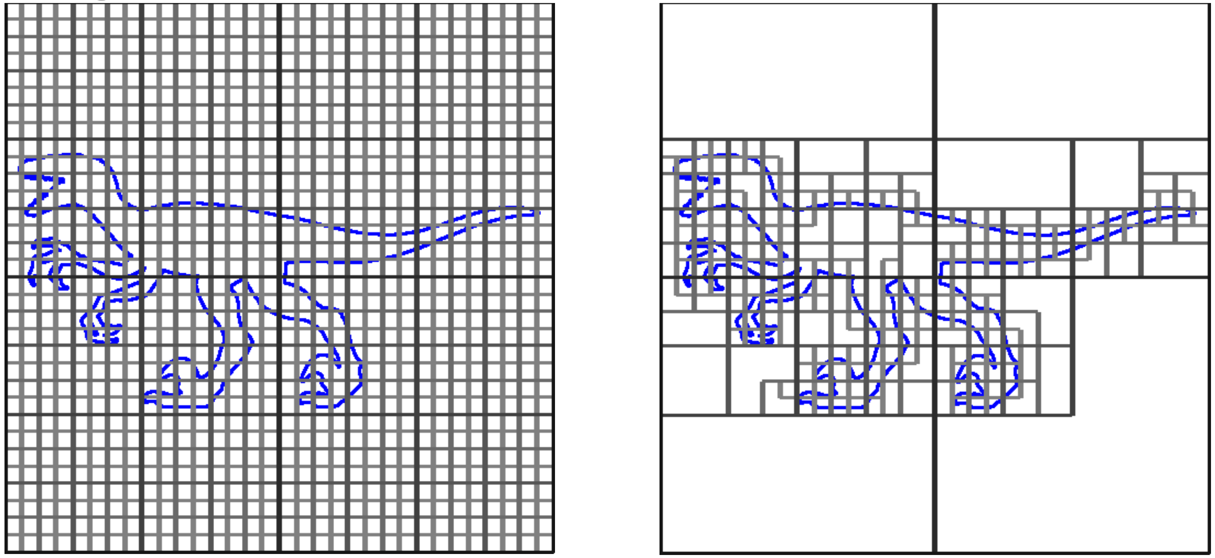
Though binary voxel grids are simplest, often represent the (signed) Euclidean Distance Transform:

$$EDT^2(p) = \min_{q \in S} \|p - q\|^2$$

Implicit Representations

<u>Adaptive Grids</u>:

In practice, we may only need a high-precision representation of the implicit function near the surface, so represent the function over an adaptive grid.



Transitioning Between Representations

Parameteric to Implicit

Assign distance values to points on a voxel grid:

Naïve:

For each voxel, find the closest point on the surface and use that to set the voxel's value.

Efficient:

For each voxel near the surface, find the closest point on the surface (using a kd-tree) and use that to set the voxel's value.

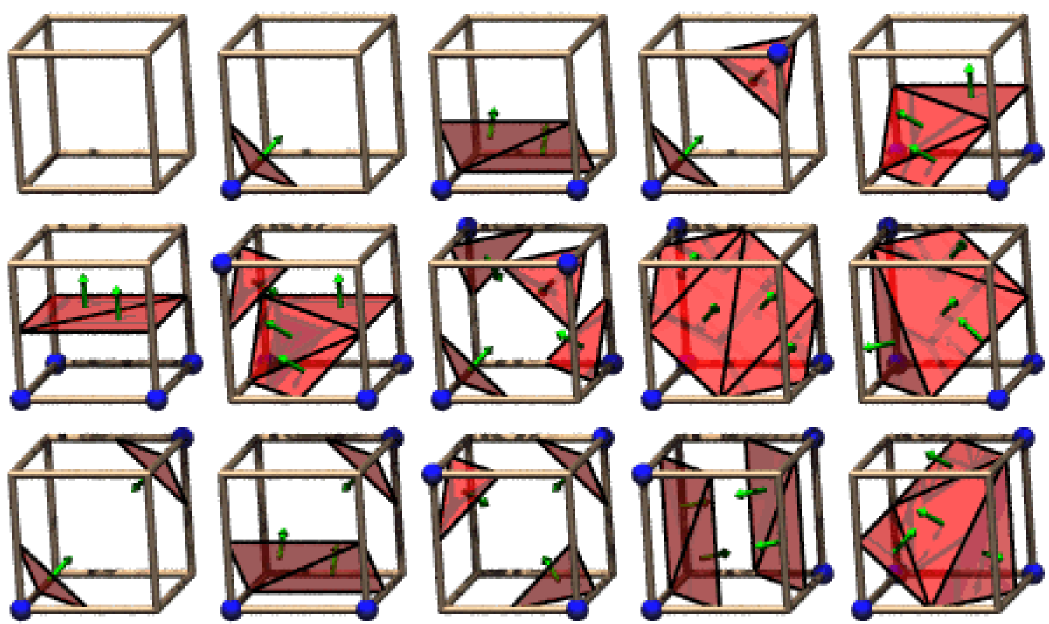
Use the *fast marching* method to define distance values away from the surface.

Transitioning Between Representations

<u>Implicit to Parameteric</u>

Extract the zero-set of the implicit function:

- Marching cubes (for voxel grids)
- Dual marching cubes, etc. (octrees)



The 15 Cube Combinations