Harmonic Coordinates for Character Articulation

PIXAR

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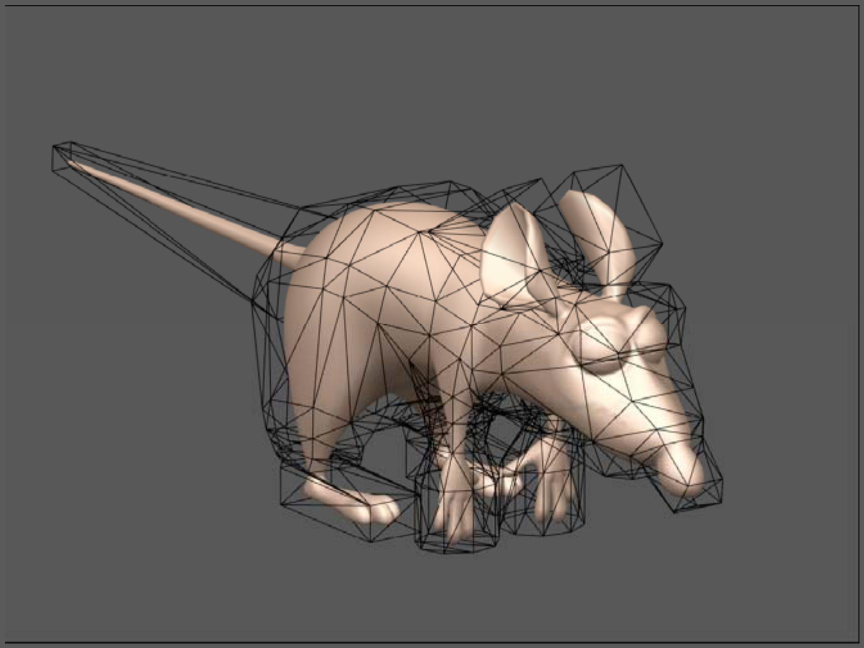
- We have a complex source mesh inside of a simpler cage mesh

- We want vertex deformations applied to the cage to be applied appropriately to the source mesh



Mean Value Coordinates

Ju, Schaefer, Warren, 2005

$$p' = \sum_i g_i(p) C_i'$$

p'	New object vertices
$g_i(p)$	Mean value coordinate Weighting functions
C_i'	Deformed cage vertices

To compute $g_i(p)$ for each interior point *p*:

- Consider each point *x* on the boundary
- Multiply $f(x)$ by the reciprocal distance from *x* to *p*
- Average over all *x*

From the Ju paper:

$$g(p) = \frac{\int_x w(x,v) f(x) dS_p}{\int_x w(x,v) dS_p}$$

where

$$w(x,p) = \frac{1}{|x-p|}$$

and

S_p is the unit sphere centered at *p*

To compute $g_i(p)$ for each interior point *p*:

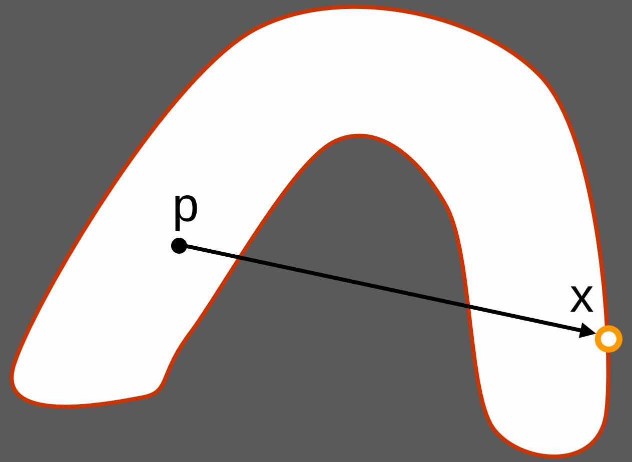
– Consider each point *x* on the boundary

– Multiply $f(x)$ by the reciprocal distance from *x* to *p*

– Average over all *x*

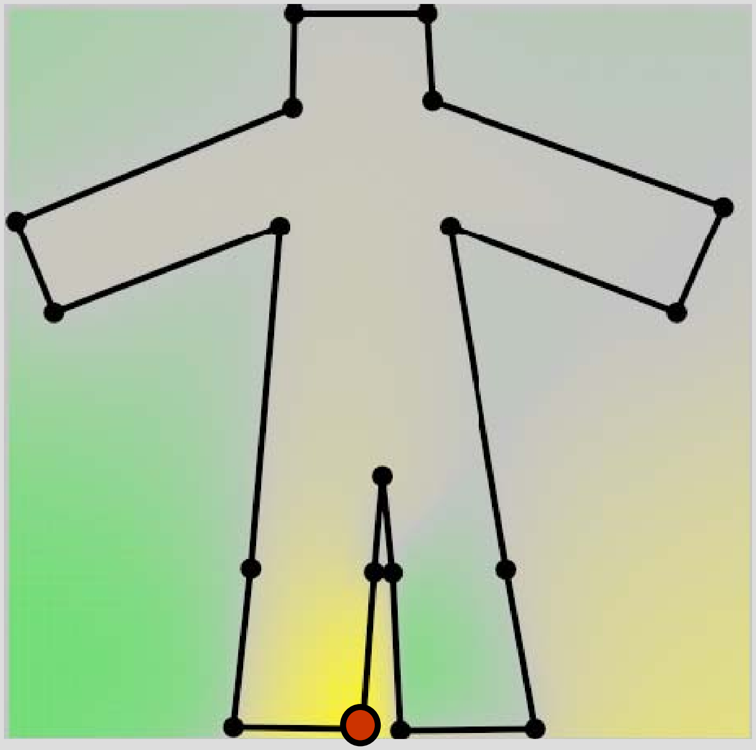
Good things:

- Topological flexibility in designing the cage (any closed tri-mesh)
- Deformations are smooth
- Functions are linear, so no popping

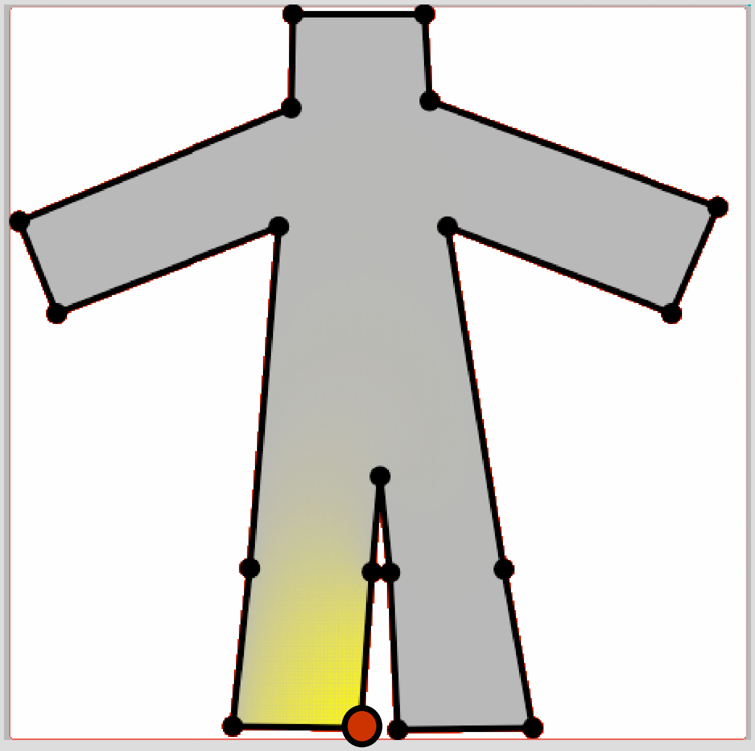


Bad things:

- Does not respect the visibility of *x* from *p*
- If a cage vertex has a negative weight associated with it, then the object vertex and cage vertex will move in opposite directions



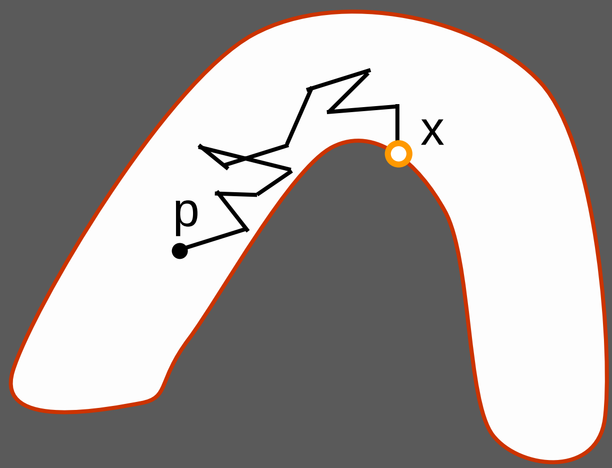
Mean Value Coordinate Field



Desired Coordinate Field

- Instead, let's average over all Brownian paths leaving *p*
  - This will consider the visibility of *x* from *p*
  - Essential for any concave mesh

- Interestingly enough…
  - This is the same as solving Laplace's equation
  - Porf, Stone 1978 & Bass 1995



$$\Delta h_i(p) = 0 \quad p \in Interior(C)$$

Solve for every cage vertex *p*

Let us first approach things in two dimensions

Boundary conditions:

- Let ∂p denote a point on the boundary ∂C of *C*
- Then:

$$h_i(\partial p) = \phi_i(\partial p), \quad \text{for all} \quad \partial p \in \partial C$$

where

$\phi_i(\partial p)$ is the piecewise linear function such that $\phi_i(C_j) = \delta_{i,j}$

Properties:

- Interpolation

$$h_i\left(C_j\right)=\delta_{i,j}$$

- Smoothness

The functions $h_i\left(p\right)$ are smooth in the interior of the cage

- Non-negativity

$h_i\left(p\right)\geq 0$ for all $p\in C$

- Interior locality

Interior locality holds if we have the non-negativity property and no interior extrema

- Linear reproduction

Given an arbitrary *f(p)*, the coordinate functions can be used to define:

$$H\left[f\right]\left(p\right)=\sum_i h_i\left(p\right)f\left(C_i\right)$$

This is the ‘no popping’ condition

Properties:

- Affine invariance

$$\sum_i h_i(p) = 1 \quad \text{for all} \quad p \in C$$

- Generalization of barycentric coordinates

$h_i(p)$ is the barycentric coordinate of *p* with respect to C_i

Interpolation:

$$h_i\left(C_j\right) = \phi_i(C_j) = \delta_{i,j}$$

Smoothness:

Away from the boundary, harmonic coordinates are solutions, so they are smooth in the cage interior

On the boundary, they are only as smooth as the boundary conditions

Non-negativity:

Harmonic functions achieve extreme at their boundaries

Boundary values are restricted to [0,1]

So interior values are restricted to [0,1]

Interior locality:

Harmonic functions possess no interior extrema

Linear reproduction:

This holds for everywhere on the boundary of *C,* by definition:

$$H\left[f\right]\left(\partial p\right)=\sum_i h_i\left(\partial p\right)f\left(C_i\right)=\sum_i \phi_i\left(\partial p\right)f\left(C_i\right)$$

Since *f(p)* is linear, second derivatives vanish, ie:

$$\nabla^2 f(p)=0$$

and *f(p)* satisfies Laplace’s equation on the interior of *C*

Since *H[f](p)* is a linear combination of harmonic functions, it also satisfies Laplace’s equation

Use proof by induction to generalize to any *n*-dimension

Results

Cage Vertices	325	112	39	27
Object Vertices	9775	8019	269	136
Grid resolution	5	5	4	5
Solve time	57.4	17.6	5.85	0.83
Pose time	0.111	0.026	0.0001	0.0007
Solution size (MB)	9.2	3.7	0.32	0.048

Error: < 0.005

Total footprint: < 90MB

Future Work:

- Compute the harmonic coordinates for each cage vertex independently and in parallel
- Better solvers (currently using MultiGrid)
- Octrees
- Localize re-solves