Inference of Surfaces, 3D Curves, and Junctions From Sparse, Noisy, 3D Data

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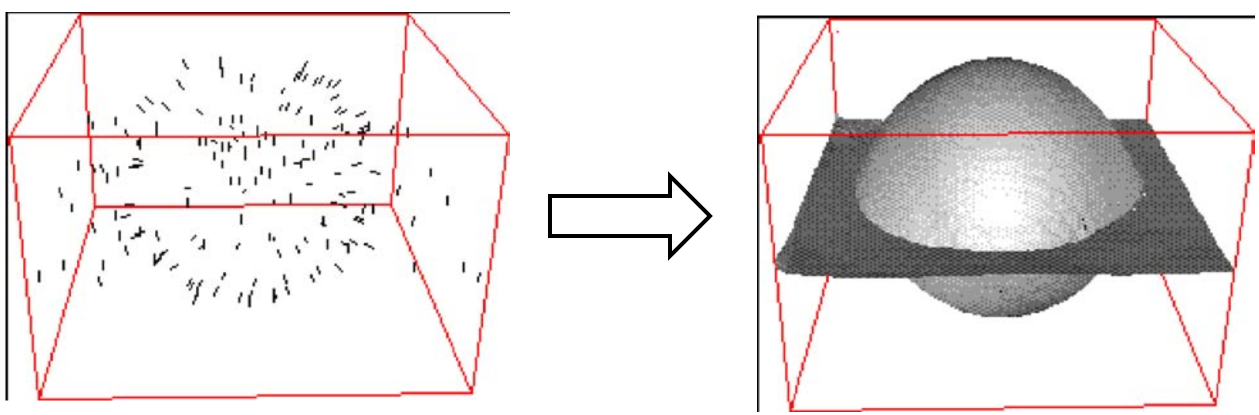
Problem Statement

- Given a collection of oriented sample points (i.e., positions and normals) in space, find the following features: continuous surfaces, boundary curves, and junctions (or corners), that are consistent with the sample.
  - Solution extended to find point normals from a non-oriented sample, and to include short-curve inputs.

Problem Statement: Ideal Example



Tang and Medioni, 1998

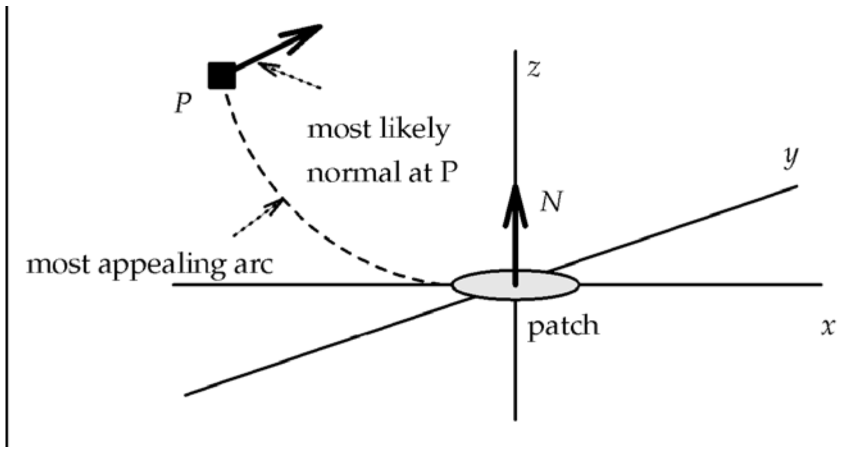
Solution Outline

- Use a voxel grid as a framework on which features are defined.
- Create *saliency maps* for the three types of features.
- Locate places where feature saliency is maximal and connect them to reconstruct the feature.

Underlying Model

- Normals of continuous surfaces vary smoothly at a rate proportional to the curvature.
- Sharp boundaries (edges and corners) between surfaces are characterized by discontinuities in normals.
- The normal field can be extrapolated from the sample points.
- Continuous surfaces, and edge and corner discontinuities can be characterized by the distribution of extrapolated normals.

Normal Extrapolation: The Diabolo Field



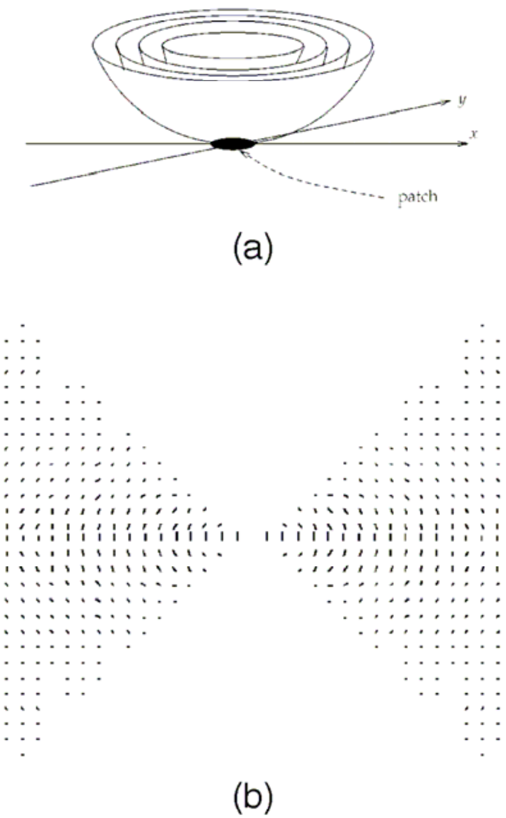
Given a surface patch located at *S* with normal *N*, and a point *P*, the most likely surface going through them includes the circular arc through *S* and *P* that is orthogonal to *N* at *S*. The normal at *P* is therefore the vector orthogonal to the arc at *P*.

Normal Extrapolation: The Diabolo Field

- The Diabolo field extends from each sample point to define its contribution to surface normals in its environments
- "In spherical coordinates, the Diabolo Field thus takes the following form:

$$\overline{DF}(r, \varphi, \theta) = e^{-Ar^2} e^{-B\varphi^2}$$

where *A* encodes the decay due to proximity, and *B* the decay due to higher curvature."



Normal Extrapolation: Vector Convolution and Vote Aggregation

- This part of the text is unclear.
- In the end, a normal vector is assigned to each voxel, based on the Diabolo fields of the sample points.
  - Probably the mean (central moment) of the Diabolo field contribution of each sample.

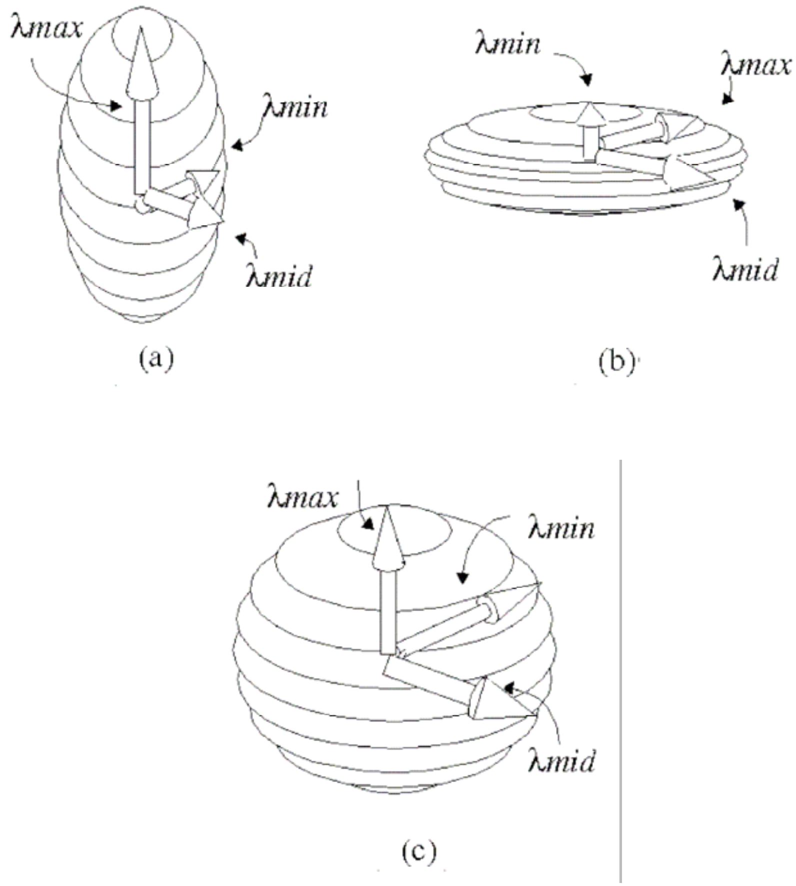
Vote Interpretation

- Second order moments are extracted for the distribution of normals in the neighborhood of each voxel.

$$\begin{bmatrix} E_{x,x} & E_{x,y} & E_{x,z} \\ E_{y,x} & E_{y,y} & E_{y,z} \\ E_{z,x} & E_{z,y} & E_{z,z} \end{bmatrix} = \begin{bmatrix} V_{\min} \\ V_{\text{mid}} \\ V_{\max} \end{bmatrix}^T \begin{bmatrix} \lambda_{\min} & 0 & 0 \\ 0 & \lambda_{\text{mid}} & 0 \\ 0 & 0 & \lambda_{\max} \end{bmatrix} \begin{bmatrix} V_{\min} \\ V_{\text{mid}} \\ V_{\max} \end{bmatrix}$$

- The values of λ can be regarded as lengths of axes of ellipsoids

Vote Interpretation: Eigenvalue Analysis



- The three important voting ellipsoids.
  - a) $\lambda_{max} >> \lambda_{mid} \approx \lambda_{min}$, high agreement in exactly one direction (a surface).
  - b) $\lambda_{max} \approx \lambda_{mid} >> \lambda_{min}$, high agreement in exactly two orientations (an intersection, or 3D curve).
  - c) $\lambda_{max} \approx \lambda_{mid} \approx \lambda_{min}$, votes are coming from all directions (a 3D junction).

Feature Reconstruction: Curve Saliency

- The voxels in the curve saliency map hold a tuple $(s, \mathbf{t})$, where $s = \lambda_{mid} - \lambda_{min}$ is the saliency, and $\mathbf{t} = V_{min}$ is the estimated tangent direction at the voxel.
- The curve is assumed to go through points where s is a local maximum in the plane normal to the tangent.
- The gradient vector $\mathbf{g}$ of s is estimated through finite differences.
- The projection of $\mathbf{g}$ on the tangent-normal plane is $\mathbf{q}=(\mathbf{t} \times \mathbf{g}) \times \mathbf{t}$
- When $\mathbf{g}$ is more-or-less aligned with $\mathbf{t}$, $\mathbf{q} \sim = \mathbf{0}$
- On a voxelized grid, finding the intersection points of a salient curve with voxel faces is done by finding sign changes of components of $\mathbf{q}$ (Fig. 10).

Main Contributions

- Defining saliency maps for features
- Reconstruction of features from saliency maps
- Recovering normals from point samples
- Extrapolation of continuous surfaces and curves from sparse samples
- Robust to outlier noise (Fig. 19)
- Integration of features into models (Tang and Medioni, 1998)

Method Limitations

- The text of the papers could be much improved if more math was used.
- Relies on voxel topology for continuity
  - Limited to voxel resolution
  - Surface reconstructed using Marching Cubes $\Rightarrow$ Marching Cubes artifacts
  - Other topologies?
- No watertight surface guarantee.
  - See the '98 paper, Fig. 16
- No measure of accuracy