Robust Creation of Implicit Surfaces from Polygon Meshes

Gary Yngve and Greg Turk

Presented by Owen Gray

Overview

- Implicit Surface Generation
- Volumetric Representation of Objects
- Implicit Surfaces
- Evaluation of surface fit
- Choice of constraints
- Eye candy

Implicit Surface Generation

- Surface is described by a weighted set of radial basis functions
  - One function for each constraint
  - Basis function (1) is chosen to minimize local curvature (2)
    - High local curvature complicates solution of linear system to determine weights of basis functions
    - Not sure why choice of $|x^3|$ minimizes the curvature

(1)

$$\phi(x)=|x^3|$$

(2)

$$\int \sum_{i,j} \left(\frac{(\delta^2 f(x))}{(\delta x_i \delta x_j)}\right) d_x$$

Implicit Surface Generation

- The implicit function is then the sum of the weighted basis functions

$$f(x)=\sum_{j=1}^{n} d_j \phi(x-c_j)+P(x)$$

- Interpolation constraints $\mathbf{h_i}$ are applied at each sample point $\mathbf{c_i}$

$$h_i=\sum_{j=1}^{k} d_j \phi(c_i-c_j)+P(c_i)$$

Implicit Surface Generation

- Specify a value for $\mathbf{h_i}$ at sample points on, in and outside the surface
  - $h_i = 0$ forces surface to pass through the point
  - $h_i > 0$ forces the point to be internal
  - $h_i < 0$ forces the point to be external

Implicit Surface Generation

- Solving the system of linear equations yields the weights **d_i** for the respective basis functions

$$
\begin{bmatrix}
\phi_{11} & \phi_{12} & \ldots & \phi_1 k & 1 & c_1^x & c_1^y & c_1^z \\
\phi_{21} & \phi_{22} & \ldots & \phi_1 k & 1 & c_2^x & c_2^y & c_2^z \\
\vdots\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\phi_{k1} & \phi_{k2} & \ldots & \phi_{kk} & 1 & c_k^x & c_k^y & c_k^z \\
1 & 1 & \ldots & 1 & 0 & 0 & 0 & 0 \\
c_1^x & c_2^x & \ldots & c_k^x & 1 & 0 & 0 & 0 \\
c_1^y & c_2^y & \ldots & c_k^y & 1 & 0 & 0 & 0 \\
c_1^z & c_2^z & \ldots & c_k^z & 1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
d_1 \\ d_2 \\ \vdots \\ d_k \\ p_0 \\ p_1 \\ p_2 \\ p_3
\end{bmatrix}
=
\begin{bmatrix}
h_1 \\ h_2 \\ \vdots \\ h_k \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

Voxelization

- Conversion from mesh to volumetric representation
  - Calculated from original polygon model
  - High sampling density (computationally intensive)
- Each voxel is assigned a weight [0, 1]
  - Weight is determined by potion of the voxel contained in the original shape
  - Weights vary continuously

Approximating signed distance

- Identify set of boundary voxels containing regions inside and outside the polygon model
  - this is the set ∂f_{goal}
- Compute approximate signed euclidean distance from each voxel to nearest boundary
  - Treat voxels as 6-connected graph, compute the shortest path to the boundary

Evaluating Isosurface Fit

The isosurface fit is evaluated by comparing the set of boundary voxels of the isosurface ∂f_{curr} and the set of boundary voxels of the original polygonal model ∂f_{goal} (Hausdorff error)

$$H = max \left[\max_{x \in \delta f_{curr}} \left[\min_{y \in \delta f_{goal}} \| x - y \| \right], \max_{y \in \delta f_{curr}} \left[\max_{x \in \delta f_{goal}} \| x - y \| \right] \right]$$

Evaluating Isosurface Fit with Original Model

New constraints are added at the voxel locations of maximum disparity between the isosurface boundary (∂f_{curr}) and the boundary of the original polygonal model (∂f_{goal}), calculated based on the signed distance value at each voxel

$$C_{new} = arg\,max\left[\left[\max_{x \in \delta f_{goal}} \left| sd_{curr}(x) \right|, \max_{x \in \delta f_{curr}} \left| sd_{goal}(x) \right|\right]\right]$$

Initial Boundary Constraints

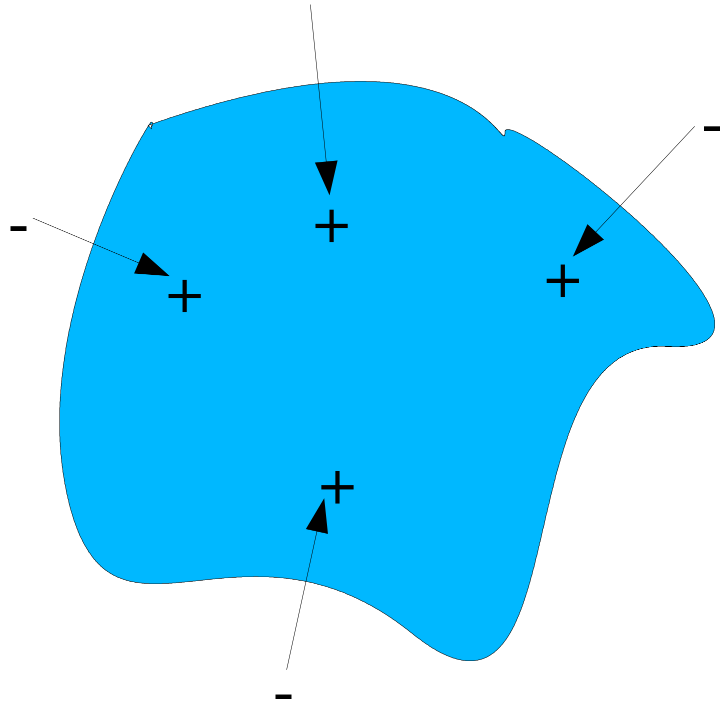
- Random choice of initial constraint locations
- Poisson distribution:
  - Points are added randomly
  - Distribution is globally uniform, but unconstrained
  - In subregions, some neighboring samples close together, some far apart

Initial Boundary Constraints

- Poisson disc distribution:
  - Same as Poisson distribution, but imposes minimum-distance constraint between samples
  - Applied using unsigned euclidean distance
  - Points are added at random, but removed if they violate distance constraint
  - Guarantees an even sample distribution

Initial Interior/Exterior Constraints

- One interior and one exterior constraint chosen for each initial boundary sample
  - Calculate the gradient of the signed distance function at each point
  - Follow gradient until we locate a local maxima or minima of the signed distance
  - Place an interior constraint (positive) or exterior constraint (negative)



Initial Interior/Exterior Constraints

- The maxima of the signed distance function will approximate the medial axis of the original shape
- The gradient at the implicit surface is an approximation of the surface normal

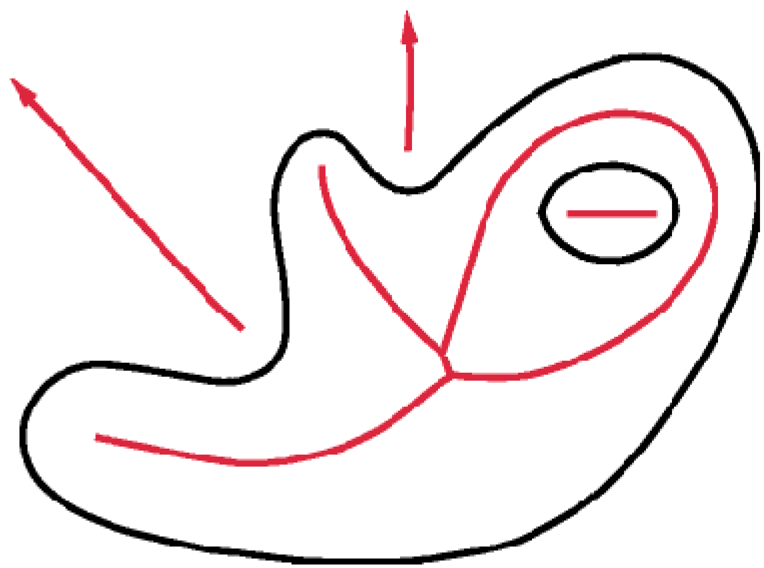


Image courtesy of N. Amenta,

Adding Constraints

- Volume is coarsely sampled
  - Interior voxels sampled randomly/sparsely
  - Boundary voxels sampled at full voxel resolution
    - Any 4x4 cube intersecting isosurface boundary or model boundary fully sampled
    - Any 4x4 cube within 8 voxels of model boundary is fully sampled

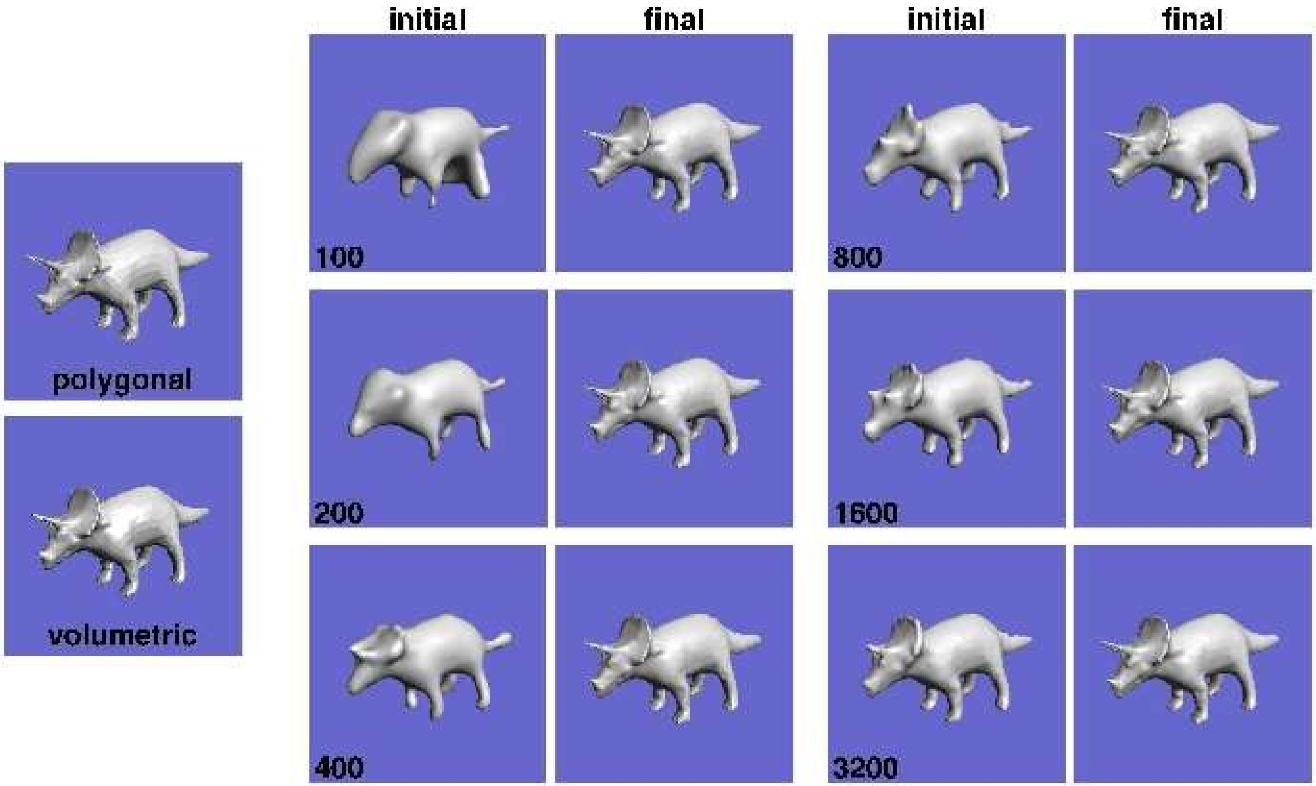
Adding Constraints

- Hausdorff Error is calculated for all voxels in the isosurface and model boundary set
  - New constraint added at voxel with maximum error (greedy algorithm)
  - Apply boundary constraint (0) if voxel is in model boundary
  - Apply interior/exterior constraint if voxel is on isosurface boundary

Adding Constraints

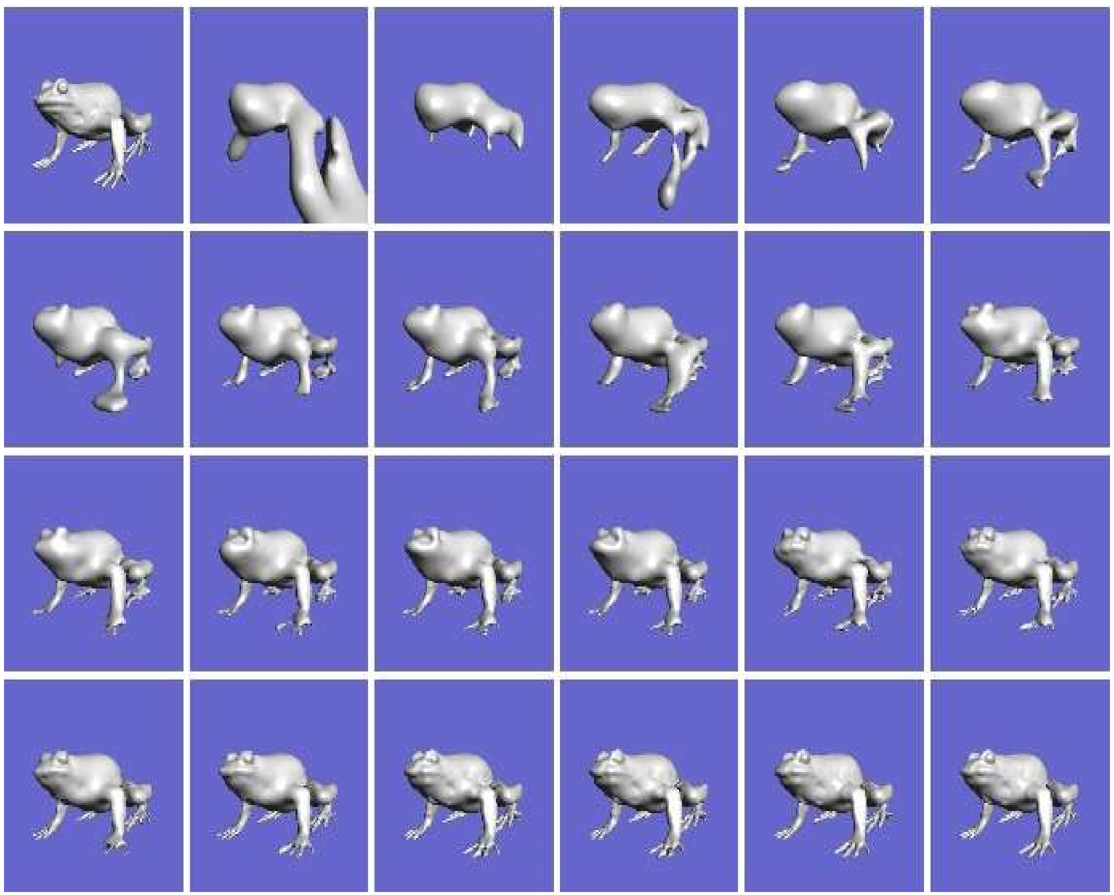
- Over-constraining the system will produce erratic results
  - Enforce a separation between constraint locations of 2x the abs. value of the signed distance
  - This ensures that spheres enclosing each constraint with radius = signed distance at x will not overlap (same idea as Poisson disc dist. in initial sampling)

Results



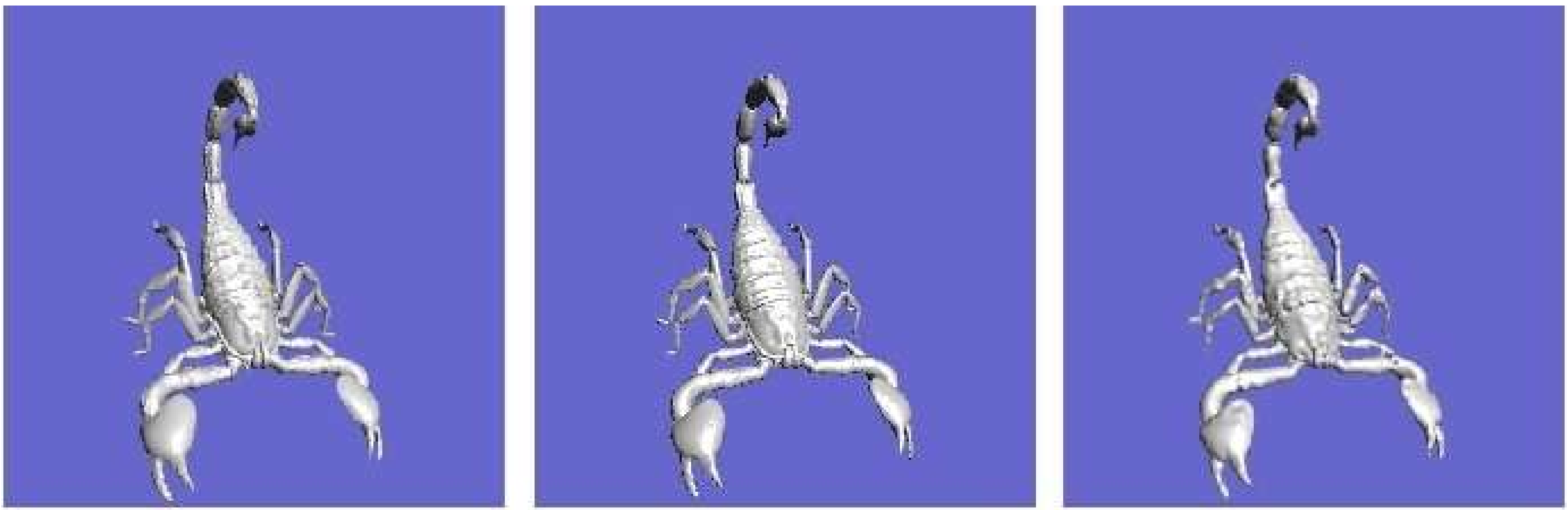
Initial and final isosurface models with varying numbers of initial constraints

Results



Original model and 22 iterations of fitting algorithm

Results



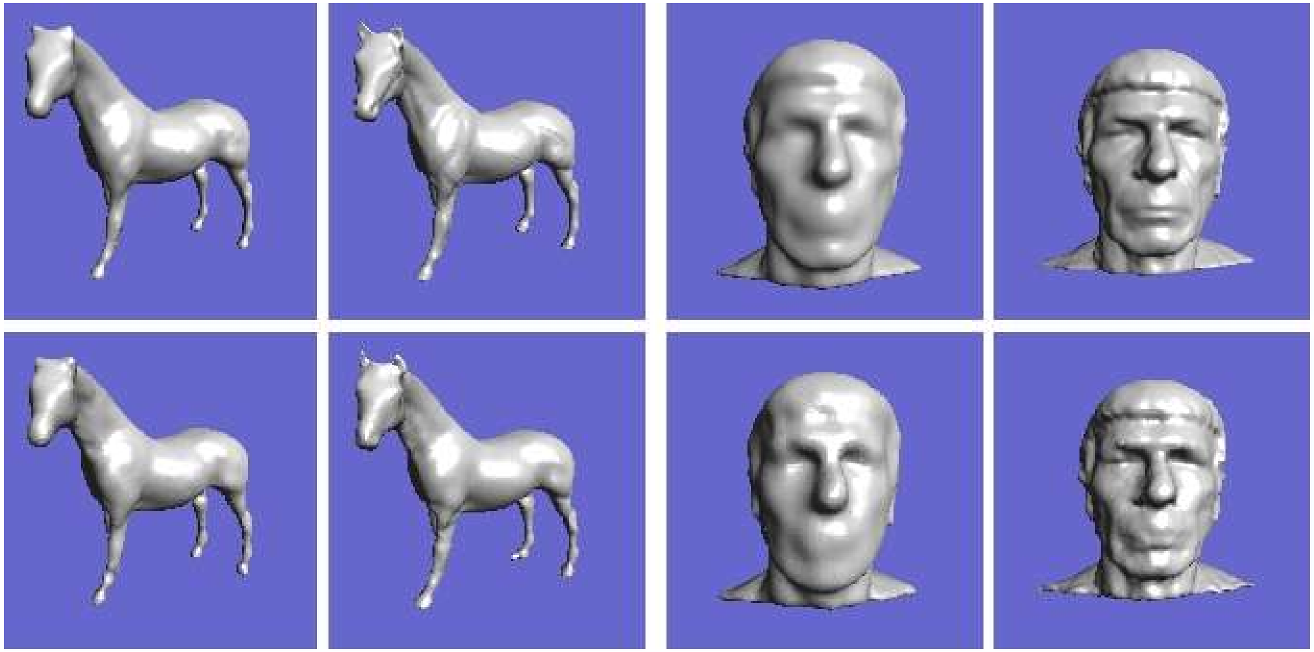
(a) (b) (c)

(a) Original polygon mesh model

(b) Volumetric model

(c) Final implicit surface (unknown number of iterations)

Sample Results



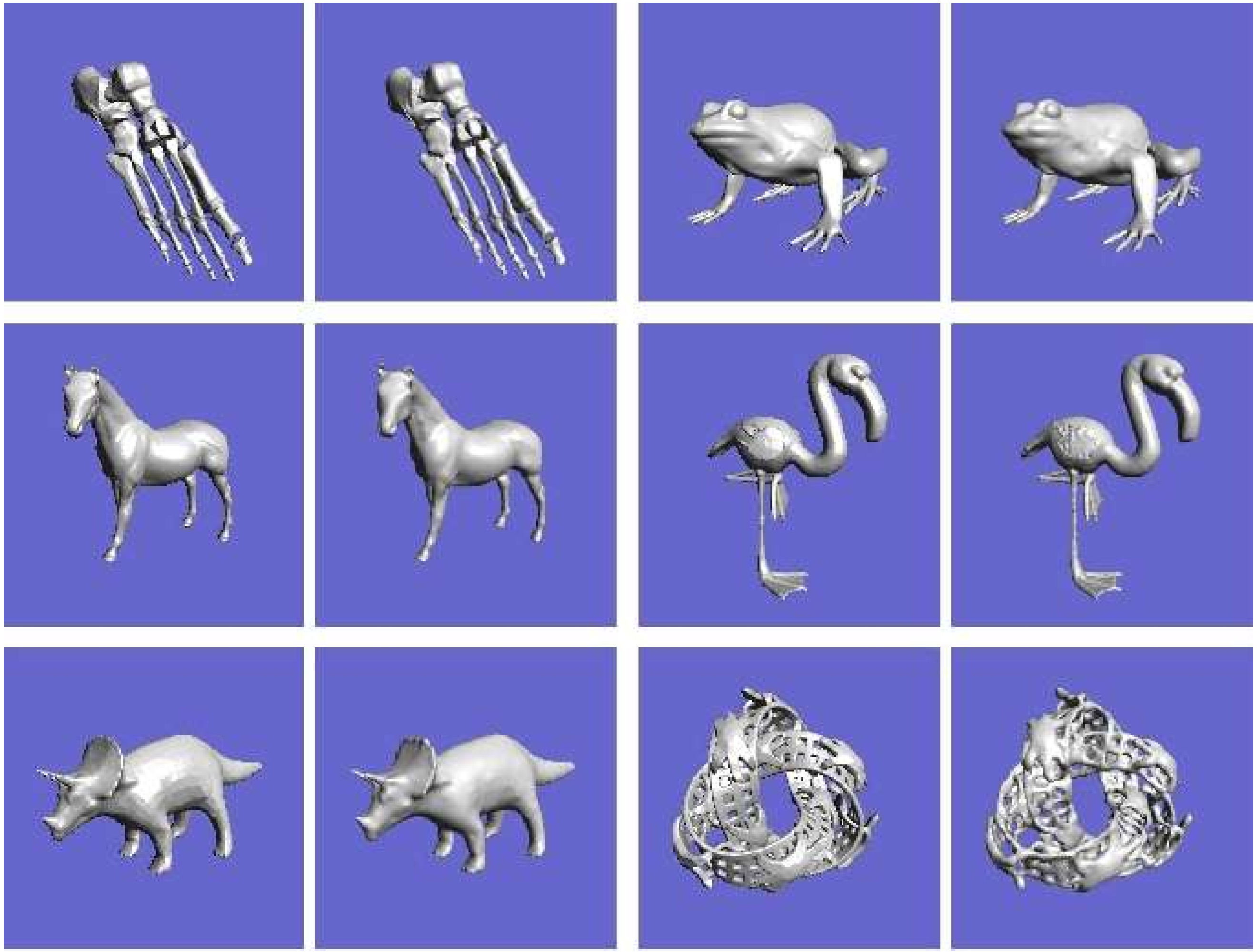
Top: Example of surface generation from low-resolution volumetric representations (large voxel grid)
Bottom: Final implicit surface

Results

Example of results with high-curvature models

Left pair: Volumetric model

Right pair implicit surface



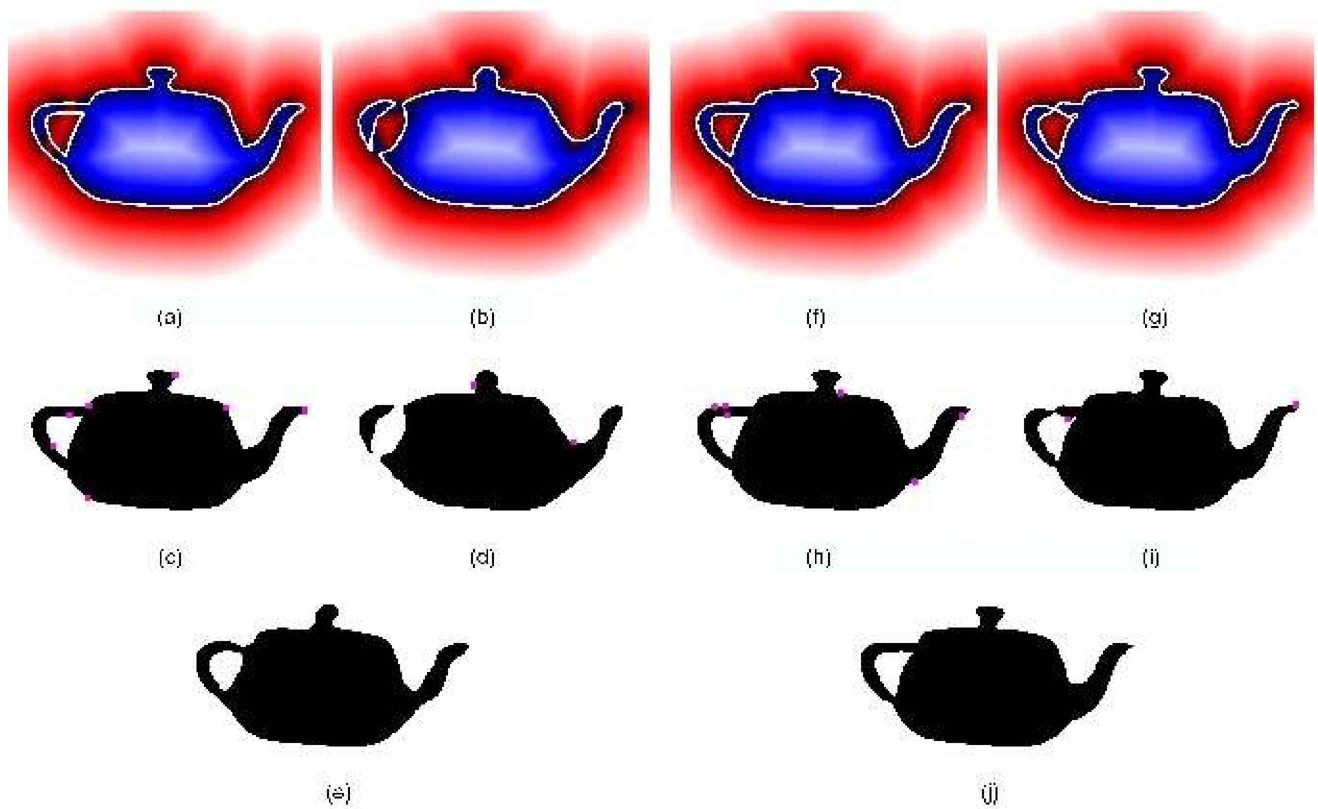
Results

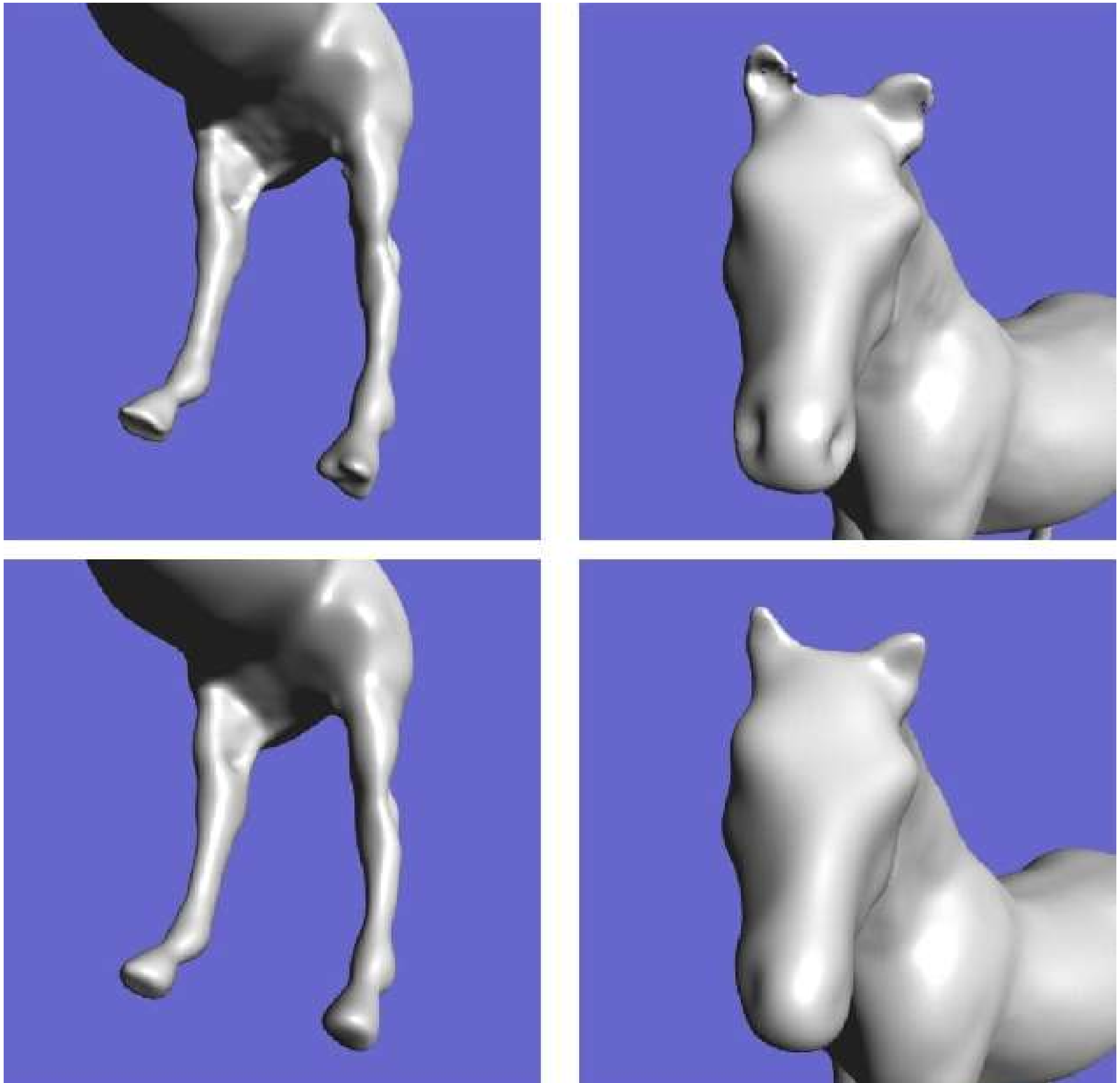
Example of results with large models

Left pair: Volumetric model

Right pair implicit surface

END


(a)
(b)
(f)
(g)
(c)
(d)
(h)
(i)
(e)
(j)



Implicit Surface Generation

$$\phi(x) = |x^3|$$

$$\int \sum_{i,j} \left(\frac{(\delta^2 f(x))}{(\delta x_i \delta x_j)} \right) d_x$$

$$f(x) = \sum_{j=1}^{n} d_j \phi(x - c_j) + P(x)$$

$$h_i = \sum_{j=1}^{k} d_j \phi(c_i - c_j) + P(c_i)$$